Chapter 25 - The Natural Logarithm.

Natural logarithm: For $r \neq -1$, $\int x^r dx = \frac{x^{r+1}}{r+1} + c$

When $r=-1$, this is finding the anti-derivative of x^{-1}.

Define the integral:

$$\ln x = \int_1^x (1/t)\,dt \quad \text{for } x > 0$$

This is the natural logarithm.

Properties: 1. $\ln 1 = 0$, since $1 = \int_1^1 (1/t)\,dt = 0$

2. If $x > 1$, then $\ln x > 0$, by the fact $\int_1^x (1/t)\,dt$

3. If $0 < x < 1$, $\ln x < 0$, as $x = \int_1^x (1/t)\,dt = -\int_x^1 (1/t)\,dt$

Now for $0 < x < 1$, if $x \leq t \leq 1$ then $1/t > 0$

4. $\ln(uv) = \ln(u) + \ln(v)$.

Because $D_x(\ln(ax)) = \frac{1}{ax} D_x(ax) = \frac{1}{ax}(a) = \frac{1}{x} = D_x(\ln x)$

Hence $\ln(ax) = \ln x + K$, and when $x=1$, $\ln a = \ln 1 + K = 0 + K = K$, so replacing a and x with u and v derives the answer.

5. $\ln(u/v) = \ln(u) - \ln(v)$, and $\ln(1/v) = -\ln(v)$.

6. $\ln(x^r) = r\ln(x)$

Formula: $$\int \frac{g'(x)}{g(x)}\,dx = \ln|g(x)| + c$$