MATH 18/19: Analysis of differences in school exams

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Read data

Read in the `data_math-1819.csv` file which contains the codes assigned to all items, and produce some summaries of this.

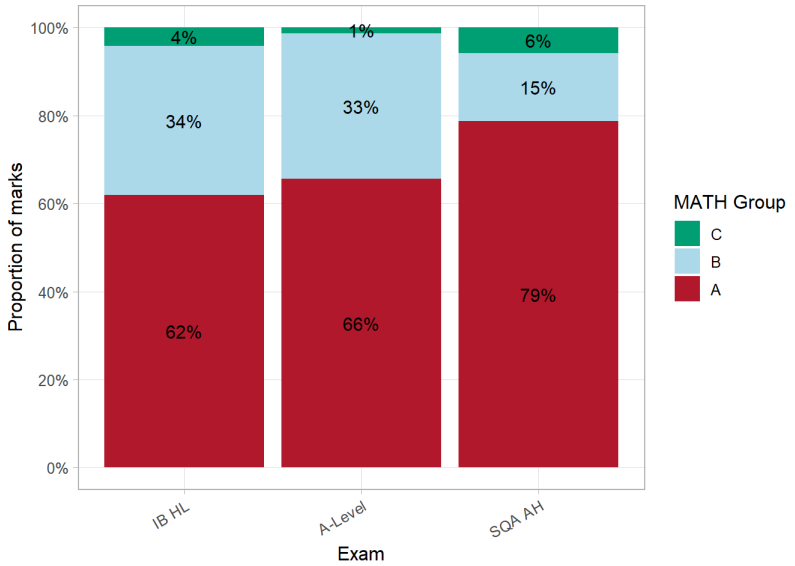
Sample of the summary data:

Course	Year	Paper	MATH_Group	total_marks_for_Group	total_marks_for_paper	prop_marks	Exam	Qual
A-Level C1	2014	C1	C	0	75	0.0000000	2014 C1	A-Level
A-Level C1	2015	C1	C	0	75	0.0000000	2015 C1	A-Level
A-Level C1	2016	C1	C	0	75	0.0000000	2016 C1	A-Level
A-Level C1	2017	C1	C	0	75	0.0000000	2017 C1	A-Level
A-Level C2	2014	C2	C	2	75	0.0266667	2014 C2	A-Level
A-Level C2	2015	C2	C	1	75	0.0133333	2015 C2	A-Level

Overall proportions

```
School_props %>%
  group_by(Qual,MATH_Group) %>%
  summarise(
    prop_marks = mean(prop_marks),
    n = n()
  ) %>%
  ungroup() %>%
  mutate(
    gpAmarks = if_else(MATH_Group=="A",prop_marks,0),
    #    Course = paste(Course,"\n(n=",n,")"),
    Qual = (reorder(Qual, gpAmarks, max))
  ) %>%
  ddply(.(Qual), transform, pos = cumsum(prop_marks) - (0.5 * prop_marks)) %>%
  ggplot(aes(x = Qual,
             y = prop_marks,
             fill = fct_rev(MATH_Group))) +
  geom_bar(stat="identity") +
  geom_text(aes(x = Qual, y = pos, label = paste0(sprintf("%.0f",100*prop_marks),"%")),
            size=4) +
  scale_y_continuous(labels = scales::percent,
                     breaks = seq(0,1,0.2)) +
  scale_fill_manual(values = palMATH) +
  theme_light(base_size = 12)+
  theme(
    panel.grid.minor = element_blank(),
    axis.text.x = element_text(angle = 30, hjust = 1)
  ) +
  labs(
    #    title = "Mean proportion of marks in each MATH Group",
    fill = "MATH Group",
    x = "Exam",
    y = "Proportion of marks"
  ) +
  ggsave("figs/ABC_school_means.pdf",width=15,height=10,units="cm",dpi=300)
```

```
## `summarise()` regrouping output by 'Qual' (override with `.groups` argument)
```

