Imbalanced kinetic Alfvén wave turbulence

George Miloshevich, Thierry Passot, Pierre-Louis Sulem and Dimitri Laveder

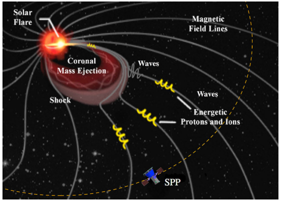
Festival de Téorie
Aix en Provance, 2019

Alfvén Wave turbulence in the Solar Wind (SW)

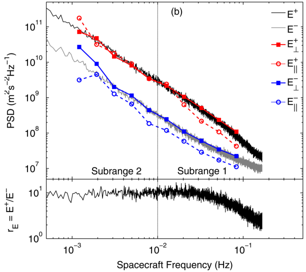
- *Alfvén Waves* (AW) play an important role in SW [1]
  - Reflection leads to backward propagating waves
  - SW turbulence results from these interactions.
  - SW turbulence has mostly been studied using MHD

Schematics of a forward cascade to small scales

- SW is collisionless so kinetical treatment is ideal
  - Microscale dispersive gyro-scales play a role
  - Imbalanced turbulence [2] is of interest for (PSP)
  - Direct Vlasov-Maxwell simulations are too costly
  - Recourse to gyrofluids is more feasible[3]
- Describe the transition to imbalanced dispersive range



Parker Solar Probe (PSP)



Wind Data.

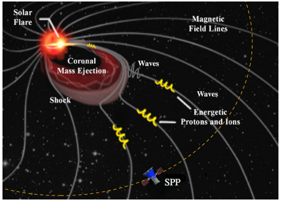
[1] F. Sahraoui et al., PRL **102**, 231102 (2009)
[2] R. T. Wicks et al., PRL **106**, 045001 (2011)
[3] T. Passot and P. L. Sulem, JPP **85**, 905850301 (2019)

Alfvén Wave turbulence in the Solar Wind (SW)

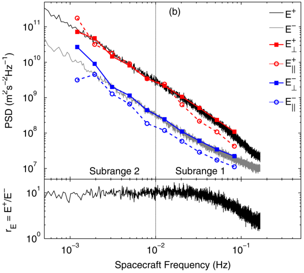
- *Alfvén Waves* (AW) play an important role in SW [1]
  - Reflection leads to backward propagating waves
  - SW turbulence results from these interactions.
  - SW turbulence has mostly been studied using MHD

Schematics of a forward cascade to small scales

- SW is collisionless so kinetical treatment is ideal
  - Microscale dispersive gyro-scales play a role
  - Imbalanced turbulence [2] is of interest for (PSP)
  - Direct Vlasov-Maxwell simulations are too costly
  - Recourse to gyrofluids is more feasible[3]
- Describe the transition to imbalanced dispersive range



Parker Solar Probe (PSP)



Wind Data.

[1] F. Sahraoui et al., PRL **102**, 231102 (2009)
[2] R. T. Wicks et al., PRL **106**, 045001 (2011)
[3] T. Passot and P. L. Sulem, JPP **85**, 905850301 (2019)

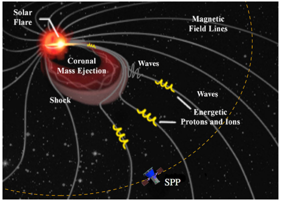
Alfvén Wave turbulence in the Solar Wind (SW)

- *Alfvén Waves* (AW) play an important role in SW [1]
  - Reflection leads to backward propagating waves
  - SW turbulence results from these interactions.
  - SW turbulence has mostly been studied using MHD

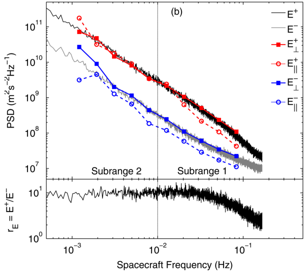
Driving Inertial Dissipative

Schematics of a forward cascade to small scales

- SW is collisionless so kinetical treatment is ideal
  - Microscale dispersive gyro-scales play a role
  - Imbalanced turbulence [2] is of interest for (PSP)
  - Direct Vlasov-Maxwell simulations are too costly
  - Recourse to gyrofluids is more feasible[3]
- Describe the transition to imbalanced dispersive range



Parker Solar Probe (PSP)

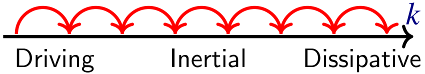


Wind Data.

[1] F. Sahraoui et al., PRL **102**, 231102 (2009)
[2] R. T. Wicks et al., PRL **106**, 045001 (2011)
[3] T. Passot and P. L. Sulem, JPP **85**, 905850301 (2019)

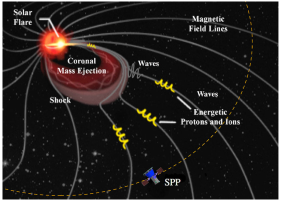
Alfvén Wave turbulence in the Solar Wind (SW)

- *Alfvén Waves* (AW) play an important role in SW [1]
  - Reflection leads to backward propagating waves
  - SW turbulence results from these interactions.
  - SW turbulence has mostly been studied using MHD

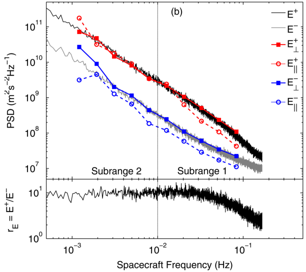


Schematics of a forward cascade to small scales

- SW is collisionless so kinetical treatment is ideal
  - Microscale dispersive gyro-scales play a role
  - Imbalanced turbulence [2] is of interest for (PSP)
  - Direct Vlasov-Maxwell simulations are too costly
  - Recourse to gyrofluids is more feasible[3]
- Describe the transition to imbalanced dispersive range



Parker Solar Probe (PSP)

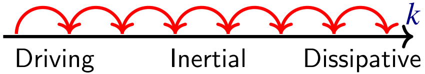


Wind Data.

[1] F. Sahraoui et al., PRL **102**, 231102 (2009)
[2] R. T. Wicks et al., PRL **106**, 045001 (2011)
[3] T. Passot and P. L. Sulem, JPP **85**, 905850301 (2019)

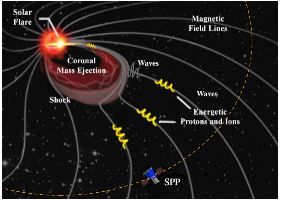
Alfvén Wave turbulence in the Solar Wind (SW)

- *Alfvén Waves* (AW) play an important role in SW [1]
  - Reflection leads to backward propagating waves
  - SW turbulence results from these interactions.
  - SW turbulence has mostly been studied using MHD

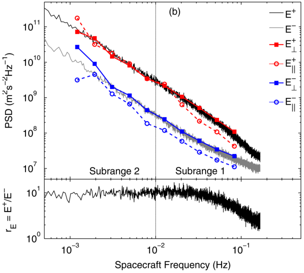


Schematics of a forward cascade to small scales

- SW is collisionless so kinetical treatment is ideal
  - Microscale dispersive gyro-scales play a role
  - Imbalanced turbulence [2] is of interest for (PSP)
  - Direct Vlasov-Maxwell simulations are too costly
  - Recourse to gyrofluids is more feasible[3]
- Describe the transition to imbalanced dispersive range



Parker Solar Probe (PSP)

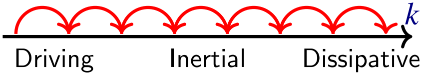


Wind Data.

[1] F. Sahraoui et al., PRL **102**, 231102 (2009)
[2] R. T. Wicks et al., PRL **106**, 045001 (2011)
[3] T. Passot and P. L. Sulem, JPP **85**, 905850301 (2019)

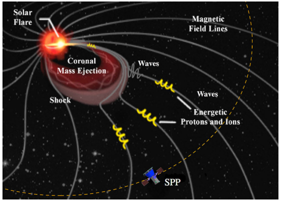
Alfvén Wave turbulence in the Solar Wind (SW)

- *Alfvén Waves* (AW) play an important role in SW [1]
  - Reflection leads to backward propagating waves
  - SW turbulence results from these interactions.
  - SW turbulence has mostly been studied using MHD

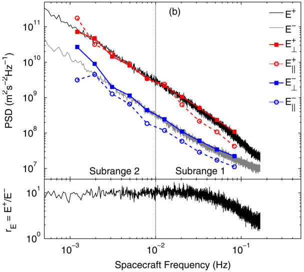


Schematics of a forward cascade to small scales

- SW is collisionless so kinetical treatment is ideal
  - Microscale dispersive gyro-scales play a role
  - Imbalanced turbulence [2] is of interest for (PSP)
  - Direct Vlasov-Maxwell simulations are too costly
  - Recourse to gyrofluids is more feasible[3]
- Describe the transition to imbalanced dispersive range



Parker Solar Probe (PSP)

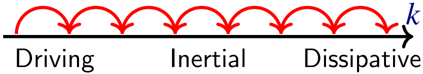


Wind Data.

[1] F. Sahraoui et al., PRL **102**, 231102 (2009)
[2] R. T. Wicks et al., PRL **106**, 045001 (2011)
[3] T. Passot and P. L. Sulem, JPP **85**, 905850301 (2019)

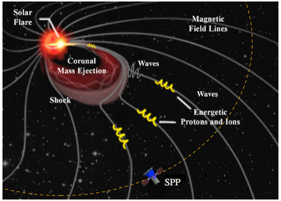
Alfvén Wave turbulence in the Solar Wind (SW)

- *Alfvén Waves* (AW) play an important role in SW [1]
  - Reflection leads to backward propagating waves
  - SW turbulence results from these interactions.
  - SW turbulence has mostly been studied using MHD

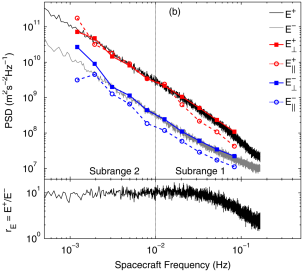


Schematics of a forward cascade to small scales

- SW is collisionless so kinetical treatment is ideal
  - Microscale dispersive gyro-scales play a role
  - Imbalanced turbulence [2] is of interest for (PSP)
  - Direct Vlasov-Maxwell simulations are too costly
  - Recourse to gyrofluids is more feasible[3]
- Describe the transition to imbalanced dispersive range



Parker Solar Probe (PSP)

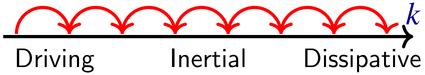


Wind Data.

[1] F. Sahraoui et al., PRL **102**, 231102 (2009)
[2] R. T. Wicks et al., PRL **106**, 045001 (2011)
[3] T. Passot and P. L. Sulem, JPP **85**, 905850301 (2019)

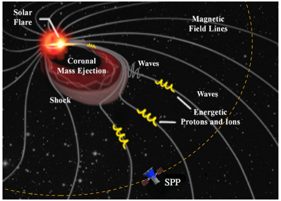
Alfvén Wave turbulence in the Solar Wind (SW)

- *Alfvén Waves* (AW) play an important role in SW [1]
  - Reflection leads to backward propagating waves
  - SW turbulence results from these interactions.
  - SW turbulence has mostly been studied using MHD

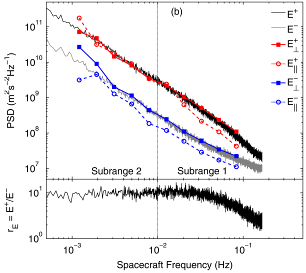


Schematics of a forward cascade to small scales

- SW is collisionless so kinetical treatment is ideal
  - Microscale dispersive gyro-scales play a role
  - Imbalanced turbulence [2] is of interest for (PSP)
  - Direct Vlasov-Maxwell simulations are too costly
  - Recourse to gyrofluids is more feasible[3]
- Describe the transition to imbalanced dispersive range



Parker Solar Probe (PSP)

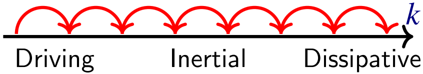


Wind Data.

[1] F. Sahraoui et al., PRL **102**, 231102 (2009)
[2] R. T. Wicks et al., PRL **106**, 045001 (2011)
[3] T. Passot and P. L. Sulem, JPP **85**, 905850301 (2019)

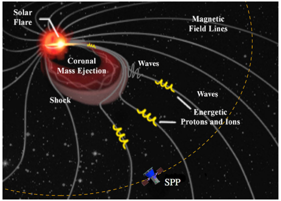
Alfvén Wave turbulence in the Solar Wind (SW)

- *Alfvén Waves* (AW) play an important role in SW [1]
  - Reflection leads to backward propagating waves
  - SW turbulence results from these interactions.
  - SW turbulence has mostly been studied using MHD

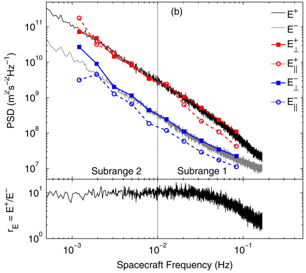


Schematics of a forward cascade to small scales

- SW is collisionless so kinetical treatment is ideal
  - Microscale dispersive gyro-scales play a role
  - Imbalanced turbulence [2] is of interest for (PSP)
  - Direct Vlasov-Maxwell simulations are too costly
  - Recourse to gyrofluids is more feasible[3]
- Describe the transition to imbalanced dispersive range



Parker Solar Probe (PSP)



Wind Data.

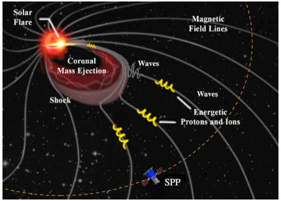
[1] F. Sahraoui et al., PRL **102**, 231102 (2009)
[2] R. T. Wicks et al., PRL **106**, 045001 (2011)
[3] T. Passot and P. L. Sulem, JPP **85**, 905850301 (2019)

Alfvén Wave turbulence in the Solar Wind (SW)

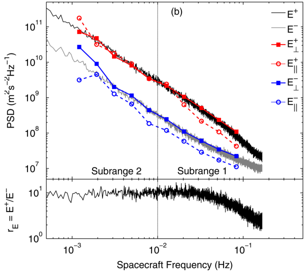
- *Alfvén Waves* (AW) play an important role in SW [1]
  - Reflection leads to backward propagating waves
  - SW turbulence results from these interactions.
  - SW turbulence has mostly been studied using MHD

Schematics of a forward cascade to small scales

- SW is collisionless so kinetical treatment is ideal
  - Microscale dispersive gyro-scales play a role
  - Imbalanced turbulence [2] is of interest for (PSP)
  - Direct Vlasov-Maxwell simulations are too costly
  - Recourse to gyrofluids is more feasible[3]
- Describe the transition to imbalanced dispersive range



Parker Solar Probe (PSP)



Wind Data.

