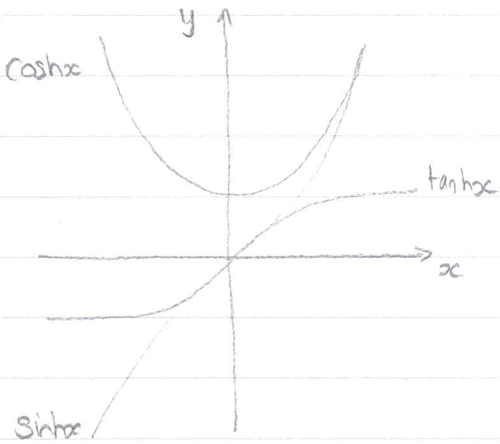
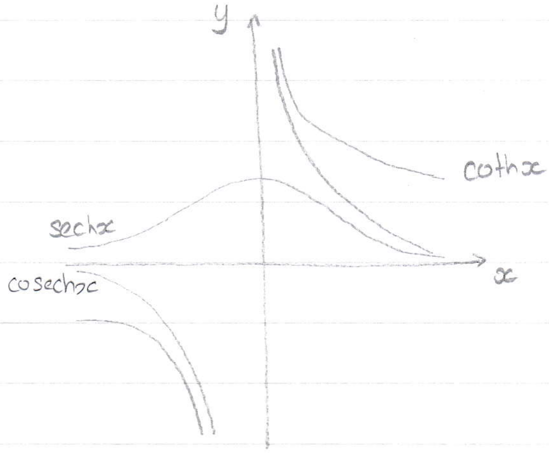
Chapter 9 – Hyperbolic Integrals.

Definition: Hypobolic functions are constructed by replacing the unit circle with the RHS branch of the unit parabola $x^2 - y^2 = 1$.





coshx: $\cosh x = (e^{x} + e^{-x})/2.$

sinhx: $\sinh x = (e^{x} - e^{-x})/2.$

Identities: $\tanh x = \dfrac{\sinh x}{\cosh x}$, $\coth x = \dfrac{1}{\tanh x}$, $\operatorname{sech} x = \dfrac{1}{\cosh x}$ and $\operatorname{cosech} x = \dfrac{1}{\sinh x}$.

Identity: $\cosh^2 x - \sinh^2 x = 1$

Proof: $\cosh^2 x - \sinh^2 x = ((e^{x} + e^{-x})/2)^2 - ((e^{x} - e^{-x})/2)^2 = 1.$

Identity: $1 - \tanh^2 x = \operatorname{sech}^2 x$

Identity: $\coth^2 x - 1 = \operatorname{cosech}^2 x$

Difference Formulae:

$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$

$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y.$

$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$

Double angle Formulae:

$$\sinh(2x) = 2\sinh x\cosh x$$

$$\cosh(2x) = \cosh^2 x + \sinh^2 x = 2\cosh^2 x - 1 = 1 + 2\sinh^2 x.$$

$$\tanh(2x) = \frac{2\tanh x}{1+\tanh^2 x}.$$

Half Angle Formulae:

$$\sinh(x/2) = \pm\sqrt{\frac{\cosh x - 1}{2}}$$

$$\cosh(x/2) = \sqrt{\frac{\cosh x + 1}{2}}$$

$$\tanh(x/2) = \pm\sqrt{\frac{\cosh x - 1}{\cosh x + 1}} = \frac{\sinh x}{\cosh x + 1}, \quad \text{positive sgn if } x \geqslant 0.$$

Derivatives:

1). $$\frac{d}{dx}(\cosh x) = \frac{d}{dx}\left(\frac{e^x + e^{-x}}{2}\right) = \frac{e^x - e^{-x}}{2} = \sinh x$$

2). $$\frac{d}{dx}(\sinh x) = \frac{d}{dx}\left(\frac{e^x - e^{-x}}{2}\right) = \frac{e^x + e^{-x}}{2} = \cosh x.$$

3). $$\frac{d}{dx}(\tanh x) = \frac{d}{dx}\left(\frac{\sinh x}{\cosh x}\right) = \frac{\cosh x\cosh x - \sinh x\sinh x}{\cosh^2 x} \quad \text{(quotient rule).}$$

$$= \frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x}$$

$$= \frac{1}{\cosh^2 x} \quad (\cosh^2 x - \sinh^2 x = 1 \text{ by identity}).$$

$$= \operatorname{sech}^2 x$$

4). $$\frac{d}{dx}(\coth x) = \frac{d}{dx}\left(\frac{\cosh x}{\sinh x}\right) = -\frac{\cosh^2 x - \sinh^2 x}{\sinh^2 x} = -\frac{1}{\sinh^2 x} = -\operatorname{cosech}^2 x.$$

5). $$\frac{d}{dx}(\operatorname{sech} x) = \frac{d}{dx}\left(\frac{1}{\cosh x}\right) = -(\cosh x)^{-2}\sinh x = -\operatorname{sech} x\tanh x$$

c). $\frac{d}{dx}(\operatorname{cosech} x) = \frac{d}{dx}\left(\frac{1}{\sinh x}\right) = -(\sinh x)^{-2} \cdot \cosh x = -\operatorname{cosech} x \coth x.$