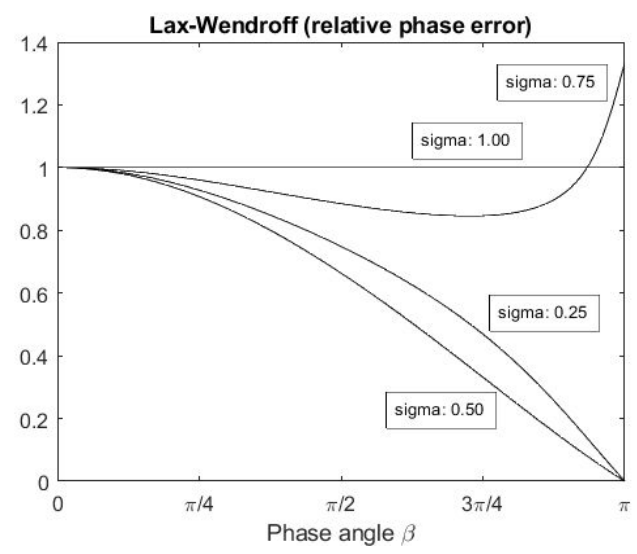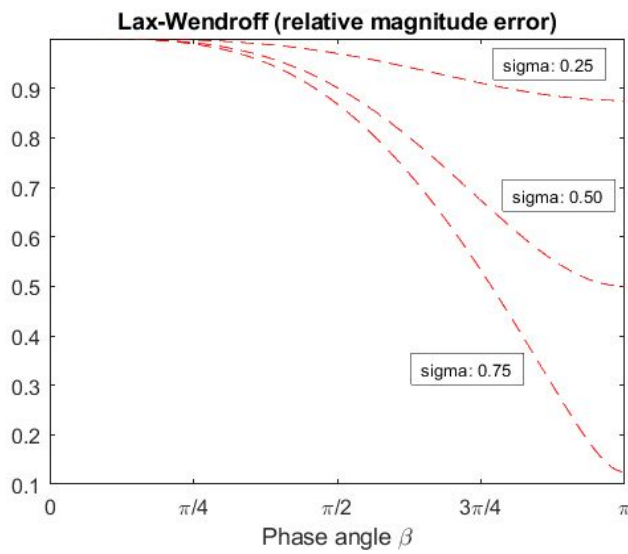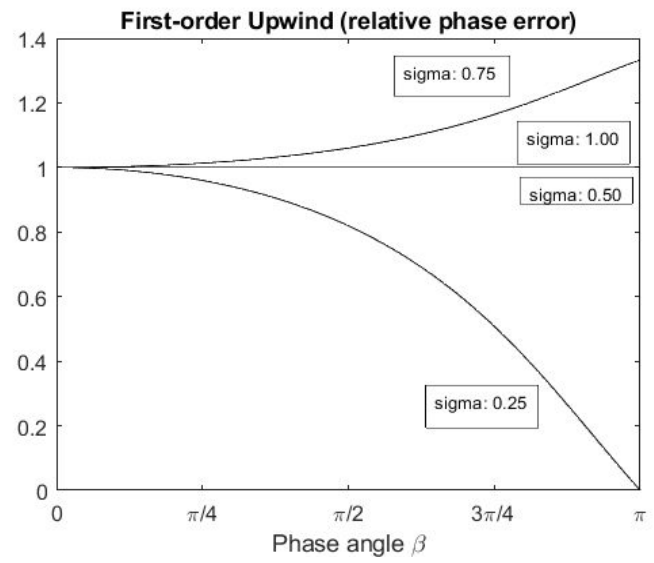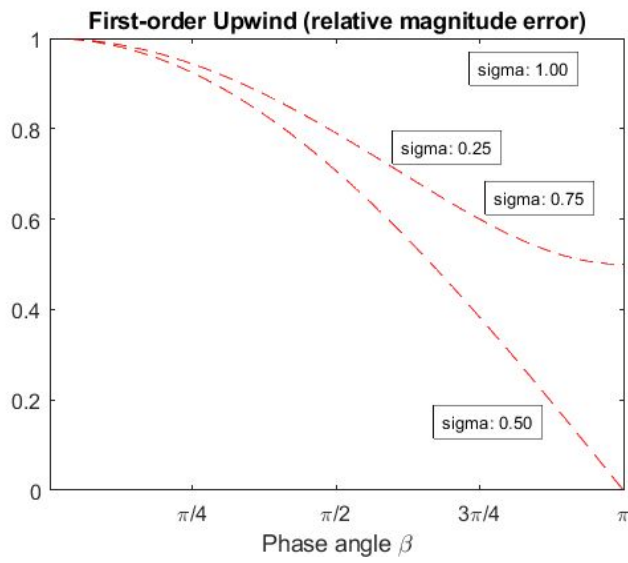2a.) $\beta = \dfrac{2\pi}{N}$, $N = \{25, 50, 100, 200, 400\}$, $\lambda = 2\pi$

N	β
25	.251
50	.126
100	.063
200	.031
400	.016

AS expected, β decreases as N increases.

First-order Upwind (relative magnitude error)

sigma: 1.00

sigma: 0.25

sigma: 0.75

sigma: 0.50

Phase angle β

First-order Upwind (relative phase error)

sigma: 0.75

sigma: 1.00

sigma: 0.50

sigma: 0.25

Phase angle β

Lax-Wendroff (relative magnitude error)

sigma: 0.25

sigma: 0.50

sigma: 0.75

Phase angle β

Lax-Wendroff (relative phase error)

sigma: 0.75

sigma: 1.00

sigma: 0.25

sigma: 0.50

Phase angle β

2c.) $\lambda = 2\pi$, $c=1$, $h=.1$, $\sigma=.5$

$1-\sigma+\sigma\cos B$
$1-.5+.5\cos.1 = .9975$

Fou: $|g| = \sqrt{|1-\sigma+\sigma\cos B|^2 + (\sigma\sin B)^2}$

$\sigma^2\sin^2 B = .00249$

$= \sqrt{(.9975)^2 + \sigma^2\sin^2 B}$

$= \sqrt{.995 + .00249}$

$|g| = .99875^N \implies N = \dfrac{\ln.5}{\ln.99875} = \boxed{555}$

$.99875^N = .5$

First order upwind requires 555 steps before the amplitude is $\frac{1}{2}$ of the exact solution.

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L-W: $|G| = 1-\sigma^2+\sigma^2\cos B + i\sigma\sin B$

$|g| = \sqrt{(1-\sigma^2+\sigma^2\cos B)^2 + (\sigma\sin B)^2}$

$|g| = .999997$

$.999997^N = .5 \implies \dfrac{\ln.5}{\ln.999997} \implies \boxed{N = 296,247}$

Lax-Wendroff requires 296247 steps before the amplitude is $\frac{1}{2}$ of the exact solution.

2d.)

FOU: $|g| = \sqrt{(1 - \sigma + \sigma \cos B)^2 + (\sigma \sin B)^2}$

LW: $|g| = \sqrt{(1 - \sigma^2 + \sigma^2 \cos B)^2 + (\sigma \sin B)^2}$

$|g| = .9999972602...$

$.9999972602 = \sqrt{(1 - .5 + .5 \cos h)^2 + (.5 \sin h)^2}$

$.9999945204 = (.5 + .5 \cos(h))^2 + (.5 \sin(h))^2$

$\boxed{h = .0043}$

This h will require $\overset{2\pi/h}{\curvearrowleft}$ 1462 points in each dimension.

In 3D, our number of points becomes N^3. Lax-Wendroff becomes advantageous as you add dimensions because the number of required points is significantly lower.