[1] F. Sahraoui et al., PRL **102**, 231102 (2009)
[2] R. T. Wicks et al., PRL **106**, 045001 (2011)
[3] T. Passot and P. L. Sulem, JPP **85**, 905850301 (2019)

Outline

Outline

Outline

Outline

Weak balanced magnetohydrodynamic (MHD) turbulence

- Before discussing dispersive effects we review what is known about MHD
- Using Elsässer variables $\boldsymbol{w}^{\pm} := \boldsymbol{v} \pm \boldsymbol{b}$, MHD can be cast:

$$\partial_t \boldsymbol{w}^{\pm} \pm V_A \partial_z \boldsymbol{w}^{\pm} = -\boldsymbol{w}^{\mp} \cdot \nabla \boldsymbol{w}^{\pm} - \nabla P, \quad \nabla \cdot \boldsymbol{w}^{\pm} = 0, \quad P = -\nabla^{-2}(\nabla \boldsymbol{w}^{+} : \nabla \boldsymbol{w}^{-}) \tag{1}$$

only counter propagating waves interact

- Assuming *weak balanced* cascade[4], many $w_{\pm}$ collisions before cascading $\tau_{nl} \gg \tau_A$

$$w^{+}_{k_\perp} = w^{-}_{k_\perp}, \qquad w^{+}_{k_\perp} w^{-}_{k_\perp} \propto k_\perp^{-1} \quad \Rightarrow w^{\pm}_{k_\perp} \propto k_\perp^{-1/2} \tag{2}$$

- Criticism of weak MHD turbulence[5]: Because AW have $\omega^{\pm}_{\boldsymbol{p}} = \pm V_A k_\parallel$

$$\left.\begin{aligned} &\boldsymbol{p} + \boldsymbol{q} = \boldsymbol{k} \\ &\omega^{\pm}_{\boldsymbol{p}} + \omega^{\mp}_{\boldsymbol{q}} = \omega^{\pm}_{\boldsymbol{k}} \Rightarrow p_\parallel - q_\parallel = k_\parallel \end{aligned}\right\} \Rightarrow p_\parallel = k_\parallel, \quad q_\parallel = 0 \tag{3}$$

- So in MHD it involves scattering of AW off a 2D perturbation ($q_\parallel = 0$)
- As weak cascade proceeds eventually $\tau_{nl} \sim k_\perp w_{k_\perp} \sim \tau_A \longrightarrow$ and becomes *strong*

[4] C. S. Ng and A. Bhattacharjee, PoP 4, 605–610 (1997)
[5] A. A. Schekochihin, MHD Turbulence: A Biased Review, submitted to JPP

Weak balanced magnetohydrodynamic (MHD) turbulence

- Before discussing dispersive effects we review what is known about MHD
- Using Elsässer variables $\boldsymbol{w}^{\pm} := \boldsymbol{v} \pm \boldsymbol{b}$, MHD can be cast:

$$\partial_t \boldsymbol{w}^{\pm} \pm V_A \partial_z \boldsymbol{w}^{\pm} = -\boldsymbol{w}^{\mp} \cdot \nabla \boldsymbol{w}^{\pm} - \nabla P, \quad \nabla \cdot \boldsymbol{w}^{\pm} = 0, \quad P = -\nabla^{-2}(\nabla \boldsymbol{w}^{+} : \nabla \boldsymbol{w}^{-}) \qquad (1)$$

only counter propagating waves interact

- Assuming *weak balanced* cascade[4], many $w_{\pm}$ collisions before cascading $\tau_{nl} \gg \tau_A$

$$w^{+}_{k_\perp} = w^{-}_{k_\perp}, \qquad w^{+}_{k_\perp} w^{-}_{k_\perp} \propto k_\perp^{-1} \quad \Rightarrow w^{\pm}_{k_\perp} \propto k_\perp^{-1/2} \qquad (2)$$

- Criticism of weak MHD turbulence[5]: Because AW have $\omega^{\pm}_{\boldsymbol{p}} = \pm V_A k_\parallel$

$$\left.\begin{aligned} &\boldsymbol{p} + \boldsymbol{q} = \boldsymbol{k} \\ &\omega^{\pm}_{\boldsymbol{p}} + \omega^{\mp}_{\boldsymbol{q}} = \omega^{\pm}_{\boldsymbol{k}} \Rightarrow p_\parallel - q_\parallel = k_\parallel \end{aligned}\right\} \Rightarrow p_\parallel = k_\parallel, \quad q_\parallel = 0 \qquad (3)$$

- So in MHD it involves scattering of AW off a 2D perturbation ($q_\parallel = 0$)
- As weak cascade proceeds eventually $\tau_{nl} \sim k_\perp w_{k_\perp} \sim \tau_A \longrightarrow$ and becomes *strong*

[4] C. S. Ng and A. Bhattacharjee, PoP 4, 605–610 (1997)
[5] A. A. Schekochihin, MHD Turbulence: A Biased Review, submitted to JPP

Weak balanced magnetohydrodynamic (MHD) turbulence

- Before discussing dispersive effects we review what is known about MHD
- Using Elsässer variables $\boldsymbol{w}^{\pm} := \boldsymbol{v} \pm \boldsymbol{b}$, MHD can be cast:

$$\partial_t \boldsymbol{w}^{\pm} \pm V_A \partial_z \boldsymbol{w}^{\pm} = -\boldsymbol{w}^{\mp} \cdot \nabla \boldsymbol{w}^{\pm} - \nabla P, \quad \nabla \cdot \boldsymbol{w}^{\pm} = 0, \quad P = -\nabla^{-2}(\nabla \boldsymbol{w}^{+} : \nabla \boldsymbol{w}^{-}) \tag{1}$$

only counter propagating waves interact

- Assuming *weak balanced* cascade[4], many $w_{\pm}$ collisions before cascading $\tau_{nl} \gg \tau_A$

$$w^{+}_{k_\perp} = w^{-}_{k_\perp}, \qquad w^{+}_{k_\perp} w^{-}_{k_\perp} \propto k_\perp^{-1} \quad \Rightarrow w^{\pm}_{k_\perp} \propto k_\perp^{-1/2} \tag{2}$$

- Criticism of weak MHD turbulence[5]: Because AW have $\omega^{\pm}_{\boldsymbol{p}} = \pm V_A k_\parallel$

$$\left.\begin{aligned} &\boldsymbol{p} + \boldsymbol{q} = \boldsymbol{k} \\ &\omega^{\pm}_{\boldsymbol{p}} + \omega^{\mp}_{\boldsymbol{q}} = \omega^{\pm}_{\boldsymbol{k}} \Rightarrow p_\parallel - q_\parallel = k_\parallel \end{aligned}\right\} \Rightarrow p_\parallel = k_\parallel, \quad q_\parallel = 0 \tag{3}$$

- So in MHD it involves scattering of AW off a 2D perturbation ($q_\parallel = 0$)
- As weak cascade proceeds eventually $\tau_{nl} \sim k_\perp w_{k_\perp} \sim \tau_A \longrightarrow$ and becomes *strong*

[4] C. S. Ng and A. Bhattacharjee, PoP 4, 605–610 (1997)
[5] A. A. Schekochihin, MHD Turbulence: A Biased Review, submitted to JPP

Weak balanced magnetohydrodynamic (MHD) turbulence

- Before discussing dispersive effects we review what is known about MHD
- Using Elsässer variables $\boldsymbol{w}^{\pm} := \boldsymbol{v} \pm \boldsymbol{b}$, MHD can be cast:

$$\partial_t \boldsymbol{w}^{\pm} \pm V_A \partial_z \boldsymbol{w}^{\pm} = -\boldsymbol{w}^{\mp} \cdot \nabla \boldsymbol{w}^{\pm} - \nabla P, \quad \nabla \cdot \boldsymbol{w}^{\pm} = 0, \quad P = -\nabla^{-2}(\nabla \boldsymbol{w}^{+} : \nabla \boldsymbol{w}^{-}) \tag{1}$$

only counter propagating waves interact

- Assuming *weak balanced* cascade[4], many $w_{\pm}$ collisions before cascading $\tau_{nl} \gg \tau_A$

$$w^{+}_{k_\perp} = w^{-}_{k_\perp}, \qquad w^{+}_{k_\perp} w^{-}_{k_\perp} \propto k_\perp^{-1} \Rightarrow w^{\pm}_{k_\perp} \propto k_\perp^{-1/2} \tag{2}$$

- Criticism of weak MHD turbulence[5]: Because AW have $\omega^{\pm}_{\boldsymbol{p}} = \pm V_A k_\parallel$

$$\left.\begin{aligned} &\boldsymbol{p} + \boldsymbol{q} = \boldsymbol{k} \\ &\omega^{\pm}_{\boldsymbol{p}} + \omega^{\mp}_{\boldsymbol{q}} = \omega^{\pm}_{\boldsymbol{k}} \Rightarrow p_\parallel - q_\parallel = k_\parallel \end{aligned}\right\} \Rightarrow p_\parallel = k_\parallel, \quad q_\parallel = 0 \tag{3}$$

- So in MHD it involves scattering of AW off a 2D perturbation ($q_\parallel = 0$)
- As weak cascade proceeds eventually $\tau_{nl} \sim k_\perp w_{k_\perp} \sim \tau_A \longrightarrow$ and becomes *strong*

[4] C. S. Ng and A. Bhattacharjee, PoP **4**, 605–610 (1997)
[5] A. A. Schekochihin, MHD Turbulence: A Biased Review, submitted to JPP

Weak balanced magnetohydrodynamic (MHD) turbulence

- Before discussing dispersive effects we review what is known about MHD
- Using Elsässer variables $\boldsymbol{w}^\pm := \boldsymbol{v} \pm \boldsymbol{b}$, MHD can be cast:

$$\partial_t \boldsymbol{w}^\pm \pm V_A \partial_z \boldsymbol{w}^\pm = -\boldsymbol{w}^\mp \cdot \nabla \boldsymbol{w}^\pm - \nabla P, \quad \nabla \cdot \boldsymbol{w}^\pm = 0, \quad P = -\nabla^{-2}(\nabla \boldsymbol{w}^+ : \nabla \boldsymbol{w}^-) \tag{1}$$

only counter propagating waves interact

- Assuming *weak balanced* cascade[4], many $w_\pm$ collisions before cascading $\tau_{nl} \gg \tau_A$

$$w^+_{k_\perp} = w^-_{k_\perp}, \qquad w^+_{k_\perp} w^-_{k_\perp} \propto k_\perp^{-1} \quad \Rightarrow w^\pm_{k_\perp} \propto k_\perp^{-1/2} \tag{2}$$

- Criticism of weak MHD turbulence[5]: Because AW have $\omega^\pm_{\boldsymbol{p}} = \pm V_A k_\parallel$

$$\left.\begin{aligned} &\boldsymbol{p} + \boldsymbol{q} = \boldsymbol{k} \\ &\omega^\pm_{\boldsymbol{p}} + \omega^\mp_{\boldsymbol{q}} = \omega^\pm_{\boldsymbol{k}} \Rightarrow p_\parallel - q_\parallel = k_\parallel \end{aligned}\right\} \Rightarrow p_\parallel = k_\parallel, \quad q_\parallel = 0 \tag{3}$$

- So in MHD it involves scattering of AW off a 2D perturbation ($q_\parallel = 0$)
- As weak cascade proceeds eventually $\tau_{nl} \sim k_\perp w_{k_\perp} \sim \tau_A \longrightarrow$ and becomes *strong*

[4] C. S. Ng and A. Bhattacharjee, PoP **4**, 605–610 (1997)
[5] A. A. Schekochihin, MHD Turbulence: A Biased Review, submitted to JPP

Weak balanced magnetohydrodynamic (MHD) turbulence

- Before discussing dispersive effects we review what is known about MHD
- Using Elsässer variables $\boldsymbol{w}^{\pm} := \boldsymbol{v} \pm \boldsymbol{b}$, MHD can be cast:

$$\partial_t \boldsymbol{w}^{\pm} \pm V_A \partial_z \boldsymbol{w}^{\pm} = -\boldsymbol{w}^{\mp} \cdot \nabla \boldsymbol{w}^{\pm} - \nabla P, \quad \nabla \cdot \boldsymbol{w}^{\pm} = 0, \quad P = -\nabla^{-2}(\nabla \boldsymbol{w}^{+} : \nabla \boldsymbol{w}^{-}) \tag{1}$$

only counter propagating waves interact

- Assuming *weak balanced* cascade[4], many $w_{\pm}$ collisions before cascading $\tau_{nl} \gg \tau_A$

$$w^{+}_{k_\perp} = w^{-}_{k_\perp}, \qquad w^{+}_{k_\perp} w^{-}_{k_\perp} \propto k_\perp^{-1} \Rightarrow w^{\pm}_{k_\perp} \propto k_\perp^{-1/2} \tag{2}$$

- Criticism of weak MHD turbulence[5]: Because AW have $\omega^{\pm}_{\boldsymbol{p}} = \pm V_A k_\parallel$

$$\left.\begin{aligned} &\boldsymbol{p} + \boldsymbol{q} = \boldsymbol{k} \\ &\omega^{\pm}_{\boldsymbol{p}} + \omega^{\mp}_{\boldsymbol{q}} = \omega^{\pm}_{\boldsymbol{k}} \Rightarrow p_\parallel - q_\parallel = k_\parallel \end{aligned}\right\} \Rightarrow p_\parallel = k_\parallel, \quad q_\parallel = 0 \tag{3}$$

- So in MHD it involves scattering of AW off a 2D perturbation ($q_\parallel = 0$)
- As weak cascade proceeds eventually $\tau_{nl} \sim k_\perp w_{k_\perp} \sim \tau_A \longrightarrow$ and becomes *strong*

[4] C. S. Ng and A. Bhattacharjee, PoP **4**, 605–610 (1997)
[5] A. A. Schekochihin, MHD Turbulence: A Biased Review, submitted to JPR

Weak balanced magnetohydrodynamic (MHD) turbulence

- Before discussing dispersive effects we review what is known about MHD
- Using Elsässer variables $\boldsymbol{w}^{\pm} := \boldsymbol{v} \pm \boldsymbol{b}$, MHD can be cast:

$$\partial_t \boldsymbol{w}^{\pm} \pm V_A \partial_z \boldsymbol{w}^{\pm} = -\boldsymbol{w}^{\mp} \cdot \nabla \boldsymbol{w}^{\pm} - \nabla P, \quad \nabla \cdot \boldsymbol{w}^{\pm} = 0, \quad P = -\nabla^{-2}(\nabla \boldsymbol{w}^{+} : \nabla \boldsymbol{w}^{-}) \tag{1}$$

only counter propagating waves interact

- Assuming *weak balanced* cascade[4], many $w_{\pm}$ collisions before cascading $\tau_{nl} \gg \tau_A$

$$w^{+}_{k_\perp} = w^{-}_{k_\perp}, \qquad w^{+}_{k_\perp} w^{-}_{k_\perp} \propto k_\perp^{-1} \Rightarrow w^{\pm}_{k_\perp} \propto k_\perp^{-1/2} \tag{2}$$

- Criticism of weak MHD turbulence[5]: Because AW have $\omega^{\pm}_{\boldsymbol{p}} = \pm V_A k_\parallel$

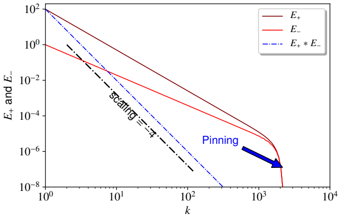
$$\left.\begin{aligned} &\boldsymbol{p} + \boldsymbol{q} = \boldsymbol{k} \\ &\omega^{\pm}_{\boldsymbol{p}} + \omega^{\mp}_{\boldsymbol{q}} = \omega^{\pm}_{\boldsymbol{k}} \Rightarrow p_\parallel - q_\parallel = k_\parallel \end{aligned}\right\} \Rightarrow p_\parallel = k_\parallel, \quad q_\parallel = 0 \tag{3}$$

- So in MHD it involves scattering of AW off a 2D perturbation ($q_\parallel = 0$)
- As weak cascade proceeds eventually $\tau_{nl} \sim k_\perp w_{k_\perp} \sim \tau_A \longrightarrow$ and becomes *strong*

[4] C. S. Ng and A. Bhattacharjee, PoP **4**, 605–610 (1997)
[5] A. A. Schekochihin, MHD Turbulence: A Biased Review, submitted to JPR

Pinning in case of imbalance and strong MHD turbulence

- *Imbalanced cascades* are much more common
- When $w^+_{k_\perp} \neq w^-_{k_\perp}$ additional criterion is required
- This is provided by the phenomenon of pinning [6][7]
- Weak turbulence proceeds to become strong
- Goldreich and Sridhar[8] introduced anisotropic theory postulating *critical balance*



Weak $E_\pm(k_\perp) := {w_{k_\perp}}^2/k_\perp$

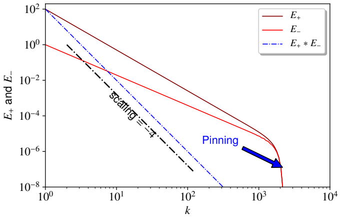
$$k_\parallel V_A \sim k_\perp v_\perp \qquad \epsilon \sim \frac{V_A^3}{L}, \quad \epsilon \sim \frac{v_\perp^2}{t_{cas}}, \quad t_{cas} \sim \frac{1}{k_\parallel V_A} \Rightarrow \qquad k_\parallel \sim k_\perp^{2/3} L^{-1/3} \tag{4}$$

- Role of dynamical alignment leading to $-3/2$ energy spectrum[9]
- High resolution MHD simulations suggest $-5/3$ energy spectrum[10]

[6] R. Grappin et al., Astron. and Astroph. **126**, 51–58 (1983)
[7] Y. Lithwick et al., ApJ **582**, 1220–1240 (2003)
[8] P. Goldreich and S. Sridhar, ApJ **438**, 763–775 (1995)
[9] S. Boldyrev et al., ApJ **699**, L39–L42 (2009)
[10] A. Beresnyak, ApJ **784**, L20 (2014)

Pinning in case of imbalance and strong MHD turbulence

- *Imbalanced cascades* are much more common
- When $w^+_{k_\perp} \neq w^-_{k_\perp}$ additional criterion is required
- This is provided by the phenomenon of pinning [6][7]
- Weak turbulence proceeds to become strong
- Goldreich and Sridhar[8] introduced anisotropic theory postulating *critical balance*



Weak $E_\pm(k_\perp) := w_{k_\perp}{}^2/k_\perp$

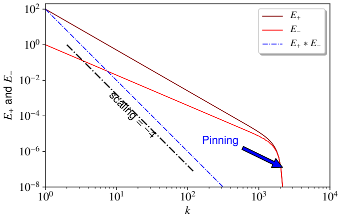
$$k_\parallel V_A \sim k_\perp v_\perp \qquad \epsilon \sim \frac{V_A^3}{L}, \quad \epsilon \sim \frac{v_\perp^2}{t_{cas}}, \quad t_{cas} \sim \frac{1}{k_\parallel V_A} \Rightarrow \qquad k_\parallel \sim k_\perp^{2/3} L^{-1/3} \tag{4}$$

- Role of dynamical alignment leading to $-3/2$ energy spectrum[9]
- High resolution MHD simulations suggest $-5/3$ energy spectrum[10]

[6] R. Grappin et al., Astron. and Astroph. **126**, 51–58 (1983)
[7] Y. Lithwick et al., ApJ **582**, 1220–1240 (2003)
[8] P. Goldreich and S. Sridhar, ApJ **438**, 763–775 (1995)
[9] S. Boldyrev et al., ApJ **699**, L39–L42 (2009)
[10] A. Beresnyak, ApJ **784**, L20 (2014)

