# Calculus - Chapter 22 - Antiderivatives

Quick formula: $\int (g(x))^r g'(x)\,dx = \frac{1}{r+1}(g(x))^{r+1} + C, \quad r \neq 1$

Examples: $\int (\frac{1}{3}x^3+7)^5 x^2\,dx = \frac{1}{6}(\frac{1}{3}x^3+7)^6 + C$

Substitution method: $\int f(g(x))\,g'(x)\,dx = \int f(u)\,du$, letting $u = g(x)$ and $du = g'(x)\,dx$

Example:

(a). $\int x\sin(x^2)\,dx$, let $u = x^2$ then $\frac{du}{dx} = 2x \Leftrightarrow dx = \frac{du}{2x}$

$$\int x\sin(x^2)\,dx = \int x\cdot\sin(x^2)\frac{du}{2x} = \int \frac{\sin(u)}{2}\,du = -\frac{1}{2}\cos(x^2) + C$$

(b). $\int \sin(x/2)\,dx$, let $u = x/2$ $\frac{du}{dx} = 1/2 \Leftrightarrow dx = 2du$.

$$\int \sin(x/2)\,dx = \int \sin(u)\cdot 2\,du = -2\cos(u) + C = -2\cos(x/2) + C.$$

Formulae:

| | |
|---|---|
| $\int \sin x\,dx = -\cos x + c$ | $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + c$ |
| $\int \cos x\,dx = \sin x + c$ | $\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + c$ |
| $\int \sec^2 x\,dx = \tan x + c$ | $\int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1}x + c$ |
| $\int \tan x\sec x\,dx = \sec x + c$ | $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}(x/a) + c$ |
| $\int \csc^2 x\,dx = -\cot x + c$ | $\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}(x/a) + c$ |
| $\int \cot x\csc x\,dx = -\csc x + c$ | $\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}(x/a) + c$ |