Calculus – Chapter 24 – Fundamental Theorem of Calculus.

Definition: **Mean-Value Theorem** states if f is continuous on $[a,b]$ then there exists $c \in [a,b]$ s.t

$$\int_a^b f(x)\,dx = (b-a)\,f(c)$$

To see this let m and M be minimum and maximum values of f in $[a,b]$ and apply:

$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a) \qquad \therefore\ m \leq \frac{1}{b-a}\int_a^b f(x)\,dx \leq M.$$

So by **intermediate value theorem** $\frac{1}{b-a}\int_a^b f(x)\,dx = f(c)$ for some $c \in [a,b]$.

Average Value of a Function: Let f be defined on $[a,b]$ and divide into n equal subintervals, then the average of the n values $f(x_1^*), f(x_2^*), \ldots, f(x_n^*)$ is:

$$\frac{f(x_1^*) + f(x_2^*) + \ldots + f(x_n^*)}{n} = \frac{1}{n}\sum_{k=1}^{n} f(x_k^*).$$

When n is large: $\frac{1}{n} = \frac{1}{b-a}\Delta x$.

However since

$$\frac{1}{n}\left(\sum_{k=1}^{n} f(x_k^*)\right) = \frac{1}{b-a}\left(\sum_{k=1}^{n} f(x_k^*)\Delta x\right)$$

The average value of f on $[a,b]$ is

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

Let f be continuous on $[a,b]$. If x is in $[a,b]$ then $\int_a^x f(t)\,dt$ is a function of x and:

$$D_x\left(\int_a^x f(t)\,dt\right) = f(x)$$

Fund. Th. Calculus: Let f be continuous on $[a,b]$ and let $F(x) = \int f(x)\,dx$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$