

Chapter 5 - Standard Forms.

Logarithmic
Integrals.

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\text{e.g. } \int \tan x dx = \int \frac{\sin x}{\cos x} dx = - \int \frac{-\sin x}{\cos x} dx = -\ln|\cos x| + c$$

$$\text{e.g. } \int \cot x dx = \int \frac{\cos x}{\sin x} dx = \ln|\sin x| + c.$$

Note.

$$\sec x = \frac{1}{\cos x}, \quad \operatorname{cosec} x = \frac{1}{\sin x}, \quad \cot x = \frac{1}{\tan x}.$$

$$\text{e.g. } \int \frac{x}{x-3} dx = \int \frac{(x-3)+3}{x-3} dx = \int 1 + \frac{3}{x-3} dx = x + 3\ln|x-3| + c$$

$$\text{e.g. } \int \frac{x^2}{1+x^2} dx = \int \frac{(1+x^2)-1}{1+x^2} dx = \int dx - \int \frac{dx}{1+x^2} = x - \tan^{-1} x + c.$$

Note :

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}, \quad x \neq \pm 1$$

$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}, \quad x \neq \pm 1$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}, \quad x \neq \pm 1, 0$$

$$\frac{d}{dx}(\operatorname{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}, \quad x \neq \pm 1, 0.$$

$$\text{e.g. } \int a^x dx = \int e^{\ln(a^x)} dx = \int e^{x \ln a} dx = \frac{1}{\ln a} e^{x \ln a} + c = \frac{a^x}{\ln a} + c.$$

$$\text{e.g. } \int \frac{\sin^2 x}{1+\cos x} dx = \int \frac{1-\cos^2 x}{1+\cos x} dx = \int \frac{(1-\cos x)(1+\cos x)}{1+\cos x} dx = x - \sin x + c.$$

$$\text{e.g. } \int \frac{(\sin x + \cos x)^3}{1+\sin 2x} dx = \int \frac{(\sin x + \cos x)^3}{(\sin^2 x + \cos^2 x) + 2\sin x \cos x} dx = \int \frac{(\sin x + \cos x)^3}{(\sin x + \cos x)^2} dx$$

$$= -\cos x + \sin x + c.$$

e.g. $\int \tan 2x \tan 3x \tan 5x \, dx$

$$\text{note } \tan 5x = \tan(2x + 3x) = \frac{\tan 2x + \tan 3x}{1 - \tan 2x \tan 3x}$$

$$\text{giving } \tan 5x - \tan 2x \tan 3x \tan 5x = \tan 2x + \tan 3x$$

$$\text{or } \tan 2x \tan 3x \tan 5x = \tan 5x - \tan 3x - \tan 2x$$

$$\int \tan 2x \tan 3x \tan 5x \, dx = \int (\tan 5x - \tan 3x - \tan 2x) \, dx$$

$$= \int \tan 5x \, dx - \int \tan 3x \, dx - \int \tan 2x \, dx$$

$$= -\frac{1}{5} \ln |\cos 5x| + \frac{1}{3} \ln |\cos 3x| + \frac{1}{2} \ln |\cos 2x| + c.$$