Calculus – Chapter 10 – Differentiation Rules.

Theorems:

1. $\dfrac{d}{dx}(c) = 0$
2. $\dfrac{d}{dx}(x) = 1$
3. $\dfrac{d}{dx}(cu) = c\dfrac{du}{dx}$
4. $\dfrac{d}{dx}(u+v+\ldots) = \dfrac{du}{dx} + \dfrac{dv}{dx} + \ldots$
5. $\dfrac{d}{dx}(u-v) = \dfrac{du}{dx} - \dfrac{dv}{dx}$
6. $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$
7. $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
8. $\dfrac{d}{dx}\left(\dfrac{1}{x}\right) = -1/x^2$
9. $\dfrac{d}{dx}(x^m) = mx^{m-1}$

Chain Rule:

$$\frac{d}{dx}(f(g(x)) = f'(g(x)) \cdot g'(x)$$

Example:

$f(x) = x^2+3$, $g(x) = 2x+1$, $f'(x) = 2x$, $g'(x) = 2$

Hence $\dfrac{d}{dx}(f(g(x)) = 2(2x+1) \cdot 2 = 8x+4$

Alternative Chain Rule:

Let $u = g(x)$ and $y = f(u)$, $y = f(u) = f(g(x))$:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Example:

Let $y = u^3$ and $u = 4x^2 - 2x + 5$ then the composite $y = (4x^2 - 2x + 5)^3$ has derivative :

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 3u^2(8x-2) = 3(4x^2-2x+5)^2(8x-2)$$

Inverse Formula: Let f be one-to-one and continuous on an interval (a,b)

$$(f^{-1})'(y_0) = 1/f'(x_0) \quad \text{where } \frac{dx}{dy} = 1/(dy/dx).$$

Example:

Let $y = f(x) = x^2, x > 0$

Then $x = f^{-1}(y) = \sqrt{y}$

Since $\frac{dy}{dx} = 2x$, $\frac{dx}{dy} = \frac{1}{2x} = \frac{1}{2\sqrt{y}}$ $\therefore D_y(\sqrt{y}) = \frac{1}{2\sqrt{y}}$