Probabilităţi şi statistică, 3

1) La o firmă sunt 10 imprimante, fiecare având probabilitatea de a se bloca 0.3. Într-o zi aglomerată toate sunt în funcţiune.

   a) **(0.5 puncte)** Să se determine probabilitatea ca exact 3 imprimante să se blocheze în acea zi.

   b) **(1 punct)** Să se determine probabilitatea ca, cel puţin jumătate din imprimante să funcţioneze.

   c) **(1 punct)** Fie X numărul de imprimante care funcţionează (nu se blochează). Să se determine distribuţia de probabilitate a variabilei aleatoare X. Ce tip de distribuţie este?

   d) **(0.5 puncte)** La câte imprimante ne putem aştepta să funcţioneze?

   e) **(1.5 puncte)** Să se arate că $30P(4 < X < 10) \geq 23$.

2) Fie $X_1, X_2, ..., X_n$ o selecţie aleatoare provenind dintr-o distribuţie $N(\mu, 1)$, cu μ necunoscut. (pentru $X \in N(\mu, \sigma)$, pdf este $f(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$, $x \in \mathbb{R}$, $M(X) = \mu$, $D(X) = \sigma^2$)

   a) **(1.5 puncte)** Să se determine estimatorul de verosimilitate maximă, $\hat{\mu}$, pentru μ.

   b) **(0.5 puncte)** Este acesta un estimator absolut corect? Justificaţi.

   c) **(1.5 puncte)** Să se determine eficienţa estimatorului $\hat{\mu}$, $e(\hat{\mu})$.

   d) **(1 punct)** La nivelul de semnificaţie $\alpha \in (0, 1)$, să se determine cel mai puternic test pentru testarea ipotezelor $H_0 : \mu = 1$, $H_1 : \mu = 2$.

Probability and Statistics Exam, 9

1) The battery of a particular car brand starts with probability 0.95. Find the probability of the following events:

   a) **(1 point)** A : the battery only starts on the 5^{th} attempt;

   b) **(2 points)** B : the battery starts on the first at least 20 consecutive attempts.

2) **(2 points)** Let $X \in Exp(\mu)$. Find the pdf of $Y = \sqrt{X}$.

3) Let $X_1, X_2, ..., X_n$ be a random sample drawn from a distribution with pdf $f(x;\theta) = \dfrac{2}{\theta^2}x$, for $0 < x < \theta$, with $\theta > 0$ unknown.

   a) **(2 points)** Find the method of moments estimator, $\hat{\theta}$, for θ.

   b) **(2 points)** Is $\hat{\theta}$ an absolutely correct estimator? Explain.

Probability and Statistics Exam, 8

1) A basketball player makes a free throw with probability 0.7. Find the probability of the following events:

   a) **(1 point)** A : the player makes his first free throw only on the 4^{th} shot;

   b) **(2 points)** B : the player makes the first at least 10 consecutive free throws.

2) **(2 points)** Let $X \in N(0,1)$. Find the pdf of $Y = X^2$. What type of distribution is it?

3) Let $X_1, X_2, ..., X_n$ be a random sample drawn from a distribution with pdf $f(x;\theta) = \frac{1}{\theta}$, for $0 < x < \theta$, with $\theta > 0$ unknown.

   a) **(2 points)** Find the method of moments estimator, $\hat{\theta}$, for θ.

   b) **(2 points)** Is $\hat{\theta}$ an absolutely correct estimator? Explain.

Probability and Statistics Exam, 13

1) A contestant participates in a game show where three important prizes are offered. His chances of winning the three prizes are $\frac{1}{6}$, $\frac{1}{3}$ and $\frac{1}{2}$, respectively.

- **a) (1 point)** Find the probability that the contestant wins exactly one prize.
- **b) (1.5 points)** Find the probability that the contestant loses at least two prizes.
- **c) (1.5 points)** Let X denote the number of prizes won by the contestant. Find the probability distribution function of X.
- **d) (1 point)** How many prizes can the contestant expect to win?

2) Let $X_1, X_2, ..., X_n$ be a random sample drawn from a $Gamma(2, 3\theta)$ distribution, with $\theta > 0$ unknown. (for $X \in Gamma(a, b)$, the pdf is $f(x; a, b) = \frac{1}{b^a\Gamma(a)} x^{a-1} e^{-x/b}$, $x > 0$, $E(X) = ab$, $V(X) = ab^2$)

- **a) (1.5 points)** Find the maximum likelihood estimator, $\overline{\theta}$, for θ.
- **b) (0.5 points)** Is it an absolutely correct estimator? Explain.
- **c) (2 points)** Find the efficiency of $\overline{\theta}$, $e(\overline{\theta})$.

Probabilităţi şi statistică, 6

1) La o cantină studenţească există 10 feluri de mâncare caldă, din care 6 sunt vegetariene. Patru studenţi care vin să mănânce, iau la întâmplare o porţie de mâncare.

   a) **(1 punct)** Să se determine probabilitatea ca un student să fi luat mâncare vegetariană.

   b) **(1.5 puncte)** Să se determine probabilitatea ca, cel puţin jumătate dintre studenţi să fi luat mâncare vegetariană.

   c) **(1.5 puncte)** Fie X numărul de studenţi care au primit mâncare vegetariană. Să se determine distribuţia de probabilitate a variabilei aleatoare X. Ce tip de distribuţie este?

   d) **(0.5 puncte)** Câţi studenţi se pot aştepta să primească mâncare vegetariană?

2) Fie $X_1, X_2, ..., X_n$ o selecţie aleatoare provenind dintr-o distribuţie cu pdf $f(x;p) = p^x(1-p)^{1-x}$, $x = 0, 1$, $M(X) = p$, $D(X) = p(1-p)$, unde $p \in (0,1)$ este necunoscut.

   a) **(1.5 puncte)** Să se determine estimatorul de verosimilitate maximă, $\overline{p}$, pentru p.

   b) **(0.5 puncte)** Este acesta un estimator absolut corect? Justificaţi.

   c) **(1.5 puncte)** Să se determine eficienţa estimatorului $\overline{p}$, $e(\overline{p})$.

   d) **(1 punct)** La nivelul de semnificaţie $\alpha \in (0,1)$, să se determine cel mai puternic test pentru testarea ipotezelor $H_0 : p = 1/2$, $H_1 : p = 1/4$.

and Statistics Exam

here are 10 hot lunches left at a cafeteria, 6 of which are vegetarian. Four students come to lun
ate and randomly pick up a plate.

a) **(0.5 points)** Find the probability that one student got a vegetarian lunch.

b) **(1 point)** Find the probability that at most half of the students got a vegetarian lunch.

c) **(1 point)** Let X denote the number of students who got a vegetarian lunch. Find the pr
bility distribution function of X. What type of distribution is it?

d) **(0.5 points)** What is the expected number of students getting a vegetarian lunch?

e) **(1.5 points)** Prove that $5P(|X| \geq 3) \leq 4$.

t $X_1, X_2, ..., X_n$ be a random sample drawn from a distribution with pdf $f(x;p) = p^x(1-$
$= 0, 1$, $E(X) = p$, $V(X) = p(1-p)$, where $p \in (0,1)$ is unknown.

a) **(1.5 points)** Find the maximum likelihood estimator, $\overline{p}$, for p.

b) **(0.5 points)** Is it an absolutely correct estimator? Explain.

c) **(1.5 points)** Find the efficiency of $\overline{p}$, $e(\overline{p})$.

d) **(1 point)** At the significance level $\alpha \in (0,1)$, find a most powerful test for testing H
against $H_1 : p = 1/4$.