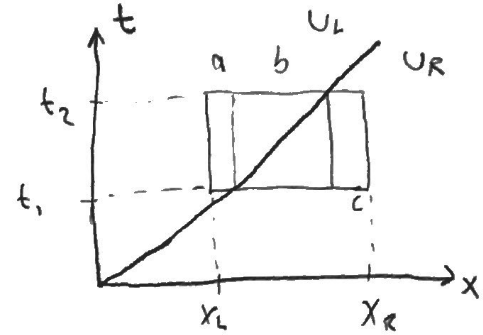
$$③\quad \underbrace{\int_{t_1}^{t_2}\int_{x_L}^{x_R} \frac{\partial}{\partial t} u(z,\tau)\,dz\,d\tau}_{①} + \underbrace{\int_{t_1}^{t_2}\int_{x_L}^{x_R} \frac{\partial}{\partial x} f(u(z,\tau))\,dz\,d\tau}_{②} = 0$$

$$① = \int_{t_1}^{t_2} \cancel{\frac{\partial}{\partial t}} \int_{x_L}^{x_R} u(z,\tau)\,dz\,d\tau = \left[\int_{x_L}^{x_R} u(z,\tau)\,dz\,d\tau\right]_{t_1}^{t_2} = \int_{x_L}^{x_R} u(z,t_2)-u(z,t_1)\,dz = ①$$



$$② = \int_{t_1}^{t_2} \underbrace{f(u(x_R,\tau))}_{\frac{1}{2}u_R^2} - \underbrace{f(u(x_L,\tau))}_{\frac{1}{2}u_L^2}\,d\tau$$

We see that ① is the difference between the top and bottom areas.

$$① = \left[(a+b)u_L + c\,u_R\right] - \left[a\,u_L + (b+c)u_R\right] = b\,u_L - b\,u_R.$$

$$b = s(t_2-t_1) = \frac{u_L+u_R}{2}(t_2-t_1) = \frac{1}{2}(u_L+u_R)(t_2-t_1).\quad ① = \frac{1}{2}(u_L-u_R)(u_L+u_R)(t_2-t_1)$$

$$② = \left(\frac{1}{2}u_R^2 - \frac{1}{2}u_L^2\right)(t_2-t_1) = \frac{1}{2}(u_R^2-u_L^2)(t_2-t_1) = \frac{1}{2}(u_R+u_L)(u_R-u_L)(t_2-t_1)$$

$$= -\frac{1}{2}(u_L+u_r)(u_L-u_R)(t_2-t_1). \rightarrow ② = -①,\quad ① + ② = 0 \quad ✓$$