10) $U_i^{n+1} = U_i^n - \frac{\sigma}{2}\left(U_{i+1}^n - U_{i-1}^n\right) + \frac{\sigma^2}{2}\left(U_{i+1}^n - 2U_i^n + U_{i-1}^n\right)$ $\sigma = \frac{k\epsilon}{h}$

i=1 $u_1^{n+1} = u_1^n - \frac{\sigma}{2}\left(u_2^n - \cancel{u_0^n}\right) + \frac{\sigma^2}{2}\left(u_2^n - 2u_1^n + \cancel{u_0^n}\right)$ $h = \frac{k}{\sigma}$ $k = \sigma h$

$\rightarrow$ $= u_1^n - \frac{\sigma}{2}\left(u_2^n - u_4^n\right) + \frac{\sigma^2}{2}\left(u_2^n - 2u_1^n + u_4^n\right)$

i=2 $u_2^{n+1} = u_2^n - \frac{\sigma}{2}\left(u_3^n - u_1^n\right) + \frac{\sigma^2}{2}\left(u_3^n - 2u_2^n + u_1^n\right)$ $\frac{k^2}{h^2}$

i=3 $u_3^{n+1} = u_3^n - \frac{\sigma}{2}\left(u_4^n - u_2^n\right) + \frac{\sigma^2}{2}\left(u_4^n - 2u_3^n + u_2^n\right)$

i=4: $u_4^{n+1} = u_4^n - \frac{\sigma}{2}\left(u_5^n - u_3^n\right) + \frac{\sigma^2}{2}\left(u_5^n - 2u_4^n + u_3^n\right)$

i=5. $u_5^{n+1} = u_5^n - \frac{\sigma}{2}\left(\cancel{u_6^n} - u_4^n\right) + \frac{\sigma^2}{2}\left(\cancel{u_6^n} - 2u_5^n + u_4^n\right)$ $\frac{u_{xxx}}{6}\left(\frac{\sigma^3 h^3}{k} - \frac{\sigma h^3}{k}\right)$

$\hookrightarrow$ $u_5^{n+1} = u_5^n - \frac{\sigma}{2}\left(u_2^n - u_4^n\right) + \frac{\sigma}{2}\left(u_2^n - 2u_5^n + u_4^n\right)$ $\frac{u_{xxxx}}{24}\left(\frac{\sigma^2 h^4}{k} - \frac{\sigma^4 h^4}{k}\right)$

error: $L_h(x,t) = -\frac{h^2}{6}\frac{h}{k}\sigma(1-\sigma^2)u_{xxx} + \frac{h^3}{24}\frac{h}{k}\sigma^2(1-\sigma^2)u_{xxxx}$

$u_{i-1} = u_i - u_x h + u_{xx}\frac{h^2}{2} - u_{xxx}\frac{h^3}{6} + u_{xxxx}\frac{h^4}{24}$, $u^{n+1} = u^n + u_t k + u_{tt}\frac{k^2}{2} + u_{ttt}\frac{k^3}{6} + u_{tttt}\frac{k^4}{24}$

$u_{i+1} = u_i + u_x h + u_{xx}\frac{h^2}{2} + u_{xxx}\frac{h^3}{6} + u_{xxxx}\frac{h^4}{24}$

$u_t = -u_x$
$u_{tt} = u_{xx}$
$u_{ttt} = -u_{xxx}$
$u_{tttt} = u_{xxxx}$

$u_i^{n+1} - u_i^n + \frac{\sigma}{2}\left(u_{i+1}^n - u_{i-1}^n\right) - \frac{\sigma^2}{2}\left(u_{i+1}^n - 2u_i^n + u_{i-1}^n\right) = 0$

$\div k$:

$\frac{1}{k}u_i^{n+1} - \frac{1}{k}u_i^n + \frac{\sigma}{2}\frac{1}{k}\left(u_{i+1}^n - u_{i-1}^n\right) - \frac{\sigma^2}{2}\frac{1}{k}\left(u_{i+1}^n - 2u_i^n + u_{i-1}^n\right) = 0$

$\frac{1}{k}\left(u + u_t k + u_{tt}\frac{k^2}{2} + u_{ttt}\frac{k^3}{6} + u_{tttt}\frac{k^4}{24}\right) - \frac{\sigma^2}{2}\frac{1}{k}\left(\cancel{-2u_i^n + 2u_i^n} + u_{xx}h^2 + \frac{1}{12}u_{xxxx}h^4\right) + \frac{\sigma}{2}\frac{1}{k}\left(2u_x h + \frac{1}{3}u_{xxx}h^3\right) - \frac{1}{k}u = 0$

$\frac{1}{k}\left(u - u_x k + u_{xx}\frac{k^2}{2} - u_{xxx}\frac{k^3}{6} + u_{xxxx}\frac{k^4}{24}\right) - \frac{\sigma^2}{2}\frac{1}{k}\left(u_{xx}h^2 + \frac{1}{12}h^4 u_{xxxx}\right) + \frac{\sigma}{2}\frac{1}{k}\left(2u_x h + \frac{1}{3}u_{xxx}h^3\right) - \frac{1}{k}u = 0$

$\frac{1}{k}\left(u - u_x k + u_{xx}\frac{k^2}{2} - u_{xxx}\frac{k^3}{6} + u_{xxxx}\frac{k^4}{24}\right) + \frac{1}{2h}\left(2u_x h + \frac{1}{3}u_{xxx}h^3\right) - \frac{k}{2h^2}\left(u_{xx}h^2 + \frac{1}{12}h^4 u_{xxxx}\right) - \frac{1}{k}u = 0$

$\frac{1}{k}\left[u - u_x k + u_{xx}\frac{k^2}{2} - u_{xxx}\frac{k^3}{6} + u_{xxxx}\frac{k^4}{24}\right] + u_x + u_{xxx}\frac{h^2}{6} - \frac{k}{2}u_{xx} - \frac{k}{24}h^2 u_{xxxx}$

$\boxed{\text{When } \sigma, \frac{h}{k} \text{ are constant, } L_h(x,t) \text{ has order } O(h^2)}$

$\frac{1}{k}u - u_x + u_{xx}\frac{k}{2} - u_{xxx}\frac{k^2}{6} + u_{xxxx}\frac{k^3}{24} + u_x + u_{xxx}\frac{h^2}{6} - u_{xx}\frac{k}{2} - u_{xxxx}\frac{kh^2}{24}$

$\frac{1}{k}u - \frac{u_{xxx}}{6}\left[k^2 - h^2\right] + \frac{u_{xxxx}}{24}\left[k^3 - kh^2\right] = 0$

$L \rightarrow$ $L(x,t) = \frac{1}{k}\left[\frac{u_{xxx}k}{6}\left[k^2 - h^2\right] - \frac{u_{xxxx}}{24}k\left[k^3 - kh^2\right]\right]$

$\boxed{L(x,t) = -\frac{h^2}{6}\frac{h}{k}\sigma(1-\sigma^2)u_{xxx} + \frac{h^3}{24}\frac{h}{k}\sigma^2(1-\sigma^2)u_{xxxx}}$ + h.o.t.

$L(x,t) = (k^3 - kh^2)\frac{u_{xxx}}{6k} - (k^4 - k^2h^2)\frac{u_{xxxx}}{24k} = (\sigma^3h^3 - \sigma h^3)\frac{u_{xxx}}{6k} - (\sigma^4h^4 - \sigma^2h^4)\frac{u_{xxxx}}{24k}$