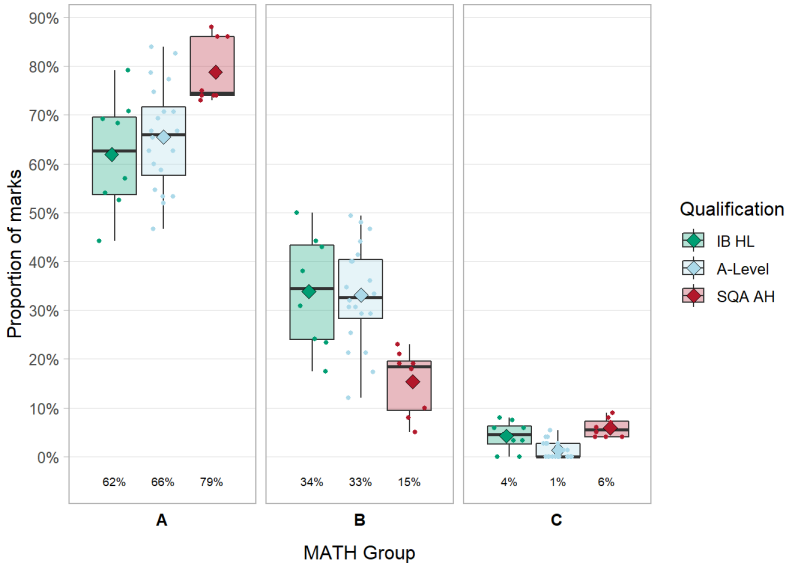


In this version, we sort by the mean % of group A, and show the means with diamonds

```
School_props_boxplot_data = School_props %>%
  # reorder the Qual factor by mean % group A
  mutate(
    gpAmarks = if_else(MATH_Group=="A",prop_marks,0),
    Qual = reorder(Qual, gpAmarks, mean)
  )
School_props_boxplot_data %>%
  ggplot(aes(x=1,y=prop_marks, fill=Qual)) +
  geom_boxplot(alpha = 0.3) +
  stat_summary(fun.y = "mean",
               geom = "point",
               shape = 23,
               size = 3,
               aes(fill = Qual),
               position = position_dodge(0.8)) +
  geom_point(aes(colour=Qual),
             position = position_jitterdodge(),
             size = 1) +
  geom_text(data = School_props_boxplot_data %>%
              group_by(Qual, MATH_Group) %>%
              summarise(prop_marks = mean(prop_marks)),
            aes(label = paste0(100*round(prop_marks,2),"%"),
                y = -0.05),
            position = position_dodge(0.75),
            size = 2.5) +
  scale_y_continuous(labels = scales::percent_format(accuracy = 1),
                     breaks = seq(0,1,0.1)) +
    scale_fill_manual(values = palMATH) +
    scale_colour_manual(values = palMATH) +
#  scale_fill_brewer(palette = "RdBu", direction = -1) +
#  scale_colour_brewer(palette = "RdBu", direction = -1) +
  theme_light(base_size = 12)+
  theme(
    panel.grid.minor = element_blank(),
    axis.text.x=element_blank(),
    axis.ticks.x=element_blank(),
    panel.grid.major.x = element_blank(),
    strip.background = element_rect(fill=NA,colour = NA),
    strip.text = element_text(size=10, face="bold", color = "black")
  ) +
  labs(
    #    title = "Proportion of marks in each MATH Group",
    #    subtitle = "Exams are ordered by proportion of Group A marks",
    fill = "Qualification",
    colour = "Qualification",
    x = "MATH Group",
    y = "Proportion of marks"
  ) +
  facet_grid( ~ MATH_Group, switch = "x") +
  ggsave("figs/ABC_school_boxplots.pdf",width=15,height=10,units="cm",dpi=300)
```

```
## Warning: `fun.y` is deprecated. Use `fun` instead.
```

```
## `summarise()` regrouping output by 'Qual' (override with `.groups` argument)
```



