

$$\textcircled{1} \quad u_{i+1} = u_i + u'(x_i)h + u''(x_i)\frac{h^2}{2!} + u^{(3)}(x_i)\frac{h^3}{3!} + u^{(4)}(x_i)\frac{h^4}{4!}$$

$$+ u_{i-1} = u_i - u'(x_i)h + u''(x_i)\frac{h^2}{2!} - u^{(3)}(x_i)\frac{h^3}{3!} + u^{(4)}(x_i)\frac{h^4}{4!}$$

$$u_{i+1} + u_{i-1} = 2u_i + \cancel{\frac{2}{2!}} u''(x_i)h^2 + \frac{2}{4!} u^{(4)}(x_i)h^4 \rightarrow \text{Rearrange to fit problem:}$$

$$u''(x_i)h^2 = u_{i+1} - 2u_i + u_{i-1} - \boxed{\frac{1}{12}} u^{(4)}(x_i)h^4$$

$\boxed{C = -\frac{1}{12}}$