(1) $u_{i+1} = u_i + u'(x_i)h + u''(x_i)\frac{h^2}{2!} + u^{(3)}(x_i)\frac{h^3}{3!} + u^{(4)}(x_i)\frac{h^4}{4!}$

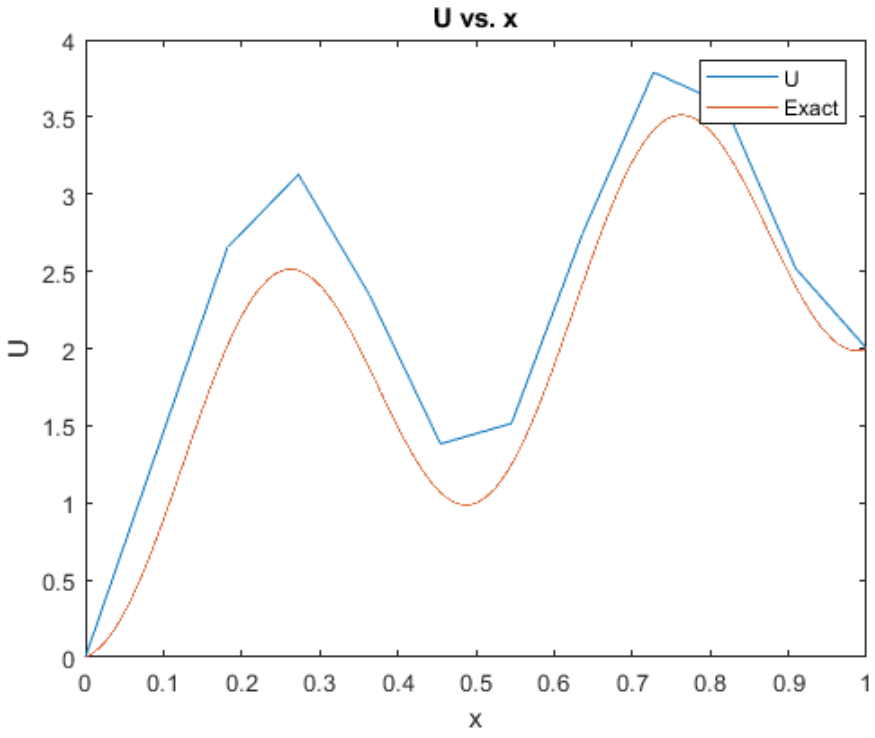
$+ u_{i-1} = u_i - u'(x_i)h + u''(x_i)\frac{h^2}{2!} - u^{(3)}(x_i)\frac{h^3}{3!} + u^{(4)}(x_i)\frac{h^4}{4!}$

$u_{i+1} + u_{i-1} = 2u_i + \cancel{\frac{2}{2!}}\, u''(x_i)h^2 + \frac{2}{4!}u^{(4)}(x_i)h^4$ → Rearrange to fit problem:

$u''(x_i)h^2 = u_{i+1} - 2u_i + u_{i-1} - \boxed{\frac{1}{12}}u^{(4)}(x_i)h^4$

$\boxed{C = -\frac{1}{12}}$

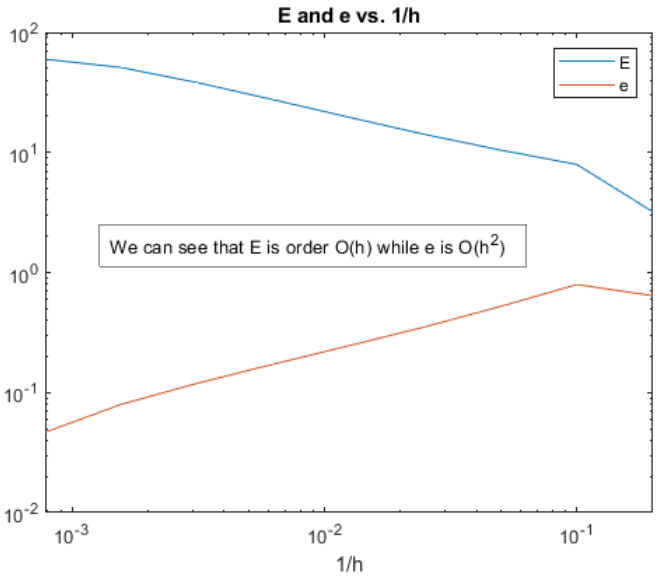
```
% Question 2

clc; close all;

n = 10;
h = 1/(n+1);
tri = full(gallery('tridiag',n,1,-2,1));

func = @(x) (4*pi)^2*cos(4*pi*x)*h^2;

f = [];
x = [];

for i=1:n
    x(i) = h*i;
    f(i) = func(x(i));
end
f(n) = f(n) - 2;
f(1) = 0;
x = [0 x 1];
x = x';
u = tri\f';
u = [0;u;2];

ax = solutionAN10000(:,1);
ay = solutionAN10000(:,2);

figure
plot(x,u);
hold on
plot(ax,ay)
title('U vs. x')
legend('U','Exact')
xlim([0 1])
xlabel('x')
ylabel('U')

fid = fopen('q2data.txt','w');
fprintf(fid, '\tx\t\t\tu\n');
fprintf(fid,'%f\t%f\n',[x,u].');
```



Data for question 2

x	u
0.000000	0.000000
0.090909	1.327214
0.181818	2.654429
0.272727	3.127003
0.363636	2.347370
0.454545	1.382006
0.545455	1.514538
0.636364	2.744966
0.727273	3.789664
0.818182	3.582154
0.909091	2.520004
1.000000	2.000000

```
% Question 3
clc; clear all; close all;

N = [5,10,20,40,80,160,320,640,1280];
f = [];

n = 10^4;
h = 1/(n+1);
tri = full(gallery('tridiag',n,1,-2,1));
func = @(x) (4*pi)^2*cos(4*pi*x)*h^2;
f1 = [];
x1 = [];
for i=1:n
    x1(i) = h*i;
    f1(i) = func(x1(i));
end
f1(n) = f1(n) - 2;
x1 = [0 x1 1];
x1 = x1';
u1 = tri\f1';
u1 = [0;u1;2];

for i=1:9
    n = N(i);
    h = 1/(n+1);
    tri = full(gallery('tridiag',n,1,-2,1));
    x = [];
    func = @(x) (4*pi)^2*cos(4*pi*x)*h^2;

    for j=1:n
        x(j) = h*j;
        f(j) = func(x(j));
    end
    f(n) = f(n) - 2;
    f(1) = 0;
    x = [0 x 1];
    x = x';
    u = tri\f';
    u = [0;u;2];
    E = [];

    for j=1:n
        E(j) = (u(j) - u1(j))^2;
    end
    E_sum = sum(E);
    E_final = sqrt(E_sum);
    e = E_final/n;
    E_vec(i) = E_final;
    e_vec(i) = e;
end

loglog(1./N,E_vec)
hold on
loglog(1./N,e_vec)
hold on

title('E and e vs. 1/h')
legend('E', 'e')
xlabel('1/h')
dim = [.2 .4 .3 .2];
str = 'We can see that E is order O(h) while e is O(h^2)';
annotation('textbox', dim, 'String', str, 'FitBoxToText', 'on');
```



4. $u_{i+1} = u_i + u'(x_i)h + u''(x_i)\frac{h^2}{2!}$

$u_{i+2} = u_i + 2hu'(x_i) + 2h^2 u''(x_i)$

$u'(x_i)h = au_i + bu_{i+1} + cu_{i+2}$

$u'(x_i)h = a u_i + b\left[u_i + u'(x_i)h + u''(x_i)\frac{h^2}{2}\right] + c\left[u_i + 2hu'(x_i) + 2h^2 u''(x_i)\right]$