Comparisons

SQA AH and A-Level

```
sp = School_props %>%
  filter(MATH_Group == "A") %>%
  select(Qual, prop_marks)

BESTout = BESTmcmc(sp %>% filter(Qual=="SQA AH") %>% select(prop_marks) %>% data.matrix,
                   sp %>% filter(Qual=="A-Level") %>% select(prop_marks) %>% data.matrix)
```

```
## Waiting for parallel processing to complete...done.
```

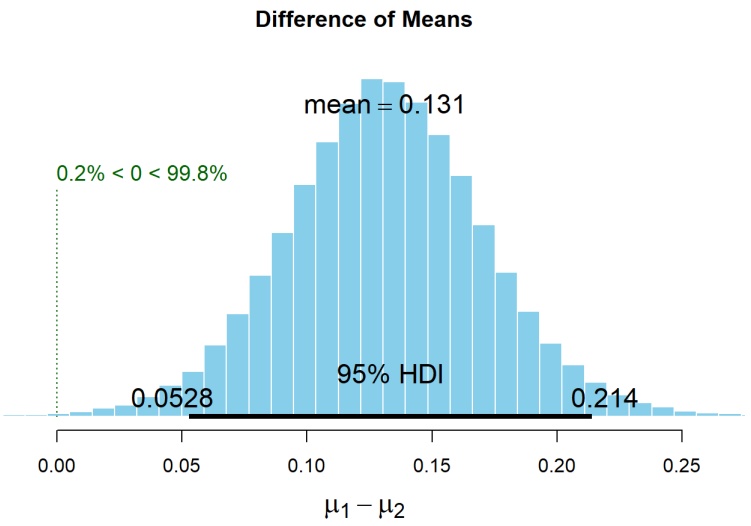
```
BESTout
```

```
## MCMC fit results for BEST analysis:
## 100002 simulations saved.
##            mean       sd   median   HDIlo   HDIup Rhat n.eff
## mu1     0.78613  0.03129  0.78598 0.72357  0.8482    1 51812
## mu2     0.65486  0.02634  0.65473 0.60293  0.7069    1 57847
## nu     38.00403 30.10413 29.70123 1.89781 98.1482    1 23271
## sigma1  0.08106  0.02989  0.07452 0.03828  0.1397    1 21064
## sigma2  0.11117  0.02091  0.10851 0.07428  0.1531    1 37921
##
## 'HDIlo' and 'HDIup' are the limits of a 95% HDI credible interval.
## 'Rhat' is the potential scale reduction factor (at convergence, Rhat=1).
## 'n.eff' is a crude measure of effective sample size.
```

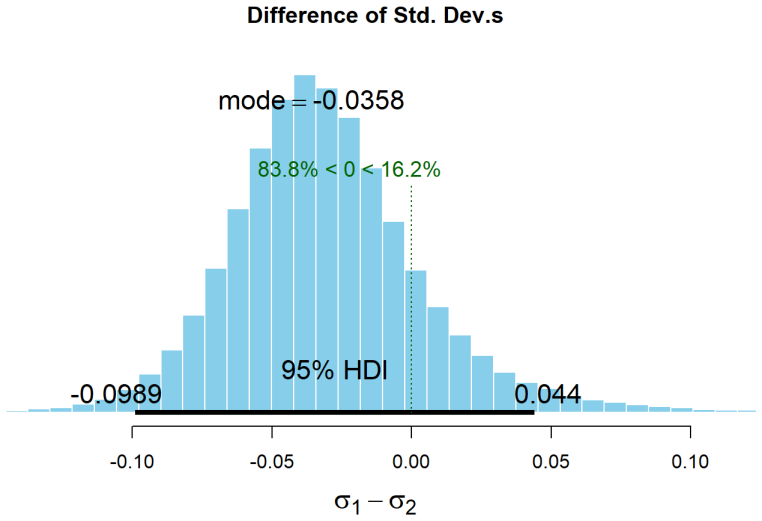
```
summary(BESTout) %>% kable(booktabs = T)
```