Pinning in case of imbalance and strong MHD turbulence

- *Imbalanced cascades* are much more common
- When $w^{+}_{k_\perp} \neq w^{-}_{k_\perp}$ additional criterion is required
- This is provided by the phenomenon of pinning [6][7]
- Weak turbulence proceeds to become strong
- Goldreich and Sridhar[8] introduced anisotropic theory postulating *critical balance*



Weak $E_{\pm}(k_\perp) := {w_{k_\perp}}^2 / k_\perp$

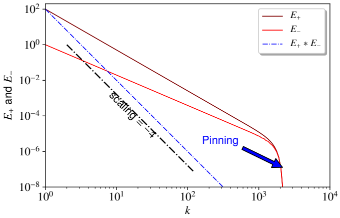
$$k_{\|} V_A \sim k_\perp v_\perp \quad \epsilon \sim \frac{V_A^3}{L}, \quad \epsilon \sim \frac{v_\perp^2}{t_{cas}}, \quad t_{cas} \sim \frac{1}{k_{\|} V_A} \Rightarrow \quad k_{\|} \sim k_\perp^{2/3} L^{-1/3} \tag{4}$$

- Role of dynamical alignment leading to $-3/2$ energy spectrum[9]
- High resolution MHD simulations suggest $-5/3$ energy spectrum[10]

[6] R. Grappin et al., Astron. and Astroph. **126**, 51–58 (1983)
[7] Y. Lithwick et al., ApJ **582**, 1220–1240 (2003)
[8] P. Goldreich and S. Sridhar, ApJ **438**, 763–775 (1995)
[9] S. Boldyrev et al., ApJ **699**, L39–L42 (2009)
[10] A. Beresnyak, ApJ **784**, L20 (2014)

Pinning in case of imbalance and strong MHD turbulence

- *Imbalanced cascades* are much more common
- When $w^+_{k_\perp} \neq w^-_{k_\perp}$ additional criterion is required
- This is provided by the phenomenon of pinning [6][7]
- Weak turbulence proceeds to become strong
- Goldreich and Sridhar[8] introduced anisotropic theory postulating *critical balance*



Weak $E_\pm(k_\perp) := w_{k_\perp}{}^2/k_\perp$

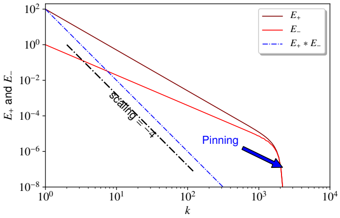
$$k_\parallel V_A \sim k_\perp v_\perp \quad \epsilon \sim \frac{V_A^3}{L}, \quad \epsilon \sim \frac{v_\perp^2}{t_{cas}}, \quad t_{cas} \sim \frac{1}{k_\parallel V_A} \Rightarrow \quad k_\parallel \sim k_\perp^{2/3} L^{-1/3} \qquad (4)$$

- Role of dynamical alignment leading to $-3/2$ energy spectrum[9]
- High resolution MHD simulations suggest $-5/3$ energy spectrum[10]

[6] R. Grappin et al., Astron. and Astroph. **126**, 51–58 (1983)
[7] Y. Lithwick et al., ApJ **582**, 1220–1240 (2003)
[8] P. Goldreich and S. Sridhar, ApJ **438**, 763–775 (1995)
[9] S. Boldyrev et al., ApJ **699**, L39–L42 (2009)
[10] A. Beresnyak, ApJ **784**, L20 (2014)

Pinning in case of imbalance and strong MHD turbulence

- *Imbalanced cascades* are much more common
- When $w^+_{k_\perp} \neq w^-_{k_\perp}$ additional criterion is required
- This is provided by the phenomenon of pinning [6][7]
- Weak turbulence proceeds to become strong
- Goldreich and Sridhar[8] introduced anisotropic theory postulating *critical balance*



Weak $E_\pm(k_\perp) := {w_{k_\perp}}^2/k_\perp$

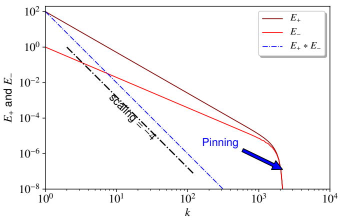
$$k_\| V_A \sim k_\perp v_\perp \quad \epsilon \sim \frac{V_A^3}{L}, \quad \epsilon \sim \frac{v_\perp^2}{t_{cas}}, \quad t_{cas} \sim \frac{1}{k_\| V_A} \Rightarrow \quad k_\| \sim k_\perp^{2/3} L^{-1/3} \tag{4}$$

- Role of dynamical alignment leading to $-3/2$ energy spectrum[9]
- High resolution MHD simulations suggest $-5/3$ energy spectrum[10]

[6] R. Grappin et al., Astron. and Astroph. **126**, 51–58 (1983)
[7] Y. Lithwick et al., ApJ **582**, 1220–1240 (2003)
[8] P. Goldreich and S. Sridhar, ApJ **438**, 763–775 (1995)
[9] S. Boldyrev et al., ApJ **699**, L39–L42 (2009)
[10] A. Beresnyak, ApJ **784**, L20 (2014)

Pinning in case of imbalance and strong MHD turbulence

- *Imbalanced cascades* are much more common
- When $w^+_{k_\perp} \neq w^-_{k_\perp}$ additional criterion is required
- This is provided by the phenomenon of pinning [6][7]
- Weak turbulence proceeds to become strong
- Goldreich and Sridhar[8] introduced anisotropic theory postulating *critical balance*



Weak $E_\pm(k_\perp) := {w_{k_\perp}}^2/k_\perp$

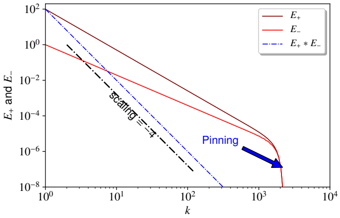
$$k_\parallel V_A \sim k_\perp v_\perp \quad \epsilon \sim \frac{V_A^3}{L}, \quad \epsilon \sim \frac{v_\perp^2}{t_{cas}}, \quad t_{cas} \sim \frac{1}{k_\parallel V_A} \Rightarrow \quad k_\parallel \sim k_\perp^{2/3} L^{-1/3} \qquad (4)$$

- Role of dynamical alignment leading to $-3/2$ energy spectrum[9]
- High resolution MHD simulations suggest $-5/3$ energy spectrum[10]

[6] R. Grappin et al., Astron. and Astroph. **126**, 51–58 (1983)
[7] Y. Lithwick et al., ApJ **582**, 1220–1240 (2003)
[8] P. Goldreich and S. Sridhar, ApJ **438**, 763–775 (1995)
[9] S. Boldyrev et al., ApJ **699**, L39–L42 (2009)
[10] A. Beresnyak, ApJ **784**, L20 (2014)

Pinning in case of imbalance and strong MHD turbulence

- *Imbalanced cascades* are much more common
- When $w^+_{k_\perp} \neq w^-_{k_\perp}$ additional criterion is required
- This is provided by the phenomenon of pinning [6][7]
- Weak turbulence proceeds to become strong
- Goldreich and Sridhar[8] introduced anisotropic theory postulating *critical balance*



Weak $E_\pm(k_\perp) := {w_{k_\perp}}^2/k_\perp$

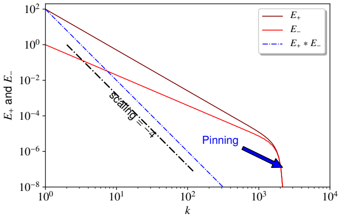
$$k_\parallel V_A \sim k_\perp v_\perp \quad \epsilon \sim \frac{V_A^3}{L}, \quad \epsilon \sim \frac{v_\perp^2}{t_{cas}}, \quad t_{cas} \sim \frac{1}{k_\parallel V_A} \Rightarrow \quad k_\parallel \sim k_\perp^{2/3} L^{-1/3} \tag{4}$$

- Role of dynamical alignment leading to $-3/2$ energy spectrum[9]
- High resolution MHD simulations suggest $-5/3$ energy spectrum[10]

[6] R. Grappin et al., Astron. and Astroph. **126**, 51–58 (1983)
[7] Y. Lithwick et al., ApJ **582**, 1220–1240 (2003)
[8] P. Goldreich and S. Sridhar, ApJ **438**, 763–775 (1995)
[9] S. Boldyrev et al., ApJ **699**, L39–L42 (2009)
[10] A. Beresnyak, ApJ **784**, L20 (2014)

Pinning in case of imbalance and strong MHD turbulence

- *Imbalanced cascades* are much more common
- When $w^+_{k_\perp} \neq w^-_{k_\perp}$ additional criterion is required
- This is provided by the phenomenon of pinning [6][7]
- Weak turbulence proceeds to become strong
- Goldreich and Sridhar[8]introduced anisotropic theory postulating *critical balance*



Weak $E_\pm(k_\perp) := {w_{k_\perp}}^2/k_\perp$

$$k_\parallel V_A \sim k_\perp v_\perp \quad \epsilon \sim \frac{V_A^3}{L}, \quad \epsilon \sim \frac{v_\perp^2}{t_{cas}}, \quad t_{cas} \sim \frac{1}{k_\parallel V_A} \Rightarrow \quad k_\parallel \sim k_\perp^{2/3} L^{-1/3} \tag{4}$$

- Role of dynamical alignment leading to $-3/2$ energy spectrum[9]
- High resolution MHD simulations suggest $-5/3$ energy spectrum[10]

[6] R. Grappin et al., Astron. and Astroph. **126**, 51–58 (1983)
[7] Y. Lithwick et al., ApJ **582**, 1220–1240 (2003)
[8] P. Goldreich and S. Sridhar, ApJ **438**, 763–775 (1995)
[9] S. Boldyrev et al., ApJ **699**, L39–L42 (2009)
[10] A. Beresnyak, ApJ **784**, L20 (2014)

Addressing the dispersive range in Solar wind

- We leave the controversy of MHD and instead address dispersive scales
- Our description must involve small β_e relevant for the solar wind near the sun
- kinetic scales of interest are: Ion ρ_i and sonic ρ_s Larmor radii

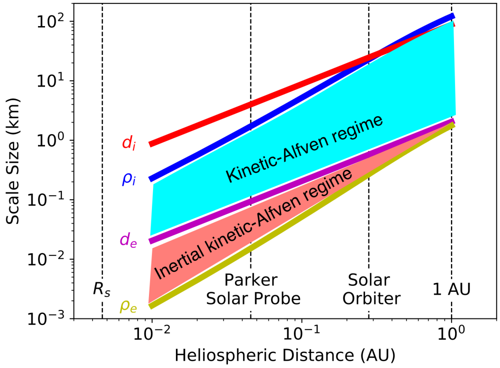
Relative kinetic plasma modes[11]

Electron beta

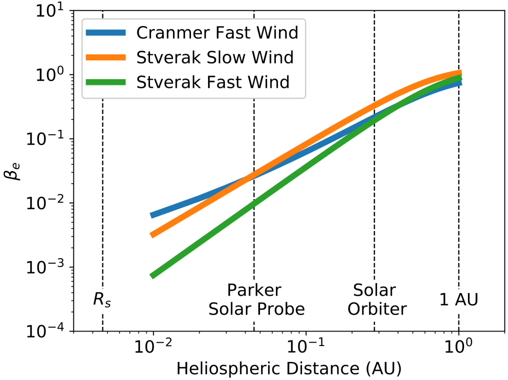
[11] V. Roytershteyn et al., ApJ **870**, 103 (2019)

Addressing the dispersive range in Solar wind

- We leave the controversy of MHD and instead address dispersive scales
- Our description must involve small β_e relevant for the solar wind near the sun
- kinetic scales of interest are: Ion ρ_i and sonic ρ_s Larmor radii



Relative kinetic plasma modes[11]

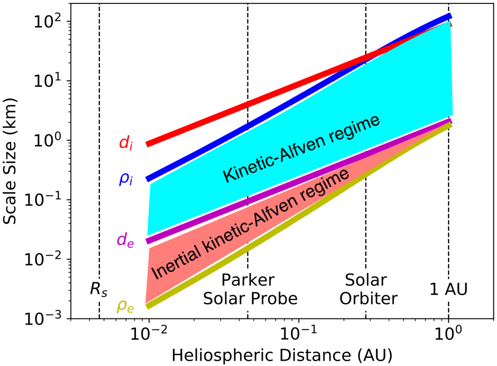


Electron beta

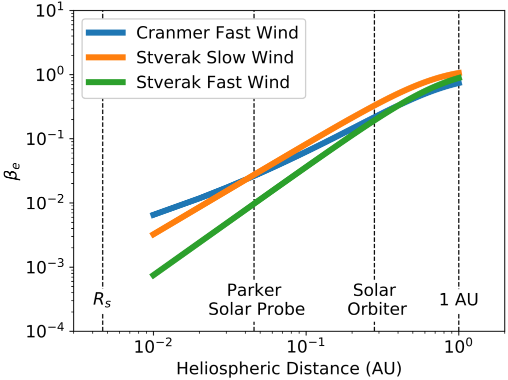
[11] V. Roytershteyn et al., ApJ **870**, 103 (2019)

Addressing the dispersive range in Solar wind

- We leave the controversy of MHD and instead address dispersive scales
- Our description must involve small β_e relevant for the solar wind near the sun
- kinetic scales of interest are: Ion ρ_i and sonic ρ_s Larmor radii



Relative kinetic plasma modes[11]

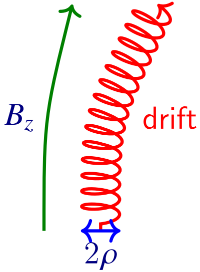


Electron beta

[11] V. Roytershteyn et al., ApJ **870**, 103 (2019)

Hamiltonian reduced gyrofluid model (GYRO)

- In order to understand gyroscale physics gyrokinetics [12](5D) is often employed
- Numerical solutions of developed spectra are too costly even for this model
- Therefore, it is desirable to find some closure for a 3D gyrofluid model
- A Hamiltonian closure of this kind has been found recently [13].
- Omitting effects of *electron inertia* $m_e/m_i = 0$ the simplified equations read:



Larmor gyration

gyrocenter density

$$\partial_t N_e + {V_A}^2 \nabla_\| \Delta_\perp A_\| = \nabla(B_z - \varphi) \times \nabla N_e \cdot \hat{z} \tag{5}$$

$$\partial_t A_\| + \nabla_\| (\varphi - N_e - B_z) = 0, \tag{6}$$

$$\nabla_\| f := \partial f / \partial z + \nabla(A_\|) \times \nabla f \cdot \hat{z} \tag{7}$$

electrostatic pot. magnetic potential

$$B_z = \hat{M}_1(\tau\Delta_\perp)[\varphi] \quad N_e = -\hat{M}_2(\tau\Delta_\perp)[\varphi] \tag{8}$$

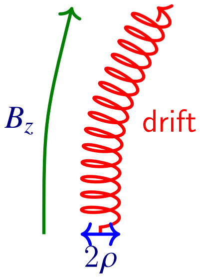
Here $\hat{M}_{1,2}$ are operators related to modified Bessel functions

$$\beta_e = 8\pi n_0 T_{0e}/B_0^2, \quad \Omega_i = eB_0/m_i c, \quad \tau = T_{0i}/T_{0e} \tag{9}$$

[12] A. Brizard, Phys. Fluids B **4**, 1213–1228 (1992)
[13] T. Passot et al., PoP **25**, 042107 (2018)

Hamiltonian reduced gyrofluid model (GYRO)

- In order to understand gyroscale physics gyrokinetics [12](5D) is often employed
- Numerical solutions of developed spectra are too costly even for this model
- Therefore, it is desirable to find some closure for a 3D gyrofluid model
- A Hamiltonian closure of this kind has been found recently [13].
- Omitting effects of *electron inertia* $m_e/m_i = 0$ the simplified equations read:



Larmor gyration

gyrocenter density

$$\partial_t N_e + {V_A}^2 \nabla_\| \Delta_\perp A_\| = \nabla(B_z - \varphi) \times \nabla N_e \cdot \hat{z} \tag{5}$$

$$\partial_t A_\| + \nabla_\| (\varphi - N_e - B_z) = 0, \tag{6}$$

$$\nabla_\| f := \partial f / \partial z + \nabla(A_\|) \times \nabla f \cdot \hat{z} \tag{7}$$

electrostatic pot. magnetic potential

$$B_z = \hat{M}_1(\tau \Delta_\perp)[\varphi] \quad N_e = -\hat{M}_2(\tau \Delta_\perp)[\varphi] \tag{8}$$

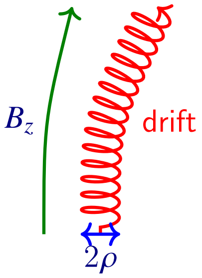
Here $\hat{M}_{1,2}$ are operators related to modified Bessel functions

$$\beta_e = 8\pi n_0 T_{0e}/B_0^2, \quad \Omega_i = eB_0/m_i c, \quad \tau = T_{0i}/T_{0e} \tag{9}$$

[12] A. Brizard, Phys. Fluids B **4**, 1213–1228 (1992)
[13] T. Passot et al., PoP **25**, 042107 (2018)

