# Design of a PID Controller for a Molten Salt Microreactor

Master's Plan


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# Xenon-135 Decay Chain

- Xenon-135 has a neutron capture cross-section of more than $2Mb$;
- Iodine-135 is a common products of Uranium-235 fission;
- $^{135}Xe$ is formed by $^{135}I$ $\beta$-decay ($t_{1/2} \approx 7\ hrs$);

$$\frac{dI}{dt} = \underbrace{\gamma_I \Sigma_f^F \phi(t)}_{\text{Fission Yield}} - \underbrace{\lambda_I I(t)}_{\text{Beta Decay}}$$

$$\frac{dXe}{dt} = \underbrace{\gamma_{Xe} \Sigma_f^F \phi(t)}_{\text{Fission Yield}} + \underbrace{\lambda_I I(t)}_{\text{Precursor Decay}} - \underbrace{\lambda_{Xe} Xe(t)}_{\text{Beta Decay}} - \underbrace{Xe(t)\sigma_a^{Xe}\phi(t)}_{\text{Radiative Capture}}$$

[? ]

# Xenon-135 Decay Chain

Table: Relevant nuclear constants [? , Ch. 7]

| | $\gamma$ (%) | $\lambda$ ($hr^{-1}$) | $t^{1/2}(hr)$ | $\sigma$ ($Mb$) |
|---|---|---|---|---|
| $^{135}I$ | 6.39 | 0.1035 | 6.57 | - |
| $^{135}Xe$ | 0.237 | 0.0753 | 9.14 | 2.65 |

$$\frac{dI}{dt} = \underbrace{\gamma_I \Sigma_f^F \phi(t)}_{\text{Fission Yield}} - \underbrace{\lambda_I I(t)}_{\text{Beta Decay}}$$

$$\frac{dXe}{dt} = \underbrace{\gamma_{Xe} \Sigma_f^F \phi(t)}_{\text{Fission Yield}} + \underbrace{\lambda_I I(t)}_{\text{Precursor Decay}} - \underbrace{\lambda_{Xe} Xe(t)}_{\text{Beta Decay}} - \underbrace{Xe(t) \sigma_a^{Xe} \phi(t)}_{\text{Radiative Capture}}$$

[? ]

# Numerical Solver (Euler's Method)

$$I[t+dt] = I[t] + \frac{dI}{dt}[t]$$

$$Xe[t+dt] = Xe[t] + \frac{dXe}{dt}[t]$$

# Numerical Solver (Euler's Method)

$$I[t+dt] = I[t] + \frac{dI}{dt}[t]$$
$$Xe[t+dt] = Xe[t] + \frac{dXe}{dt}[t]$$

- 1 second time steps;
- Able to change neutron flux (i.e. scram) at any time step;
- Tracks absolute concentration as well as the contribution to the overall rate of change by each term in the differential equations vs. time;

# Xenon-135 Decay Chain: Start-up

Nuclide Concentration Rates of Change - Reactor Start-up

Nuclide Concentration - Reactor Start-up

# Xenon-135 Decay Chain: Start-up

$$I_\infty(\phi) = \frac{\gamma_I \Sigma_f^F}{\lambda_I}\phi$$

$$Xe_\infty(\phi) = \frac{(\gamma_I + \gamma_{Xe})\Sigma_f^F}{\lambda_I + \sigma_a^{Xe}\phi}\phi$$

Nuclide Concentration - Reactor Start-up

[? ]

# Xenon-135 Decay Chain: Scram

$$\frac{dI}{dt} = \underbrace{\cancelto{0}{\gamma_I \Sigma_f^F \phi(t)}}_{\text{Fission Yield}} - \underbrace{\lambda_I I(t)}_{\text{Beta Decay}}$$

$$\frac{dXe}{dt} = \underbrace{\cancelto{0}{\gamma_{Xe} \Sigma_f^F \phi(t)}}_{\text{Fission Yield}} + \underbrace{\lambda_I I(t)}_{\text{Precursor Decay}} - \underbrace{\lambda_{Xe} Xe(t)}_{\text{Beta Decay}} - \underbrace{\cancelto{0}{Xe(t) \sigma_a^{Xe} \phi(t)}}_{\text{Radiative Capture}}$$

# Xenon-135 Decay Chain: Scram

$$\frac{dI}{dt} = -\underbrace{\lambda_I I(t)}_{\text{Beta Decay}}$$

$$\frac{dXe}{dt} = \underbrace{\lambda_I I(t)}_{\text{Precursor Decay}} - \underbrace{\lambda_{Xe} Xe(t)}_{\text{Beta Decay}}$$