	mean	median	mode	HDI%	HDIlo	HDIup	compVal	%>compVal	ROPElow	ROPEhigh	%InROPE
mu1	0.7861298	0.7859754	0.7869302	95	0.7235724	0.8481833	NA	NA	NA	NA	NA
mu2	0.6548589	0.6547312	0.6520399	95	0.6029280	0.7068664	NA	NA	NA	NA	NA
muDiff	0.1312709	0.1310808	0.1291982	95	0.0527590	0.2138857	0	99.77300	NA	NA	NA
sigma1	0.0810632	0.0745176	0.0658942	95	0.0382839	0.1396700	NA	NA	NA	NA	NA
sigma2	0.1111704	0.1085090	0.1030438	95	0.0742817	0.1530879	NA	NA	NA	NA	NA
sigmaDiff	-0.0301071	-0.0331474	-0.0358449	95	-0.0989347	0.0440493	0	16.16568	NA	NA	NA
nu	38.0040324	29.7012291	14.3893449	95	1.8978080	98.1482390	NA	NA	NA	NA	NA
log10nu	1.4483441	1.4727744	1.5406860	95	0.7335559	2.0940458	NA	NA	NA	NA	NA
effSz	1.3701428	1.3684369	1.3732513	95	0.4613812	2.2889970	0	99.77300	NA	NA	NA