Hamiltonian reduced gyrofluid model (GYRO)

- In order to understand gyroscale physics gyrokinetics [12](5D) is often employed
- Numerical solutions of developed spectra are too costly even for this model
- Therefore, it is desirable to find some closure for a 3D gyrofluid model
- A Hamiltonian closure of this kind has been found recently [13].
- Omitting effects of *electron inertia* $m_e/m_i = 0$ the simplified equations read:



Larmor gyration

$$\partial_t N_e + {V_A}^2 \nabla_\| \Delta_\perp A_\| = \nabla(B_z - \varphi) \times \nabla N_e \cdot \hat{z} \tag{5}$$

gyrocenter density

$$\partial_t A_\| + \nabla_\| (\varphi - N_e - B_z) = 0, \tag{6}$$

$$\nabla_\| f := \partial f / \partial z + \nabla(A_\|) \times \nabla f \cdot \hat{z} \tag{7}$$

electrostatic pot. magnetic potential

$$B_z = \hat{M}_1(\tau \Delta_\perp)[\varphi] \quad N_e = -\hat{M}_2(\tau \Delta_\perp)[\varphi] \tag{8}$$

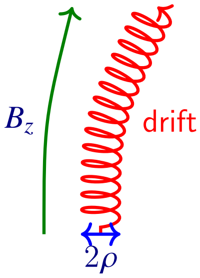
Here $\hat{M}_{1,2}$ are operators related to modified Bessel functions

$$\beta_e = 8\pi n_0 T_{0e}/B_0^2, \quad \Omega_i = eB_0/m_i c, \quad \tau = T_{0i}/T_{0e} \tag{9}$$

[12] A. Brizard, Phys. Fluids B **4**, 1213–1228 (1992)
[13] T. Passot et al., PoP **25**, 042107 (2018)

Hamiltonian reduced gyrofluid model (GYRO)

- In order to understand gyroscale physics gyrokinetics [12](5D) is often employed
- Numerical solutions of developed spectra are too costly even for this model
- Therefore, it is desirable to find some closure for a 3D gyrofluid model
- A Hamiltonian closure of this kind has been found recently [13].
- Omitting effects of *electron inertia* $m_e/m_i = 0$ the simplified equations read:



Larmor gyration

$$\partial_t N_e + {V_A}^2 \nabla_\| \Delta_\perp A_\| = \nabla(B_z - \varphi) \times \nabla N_e \cdot \hat{z} \tag{5}$$

gyrocenter density

$$\partial_t A_\| + \nabla_\| (\varphi - N_e - B_z) = 0, \tag{6}$$

$$\nabla_\| f := \partial f / \partial z + \nabla(A_\|) \times \nabla f \cdot \hat{z} \tag{7}$$

electrostatic pot. magnetic potential

$$B_z = \hat{M}_1(\tau\Delta_\perp)[\varphi] \quad N_e = -\hat{M}_2(\tau\Delta_\perp)[\varphi] \tag{8}$$

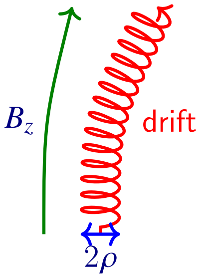
Here $\hat{M}_{1,2}$ are operators related to modified Bessel functions

$$\beta_e = 8\pi n_0 T_{0e}/B_0^2, \quad \Omega_i = eB_0/m_i c, \quad \tau = T_{0i}/T_{0e} \tag{9}$$

[12] A. Brizard, Phys. Fluids B **4**, 1213–1228 (1992)
[13] T. Passot et al., PoP **25**, 042107 (2018)

Hamiltonian reduced gyrofluid model (GYRO)

- In order to understand gyroscale physics gyrokinetics [12](5D) is often employed
- Numerical solutions of developed spectra are too costly even for this model
- Therefore, it is desirable to find some closure for a 3D gyrofluid model
- A Hamiltonian closure of this kind has been found recently [13].
- Omitting effects of *electron inertia* $m_e/m_i = 0$ the simplified equations read:



Larmor gyration

$$\partial_t N_e + {V_A}^2 \nabla_\parallel \Delta_\perp A_\parallel = \nabla(B_z - \varphi) \times \nabla N_e \cdot \hat{z} \tag{5}$$

gyrocenter density

$$\partial_t A_\parallel + \nabla_\parallel(\varphi - N_e - B_z) = 0, \tag{6}$$

$$\nabla_\parallel f := \partial f/\partial z + \nabla(A_\parallel) \times \nabla f \cdot \hat{z} \tag{7}$$

electrostatic pot. magnetic potential

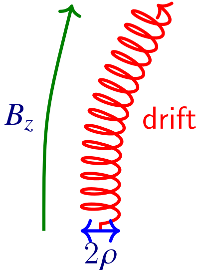
$$B_z = \hat{M}_1(\tau\Delta_\perp)[\varphi] \quad N_e = -\hat{M}_2(\tau\Delta_\perp)[\varphi] \tag{8}$$

Here $\hat{M}_{1,2}$ are operators related to modified Bessel functions

$$\beta_e = 8\pi n_0 T_{0e}/B_0^2, \quad \Omega_i = eB_0/m_i c, \quad \tau = T_{0i}/T_{0e} \tag{9}$$

[12] A. Brizard, Phys. Fluids B **4**, 1213–1228 (1992)
[13] T. Passot et al., PoP **25**, 042107 (2018)

- In order to understand gyroscale physics gyrokinetics [12](5D) is often employed
- Numerical solutions of developed spectra are too costly even for this model
- Therefore, it is desirable to find some closure for a 3D gyrofluid model
- A Hamiltonian closure of this kind has been found recently [13].
- Omitting effects of *electron inertia* $m_e/m_i = 0$ the simplified equations read:



Larmor gyration

$$\partial_t N_e + {V_A}^2 \nabla_\| \Delta_\perp A_\| = \nabla(B_z - \varphi) \times \nabla N_e \cdot \hat{z} \qquad (5)$$

gyrocenter density

$$\partial_t A_\| + \nabla_\|(\varphi - N_e - B_z) = 0, \qquad (6)$$

$$\nabla_\| f := \partial f/\partial z + \nabla(A_\|) \times \nabla f \cdot \hat{z} \qquad (7)$$

electrostatic pot. magnetic potential

$$B_z = \hat{M}_1(\tau\Delta_\perp)[\varphi] \quad N_e = -\hat{M}_2(\tau\Delta_\perp)[\varphi] \qquad (8)$$

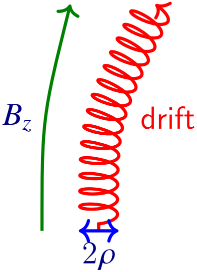
Here $\hat{M}_{1,2}$ are operators related to modified Bessel functions

$$\beta_e = 8\pi n_0 T_{0e}/B_0^2, \quad \Omega_i = eB_0/m_i c, \quad \tau = T_{0i}/T_{0e} \qquad (9)$$

[12] A. Brizard, Phys. Fluids B **4**, 1213–1228 (1992)
[13] T. Passot et al., PoP **25**, 042107 (2018)

Hamiltonian reduced gyrofluid model (GYRO)

- In order to understand gyroscale physics gyrokinetics [12](5D) is often employed
- Numerical solutions of developed spectra are too costly even for this model
- Therefore, it is desirable to find some closure for a 3D gyrofluid model
- A Hamiltonian closure of this kind has been found recently [13].
- Omitting effects of *electron inertia* $m_e/m_i = 0$ the simplified equations read:



Larmor gyration

$$\partial_t N_e + {V_A}^2 \nabla_\parallel \Delta_\perp A_\parallel = \nabla(B_z - \varphi) \times \nabla N_e \cdot \hat{z} \tag{5}$$

gyrocenter density

$$\partial_t A_\parallel + \nabla_\parallel(\varphi - N_e - B_z) = 0, \tag{6}$$

$$\nabla_\parallel f := \partial f/\partial z + \nabla(A_\parallel) \times \nabla f \cdot \hat{z} \tag{7}$$

electrostatic pot.   magnetic potential

$$B_z = \hat{M}_1(\tau\Delta_\perp)[\varphi] \quad N_e = -\hat{M}_2(\tau\Delta_\perp)[\varphi] \tag{8}$$

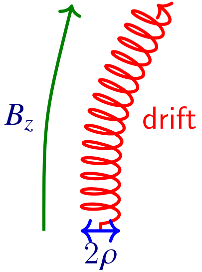
Here $\hat{M}_{1,2}$ are operators related to modified Bessel functions

$$\beta_e = 8\pi n_0 T_{0e}/B_0^2, \quad \Omega_i = eB_0/m_i c, \quad \tau = T_{0i}/T_{0e} \tag{9}$$

[12] A. Brizard, Phys. Fluids B **4**, 1213–1228 (1992)
[13] T. Passot et al., PoP **25**, 042107 (2018)

Hamiltonian reduced gyrofluid model (GYRO)

- In order to understand gyroscale physics gyrokinetics [12](5D) is often employed
- Numerical solutions of developed spectra are too costly even for this model
- Therefore, it is desirable to find some closure for a 3D gyrofluid model
- A Hamiltonian closure of this kind has been found recently [13].
- Omitting effects of *electron inertia* $m_e/m_i = 0$ the simplified equations read:



Larmor gyration

gyrocenter density

$$\partial_t N_e + {V_A}^2 \nabla_\| \Delta_\perp A_\| = \nabla(B_z - \varphi) \times \nabla N_e \cdot \hat{z} \tag{5}$$

$$\partial_t A_\| + \nabla_\|(\varphi - N_e - B_z) = 0, \tag{6}$$

$$\nabla_\| f := \partial f/\partial z + \nabla(A_\|) \times \nabla f \cdot \hat{z} \tag{7}$$

electrostatic pot. magnetic potential

$$B_z = \hat{M}_1(\tau\Delta_\perp)[\varphi] \quad N_e = -\hat{M}_2(\tau\Delta_\perp)[\varphi] \tag{8}$$

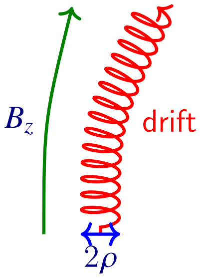
Here $\hat{M}_{1,2}$ are operators related to modified Bessel functions

$$\beta_e = 8\pi n_0 T_{0e}/B_0^2, \quad \Omega_i = eB_0/m_i c, \quad \tau = T_{0i}/T_{0e} \tag{9}$$

[12] A. Brizard, Phys. Fluids B **4**, 1213–1228 (1992)
[13] T. Passot et al., PoP **25**, 042107 (2018)

Hamiltonian reduced gyrofluid model (GYRO)

- In order to understand gyroscale physics gyrokinetics [12](5D) is often employed
- Numerical solutions of developed spectra are too costly even for this model
- Therefore, it is desirable to find some closure for a 3D gyrofluid model
- A Hamiltonian closure of this kind has been found recently [13].
- Omitting effects of *electron inertia* $m_e/m_i = 0$ the simplified equations read:



Larmor gyration

gyrocenter density

$$\partial_t N_e + {V_A}^2 \nabla_\| \Delta_\perp A_\| = \nabla(B_z - \varphi) \times \nabla N_e \cdot \hat{z} \tag{5}$$

$$\partial_t A_\| + \nabla_\| (\varphi - N_e - B_z) = 0, \tag{6}$$

$$\nabla_\| f := \partial f / \partial z + \nabla(A_\|) \times \nabla f \cdot \hat{z} \tag{7}$$

electrostatic pot. magnetic potential

$$B_z = \hat{M}_1(\tau \Delta_\perp)[\varphi] \quad N_e = -\hat{M}_2(\tau \Delta_\perp)[\varphi] \tag{8}$$

Here $\hat{M}_{1,2}$ are operators related to modified Bessel functions

$$\beta_e = 8\pi n_0 T_{0e}/B_0^2, \quad \Omega_i = eB_0/m_i c, \quad \tau = T_{0i}/T_{0e} \tag{9}$$

[12] A. Brizard, Phys. Fluids B **4**, 1213–1228 (1992)
[13] T. Passot et al., PoP **25**, 042107 (2018)

Kinetic Alfvén Waves & invariants of a gyrofluid model

- In nondimensional form the length is normalized to sonic Larmor radius ρ_s.

$$\rho_s = c_s/\Omega_i, \quad c_s := \sqrt{T_{oe}/m_i}, \quad \rho_i = \sqrt{2\tau}\rho_s, \quad \tau = T_{0i}/T_{0e} \tag{10}$$

- Introducing $\mu^\pm := \hat{\Lambda}\phi \pm V_A A_\parallel$ - Elsässer potentials, where $\hat{\Lambda} := \hat{\Lambda}(\Delta_\perp, \hat{M}_1, \hat{M}_2)$
- In MHD $\tau \ll 1, \Delta_\perp \ll 1 \Rightarrow \hat{\Lambda} \to 1 \Rightarrow w^\pm \to \nabla\mu^\pm \times \hat{z}$ are like MHD Elsässer fields
- Model describes Kinetic Alfvén Waves (KAW): $\boxed{\omega \to \alpha(\beta_e, \tau)\, k_\parallel k_\perp \text{ as } k \to \infty}$
- Energy and generalized cross-helicity conserved:

$$\mathcal{E} = \frac{1}{2}\int \Big(V_A^2 |\nabla_\perp A_\parallel|^2 - N_e(\varphi - N_e - B_z) \Big) d^3x, \tag{11}$$

$$\mathcal{E}_C = -\int N_e A_\parallel \, d^3x \to \begin{cases} -\int \omega_{iz} A_\parallel \, d^3x, & \text{In MHD} \\ \left(\frac{2}{\beta_e + \beta_i} + 1\right) \int B_z A_\parallel \, d^3x, & \text{sub-ion} \end{cases} \tag{12}$$

KAW have V_{ph}

We can also introduce energies associated with "backward" and "forward" propagating waves: $\mathcal{E}_\pm := \frac{1}{2}\int (\mu^\pm)^2 \, d^3x$

Kinetic Alfvén Waves & invariants of a gyrofluid model

- In nondimensional form the length is normalized to sonic Larmor radius ρ_s.

$$\rho_s = c_s/\Omega_i, \quad c_s := \sqrt{T_{oe}/m_i}, \quad \rho_i = \sqrt{2\tau}\rho_s, \quad \tau = T_{0i}/T_{0e} \tag{10}$$

- Introducing $\mu^{\pm} := \hat{\Lambda}\phi \pm V_A A_{\parallel}$ - Elsässer potentials, where $\hat{\Lambda} := \hat{\Lambda}(\Delta_{\perp}, \hat{M}_1, \hat{M}_2)$
- In MHD $\tau \ll 1, \Delta_{\perp} \ll 1 \Rightarrow \hat{\Lambda} \to 1 \Rightarrow w^{\pm} \to \nabla\mu^{\pm} \times \hat{z}$ are like MHD Elsässer fields
- Model describes Kinetic Alfvén Waves (KAW): $\boxed{\omega \to \alpha(\beta_e, \tau)\, k_{\parallel} k_{\perp} \text{ as } k \to \infty}$
- Energy and generalized cross-helicity conserved:

$$\mathcal{E} = \frac{1}{2}\int \Big(V_A^2|\nabla_{\perp}A_{\parallel}|^2 - N_e(\varphi - N_e - B_z)\Big)\, d^3x, \tag{11}$$

$$\mathcal{E}_C = -\int N_e A_{\parallel}\, d^3x \to \begin{cases} -\int \omega_{iz} A_{\parallel}\, d^3x, & \text{In MHD} \\ \left(\frac{2}{\beta_e + \beta_i} + 1\right)\int B_z A_{\parallel}\, d^3x, & \text{sub-ion} \end{cases} \tag{12}$$

KAW have V_{ph}

We can also introduce energies associated with "backward" and "forward" propagating waves: $\mathcal{E}_{\pm} := \frac{1}{2}\int (\mu^{\pm})^2\, d^3x$

Kinetic Alfvén Waves & invariants of a gyrofluid model

- In nondimensional form the length is normalized to sonic Larmor radius ρ_s.

$$\rho_s = c_s/\Omega_i, \quad c_s := \sqrt{T_{oe}/m_i}, \quad \rho_i = \sqrt{2\tau}\rho_s, \quad \tau = T_{0i}/T_{0e} \tag{10}$$

- Introducing $\mu^{\pm} := \hat{\Lambda}\phi \pm V_A A_{\|}$ - Elsässer potentials, where $\hat{\Lambda} := \hat{\Lambda}(\Delta_{\perp}, \hat{M}_1, \hat{M}_2)$
- In MHD $\tau \ll 1, \Delta_{\perp} \ll 1 \Rightarrow \hat{\Lambda} \to 1 \Rightarrow \boldsymbol{w}^{\pm} \to \nabla\mu^{\pm} \times \hat{z}$ are like MHD Elsässer fields
- Model describes Kinetic Alfvén Waves (KAW): $\boxed{\omega \to \alpha(\beta_e, \tau)\, k_{\|} k_{\perp} \text{ as } k \to \infty}$
- Energy and generalized cross-helicity conserved:

$$\mathcal{E} = \frac{1}{2}\int \Big(V_A^2 |\nabla_{\perp} A_{\|}|^2 - N_e(\varphi - N_e - B_z)\Big) d^3x, \tag{11}$$

$$\mathcal{E}_C = -\int N_e A_{\|}\, d^3x \to \begin{cases} -\int \omega_{iz} A_{\|}\, d^3x, & \text{In MHD} \\ \left(\frac{2}{\beta_e + \beta_i} + 1\right) \int B_z A_{\|}\, d^3x, & \text{sub-ion} \end{cases} \tag{12}$$

