

① Forward Euler: $R(z) = 1 + z$
 $|1 - 10h| \leq 1 \Rightarrow \boxed{h \leq .2}$

★ Assuming h to be positive ★

Backward Euler: $R(z) = \frac{1}{1-z}$
 $\left| \frac{1}{1+10h} \right| \leq 1 \Rightarrow \boxed{h \geq 0}$

Trapezoidal/midpoint: $R(z) = \frac{1 + \frac{z}{2}}{1 - \frac{z}{2}}$
 $\left| \frac{1 - \frac{10h}{2}}{1 + \frac{10h}{2}} \right| \leq 1 \Rightarrow \left| \frac{1-5h}{1+5h} \right| \leq 1 \Rightarrow \boxed{h \geq 0}$

RK2: $R(z) = 1 + z + \frac{z^2}{2}$
 $\left| 1 - 10h + \frac{100h^2}{2} \right| \Rightarrow \left| 1 - 10h + 50h^2 \right| \leq 1 \Rightarrow \boxed{h \leq .2}$

RK4: $R(z) = 1 + z + \frac{z^2}{2} + \frac{z^3}{6} + \frac{z^4}{24}$
 $\left| 1 - 10h + 50h^2 - \frac{500}{3}h^3 + \frac{10,000}{24}h^4 \right| \leq 1 \Rightarrow \boxed{h \leq .279}$

AB2: $R^2 - (1 + \frac{3z}{2})R + \frac{z}{2} = 0$
 $R^2 - (1 - 15h)R - 5h = 0 \Rightarrow R = \frac{(1-15h) \pm \sqrt{(1-15h)^2 + 20h}}{2}$

For positive root: $\boxed{h \geq 0}$

For negative root: $\boxed{h \leq .1}$

Method	Stability Range
Forward Euler	$h \leq .2$
Backward Euler	$h \geq 0$
Trapezoidal/midpoint	$h \geq 0$
RK2	$h \leq .2$
RK4	$h \leq .279$
AB2	$h \leq .1$

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clc; clear all; close all;

h = linspace(0,.3,100);
z = -10.*h;

r = {
    1+z,                                     ... % fwd
    1./(1-z),                               ... % bwd
    (1+z./2)./(1-z./2),                     ... % trap and mid
    1+z+(z.^2)./2,                           ... % rk2
    1+z+(z.^2)./2+(z.^3)./6+(z.^4)./24,       ... % rk4
    ((1-15.*h)+sqrt((1-15.*h).^2+20.*h))./2, ... % ab2
    ((1-15.*h)-sqrt((1-15.*h).^2+20.*h))./2 ... % ab2
};

figure(1)
for i = 1:length(r)
    x = h;
    y = abs(r{i});
    plot(x,y)
    xlim([0 0.3])
    ylim([0 1])
    hold on
end
legend('Forward','Backward','Trapezoidal and Midpoint','RK2','RK4','AB2','AB2')
title('R Value with Increasing h')
xlabel('Step size (h)')
ylabel('|R(-10h)|')

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