# Xenon-135 Decay Chain: Scram

Nuclide Concentration Rates of Change - Reactor Scram

Nuclide Concentration - Reactor Scram

# Xenon-135 Decay Chain: Scram

Bateman Equations [? ]

$$I(t) = I_0 e^{-\lambda_I t}$$
$$Xe(t) = Xe_0 e^{-\lambda_{Xe} t} + \frac{\lambda_I I_0}{\lambda_I - \lambda_{Xe}} (e^{\lambda_{Xe} t} - e^{\lambda_I t})$$

Nuclide Concentration - Reactor Scram

[? ]

# Xenon-135 Decay Chain: Scram

- Restarting may be impossible during peak if reactor does not have enough excess control reactivity;
- Burn-off during premature restart is akin to positive reactivity insertion [**?** ]

Nuclide Concentration - Reactor Scram

[**?** ]

# Molten Salt Reactor

Molten Salt Microreactor with Xenon Stripping Module

[? ]
[? ]
[? ]

# Henry's Law

- Applies to dissolved gasses at low concentration
- Temperature dependent relationship between the concentration of the solute in the liquid phase and the partial pressure of the solute in the cover gas.

$$c = Hp$$

[? ]

# Henry's Law

By considering total pressure of the cover gas and assuming an equation of state, Henry's law can be modified to be expressed in terms of gas phase concentration instead of partial pressure.

$$c_\ell = H^* c_g$$

[? ]

# Xenon Stripping Module

Schematic Drawing of Xenon Stripping Module

# Xenon Stripping Module

Schematic Drawing of Xenon Stripping Module

$$C_{salt}^{out} = C_{salt}^{in}\left(1 + \frac{1}{H^*}\frac{\dot{V}_{He}}{\dot{V}_{salt}}\right)^{-1}$$

# Xenon Stripping Module

Schematic Drawing of Xenon Stripping Module

$$C_{salt}^{out} = C_{salt}^{in} \left( 1 + \frac{1}{H^*} \frac{\dot{V}_{He}}{\dot{V}_{salt}} \right)^{-1}$$

$$\begin{aligned} \frac{dC_{salt}}{dt} &= C_{He}^{out} \frac{\dot{V}_{He}}{\dot{V}_{salt}} = \frac{C_{salt}^{out}}{H^*} \frac{\dot{V}_{He}}{\dot{V}_{salt}} \\ &= C_{salt}^{in} \left( H^* \frac{\dot{V}_{salt}}{\dot{V}_{He}} + 1 \right)^{-1} \end{aligned}$$

# Xenon-135 Decay Chain: With Stripping

$$\frac{dI}{dt} = \underbrace{\gamma_I \Sigma_f^F \phi(t)}_{\text{Fission Yield}} - \underbrace{\lambda_I I(t)}_{\text{Beta Decay}} - \underbrace{I(t)\sigma_a^I \phi(t)}_{\text{Radiative Capture}}$$

$$\frac{dXe}{dt} = \underbrace{\gamma_{Xe} \Sigma_f^F \phi(t)}_{\text{Fission Yield}} + \underbrace{\lambda_{Xe} Xe(t)}_{\text{Precursor Decay}} - \underbrace{\lambda_{Xe} Xe(t)}_{\text{Beta Decay}}$$

$$- \underbrace{Xe(t)\sigma_a^{Xe} \phi(t)}_{\text{Radiative Capture}} - \underbrace{\frac{Xe(t)}{\tau_{salt}} \left( H^* \frac{\dot{V}_{salt}}{\dot{V}_{He}} + 1 \right)^{-1}}_{\text{Stripping}}$$

- Cold Clean Start
- 12-hr restart
- 8.26 $L/s$, $\tau_{salt} \approx 130$ $s$
- $10^{14} n/cm^2 \cdot s$

Nuclide Concentration - 12-hr Restart

# Results and Discussion

- Flowrate Ratio $1.1 \times 10^{-6}$
- 33 $mL/hr$ Helium, 392 $mL$ total
- Such a small amount, it may be more practical to release a higher flow rate in a shorter amount of time - *On-Demand*

Nuclide Concentration - 12-hr Restart

# Results and Discussion

- Call for restart at peak (10.3*hrs* after scram)
- Start-up in 260 seconds, or two flow periods - on par with natural gas peaking generators

Nuclide Concentration - Standby Restart

# Results and Discussion

Nuclide Concentration - Standby Restart

- 2.64 $mL/s$ Helium, or 687 $mL$ total
- 30 moles (modestly sized cylinder) over entire decade if restarted in this manner every day

# Final Remarks

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# References