KAW have V_{ph}

We can also introduce energies associated with "backward" and "forward" propagating waves: $\mathcal{E}_{\pm} := \frac{1}{2}\int (\mu^{\pm})^2\, d^3x$

Kinetic Alfvén Waves & invariants of a gyrofluid model

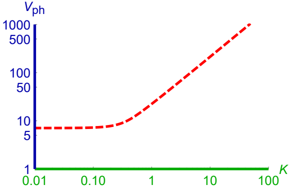
- In nondimensional form the length is normalized to sonic Larmor radius ρ_s.

$$\rho_s = c_s/\Omega_i, \quad c_s := \sqrt{T_{oe}/m_i}, \quad \rho_i = \sqrt{2\tau}\rho_s, \quad \tau = T_{0i}/T_{0e} \tag{10}$$

- Introducing $\mu^\pm := \hat{\Lambda}\phi \pm V_A A_\|$ - Elsässer potentials, where $\hat{\Lambda} := \hat{\Lambda}(\Delta_\perp, \hat{M}_1, \hat{M}_2)$
- In MHD $\tau \ll 1, \Delta_\perp \ll 1 \Rightarrow \hat{\Lambda} \to 1 \Rightarrow \boldsymbol{w}^\pm \to \nabla\mu^\pm \times \hat{z}$ are like MHD Elsässer fields
- Model describes Kinetic Alfvén Waves (KAW): $\boxed{\omega \to \alpha(\beta_e, \tau)\, k_\| k_\perp \text{ as } k \to \infty}$
- Energy and generalized cross-helicity conserved:

$$\mathcal{E} = \frac{1}{2}\int \Big(V_A^2 |\nabla_\perp A_\||^2 - N_e(\varphi - N_e - B_z)\Big) d^3x, \tag{11}$$

$$\mathcal{E}_C = -\int N_e A_\|\, d^3x \to \begin{cases} -\int \omega_{iz} A_\|\, d^3x, & \text{In MHD} \\ \left(\frac{2}{\beta_e+\beta_i} + 1\right) \int B_z A_\|\, d^3x, & \text{sub-ion} \end{cases} \tag{12}$$



KAW have V_{ph}

We can also introduce energies associated with “backward” and “forward” propagating waves: $\mathcal{E}_\pm := \frac{1}{2}\int (\mu^\pm)^2\, d^3x$

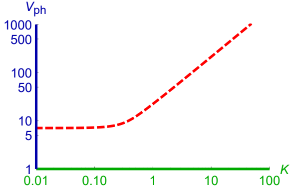
Kinetic Alfvén Waves & invariants of a gyrofluid model

- In nondimensional form the length is normalized to sonic Larmor radius ρ_s.

$$\rho_s = c_s/\Omega_i, \quad c_s := \sqrt{T_{oe}/m_i}, \quad \rho_i = \sqrt{2\tau}\rho_s, \quad \tau = T_{0i}/T_{0e} \tag{10}$$

- Introducing $\mu^{\pm} := \hat{\Lambda}\phi \pm V_A A_{\|}$ - Elsässer potentials, where $\hat{\Lambda} := \hat{\Lambda}(\Delta_\perp, \hat{M}_1, \hat{M}_2)$
- In MHD $\tau \ll 1, \Delta_\perp \ll 1 \Rightarrow \hat{\Lambda} \to 1 \Rightarrow \boldsymbol{w}^{\pm} \to \nabla\mu^{\pm} \times \hat{z}$ are like MHD Elsässer fields
- Model describes Kinetic Alfvén Waves (KAW): $\boxed{\omega \to \alpha(\beta_e, \tau)\, k_{\|} k_\perp \text{ as } k \to \infty}$
- Energy and generalized cross-helicity conserved:

$$\mathcal{E} = \frac{1}{2}\int \Big(V_A^2|\nabla_\perp A_{\|}|^2 - N_e(\varphi - N_e - B_z)\Big) d^3x, \tag{11}$$



KAW have V_{ph}

$$\mathcal{E}_C = -\int N_e A_{\|}\, d^3x \to \begin{cases} -\int \omega_{iz} A_{\|}\, d^3x, & \text{In MHD} \\ \left(\frac{2}{\beta_e+\beta_i} + 1\right) \int B_z A_{\|}\, d^3x, & \text{sub-ion} \end{cases} \tag{12}$$

We can also introduce energies associated with "backward" and "forward" propagating waves: $\mathcal{E}_{\pm} := \frac{1}{2}\int (\mu^{\pm})^2\, d^3x$

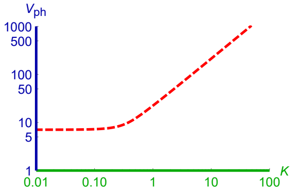
Kinetic Alfvén Waves & invariants of a gyrofluid model

- In nondimensional form the length is normalized to sonic Larmor radius ρ_s.

$$\rho_s = c_s/\Omega_i, \quad c_s := \sqrt{T_{oe}/m_i}, \quad \rho_i = \sqrt{2\tau}\rho_s, \quad \tau = T_{0i}/T_{0e} \tag{10}$$

- Introducing $\mu^\pm := \hat{\Lambda}\phi \pm V_A A_\parallel$ - Elsässer potentials, where $\hat{\Lambda} := \hat{\Lambda}(\Delta_\perp, \hat{M}_1, \hat{M}_2)$
- In MHD $\tau \ll 1, \Delta_\perp \ll 1 \Rightarrow \hat{\Lambda} \to 1 \Rightarrow \boldsymbol{w}^\pm \to \nabla\mu^\pm \times \hat{z}$ are like MHD Elsässer fields
- Model describes Kinetic Alfvén Waves (KAW): $\boxed{\omega \to \alpha(\beta_e, \tau)\, k_\parallel k_\perp \text{ as } k \to \infty}$
- Energy and generalized cross-helicity conserved:

$$\mathcal{E} = \frac{1}{2}\int \Big(V_A^2|\nabla_\perp A_\parallel|^2 - N_e(\varphi - N_e - B_z)\Big)d^3x, \tag{11}$$



KAW have V_{ph}

$$\mathcal{E}_C = -\int N_e A_\parallel \, d^3x \to \begin{cases} -\int \omega_{iz} A_\parallel \, d^3x, & \text{In MHD} \\ \left(\frac{2}{\beta_e+\beta_i} + 1\right) \int B_z A_\parallel \, d^3x, & \text{sub-ion} \end{cases} \tag{12}$$

We can also introduce energies associated with "backward" and "forward" propagating waves: $\mathcal{E}_\pm := \frac{1}{2}\int (\mu^\pm)^2 \, d^3x$

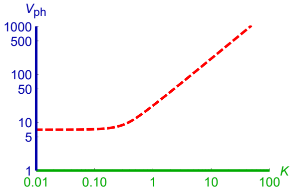
Kinetic Alfvén Waves & invariants of a gyrofluid model

- In nondimensional form the length is normalized to sonic Larmor radius ρ_s.

$$\rho_s = c_s/\Omega_i, \quad c_s := \sqrt{T_{oe}/m_i}, \quad \rho_i = \sqrt{2\tau}\rho_s, \quad \tau = T_{0i}/T_{0e} \tag{10}$$

- Introducing $\mu^{\pm} := \hat{\Lambda}\phi \pm V_A A_{\|}$ - Elsässer potentials, where $\hat{\Lambda} := \hat{\Lambda}(\Delta_{\perp}, \hat{M}_1, \hat{M}_2)$
- In MHD $\tau \ll 1, \Delta_{\perp} \ll 1 \Rightarrow \hat{\Lambda} \to 1 \Rightarrow \boldsymbol{w}^{\pm} \to \nabla\mu^{\pm} \times \hat{z}$ are like MHD Elsässer fields
- Model describes Kinetic Alfvén Waves (KAW): $\boxed{\omega \to \alpha(\beta_e, \tau)\, k_{\|} k_{\perp} \text{ as } k \to \infty}$
- Energy and generalized cross-helicity conserved:

$$\mathcal{E} = \frac{1}{2}\int \left(V_A^2|\nabla_{\perp}A_{\|}|^2 - N_e(\varphi - N_e - B_z)\right)d^3x, \tag{11}$$



KAW have V_{ph}

$$\mathcal{E}_C = -\int N_e A_{\|}\, d^3x \to \begin{cases} -\int \omega_{iz} A_{\|}\, d^3x, & \text{In MHD} \\ \left(\frac{2}{\beta_e+\beta_i} + 1\right)\int B_z A_{\|}\, d^3x, & \text{sub-ion} \end{cases} \tag{12}$$

We can also introduce energies associated with "backward" and "forward" propagating waves: $\mathcal{E}_{\pm} := \frac{1}{2}\int (\mu^{\pm})^2\, d^3x$

Outline

Steps to derive wave kinetic equation: Ask me later?

- Using Fourier decomposition, $a_{\boldsymbol{k}}^{\sigma_k} := e^{i\omega_k^{\sigma_k} t} k\mu_{\boldsymbol{k}}^{\sigma_k}$, $\tau_{NL} \gg \omega^{-1}$, $\sigma = \pm$ and

$$\Omega_{\boldsymbol{k};\boldsymbol{pq}}^{\sigma_k\sigma_p\sigma_q} = \omega_{\boldsymbol{k}}^{\sigma_k} - \omega_{\boldsymbol{p}}^{\sigma_p} - \omega_{\boldsymbol{q}}^{\sigma_q} = \sigma_k v_{ph}(k_\perp)k_\parallel - \sigma_p v_{ph}(p_\perp)p_\parallel - \sigma_q v_{ph}(q_\perp)q_\parallel. \qquad (13)$$

resonance condition one obtains wave kinetic equation:

$$\partial_t Q_{\boldsymbol{k}}^{\sigma} = 4\pi \int \sum_{\sigma_p,\sigma_q} \delta(\boldsymbol{k}+\boldsymbol{p}+\boldsymbol{q})\delta(\Omega_{\boldsymbol{kpq}}^{\sigma\sigma_p\sigma_q})$$

$$V_{\boldsymbol{kpq}}^{\sigma\sigma_p\sigma_q}\left\{\left(V_{\boldsymbol{pqk}}^{\sigma_p\sigma_q\sigma} Q_{\boldsymbol{q}}^{\sigma_q} + V_{\boldsymbol{qpk}}^{\sigma_q\sigma_p\sigma} Q_{\boldsymbol{p}}^{\sigma_p}\right) Q_{\boldsymbol{k}}^{\sigma} + V_{\boldsymbol{kpq}}^{\sigma\sigma_p\sigma_q} Q_{\boldsymbol{p}}^{\sigma_p} Q_{\boldsymbol{q}}^{\sigma_q}\right\} d\boldsymbol{p}d\boldsymbol{q}., \qquad (14)$$

where

$$\langle a_{\boldsymbol{k}}^{\sigma_k} a_{\boldsymbol{k}'}^{\sigma_{k'}}\rangle =: Q_{\boldsymbol{k}}^{\sigma_k\sigma_{k'}}\delta(\boldsymbol{k}+\boldsymbol{k}') \rightleftarrows Q_{\boldsymbol{k}}^{\sigma} = \frac{1}{\pi k_\perp}\left(E(k_\perp,k_\parallel) + \sigma v_{ph}(k_\perp)E_C(k_\perp,k_\parallel)\right). \qquad (15)$$

and introducing notation $\xi := V_A/V_{ph}$, the vertex is defined as

$$V_{\boldsymbol{kpq}}^{\sigma_k\sigma_p\sigma_q} := \frac{\widehat{\boldsymbol{z}}\cdot(\boldsymbol{p}\times\boldsymbol{q})}{8\,\xi(k_\perp)}\left(\frac{\sigma_p}{\xi(p_\perp)} - \frac{\sigma_q}{\xi(q_\perp)}\right)\frac{\sigma_p\sigma_q}{k_\perp p_\perp q_\perp}\left(\sigma_k k_\perp^2\xi(k_\perp) + \underset{p.q.k}{\circlearrowleft}\right) \qquad (16)$$

Nonlinear diffusion equations (NDE) for weak turbulence

- Assuming isotropy in the transverse plane we can collapse

$$\int d\boldsymbol{p}d\boldsymbol{q}\delta(\boldsymbol{k}+\boldsymbol{p}+\boldsymbol{q}) \to 2\pi \int dp_{\parallel}dq_{\parallel}\delta(k_{\parallel}+p_{\parallel}+q_{\parallel}) \int_{\Delta_{k_\perp}} (1/\sin\alpha)dp_\perp dq_\perp \quad (17)$$

- In addition, assuming local interactions $k_\perp \approx p_\perp \approx q_\perp$ and $k_\parallel \approx p_\parallel \approx q_\parallel$

$E^{\pm} = (E \pm V_{ph}E_C)/2$

- Under these assumptions nonlinear diffusion equation for energy and cross-helicity has been derived[14]

$$\frac{\partial}{\partial t}\frac{E}{2} = \frac{\partial}{\partial k_\perp}\left\{k_\perp^6 V_{ph}\sum_{r=\pm 1} E^{(-r)}\frac{\partial}{\partial k_\perp}\left(\frac{E^{(r)}}{k_\perp}\right)\right\} \quad (18)$$

$$\frac{\partial}{\partial t}\frac{E_C}{2} = \frac{\partial}{\partial k_\perp}\left\{k_\perp^6 \sum_{r=\pm 1}(-1)^r E^{(-r)}\frac{\partial}{\partial k_\perp}\left(\frac{E^{(r)}}{k_\perp}\right)\right\} \quad (19)$$

[14] T. Passot and P. L. Sulem, JPP **85**, 905850301 (2019)

Nonlinear diffusion equations (NDE) for weak turbulence

- Assuming isotropy in the transverse plane we can collapse

$$\int d\boldsymbol{p}d\boldsymbol{q}\delta(\boldsymbol{k}+\boldsymbol{p}+\boldsymbol{q}) \to 2\pi \int dp_\parallel dq_\parallel \delta(k_\parallel + p_\parallel + q_\parallel) \int_{\Delta_{k_\perp}} (1/\sin\alpha) dp_\perp dq_\perp \tag{17}$$

- In addition, assuming local interactions $k_\perp \approx p_\perp \approx q_\perp$ and $k_\parallel \approx p_\parallel \approx q_\parallel$

$E^\pm = (E \pm V_{ph} E_C)/2$

- Under these assumptions nonlinear diffusion equation for energy and cross-helicity has been derived[14]

$$\frac{\partial}{\partial t}\frac{E}{2} = \frac{\partial}{\partial k_\perp}\left\{ k_\perp^6 V_{ph} \sum_{r=\pm 1} E^{(-r)} \frac{\partial}{\partial k_\perp}\left(\frac{E^{(r)}}{k_\perp}\right)\right\} \tag{18}$$

$$\frac{\partial}{\partial t}\frac{E_C}{2} = \frac{\partial}{\partial k_\perp}\left\{ k_\perp^6 \sum_{r=\pm 1} (-1)^r E^{(-r)} \frac{\partial}{\partial k_\perp}\left(\frac{E^{(r)}}{k_\perp}\right)\right\} \tag{19}$$

[14] T. Passot and P. L. Sulem, JPP **85**, 905850301 (2019)

Nonlinear diffusion equations (NDE) for weak turbulence

- Assuming isotropy in the transverse plane we can collapse

$$\int d\boldsymbol{p}d\boldsymbol{q}\delta(\boldsymbol{k}+\boldsymbol{p}+\boldsymbol{q}) \to 2\pi \int dp_{\|}dq_{\|}\delta(k_{\|}+p_{\|}+q_{\|}) \int_{\Delta_{k_\perp}} (1/\sin\alpha)dp_\perp dq_\perp \qquad (17)$$

- In addition, assuming local interactions $k_\perp \approx p_\perp \approx q_\perp$ and $k_{\|} \approx p_{\|} \approx q_{\|}$

$E^{\pm} = (E \pm V_{ph}E_C)/2$

- Under these assumptions nonlinear diffusion equation for energy and cross-helicity has been derived[14]

$$\boxed{\frac{\partial}{\partial t}\frac{E}{2} = \frac{\partial}{\partial k_\perp}\left\{k_\perp^6 V_{ph} \sum_{r=\pm 1} E^{(-r)} \frac{\partial}{\partial k_\perp}\left(\frac{E^{(r)}}{k_\perp}\right)\right\}} \qquad (18)$$

$$\boxed{\frac{\partial}{\partial t}\frac{E_C}{2} = \frac{\partial}{\partial k_\perp}\left\{k_\perp^6 \sum_{r=\pm 1} (-1)^r E^{(-r)} \frac{\partial}{\partial k_\perp}\left(\frac{E^{(r)}}{k_\perp}\right)\right\}} \qquad (19)$$

[14] T. Passot and P. L. Sulem, JPP **85**, 905850301 (2019)