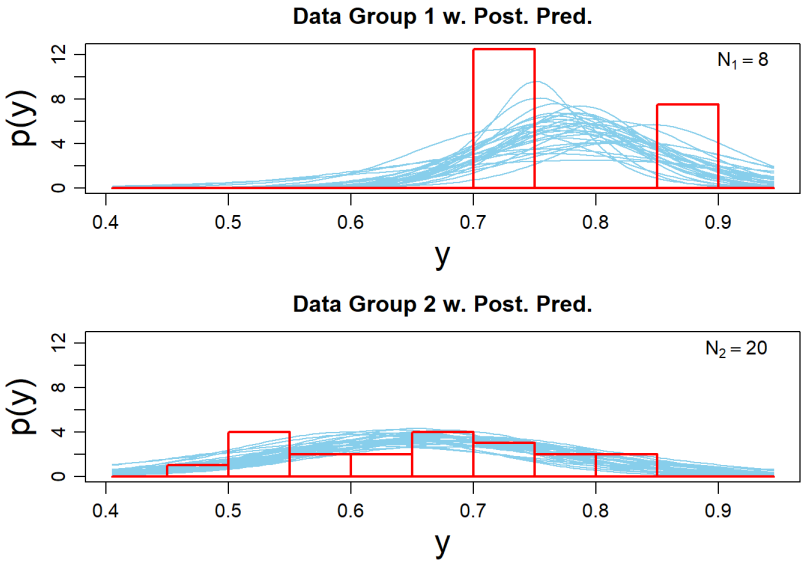
```
plot(BESTout)
```



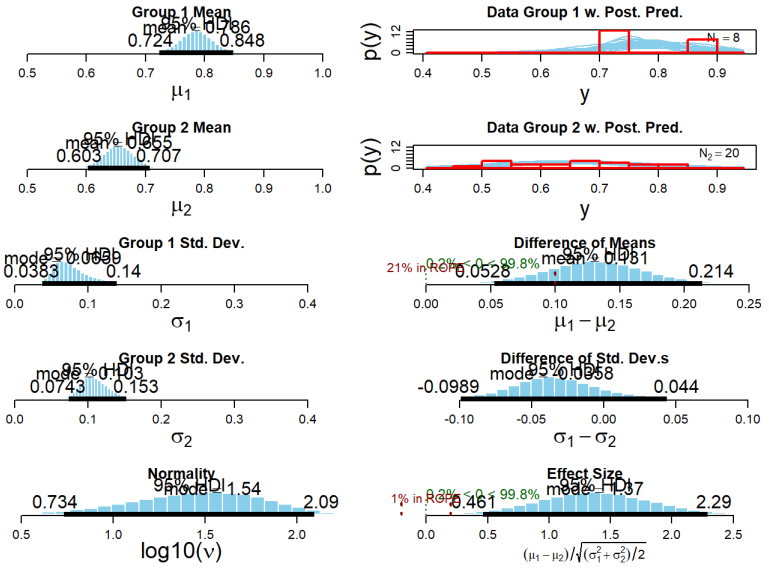

```
plot(BESTout, "sd")
```



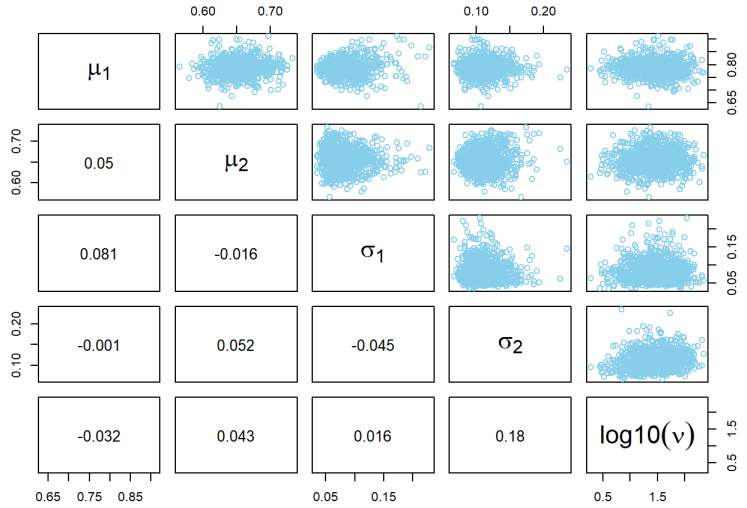

```
plotPostPred(BESTout)
```




```
plotAll(BESTout, credMass=0.95, ROPEm=c(-0.1,0.1),
        ROPEeff=c(-0.2,0.2), compValm=0.5)
```




```
pairs(BESTout)
```



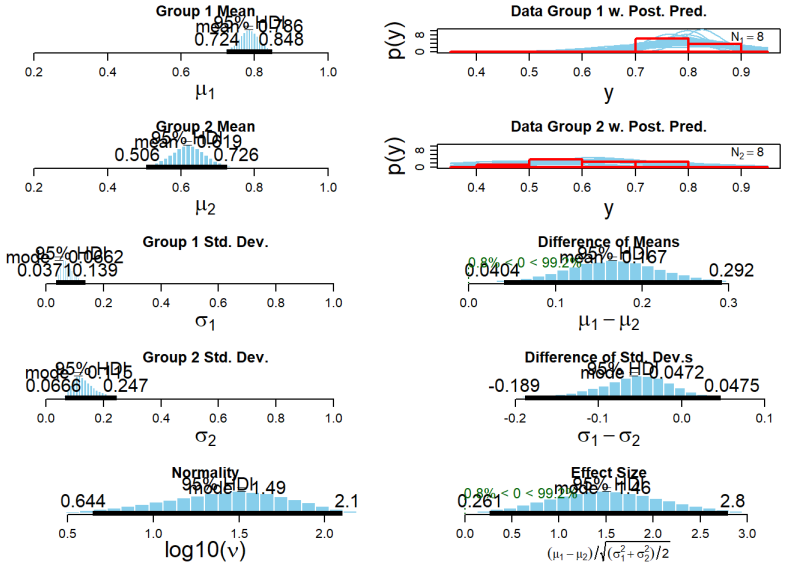
This shows a mean difference of 0.13 (95% HDI [0.05, 0.21]).

SQA AH and IB

```
BESToutIB = BESTmcmc(sp %>% filter(Qual=="SQA AH") %>% select(prop_marks) %>% data.matrix,
                     sp %>% filter(Qual=="IB HL") %>% select(prop_marks) %>% data.matrix)
```

```
## Waiting for parallel processing to complete...done.
```

```
plotAll(BESToutIB, credMass=0.95)
```



This shows a mean difference of 0.17 (95% HDI [0.04, 0.29]).

