(1.) $U_t + F(U)_x = 0 \qquad a = \sqrt{\gamma R T}$

$$U = \begin{vmatrix} u_1 \\ u_2 \end{vmatrix} = \begin{vmatrix} \rho \\ \rho u \end{vmatrix}$$

$$F = \begin{vmatrix} f_1 \\ f_2 \end{vmatrix} = \begin{vmatrix} \rho u \\ \rho u^2 + \rho a^2 \end{vmatrix} = \begin{vmatrix} \rho u \\ \rho u^2 + \rho \gamma R T \end{vmatrix}$$

1a) $U = \begin{vmatrix} u_1 \\ u_2 \end{vmatrix}. \quad F = \begin{vmatrix} \rho u \\ \rho u^2 + \rho a^2 \end{vmatrix} \Rightarrow \boxed{F = \begin{vmatrix} u_2 \\ u_2^2/u_1 + u_1 a^2 \end{vmatrix}}$

1b.) $A(U) = \partial F / \partial U = \begin{bmatrix} 0 & 1 \\ -\frac{u_2^2}{u_1^2} + a^2 & \frac{2u_2}{u_1} \end{bmatrix}$

1c.) $|A - \lambda I| = 0 \Rightarrow \det \begin{bmatrix} -\lambda & 1 \\ -u^2 + a^2 & 2u - \lambda \end{bmatrix} = 0 \Rightarrow$

$$-\lambda(2u-\lambda) - (-u^2 + a^2) = 0$$

$$-2\lambda u + \lambda^2 + u^2 + a^2 - 0$$

$$(\lambda - u + a)(\lambda - u - a) = 0$$

$$\boxed{\begin{matrix} \lambda_1 = u - a \\ \lambda_2 = u + a \end{matrix}} \quad \checkmark$$

1d) $\begin{bmatrix} -u+a & 1 \\ -u^2 + a^2 & u+a \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$X_1(a-u) + X_2 = 0$$
$$X_1(a^2 - u^2) + X_2(a+u) = 0$$

$X_2 = -X_1(a-u)$

$X_2 = \frac{-X_1(a^2-u^2)}{(a+u)}$;

$X_1(a-u) = \frac{X_1(a^2-u^2)}{(a+u)} \rightarrow X_1(a^2-u^2) = X_1(a^2-u^2)$

$X_1 = 1, \; X_2 = u - a \qquad \boxed{K^{(1)} = \begin{bmatrix} 1 \\ u-a \end{bmatrix}} \quad \checkmark$

$\begin{bmatrix} -u-a & 1 \\ -u^2 + a^2 & u-a \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$X_1(-u-a) + X_2 = 0$$
$$X_2 = -X_1(-u-a)$$

$$-X_1(-u^2 + a^2) = X_2(u-a)$$
$$X_2 = -\frac{X_1(-u^2 + a^2)}{(u-a)}$$

$X_1(-u-a) = \frac{X_1(-u^2 + a^2)}{(u-a)} \quad \checkmark$

$X_1 = 1, X_2 = u + a \qquad \boxed{K^{(2)} = \begin{bmatrix} 1 \\ u+a \end{bmatrix}}$