Phenomenological extension to strong turbulence

- As turbulence proceeds to small scales it ceases to be weak: $\boxed{\tau_{NL} \sim \tau_L}$
- In strong turbulence parallel transfer is non-negligible $\widetilde{k}_{\parallel} \neq$ const
- In weak turbulence the transfer time is: $\tau^{\pm}_{tr,w} = (k_{\perp}^3 V_{ph} E^{\mp})^{-1}, \quad \omega_L := V_{ph} \widetilde{k}_{\parallel}$
- In strong turbulence transfer time is modified consistently with *critical balance*

$$\tau^{\pm}_{tr,w} = (\tau^{\pm}_{NL})^2 \omega_L \Rightarrow \tau^{\pm}_{NL} \sim (k_{\perp}^3 V_{ph}^2 E^{\mp})^{-1/2} \sim \tau_L = (V_{ph} \widetilde{k}_{\parallel})^{-1} \tag{20}$$

- The + wave affects the correlation length of − so

$$\boxed{\widetilde{k}^{(r)}_{\parallel} = (k_{\perp}^3 E^{(r)})^{1/2} \left(\frac{E^{+}}{E^{-}}\right)^{(1-r)\chi/4}, \quad r = \pm 1} \tag{21}$$

- When $\chi = 0$ both E_+ and E_- cascades are strong.
- It is reasonable to treat E_+ weakly [15], where $\chi = 1$
- DNS MHD simulations [16]are consistent with $\chi \sim 1/4$

The effect of χ in MHD

[15] B. D. G. Chandran, The Astrophysical Journal **685**, 646–658 (2008)
[16] A. Beresnyak and A. Lazarian, ApJ **682**, 1070–1075 (2008)

Phenomenological extension to strong turbulence

- As turbulence proceeds to small scales it ceases to be weak: $\boxed{\tau_{NL} \sim \tau_L}$
- In strong turbulence parallel transfer is non-negligible $\widetilde{k}_\parallel \neq$ const
- In weak turbulence the transfer time is: $\tau^{\pm}_{tr,w} = (k_\perp^3 V_{ph} E^{\mp})^{-1}, \quad \omega_L := V_{ph}\widetilde{k}_\parallel$
- In strong turbulence transfer time is modified consistently with *critical balance*

$$\tau^{\pm}_{tr,w} = (\tau^{\pm}_{NL})^2 \omega_L \Rightarrow \tau^{\pm}_{NL} \sim (k_\perp^3 V_{ph}^2 E^{\mp})^{-1/2} \sim \tau_L = (V_{ph}\widetilde{k}_\parallel)^{-1} \tag{20}$$

The effect of χ in MHD

- The + wave affects the correlation length of − so

$$\boxed{\widetilde{k}^{(r)}_\parallel = (k_\perp^3 E^{(r)})^{1/2} \left(\frac{E^+}{E^-}\right)^{(1-r)\chi/4}, \quad r = \pm 1} \tag{21}$$

- When $\chi = 0$ both E_+ and E_- cascades are strong.
- It is reasonable to treat E_+ weakly [15], where $\chi = 1$
- DNS MHD simulations [16] are consistent with $\chi \sim 1/4$

[15] B. D. G. Chandran, The Astrophysical Journal **685**, 646–658 (2008)
[16] A. Beresnyak and A. Lazarian, ApJ **682**, 1070–1075 (2008)

Phenomenological extension to strong turbulence

- As turbulence proceeds to small scales it ceases to be weak: $\boxed{\tau_{NL} \sim \tau_L}$
- In strong turbulence parallel transfer is non-negligible $\widetilde{k}_{\parallel} \neq \text{const}$
- In weak turbulence the transfer time is: $\tau^{\pm}_{tr,w} = (k_{\perp}^3 V_{ph} E^{\mp})^{-1}, \quad \omega_L := V_{ph} \widetilde{k}_{\parallel}$
- In strong turbulence transfer time is modified consistently with *critical balance*

$$\tau^{\pm}_{tr,w} = (\tau^{\pm}_{NL})^2 \omega_L \Rightarrow \tau^{\pm}_{NL} \sim (k_{\perp}^3 V_{ph}^2 E^{\mp})^{-1/2} \sim \tau_L = (V_{ph} \widetilde{k}_{\parallel})^{-1} \tag{20}$$

The effect of χ in MHD

- The $+$ wave affects the correlation length of $-$ so

$$\boxed{\widetilde{k}^{(r)}_{\parallel} = (k_{\perp}^3 E^{(r)})^{1/2} \left(\frac{E^{+}}{E^{-}}\right)^{(1-r)\chi/4}, \quad r = \pm 1} \tag{21}$$

- When $\chi = 0$ both E_+ and E_- cascades are strong.
- It is reasonable to treat E_+ weakly [15], where $\chi = 1$
- DNS MHD simulations [16] are consistent with $\chi \sim 1/4$

[15] B. D. G. Chandran, The Astrophysical Journal **685**, 646–658 (2008)
[16] A. Beresnyak and A. Lazarian, ApJ **682**, 1070–1075 (2008)

Phenomenological extension to strong turbulence

- As turbulence proceeds to small scales it ceases to be weak: $\boxed{\tau_{NL} \sim \tau_L}$
- In strong turbulence parallel transfer is non-negligible $\widetilde{k}_{\|} \neq \text{const}$
- In weak turbulence the transfer time is: $\tau^{\pm}_{tr,w} = (k_\perp^3 V_{ph} E^{\mp})^{-1}, \quad \omega_L := V_{ph}\widetilde{k}_{\|}$
- In strong turbulence transfer time is modified consistently with *critical balance*

$$\tau^{\pm}_{tr,w} = (\tau^{\pm}_{NL})^2 \omega_L \Rightarrow \tau^{\pm}_{NL} \sim (k_\perp^3 V_{ph}^2 E^{\mp})^{-1/2} \sim \tau_L = (V_{ph}\widetilde{k}_{\|})^{-1} \tag{20}$$

The effect of χ in MHD

- The + wave affects the correlation length of − so

$$\widetilde{k}^{(r)}_{\|} = (k_\perp^3 E^{(r)})^{1/2} \left(\frac{E^+}{E^-}\right)^{(1-r)\chi/4}, \quad r = \pm 1 \tag{21}$$

- When $\chi = 0$ both E_+ and E_- cascades are strong.
- It is reasonable to treat E_+ weakly [15], where $\chi = 1$
- DNS MHD simulations [16] are consistent with $\chi \sim 1/4$

[15] B. D. G. Chandran, The Astrophysical Journal **685**, 646–658 (2008)

[16] A. Beresnyak and A. Lazarian, ApJ **682**, 1070–1075 (2008)

- As turbulence proceeds to small scales it ceases to be weak: $\boxed{\tau_{NL} \sim \tau_L}$
- In strong turbulence parallel transfer is non-negligible $\widetilde{k}_\parallel \neq$ const
- In weak turbulence the transfer time is: $\tau^{\pm}_{tr,w} = (k_\perp^3 V_{ph} E^{\mp})^{-1}, \quad \omega_L := V_{ph}\widetilde{k}_\parallel$
- In strong turbulence transfer time is modified consistently with *critical balance*

$$\tau^{\pm}_{tr,w} = (\tau^{\pm}_{NL})^2 \omega_L \Rightarrow \tau^{\pm}_{NL} \sim (k_\perp^3 V_{ph}^2 E^{\mp})^{-1/2} \sim \tau_L = (V_{ph}\widetilde{k}_\parallel)^{-1} \tag{20}$$

- The + wave affects the correlation length of − so

$$\boxed{\widetilde{k}_\parallel^{(r)} = (k_\perp^3 E^{(r)})^{1/2} \left(\frac{E^+}{E^-}\right)^{(1-r)\chi/4}, \quad r = \pm 1} \tag{21}$$

- When $\chi = 0$ both E_+ and E_- cascades are strong.
- It is reasonable to treat E_+ weakly [15], where $\chi = 1$
- DNS MHD simulations [16] are consistent with $\chi \sim 1/4$

The effect of χ in MHD

[15] B. D. G. Chandran, The Astrophysical Journal **685**, 646–658 (2008)
[16] A. Beresnyak and A. Lazarian, ApJ **682**, 1070–1075 (2008)

Phenomenological extension to strong turbulence

- As turbulence proceeds to small scales it ceases to be weak: $\boxed{\tau_{NL} \sim \tau_L}$
- In strong turbulence parallel transfer is non-negligible $\widetilde{k}_\parallel \neq$ const
- In weak turbulence the transfer time is: $\tau^{\pm}_{tr,w} = (k_\perp^3 V_{ph} E^{\mp})^{-1}, \quad \omega_L := V_{ph}\widetilde{k}_\parallel$
- In strong turbulence transfer time is modified consistently with *critical balance*

$$\tau^{\pm}_{tr,w} = (\tau^{\pm}_{NL})^2 \omega_L \Rightarrow \tau^{\pm}_{NL} \sim (k_\perp^3 V_{ph}^2 E^{\mp})^{-1/2} \sim \tau_L = (V_{ph}\widetilde{k}_\parallel)^{-1} \tag{20}$$

- The $+$ wave affects the correlation length of $-$ so

$$\boxed{\widetilde{k}_\parallel^{(r)} = (k_\perp^3 E^{(r)})^{1/2} \left(\frac{E^+}{E^-}\right)^{(1-r)\chi/4}, \quad r = \pm 1} \tag{21}$$

- When $\chi = 0$ both E_+ and E_- cascades are strong.
- It is reasonable to treat E_+ weakly [15], where $\chi = 1$
- DNS MHD simulations [16] are consistent with $\chi \sim 1/4$

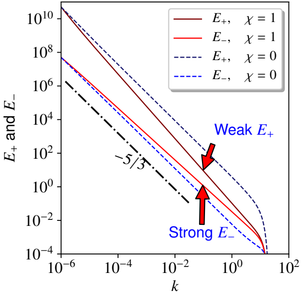
The effect of χ in MHD

[15] B. D. G. Chandran, The Astrophysical Journal **685**, 646–658 (2008)
[16] A. Beresnyak and A. Lazarian, ApJ **682**, 1070–1075 (2008)

Phenomenological extension to strong turbulence

- As turbulence proceeds to small scales it ceases to be weak: $\boxed{\tau_{NL} \sim \tau_L}$
- In strong turbulence parallel transfer is non-negligible $\widetilde{k}_\parallel \neq$ const
- In weak turbulence the transfer time is: $\tau^{\pm}_{tr,w} = (k_\perp^3 V_{ph} E^{\mp})^{-1}, \quad \omega_L := V_{ph}\widetilde{k}_\parallel$
- In strong turbulence transfer time is modified consistently with *critical balance*

$$\tau^{\pm}_{tr\ w} = (\tau^{\pm}_{NL})^2 \omega_L \Rightarrow \tau^{\pm}_{NL} \sim (k_\perp^3 V_{ph}^2 E^{\mp})^{-1/2} \sim \tau_L = (V_{ph}\widetilde{k}_\parallel)^{-1} \tag{20}$$



The effect of χ in MHD

- The $+$ wave affects the correlation length of $-$ so

$$\boxed{\widetilde{k}_\parallel^{(r)} = (k_\perp^3 E^{(r)})^{1/2} \left(\frac{E^+}{E^-}\right)^{(1-r)\chi/4}, \quad r = \pm 1} \tag{21}$$

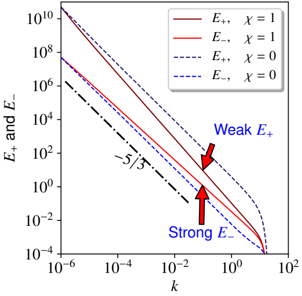
- When $\chi = 0$ both E_+ and E_- cascades are strong.
- It is reasonable to treat E_+ weakly [15], where $\chi = 1$
- DNS MHD simulations [16] are consistent with $\chi \sim 1/4$

[15] B. D. G. Chandran, The Astrophysical Journal **685**, 646–658 (2008)
[16] A. Beresnyak and A. Lazarian, ApJ **682**, 1070–1075 (2008)

Phenomenological extension to strong turbulence

- As turbulence proceeds to small scales it ceases to be weak: $\boxed{\tau_{NL} \sim \tau_L}$
- In strong turbulence parallel transfer is non-negligible $\widetilde{k}_{\parallel} \neq$ const
- In weak turbulence the transfer time is: $\tau^{\pm}_{tr,w} = (k_\perp^3 V_{ph} E^{\mp})^{-1}, \quad \omega_L := V_{ph}\widetilde{k}_{\parallel}$
- In strong turbulence transfer time is modified consistently with *critical balance*

$$\tau^{\pm}_{tr,w} = (\tau^{\pm}_{NL})^2 \omega_L \Rightarrow \tau^{\pm}_{NL} \sim (k_\perp^3 V_{ph}^2 E^{\mp})^{-1/2} \sim \tau_L = (V_{ph}\widetilde{k}_{\parallel})^{-1} \tag{20}$$



The effect of χ in MHD

- The $+$ wave affects the correlation length of $-$ so

$$\boxed{\widetilde{k}_{\parallel}^{(r)} = (k_\perp^3 E^{(r)})^{1/2} \left(\frac{E^+}{E^-}\right)^{(1-r)\chi/4}, \quad r = \pm 1} \tag{21}$$

- When $\chi = 0$ both E_+ and E_- cascades are strong.
- It is reasonable to treat E_+ weakly [15], where $\chi = 1$
- DNS MHD simulations [16] are consistent with $\chi \sim 1/4$

[15] B. D. G. Chandran, The Astrophysical Journal **685**, 646–658 (2008)
[16] A. Beresnyak and A. Lazarian, ApJ **682**, 1070–1075 (2008)

Numerical scheme for NDE - Ask me later?

$$\frac{1}{2}\frac{\partial}{\partial t}\begin{pmatrix}E\\E_C\end{pmatrix} = \frac{\partial}{\partial k_\perp}\left\{k_\perp^6\begin{pmatrix}V_{ph}\\1\end{pmatrix}\sum_{r=\pm1}\begin{pmatrix}1\\(-1)^r\end{pmatrix}\frac{E^{(-r)}}{\widetilde{k}_\parallel^\pm}\frac{\partial}{\partial k_\perp}\left(\frac{E^{(r)}}{k_\perp}\right)\right\} =: \frac{\partial}{\partial k_\perp}\begin{pmatrix}\varepsilon\\\eta\end{pmatrix} \tag{22}$$

- We are modeling coupled nonlinear diffusion equations of Leith type.
- Code for Gravitational wave turbulence [17] was used with modified the scheme

Fields are evaluated on the expanding grid.

$$k_i = k_0 \cdot \lambda^i \tag{23}$$

Coupling the grid using interpolated values

Changes that were made:

1. Modification of the finite differencing
2. The addition for dispersive effects
3. Support for strong turbulence
4. Implementation of Landau damping

[17] S. Galtier et al., Phys. D: Nonl. Phen. **390**, 84 –88 (2019)

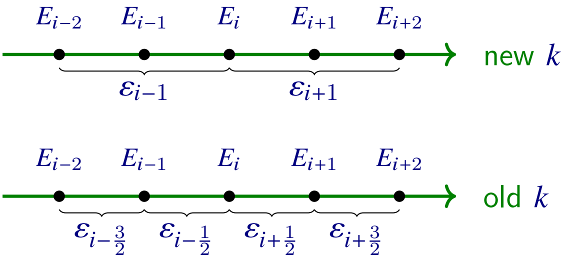
Numerical scheme for NDE - Ask me later?

$$\boxed{\frac{1}{2}\frac{\partial}{\partial t}\begin{pmatrix} E \\ E_C \end{pmatrix} = \frac{\partial}{\partial k_\perp}\left\{ k_\perp^6 \begin{pmatrix} V_{ph} \\ 1 \end{pmatrix} \sum_{r=\pm 1} \begin{pmatrix} 1 \\ (-1)^r \end{pmatrix} \frac{E^{(-r)}}{\widetilde{k}_\parallel^\pm} \frac{\partial}{\partial k_\perp}\left(\frac{E^{(r)}}{k_\perp}\right)\right\} =: \frac{\partial}{\partial k_\perp}\begin{pmatrix} \varepsilon \\ \eta \end{pmatrix}} \qquad (22)$$

- We are modeling coupled nonlinear diffusion equations of Leith type.
- Code for Gravitational wave turbulence [17] was used with modified the scheme

Fields are evaluated on the expanding grid.

$$k_i = k_0 \cdot \lambda^i \qquad (23)$$



Coupling the grid using interpolated values

Changes that were made:

1. Modification of the finite differencing
2. The addition for dispersive effects
3. Support for strong turbulence
4. Implementation of Landau damping

[17] S. Galtier et al., Phys. D: Nonl. Phen. **390**, 84 –88 (2019)

Numerical scheme for NDE - Ask me later?

$$\frac{1}{2}\frac{\partial}{\partial t}\begin{pmatrix}E\\ E_C\end{pmatrix} = \frac{\partial}{\partial k_\perp}\left\{k_\perp^6 \begin{pmatrix}V_{ph}\\ 1\end{pmatrix}\sum_{r=\pm1}\begin{pmatrix}1\\ (-1)^r\end{pmatrix}\frac{E^{(-r)}}{\widetilde{k}_\parallel^\pm}\frac{\partial}{\partial k_\perp}\left(\frac{E^{(r)}}{k_\perp}\right)\right\} =: \frac{\partial}{\partial k_\perp}\begin{pmatrix}\varepsilon\\ \eta\end{pmatrix} \tag{22}$$

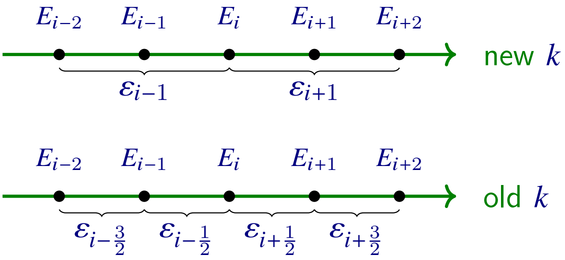
- We are modeling coupled nonlinear diffusion equations of Leith type.
- Code for Gravitational wave turbulence [17] was used with modified the scheme

Fields are evaluated on the expanding grid.

$$k_i = k_0 \cdot \lambda^i \tag{23}$$

Changes that were made:

1. Modification of the finite differencing
2. The addition for dispersive effects
3. Support for strong turbulence
4. Implementation of Landau damping



Coupling the grid using interpolated values

[17] S. Galtier et al., Phys. D: Nonl. Phen. **390**, 84 –88 (2019)

Value of the correlation coefficient in the dispersive case?

