Using Fermi Function in the Decision-Making Process to Explore Diffusion in Networks

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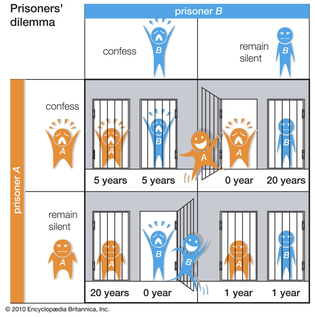
Introduction

The phrase *"It's a small world"* is a widely used idiom that is used to describe the chance of connecting with a person, place, or event that seems unrelated at first glance. It has been observed that most real-world networks have small-world properties. The Watts-Strogatz network is a model for generating small-world networks. At a glance, the Watts-Strogatz network and the real-world networks seem unrelated, but they both share the same common thread: the concept of the small-world network.

This study uses the Watts-Strogatz and Facebook network and game theory approach to analyze real-world networks' structure, dynamics, and evolution.

Game Setup and Simulation Model

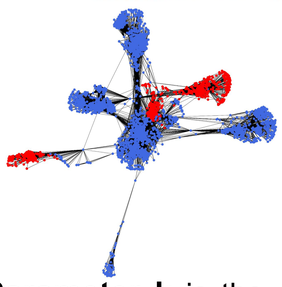


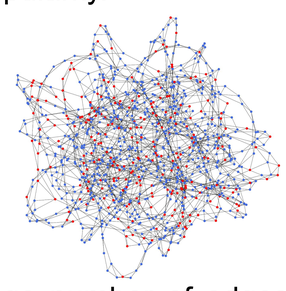
$$\begin{bmatrix} b-c & -c \\ b & 0 \end{bmatrix}$$

Variables: b = 1.8, c = 0.3
and b = 15 c = 0.5

The Fermi Update Rule is used to find the probability of a node updating its strategy based on its neighbors' payoff and popularity.

$$P_{i\to j} = \frac{1}{1+e^{-\beta(wj-wi)}}$$





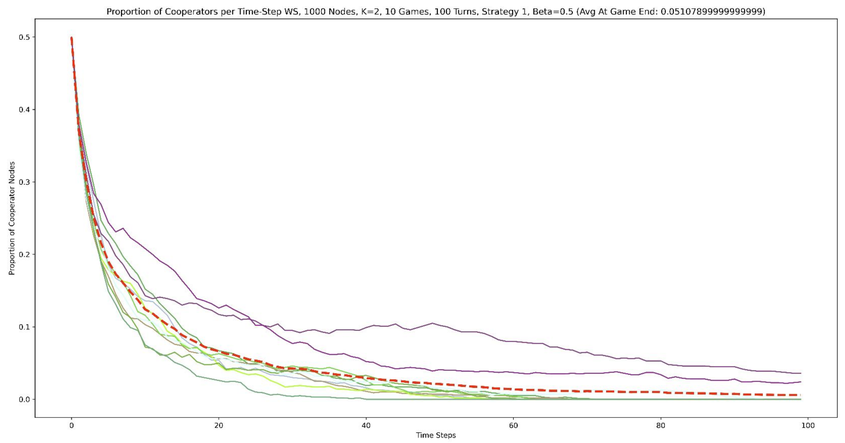
Parameter k is the average number of edges per node in a network, or degree centrality.

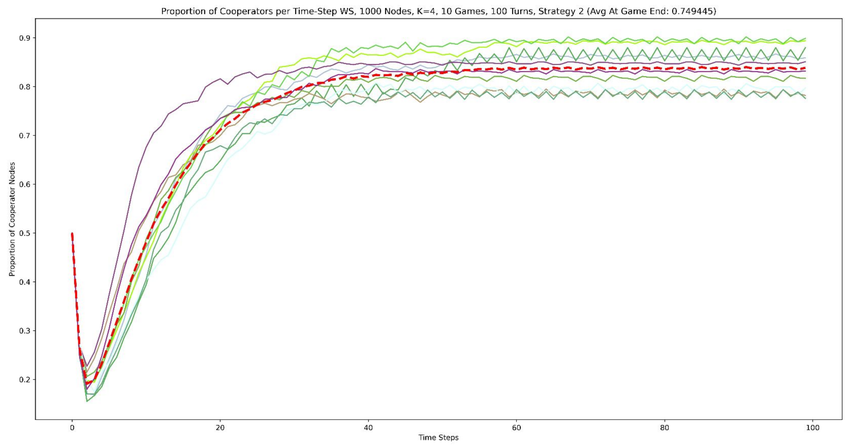
Parameter β is used to measure the popularity of a node. It is set to the neighbor node's degree centrality, or number of nodes / total nodes.

