Calculus - Chapter 26 - Exponential and Log Functions.

Definition: e^x is the inverse of $\ln(x)$

Properties:

1. $e^x > 0 \ \forall x$
2. $\ln(e^x) = x$
3. $e^{\ln x} = x$
4. e^x is an increasing function.

   Assume $u<v$, since $u = \ln(e^u)$ and $v = \ln(e^v)$, $\ln(e^u) < \ln(e^v)$

   But as $\ln(x)$ is increasing, $e^u < e^v$
5. $D_x(e^x) = e^x$

   Let $y = e^x$, then $\ln y = x$ and implicit differentiation gives:

   $$\frac{1}{y} y' = 1 \quad \therefore \quad y' = y = e^x.$$
6. $\int e^x = e^x + c$
7. $e^0 = 1$
8. $e^{u+v} = e^u e^v$
9. $e^{u-v} = e^u / e^v$.
10. $\lim_{x \to +\infty} e^x = +\infty$ and $\lim_{x \to -\infty} e^x = 0$.

    Let $u = -x$, as $x \to \infty$, $u \to +\infty$ and $e^u \to +\infty$

    Hence, $e^x = e^{-u} = \frac{1}{e^u} \to 0$.
11. $\ln e = 1$

Definition: $e^x = \lim_{n \to +\infty} (1 + (x/n))^n$

Definition: $e = \lim_{n \to +\infty} (1 + \frac{1}{n})^n$

Definition: $a^x = e^{x \ln a}$

Definition: $\int a^x dx = \frac{1}{\ln a} a^x + c$

Example: $\int 10^x dx = \frac{1}{\ln 10} 10^x + c$

Definition: If $y = \log_a x$ then $a^y = x$, therefore $\ln(a^y) = \ln(x)$, $y\ln(a) = \ln(x)$

e.g. $y = \log_a x \Leftrightarrow a^y = x$

Definition: $\log_a(x) = \ln(x)/\ln(a)$

Definition: $\log_e(x) = \ln(x)/\ln(e) = \ln(x)/1 = \ln(x)$.