Weak turbulence analytics

Strong turbulence simulation

We can rewrite the stationary weak diffusive equations using

$$E^{\pm}(k_\perp) =: \frac{1}{2} k_\perp \rho(k_\perp) e^{\pm\phi(k_\perp)}. \tag{24}$$

so that $|E_C(k_\perp)| \leq E(k_\perp)/v_{ph}(k_\perp)$. NDE rewrites

$$\frac{\partial}{\partial k_\perp}\rho^2(k_\perp) = -\frac{2\varepsilon}{C k_\perp^7 v_{ph}(k_\perp)} \tag{25}$$

$$\rho^2(k_\perp)\frac{\partial}{\partial k_\perp}\phi(k_\perp) = -\frac{\eta}{C k_\perp^7}. \tag{26}$$

- $v_{ph}(k_\perp) \to k_\perp \Rightarrow \rho^2(k_\perp) \sim \varepsilon k_\perp^{-7} \Rightarrow \phi(k_\perp) \sim a + b k_\perp$ where $0 > b \propto \eta/\varepsilon \Rightarrow \boxed{E^{\pm}(k_\perp) \sim k_\perp^{-5/2} e^{\pm b k_\perp}}$
- So one solutions exponentially diverge, unless we make modifications (confirmed in strong NDE simulation)
- When we choose $\chi = 1$ this problem is cured.

Value of the correlation coefficient in the dispersive case?

We can rewrite the stationary weak diffusive equations using

$$E^{\pm}(k_\perp) =: \frac{1}{2}k_\perp \rho(k_\perp)e^{\pm\phi(k_\perp)}. \tag{24}$$

so that $|E_C(k_\perp)| \leq E(k_\perp)/v_{ph}(k_\perp)$. NDE rewrites

$$\frac{\partial}{\partial k_\perp}\rho^2(k_\perp) = -\frac{2\varepsilon}{Ck_\perp^7 v_{ph}(k_\perp)} \tag{25}$$

$$\rho^2(k_\perp)\frac{\partial}{\partial k_\perp}\phi(k_\perp) = -\frac{\eta}{Ck_\perp^7}. \tag{26}$$

Weak turbulence analytics

- $v_{ph}(k_\perp) \to k_\perp \Rightarrow \rho^2(k_\perp) \sim \varepsilon k_\perp^{-7} \Rightarrow \phi(k_\perp) \sim a + bk_\perp$ where $0 > b \propto \eta/\varepsilon \Rightarrow \boxed{E^{\pm}(k_\perp) \sim k_\perp^{-5/2}e^{\pm bk_\perp}}$
- So one solutions exponentially diverge, unless we make modifications (confirmed in strong NDE simulation)
- When we choose $\chi = 1$ this problem is cured.

Strong turbulence simulation

Value of the correlation coefficient in the dispersive case?

Weak turbulence analytics

Strong turbulence simulation

We can rewrite the stationary weak diffusive equations using

$$E^{\pm}(k_\perp) =: \frac{1}{2}k_\perp \rho(k_\perp)e^{\pm\phi(k_\perp)}. \tag{24}$$

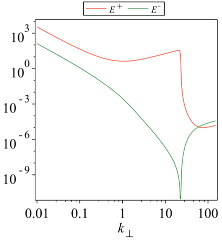
so that $|E_C(k_\perp)| \leq E(k_\perp)/v_{ph}(k_\perp)$. NDE rewrites

$$\frac{\partial}{\partial k_\perp}\rho^2(k_\perp) = -\frac{2\varepsilon}{Ck_\perp^7 v_{ph}(k_\perp)} \tag{25}$$

$$\rho^2(k_\perp)\frac{\partial}{\partial k_\perp}\phi(k_\perp) = -\frac{\eta}{Ck_\perp^7}. \tag{26}$$

- $v_{ph}(k_\perp) \to k_\perp \Rightarrow \rho^2(k_\perp) \sim \varepsilon k_\perp^{-7} \Rightarrow \phi(k_\perp) \sim a + bk_\perp$ where $0 > b \propto \eta/\varepsilon \Rightarrow \boxed{E^{\pm}(k_\perp) \sim k_\perp^{-5/2}e^{\pm bk_\perp}}$
- So one solutions exponentially diverge, unless we make modifications (confirmed in strong NDE simulation)
- When we choose $\chi = 1$ this problem is cured.

Value of the correlation coefficient in the dispersive case?



Weak turbulence analytics

Strong turbulence simulation

We can rewrite the stationary weak diffusive equations using

$$E^\pm(k_\perp) =: \frac{1}{2} k_\perp \rho(k_\perp) e^{\pm\phi(k_\perp)}. \tag{24}$$

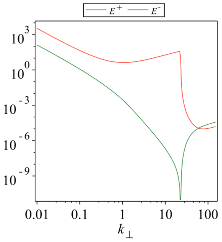
so that $|E_C(k_\perp)| \leq E(k_\perp)/v_{ph}(k_\perp)$. NDE rewrites

$$\frac{\partial}{\partial k_\perp}\rho^2(k_\perp) = -\frac{2\varepsilon}{C k_\perp^7 v_{ph}(k_\perp)} \tag{25}$$

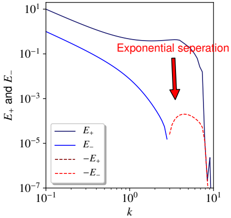
$$\rho^2(k_\perp)\frac{\partial}{\partial k_\perp}\phi(k_\perp) = -\frac{\eta}{C k_\perp^7}. \tag{26}$$

- $v_{ph}(k_\perp) \rightarrow k_\perp \Rightarrow \rho^2(k_\perp) \sim \varepsilon k_\perp^{-7} \Rightarrow \phi(k_\perp) \sim a + b k_\perp$ where $0 > b \propto \eta/\varepsilon \Rightarrow \boxed{E^\pm(k_\perp) \sim k_\perp^{-5/2} e^{\pm b k_\perp}}$
- So one solutions exponentially diverge, unless we make modifications (confirmed in strong NDE simulation)
- When we choose $\chi = 1$ this problem is cured.

Value of the correlation coefficient in the dispersive case?



Weak turbulence analytics



Strong turbulence simulation

We can rewrite the stationary weak diffusive equations using

$$E^\pm(k_\perp) =: \frac{1}{2} k_\perp \rho(k_\perp) e^{\pm\phi(k_\perp)}. \tag{24}$$

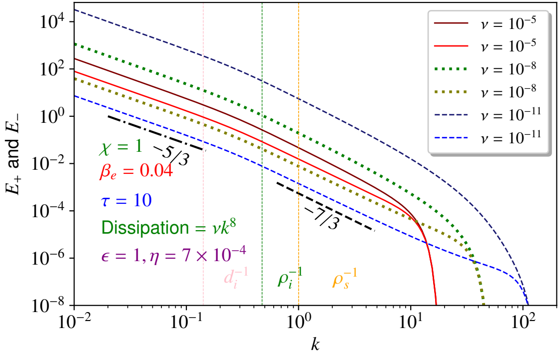
so that $|E_C(k_\perp)| \leq E(k_\perp)/v_{ph}(k_\perp)$. NDE rewrites

$$\frac{\partial}{\partial k_\perp}\rho^2(k_\perp) = -\frac{2\varepsilon}{Ck_\perp^7 v_{ph}(k_\perp)} \tag{25}$$

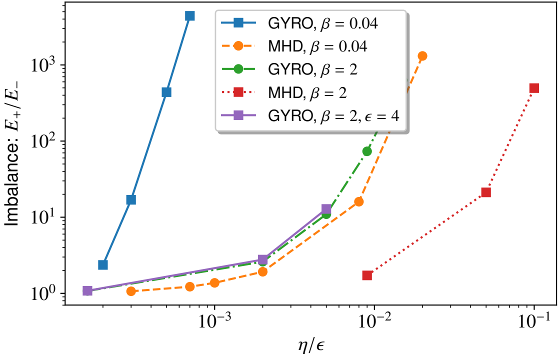
$$\rho^2(k_\perp)\frac{\partial}{\partial k_\perp}\phi(k_\perp) = -\frac{\eta}{Ck_\perp^7}. \tag{26}$$

- $v_{ph}(k_\perp) \to k_\perp \Rightarrow \rho^2(k_\perp) \sim \varepsilon k_\perp^{-7} \Rightarrow \phi(k_\perp) \sim a + bk_\perp$ where $0 > b \propto \eta/\varepsilon \Rightarrow \boxed{E^\pm(k_\perp) \sim k_\perp^{-5/2} e^{\pm bk_\perp}}$
- So one solutions exponentially diverge, unless we make modifications (confirmed in strong NDE simulation)
- When we choose $\chi = 1$ this problem is cured.

Presence of dispersion range increases imbalance



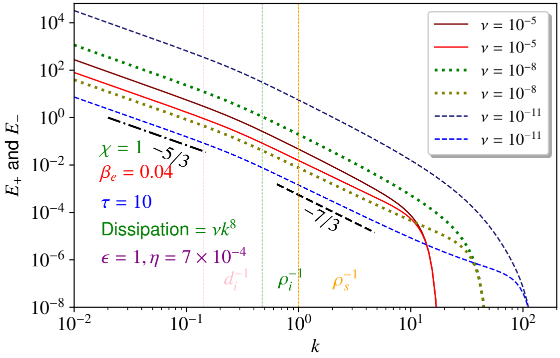
Dispersive case, $k_d = k_d(\nu)$



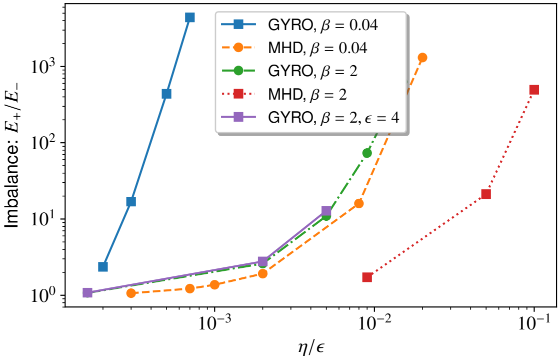
Imbalance vs. cross-helicity to energy flux ratio.

- The size of the dispersive range is determined by k_d, which is set by ν
- This size has significant impact on the size of imbalance: E_+/E_-!
- It appears that imbalance strongly depends on the cross-helicity flux η
- The imbalance is also more pronounced for lower values of β_e

Presence of dispersion range increases imbalance



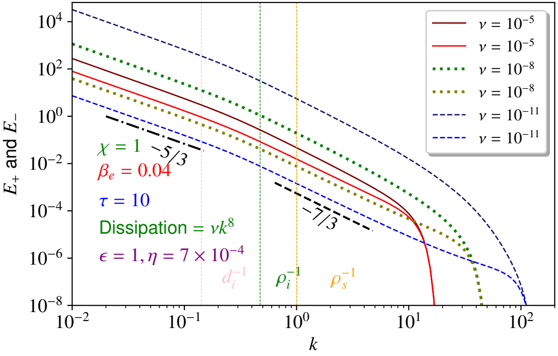
Dispersive case, $k_d = k_d(\nu)$



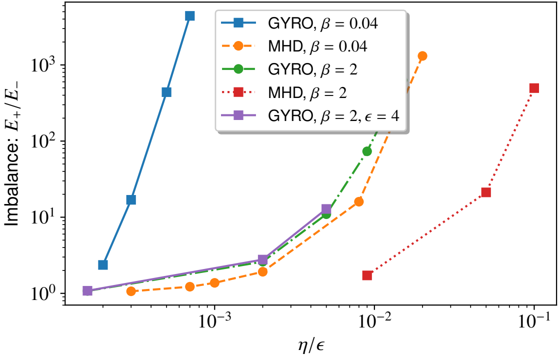
Imbalance vs. cross-helicity to energy flux ratio.

- The size of the dispersive range is determined by k_d, which is set by ν
- This size has significant impact on the size of imbalance: E_+/E_-!
- It appears that imbalance strongly depends on the cross-helicity flux η
- The imbalance is also more pronounced for lower values of β_e

Presence of dispersion range increases imbalance



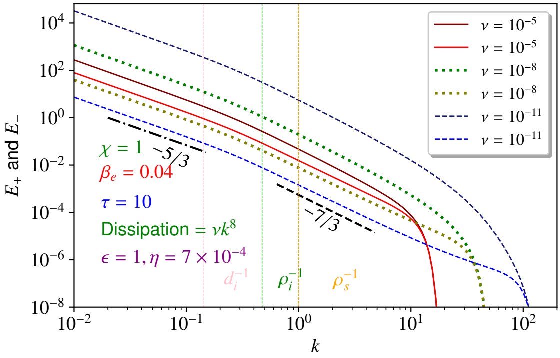
Dispersive case, $k_d = k_d(\nu)$



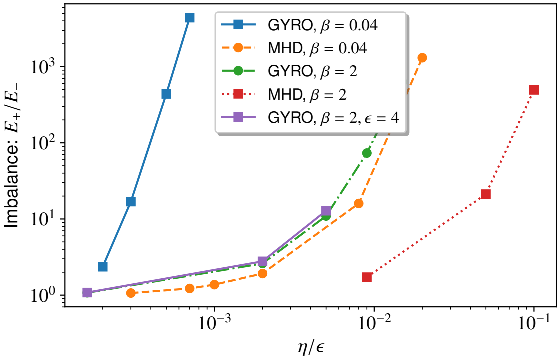
Imbalance vs. cross-helicity to energy flux ratio.

- The size of the dispersive range is determined by k_d, which is set by ν
- This size has significant impact on the size of imbalance: E_+/E_-!
- It appears that imbalance strongly depends on the cross-helicity flux η
- The imbalance is also more pronounced for lower values of β_e

Presence of dispersion range increases imbalance



Dispersive case, $k_d = k_d(\nu)$



Imbalance vs. cross-helicity to energy flux ratio.

- The size of the dispersive range is determined by k_d, which is set by ν
- This size has significant impact on the size of imbalance: E_+/E_-!
- It appears that imbalance strongly depends on the cross-helicity flux η
- The imbalance is also more pronounced for lower values of β_e

Landau Damping as a turbulence sink

- In solar wind plasmas are collisionless so (hyper)dissipation is unphysical
- We confirm that in case of low beta $\beta_e = 0.04$ electron contribution is stronger
- Terms $-2\gamma E$ and $-2\gamma E_C$ are introduced in spectral equations
- The expression for damping[18]is derived from linearized of gyrokinetics

$$\boxed{\gamma = \sqrt{\frac{\pi}{2}}\frac{\delta}{\beta_e}\frac{1}{1+4(1+\tau)^{-2}\beta_e^{-2}}\widetilde{k}_{\parallel}k_{\perp}^2,} \tag{27}$$

Wave-particle resonance

- Transfer time is also modified however the contribution is negligible.

[18] G. G. Howes et al., ApJ **651**, 590–614 (2006)

Landau Damping as a turbulence sink

- In solar wind plasmas are collisionless so (hyper)dissipation is unphysical
- We confirm that in case of low beta $\beta_e = 0.04$ electron contribution is stronger
- Terms $-2\gamma E$ and $-2\gamma E_C$ are introduced in spectral equations
- The expression for damping[18] is derived from linearized of gyrokinetics

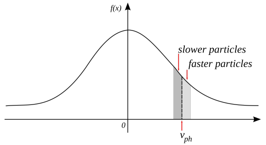
$$\boxed{\gamma = \sqrt{\frac{\pi}{2}}\frac{\delta}{\beta_e}\frac{1}{1+4(1+\tau)^{-2}\beta_e^{-2}}\widetilde{k}_{\|}k_{\perp}^2,} \quad (27)$$

Wave-particle resonance

- Transfer time is also modified however the contribution is negligible.