$u'(x_i)h = 1a u_i + 1b u_i + (b)u'(x_i)h + 1b u''(x_i)\frac{h^2}{2} + (c)u_i + (c)2hu'(x_i) + (c)2h^2 u''(x_i)$

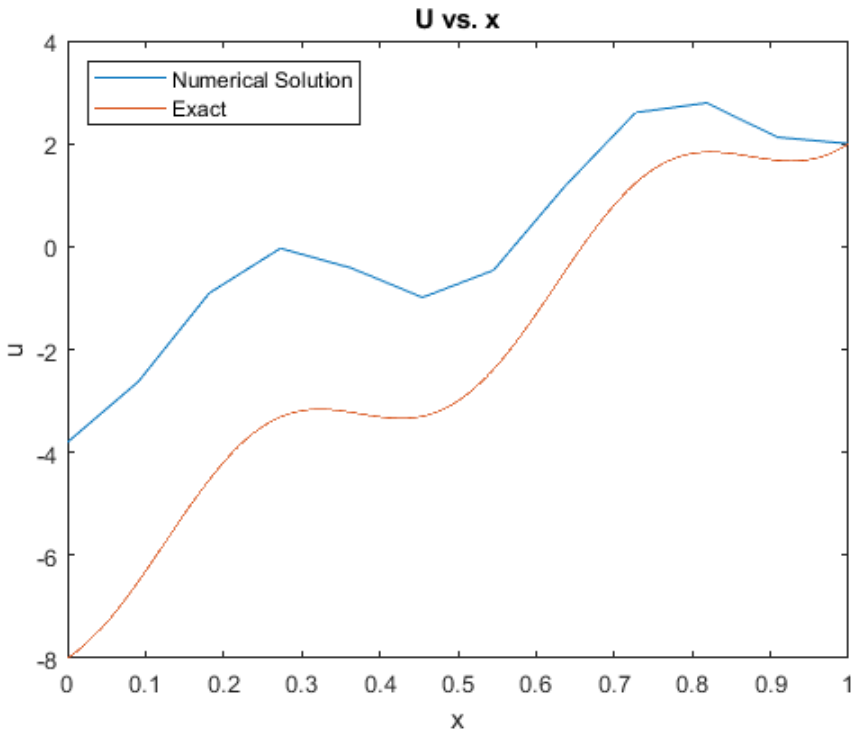
$a + b + c = 0$

$b + 2c = 1$

$\frac{b}{2} + 2c = 0$

$a = -3/2,\ b = 2,\ c = -1/2$

```
% Question 5
clc; close all;

n = 10;
h = 1/(n+1);
tri = full(gallery('tridiag',n,1,-2,1));
tri(1,1) = -2/3;
tri(1,2) = 2/3;

func = @(x) (4*pi)^2*cos(4*pi*x)*h^2;
f = [];
x = [];

for i=1:n
    x(i) = h*i;
    f(i) = func(x(i));
end
f(1) = f(1) + (20/3)*h;
f(n) = f(n) - 2;

x = [0 x 1];
x = x';
u = tri\f';
u0 = -(20/3)*h+(4/3)*u(1)-(1/3)*u(2);
u = [u0;u;2];

ax = solutionBN10000(:,1);
ay = solutionBN10000(:,2);

figure
plot(x,u)
hold on
plot(ax,ay)
title('U vs. x')
legend('Numerical Solution','Exact', 'Location', 'northwest')
xlabel('x')
ylabel('u')

fid = fopen('q5data.txt','w');
fprintf(fid, '\tx\t\t\tu\n');
fprintf(fid,'%f\t%f\n',[x,u].');
```



Data for qeustion 5

x	u
0.000000	-3.803911
0.090909	-2.623747
0.181818	-0.901436
0.272727	-0.033766
0.363636	-0.418303
0.454545	-0.988571
0.545455	-0.460943
0.636364	1.164582
0.727273	2.604375
0.818182	2.791961
0.909091	2.124908
1.000000	2.000000