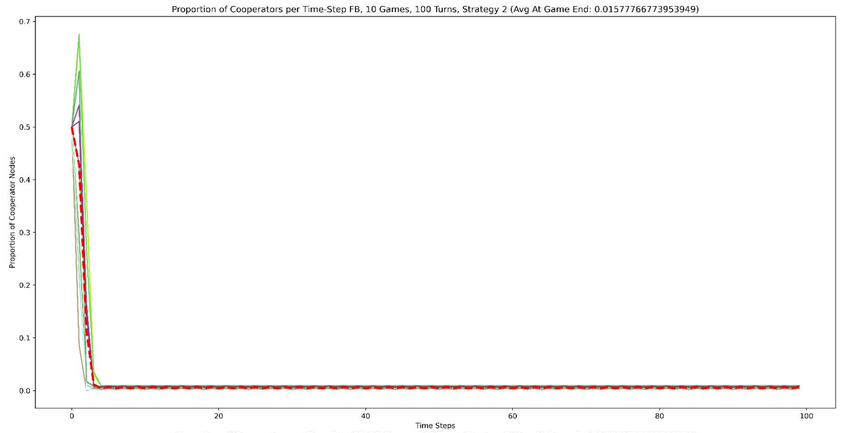
The Initial percentage of cooperators is set to 50% to allow an even and fair start for both cooperators and defectors.

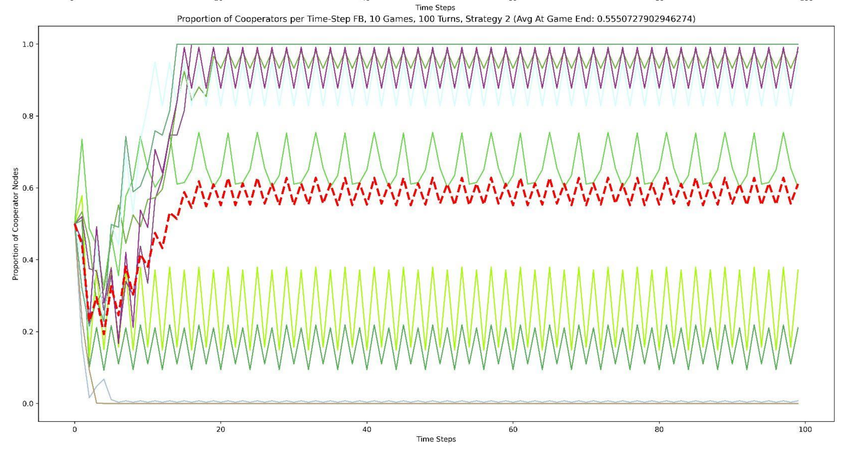
To determine the dominant strategy, we ran the simulation 100 times with 10 rounds for each game and found the mean ratio of cooperators to defectors in each network.

Experimental Results









Analysis and Conclusion

Cooperation cannot exist in any network without special rules. Natural selection in the Prisoner's Dilemma game favors defectors due to the payoff structure. It remains this way until certain rules are made to encourage cooperation.

The higher payoff for defectors cause players to have the temptation to defect; therefore, initially we see a decline in cooperation.

Cooperators form clusters around popular nodes and those clusters effect the defectors locally. Over time, as a result of growth in those clusters, defectors can't resist and more players start to become cooperators.

The increase in cooperators not only depends on the parameter **β** it also depends on the payoff values which are set at the beginning of the game.

Watts Strogatz with Popularity Strategy resulted cooperators to take over the the population.

In the Facebook network, when b/c < k the defectors take over the population; however, when the payoff values are chosen to satisfy the condition b/c > k cooperators start to increase, and as a result on average more than 50% of cooperators exist in the population.

Future Research

The studies will be done in different real life networks to draw comparison and create a general model.

We used 2 player game in this study, Creating a 3 player version of the Prisoner's Dilemma might be more interesting and we can get different insights on group decisions.

References

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