[18] G. G. Howes et al., ApJ **651**, 590–614 (2006)

- In solar wind plasmas are collisionless so (hyper)dissipation is unphysical
- We confirm that in case of low beta $\beta_e = 0.04$ electron contribution is stronger
- Terms $-2\gamma E$ and $-2\gamma E_C$ are introduced in spectral equations
- The expression for damping[18] is derived from linearized of gyrokinetics



Wave-particle resonance

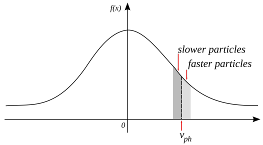
$$\gamma = \sqrt{\frac{\pi}{2}\frac{\delta}{\beta_e}} \frac{1}{1+4(1+\tau)^{-2}\beta_e^{-2}} \widetilde{k}_{\parallel} k_{\perp}^2, \quad (27)$$

- Transfer time is also modified however the contribution is negligible.

[18] G. G. Howes et al., ApJ **651**, 590–614 (2006)

Landau Damping as a turbulence sink

- In solar wind plasmas are collisionless so (hyper)dissipation is unphysical
- We confirm that in case of low beta $\beta_e = 0.04$ electron contribution is stronger
- Terms $-2\gamma E$ and $-2\gamma E_C$ are introduced in spectral equations
- The expression for damping[18] is derived from linearized of gyrokinetics



Wave-particle resonance

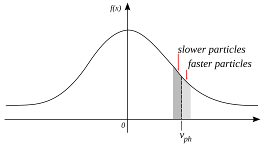
$$\boxed{\gamma = \sqrt{\frac{\pi}{2}} \frac{\delta}{\beta_e} \frac{1}{1 + 4(1+\tau)^{-2}\beta_e^{-2}} \widetilde{k}_{\|} k_\perp^2,} \tag{27}$$

- Transfer time is also modified however the contribution is negligible.

[18] G. G. Howes et al., ApJ **651**, 590–614 (2006)

Landau Damping as a turbulence sink

- In solar wind plasmas are collisionless so (hyper)dissipation is unphysical
- We confirm that in case of low beta $\beta_e = 0.04$ electron contribution is stronger
- Terms $-2\gamma E$ and $-2\gamma E_C$ are introduced in spectral equations
- The expression for damping[18]is derived from linearized of gyrokinetics



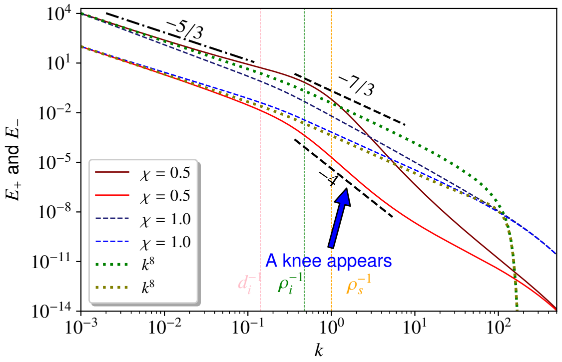
Wave-particle resonance

$$\boxed{\gamma = \sqrt{\frac{\pi}{2}}\frac{\delta}{\beta_e}\frac{1}{1+4(1+\tau)^{-2}\beta_e^{-2}}\widetilde{k}_{\parallel}k_{\perp}^{2},} \tag{27}$$

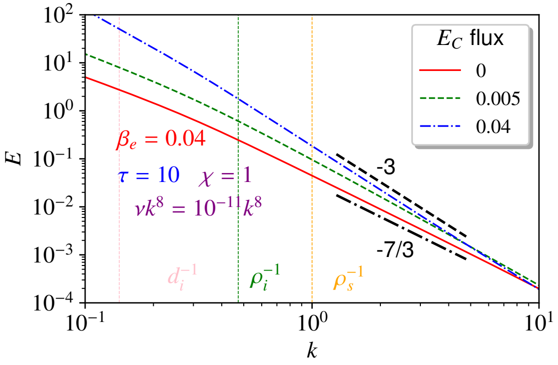
- Transfer time is also modified however the contribution is negligible.

[18] G. G. Howes et al., ApJ **651**, 590–614 (2006)

Landau damping effects



Different models of dissipation

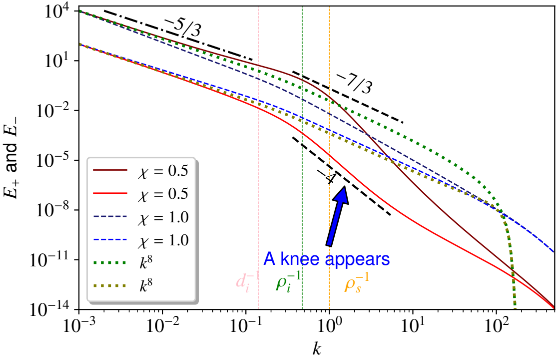


Steepening of the energy spectra vs η.

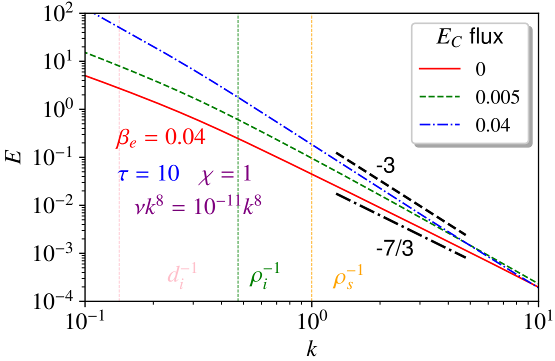
Results

- Correct mechanism of dissipation has a profound effect on the dispersion range
- When $\chi < 1$ we see the possibility of having in a knee in the spectrum
- Non-universality: for larger values η injection the spectrum becomes steeper.

Landau damping effects



Different models of dissipation

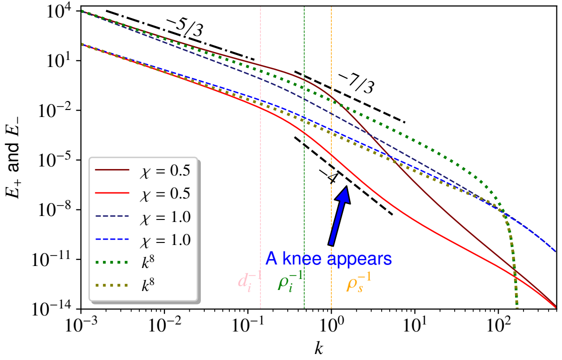


Steepening of the energy spectra vs η.

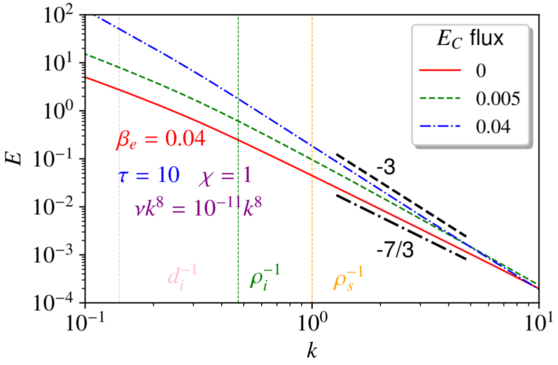
Results

- Correct mechanism of dissipation has a profound effect on the dispersion range
- When $\chi < 1$ we see the possibility of having in a knee in the spectrum
- Non-universality: for larger values η injection the spectrum becomes steeper.

Landau damping effects



Different models of dissipation

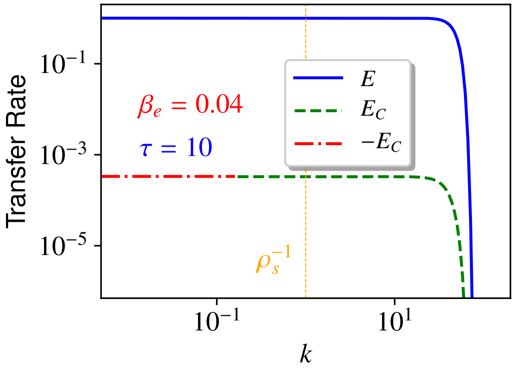


Steepening of the energy spectra vs η.

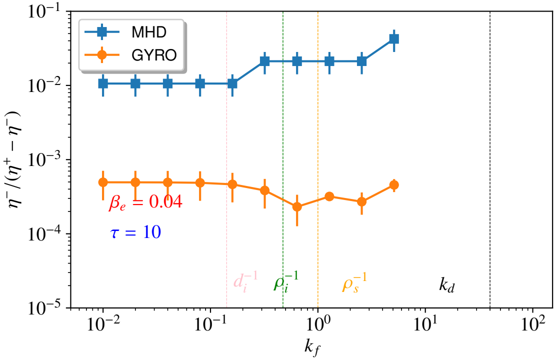
Results

- Correct mechanism of dissipation has a profound effect on the dispersion range
- When $\chi < 1$ we see the possibility of having in a knee in the spectrum
- Non-universality: for larger values η injection the spectrum becomes steeper.

Inverse cascade of cross-helicity and forward of energy



Dispersive case: Fluxes for $k_f \sim 0.016$

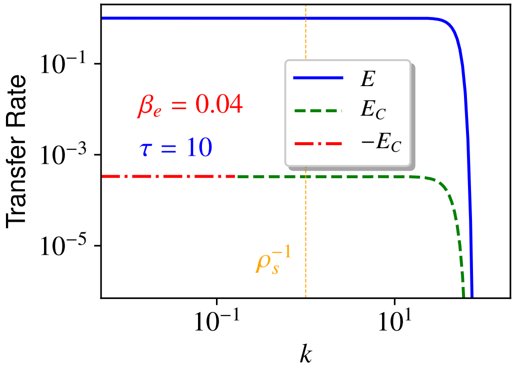


Small scale to large scale cross-helicity injection ratio needed to drive the inverse cascade

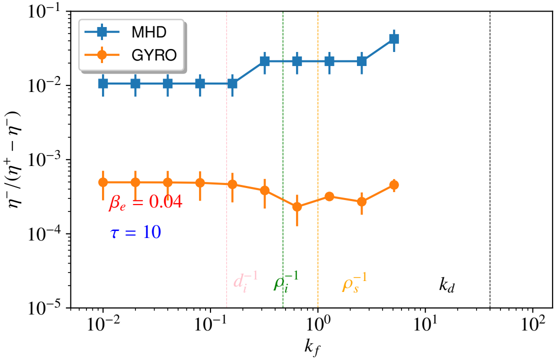
- In presence of reservoir $E_\pm$ it is possible to drive η_- from small scales
- Inverse cascade exists even in MHD but is stronger when dispersive range is present
- In sub-ion range E_C approaches magnetic-helicity so is perhaps related to Ref. [19]

[19] U. Frisch et al., J. of Fluid Mech. **68**, 769–778 (1975)

Inverse cascade of cross-helicity and forward of energy



Dispersive case: Fluxes for $k_f \sim 0.016$



Small scale to large scale cross-helicity injection ratio needed to drive the inverse cascade

- In presence of reservoir $E_{\pm}$ it is possible to drive η_{-} from small scales
- Inverse cascade exists even in MHD but is stronger when dispersive range is present
- In sub-ion range E_C approaches magnetic-helicity so is perhaps related to Ref. [19]

[19] U. Frisch et al., J. of Fluid Mech. **68**, 769–778 (1975)

Outline

Comparison with direct 3D gyrofluid simulations

- We have analysed consequences of NDEs that describe imbalanced KAW turbulence
- We conclude that dispersive range significantly affects the imbalance
- The dissipation mechanism is important, e.g. Landau damping

Preliminary results from 3D DNS are supporting some of the claims

- In particular the amount of imbalance strongly depends on the size of k_d
- ToDo: Measure correlation lengths $\tilde{k}_{\parallel}^{\pm}$ and estimate χ

Turbulent current j

Comparison with direct 3D gyrofluid simulations

- We have analysed consequences of NDEs that describe imbalanced KAW turbulence
- We conclude that dispersive range significantly affects the imbalance
- The dissipation mechanism is important, e.g. Landau damping

Preliminary results from 3D DNS are supporting some of the claims

- In particular the amount of imbalance strongly depends on the size of k_d
- ToDo: Measure correlation lengths $\tilde{k}_{\|}^{\pm}$ and estimate χ

Turbulent current j

Comparison with direct 3D gyrofluid simulations

- We have analysed consequences of NDEs that describe imbalanced KAW turbulence
- We conclude that dispersive range significantly affects the imbalance
- The dissipation mechanism is important, e.g. Landau damping

Preliminary results from 3D DNS are supporting some of the claims

- In particular the amount of imbalance strongly depends on the size of k_d
- ToDo: Measure correlation lengths $\tilde{k}_{\parallel}^{\pm}$ and estimate χ

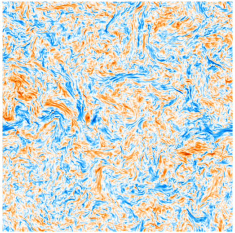
Turbulent current j

Comparison with direct 3D gyrofluid simulations

- We have analysed consequences of NDEs that describe imbalanced KAW turbulence
- We conclude that dispersive range significantly affects the imbalance
- The dissipation mechanism is important, e.g. Landau damping

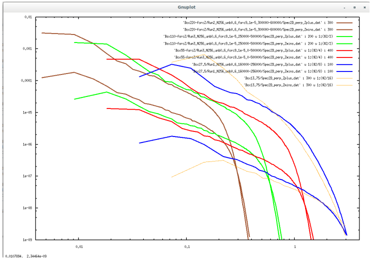
Preliminary results from 3D DNS are supporting some of the claims

- In particular the amount of imbalance strongly depends on the size of k_d
- ToDo: Measure correlation lengths $\tilde{k}_{\parallel}^{\pm}$ and estimate χ

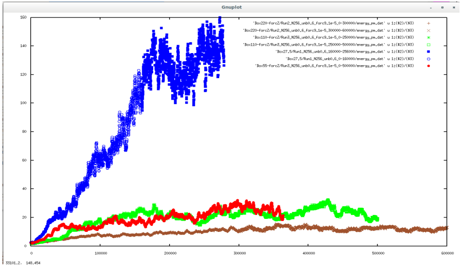


Turbulent current j_z

.



Spectra $E_{\pm}$ for different box sizes



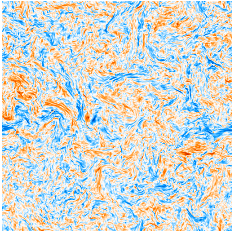
Growth of imbalance E_+/E_- vs. time

Comparison with direct 3D gyrofluid simulations

- We have analysed consequences of NDEs that describe imbalanced KAW turbulence
- We conclude that dispersive range significantly affects the imbalance
- The dissipation mechanism is important, e.g. Landau damping

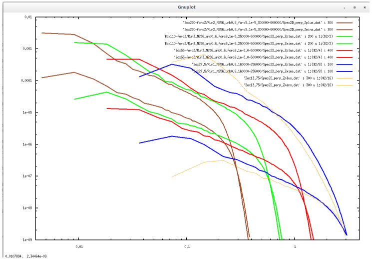
Preliminary results from 3D DNS are supporting some of the claims

- In particular the amount of imbalance strongly depends on the size of k_d
- ToDo: Measure correlation lengths $\tilde{k}_{\parallel}^{\pm}$ and estimate χ

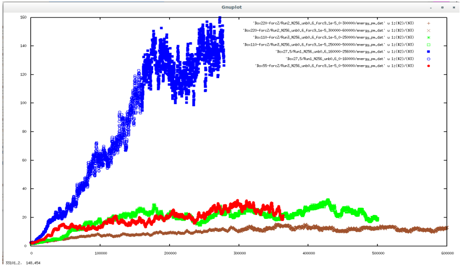


Turbulent current j_z

.



Spectra $E_{\pm}$ for different box sizes



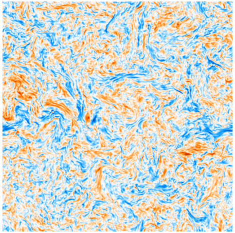
Growth of imbalance E_{+}/E_{-} vs. time

Comparison with direct 3D gyrofluid simulations

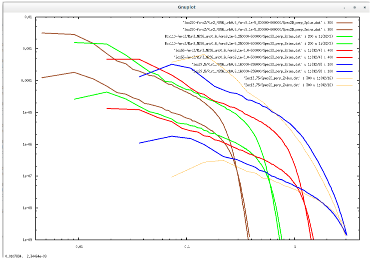
- We have analysed consequences of NDEs that describe imbalanced KAW turbulence
- We conclude that dispersive range significantly affects the imbalance
- The dissipation mechanism is important, e.g. Landau damping

Preliminary results from 3D DNS are supporting some of the claims

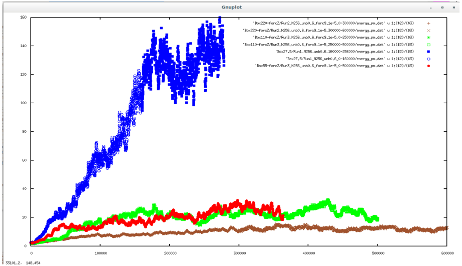
- In particular the amount of imbalance strongly depends on the size of k_d
- ToDo: Measure correlation lengths $\tilde{k}_{\|}^{\pm}$ and estimate χ



Turbulent current j_z
.



Spectra $E_{\pm}$ for different box sizes



Growth of imbalance E_{+}/E_{-} vs. time