6.541/18.405 Final Project, Progress Report #2

Subject: The Computational Complexity of Probabilistic Inference

Proposed project title: Progress Toward Characterizing the Complexity of Inference in Probabilistic Graphical Models on Typical-Case Observations

George Matheos, April 30, 2024

1 Progress report

Papers read. I have read a range of papers surveying the study of the complexity of probabilistic inference, including [1, 2, 3, 4, 5, 6, 7, 8, 9], and read the abstract and introduction of several others, including [10, 11, 12].

New results. I've also spent a while trying to derive new results, extending the study of inference in probabilistic graphical models by Dagum and Luby [3, 4] from doing a worst-case analysis to doing a specific type of typical-case analysis which I believe captures the hardness of probabilistic inference in graphical models in many practical settings.

I have discovered that the results I was initially hoping to prove are more subtle than I had anticipated. I am not entirely certain the main result I wanted to prove is even true! However, I believe I have managed to prove weaker versions of these new results. I also think the difficulty that arose in proving the main result is pretty interesting, and worth writing up.

My plan for the report. I think that I have enough content that it is worth writing a report on my progress toward new results, rather than just doing a literature review. (I will also see if I can make more progress over the coming week; I don't think I've hit a wall yet, but I don't know exactly how far I can get by the project deadline.) **It would help me if you could please sign off that writing a project report like this will fulfill the course requirements!** (I'll ask Prof. W about this in OH.)

Potential rough report outline. Here is a rough outline for the project report I am planning to write, assuming I don't manage to derive new results beyond what I believe I can already prove.

1. Introduce probabilistic graphical models, and define several different inference problems we may want an algorithm to solve (including sampling from a posterior distribution, computing the posterior probability mass function up to an additive error, and computing it up to a relative error). State a couple results from the literature on the relations between the hardness of these problems.

2. Introduce Dagum and Luby's result about hardness of inference in graphical models [3], for posterior PMF (probability mass function) approximation up to additive error.

3. Define the problem of "doing inference in graphical models on typical-case inputs", and motivate why I believe this is a relevant problem.

4. State and prove a new hardness result, extending Dagum and Luby's approach to do typical-case analysis (but with weaker aspects in the theorem statement than I would like). This result will show that a polynomial time algorithm for typical-case relative-approximation to posterior PMF computation implies the nonexistance of one-way functions.

5. Discuss the difficulties encountered in strengthening this result, and state a couple of questions regarding the existance of efficient inference algorithms that succeed on typical-case observations. (One example strengthening of the hardness result I am not sure is possible: showing that not only is relative PMF approximation hard, but additive PMF approximation is hard as well.)

If it would help the report fulfill the course requirements, I could also include a related work section at the end where I give a terse summary of the topics covered in the differnet papers I read (which were very interesting, but not all directly relevant to the area of inference in graphical models where I have made a little bit of new research progress).

References

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