Chapter 3 – Standard Forms.

Standard Integrals:

1. $\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \; n \neq 1, \; x \neq 0 \text{ if } n < 0.$

2. $\int \frac{dx}{x} = \ln x, \; x > 0.$

3. $\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \; a \neq 0$

4. $\int \cos ax \, dx = \frac{1}{a} \sin ax, \; a \neq 0$

5. $\int \sin ax \, dx = -\frac{1}{a} \cos x, \; a \neq 0$

6. $\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \; a \neq 0$

7. $\int \sec ax \tan ax = \frac{1}{a} \sec ax, \; a \neq 0$

8. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}(x/a), \; a \neq 0$

9. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}(x/a), \; a > 0, \; |x| < a$

10. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln \left| x + \sqrt{x^2 - a^2} \right|, \; 0 < a < x.$

11. $\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln \left| x + \sqrt{x^2 + a^2} \right|$

Example:

$\int \sin(7x - 2) \, dx$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\therefore \sin(7x - 2) = \sin 7x \cos 2 - \cos 7x \sin 2$

$$\therefore \int \sin(7x-2) \, dx = \int (\sin 7x \cos 2 - \cos 7x \sin 2) \, dx$$

$$= \cos 2 \int \sin 7x - \sin 2 \int \cos 7x \, dx$$

$$= -\frac{1}{7} \cos 2 \cos 7x - \frac{1}{7} \sin 2 \sin 7x + C.$$

But $\cos 7x \cos 2 + \sin 7x \sin 2 = \cos(7x-2)$

$\therefore \int \sin(7x-2)\,dx = -\frac{1}{7}\cos(7x-2) + c.$

Note: $\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + c,$

$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + c.$

Example: $\int \frac{dx}{11+x^2} = \int \frac{dx}{(\sqrt{11})^2+x^2} = \frac{1}{\sqrt{11}}\tan^{-1}(x/\sqrt{11}) + c.$

Example: $\int \frac{dx}{1+16x^2} = \frac{1}{16}\int \frac{dx}{\frac{1}{16}+x^2} = \frac{1}{16}\int \frac{dx}{(1/4)^2+x^2} = \frac{1}{16}\cdot 4\tan^{-1}(4x) + c$

$= \frac{1}{4}\tan^{-1}(4x) + c.$

Example: $f(x) = \frac{1}{\sqrt{1-x^2}}$, f is increasing on interval $[0, 1/2]$.

Proof: $f'(x) = \frac{x}{(1-x^2)^{3/2}} > 0 \quad \forall x \in [0, 1/2]$.