Calculus - Chapter 48 - Partial Derivatives.

Limits example: $\lim_{(x,y)\to(3,1)} \left(\frac{3xy^2}{7+y} + \frac{1}{2}xy \right) = \frac{3(3)(1)}{7+1} = \frac{1}{2}(3)(1) = 21/8.$

Example with issue. $\lim_{(x,y)\to(0,0)} \frac{3xy^2}{x^2+y^2} = 0$ can't be as easy solved

Assume $\varepsilon > 0$, $\left| \frac{3xy^2}{x^2+y^2} - 0 \right| = \left| \frac{3xy^2}{x^2+y^2} \right| = 3|x| \left| \frac{y^2}{x^2+y^2} \right| \leq 3|x| \leq 3\sqrt{x^2+y^2} < 3\delta = \varepsilon$

Choose $\delta = \varepsilon/3$ and assume $0 < \sqrt{x^2+y^2} < \delta$

Example. Show that the following doesnt exist:

$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$, along x axis $\frac{x^2-y^2}{x^2+y^2} = \frac{x^2}{x^2} = 1$ if $y=0$

and $\frac{x^2-y^2}{x^2+y^2} = -1$ if $x=0$, hence no common limit.

Continuity: f is continuous at (a,b) iff f is defined at (a,b)

$\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$

Partial Derivative: $\lim_{\Delta x \to 0} \frac{f(x+\Delta x, y) - f(x,y)}{\Delta x}$, $f_x(x,y) = \frac{\partial f}{\partial x}$ and $f_y(x,y) = \frac{\partial f}{\partial y}$

Example: $f(x,y) = x^2 \sin y$, $\frac{\partial f}{\partial x} = 2x\sin y$, $\frac{\partial f}{\partial y} = x^2 \cos y$

Higher Orders: $\frac{\partial^2 f}{\partial x^2} = f_{xx}(x,y) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)$, $\frac{\partial^2 f}{\partial y \partial x} = f_{xy}(x,y) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)$

Similarly, $\frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\right)$.

Example: $f(x,y) = x^2(\sin yx)$, $f_x(x,y) = \frac{\partial}{\partial x}(x^2 \sin(yx))$

note $\frac{\partial}{\partial x} \sin(yx) = y \cdot \cos(yx)$

$= x^2(\cos(yx))y + 2x(\sin yx)$

$= x[xy(\cos yx) + 2\sin yx]$.

like $\frac{d}{dx}(\sin(2x)) = 2\cos(2x)$.