~~8~~ - GAUSSIAN ELIMINATION

Solve the system of LINEAR EQUATIONS USING GAUSSIAN ELIMINATION

① $\begin{aligned} X + 2Y &= 0 \\ -X + Y &= 10 \end{aligned}$

$$\begin{array}{r} X + 2Y = 0 \\ -X + Y = 10 \\ \hline 0X + 3Y = 10 \end{array} \qquad Y = \frac{10}{3}$$

$$X + 2\left(\frac{10}{3}\right) = 0$$

$$X + \frac{20}{3} = 0 \qquad X = -\frac{20}{3}$$

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② $\begin{aligned} 3X - Y &= 3 \\ -4X + 11Y &= 7 \end{aligned}$

$11\,\mathcal{E}1 + \mathcal{E}2$

$$\begin{array}{r} 33X - 11Y = 33 \\ -4X + 11Y = 7 \\ \hline 29X + 0Y = 40 \end{array}$$

$$X = \frac{40}{29}$$

$$3\left(\frac{40}{29}\right) - Y = 3 \quad \leftarrow$$

$$-Y = 3 - (3)\left(\frac{40}{29}\right)$$

$$\frac{-Y}{-1} = \frac{3}{-1} - \frac{120}{29} \quad 4\frac{4}{29}$$

$$Y = -3 + 4\frac{4}{29} \qquad Y = 1\frac{4}{29} = \frac{33}{29}$$

$$\begin{array}{r} 4 \\ 29\overline{)120} \\ \underline{116} \\ 4 \end{array}$$

$$Y = \frac{33}{29}$$

(3) $8x - 5y = 20$
$-16x + 10y = -40$

$\rightarrow$ $2E1 + E2$

$\downarrow$

$16x - 10y = 40$
$-16x + 10y = -40$
$0x + 0y = 0$

let $y = t$, solve for x

$8x - 5t = 20$

$8x = 5t + 20$

$x = \frac{5t}{8} + \frac{20}{8}$ $= \boxed{\frac{5t}{8} + \frac{5}{2} = x}$

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(4) $2x + y = 13$
$-4x - 2y = 4$

$2E1 + E2$

$\downarrow$

$4x + 2y = 26$
$-4x - 2y = 4$
$0x + 0y = 30$

CAN't Be solved.

(5) $x - y - z = 1$
$2x + y + 3z = 0$
$3x - y + z = -1$

$-2E1 + E2$

$-2x + 2y + 2z = -2$
$2x + y + 3z = 0$
$0 + 3y + 5z = -2$

$x - y - z = 1$ (same)
$+3y + 5z = -2$
$3x - y + z = -1$

get rid of $3x$ in $E3$

$-3E1 + E3$

$-3x + 3y + 3z = -3$
$3x - y + z = -1$
$0 + 2y + 4z = -4$

$x - y - z = 1$ (same)
$+3y + 5z = -2$ (same)
$2y + 4z = -4$

$-2E2 + 3E3$

$-6y - 10z = 4$
$6y + 12z = -12$
$0 + 2z = -8$

$z = \frac{-8}{2} = -4$

$E2$: $3y + 5z = -2$
$3y + 5(-4) = -2$
$3y - 20 = -2$
$+20$
$3y = 18$
$y = 6$

$x - y - z = 1$
$x - (6) - (-4) = 1$
$x - 2 = 1$
$x = 3$