Overall summary

```
School_props %>%
  group_by(Qual, MATH_Group) %>%
  summarise(
    m = mean(prop_marks),
    sd = sd(prop_marks),
    n = n()
  ) %>%
  ungroup() %>%
  arrange(MATH_Group) %>%
  kable(booktabs = TRUE)
```

```
## `summarise()` regrouping output by 'Qual' (override with `.groups` argument)
```

Qual	MATH_Group	m	sd	n
A-Level	A	0.6553333	0.1063394	20
IB HL	A	0.6189583	0.1171925	8
SQA AH	A	0.7875000	0.0660627	8
A-Level	B	0.3313333	0.1034855	20

Qual	MATH_Group	m	sd	n
IB HL	B	0.3387500	0.1164343	8
SQA AH	B	0.1537500	0.0669621	8
A-Level	C	0.0133333	0.0173036	20
IB HL	C	0.0422917	0.0310394	8
SQA AH	C	0.0587500	0.0195941	8

SQA AH and A-Level, Group C

```
school_gpC = School_props %>%
  filter(MATH_Group == "C") %>%
  select(Qual, prop_marks)

BESTout = BESTmcmc(school_gpC %>% filter(Qual=="SQA AH") %>% select(prop_marks) %>% data.matrix,
                   school_gpC %>% filter(Qual=="A-Level") %>% select(prop_marks) %>% data.matrix)
```

```
## Waiting for parallel processing to complete...done.
```

```
BESTout
```

```
## MCMC fit results for BEST analysis:
## 100002 simulations saved.
##            mean        sd   median    HDIlo    HDIup  Rhat n.eff
## mu1     0.05846  0.009267  0.05838 0.039675  0.07645 1.000 48190
## mu2     0.01262  0.004290  0.01262 0.004166  0.02112 1.000 55060
## nu     36.56125 30.555653 27.76187 1.438257 97.66724 1.000 21203
## sigma1  0.02388  0.008847  0.02196 0.011064  0.04080 1.001 19381
## sigma2  0.01786  0.003418  0.01743 0.011811  0.02480 1.000 38132
##
## 'HDIlo' and 'HDIup' are the limits of a 95% HDI credible interval.
## 'Rhat' is the potential scale reduction factor (at convergence, Rhat=1).
## 'n.eff' is a crude measure of effective sample size.
```

```
summary(BESTout) %>% kable(booktabs = T)
```

	mean	median	mode	HDI%	HDIlo	HDIup	compVal	%>compVal	ROPElow	ROPEhigh	%InROPE
mu1	0.0584631	0.0583772	0.0582065	95	0.0396746	0.0764465	NA	NA	NA	NA	NA
mu2	0.0126205	0.0126245	0.0129809	95	0.0041660	0.0211194	NA	NA	NA	NA	NA
muDiff	0.0458426	0.0457658	0.0455018	95	0.0256539	0.0659695	0	99.9580	NA	NA	NA
sigma1	0.0238793	0.0219618	0.0189362	95	0.0110637	0.0407974	NA	NA	NA	NA	NA
sigma2	0.0178564	0.0174332	0.0166566	95	0.0118109	0.0247982	NA	NA	NA	NA	NA
sigmaDiff	0.0060229	0.0044978	0.0025263	95	-0.0098131	0.0247725	0	74.7645	NA	NA	NA
nu	36.5612502	27.7618671	11.7961600	95	1.4382570	97.6672377	NA	NA	NA	NA	NA
log10nu	1.4191210	1.4434487	1.5816324	95	0.6929716	2.1124459	NA	NA	NA	NA	NA
effSz	2.2533815	2.2522382	2.3306876	95	0.9668309	3.5599477	0	99.9580	NA	NA	NA

This shows a mean difference of 0.05 (95% HDI [0.03, 0.07]).