1a.) $x_1 = 0 \quad x_5 = 2\pi$

$$A\,U^{N+1} = b$$

$$u_1^{n+1} = u_1^n - \frac{\sigma}{2}\left(u_2^{n+1} - u_0^{n+1}\right)$$

$$\hookrightarrow = u_1^n - \frac{\sigma}{2}\left(u_2^{n+1} - u_4^{n+1}\right) \rightsquigarrow$$

$$u_1^{n+1} + \frac{\sigma}{2}\left[u_2^{n+1} - \cancel{\tfrac{\sigma}{2}}\, u_4^{n+1}\right] = u_1^n$$

$$u_2^{n+1} = u_2^n - \frac{\sigma}{2}\left(u_3^{n+1} - u_1^{n+1}\right)$$

$$u_2^{n+1} + \frac{\sigma}{2}\left[u_3^{N+1} - u_1^{N+1}\right] = u_1^n$$

$$u_3^{n+1} = u_3^n - \frac{\sigma}{2}\left(u_4^{n+1} - u_2^{n+1}\right)$$

$$u_4^{n+1} = u_4^n - \frac{\sigma}{2}\left(u_5^{n+1} - u_3^{n+1}\right)$$

$$u_5^{n+1} = u_5^n - \frac{\sigma}{2}\left(u_2^{n+1} - u_4^{n+1}\right)$$

$$\begin{bmatrix} 1 & \sigma/2 & 0 & -\sigma/2 & 0 \\ -\frac{\sigma}{2} & 1 & \sigma/2 & 0 & 0 \\ & -\sigma/2 & 1 & & \\ & & & 1 & \\ & & & & 1 \end{bmatrix} \begin{bmatrix} u_1^{N+1} \\ u_2^{N+1} \\ \\ \cdots \\ u_3^{N+1} \end{bmatrix} = \begin{bmatrix} u_1^N \\ u_2^N \\ \cdots \\ \\ u_5^N \end{bmatrix}$$

$$\frac{\sigma}{2}u_2^{N+1} \qquad u_5^{N+1} - \frac{\sigma}{2}u_4^{N+1} = \ldots$$

$$\underline{A U^{N+1} = b}$$

$$\begin{bmatrix} 1 & \sigma/2 & 0 & -\sigma/2 & 0 \\ -\sigma/2 & 1 & \sigma/2 & 0 & 0 \\ 0 & -\sigma/2 & 1 & \sigma/2 & 0 \\ 0 & 0 & -\sigma/2 & 1 & \sigma/2 \\ 0 & \sigma/2 & 0 & -\sigma/2 & 1 \end{bmatrix} \begin{bmatrix} u_1^{N+1} \\ u_2^{N+1} \\ u_3^{N+1} \\ U_4^{N+1} \\ U_5^{N+1} \end{bmatrix} = \begin{bmatrix} u_1^N \\ U_2^N \\ U_3^N \\ U_4^N \\ U_5^N \end{bmatrix}.$$