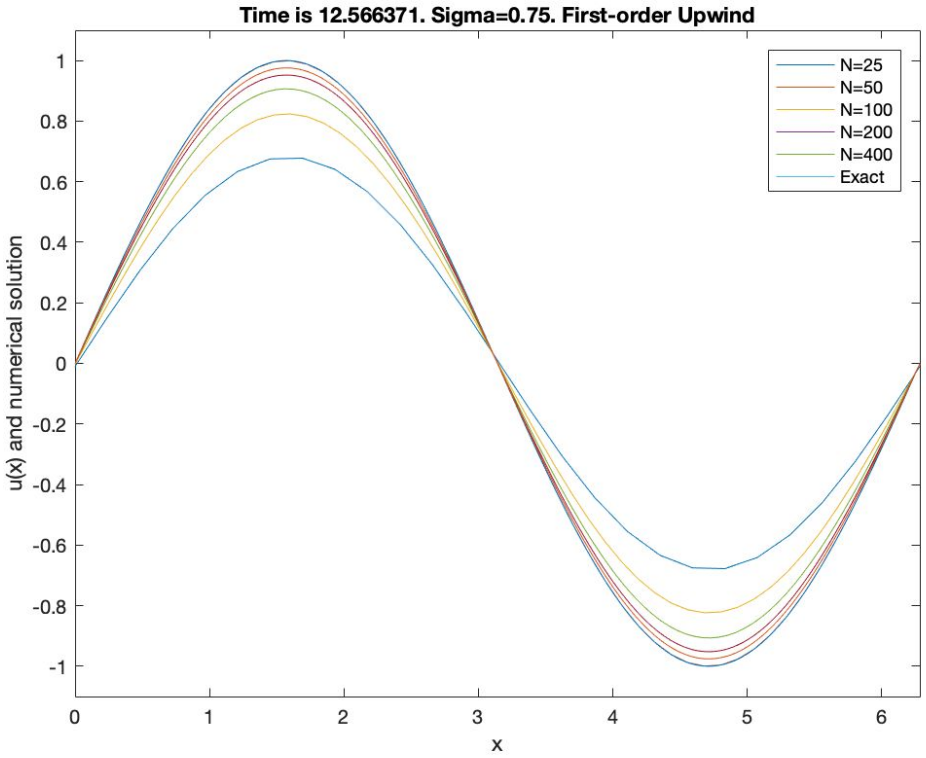
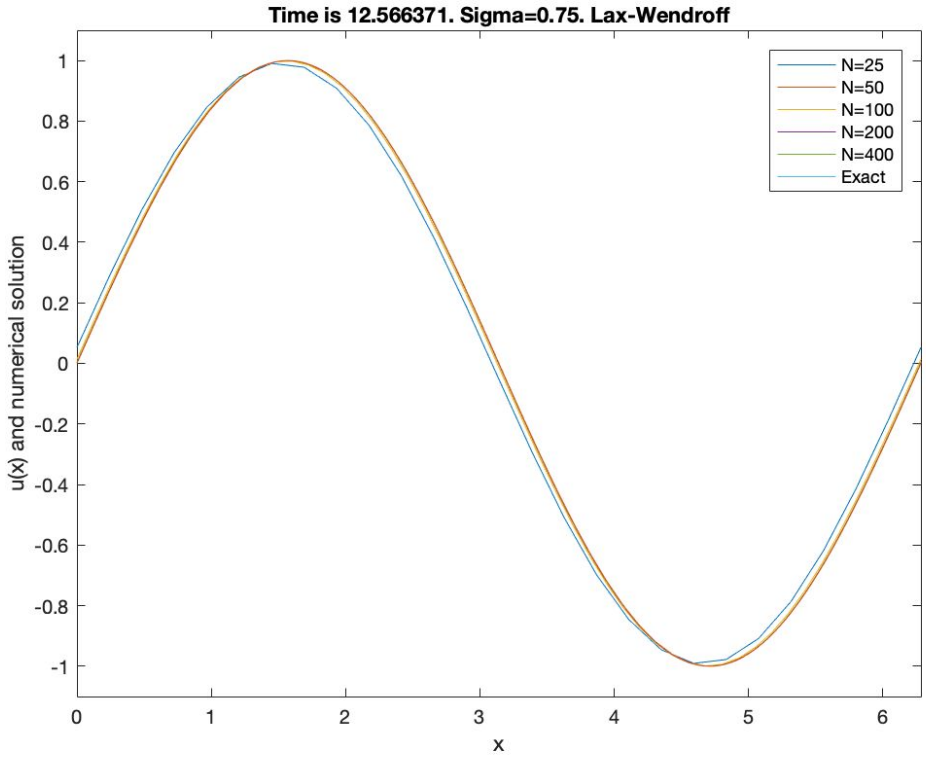
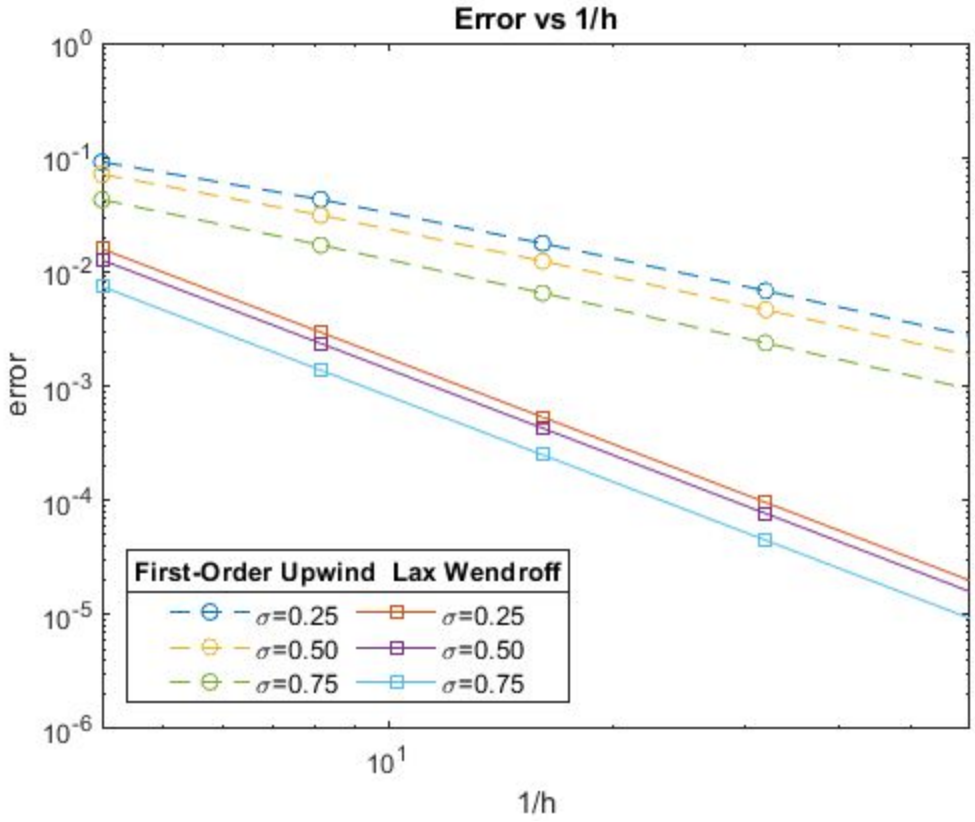
1a:


Time is 12.566371. Sigma=0.75. First-order Upwind
N=25
N=50
N=100
N=200
N=400
Exact
u(x) and numerical solution
x
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
1
2
3
4
5
6


Time is 12.566371. Sigma=0.75. Lax-Wendroff
N=25
N=50
N=100
N=200
N=400
Exact
u(x) and numerical solution
x
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
1
2
3
4
5
6

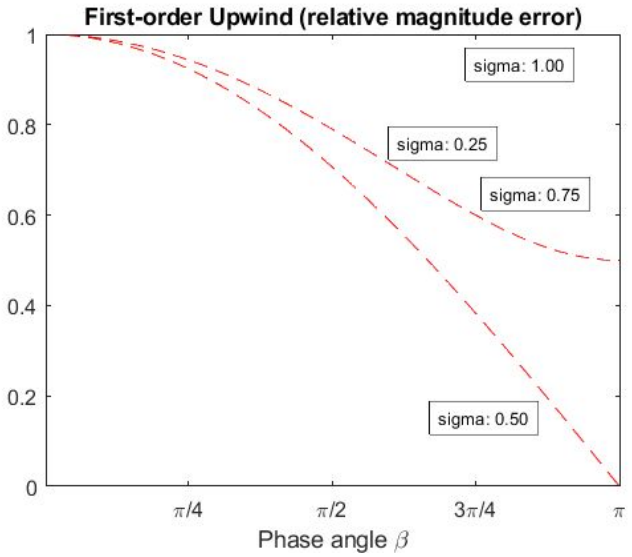
1b:

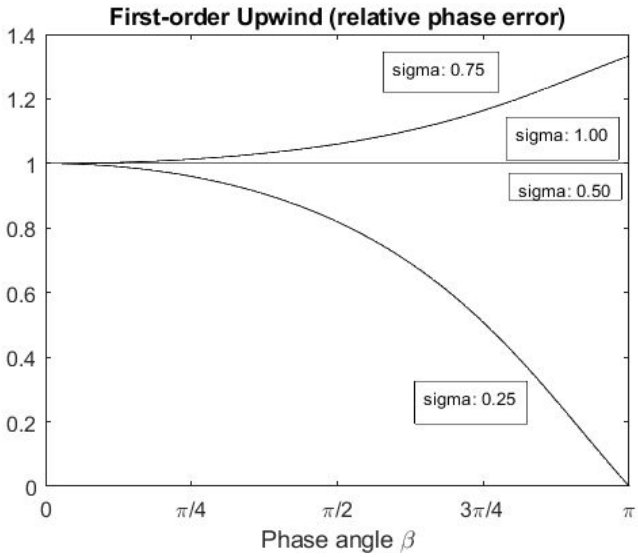

Error vs 1/h
error
1/h
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{1}
First-Order Upwind
Lax Wendroff
σ=0.25
σ=0.50
σ=0.75
σ=0.25
σ=0.50
σ=0.75

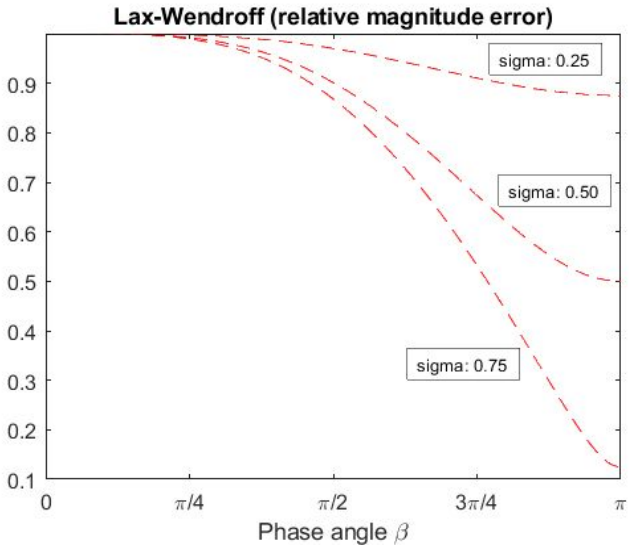
2a) $\beta = \frac{2\pi}{N}$, $N = \{25, 50, 100, 200, 400\}$, $\lambda = 2\pi$

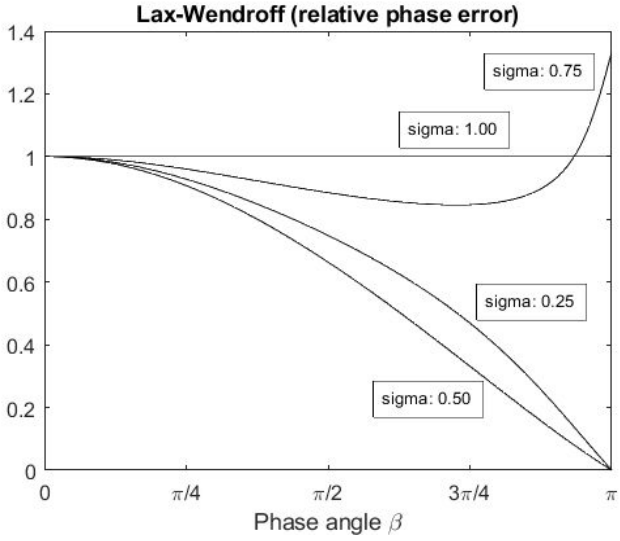
N	β
25	.251
50	.126
100	.063
200	.031
400	.016

As expected, β decreases as N increases.


First-order Upwind (relative magnitude error)
sigma: 1.00
sigma: 0.25
sigma: 0.75
sigma: 0.50
1
0.8
0.6
0.4
0.2
0
π/4
π/2
3π/4
π
Phase angle β


First-order Upwind (relative phase error)
sigma: 0.75
sigma: 1.00
sigma: 0.50
sigma: 0.25
1.4
1.2
1
0.8
0.6
0.4
0.2
0
π/4
π/2
3π/4
π
Phase angle β


Lax-Wendroff (relative magnitude error)
sigma: 0.25
sigma: 0.50
sigma: 0.75
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
π/4
π/2
3π/4
π
Phase angle β


Lax-Wendroff (relative phase error)
sigma: 0.75
sigma: 1.00
sigma: 0.25
sigma: 0.50
1.4
1.2
1
0.8
0.6
0.4
0.2
0
π/4
π/2
3π/4
π
Phase angle β

2c.) $\lambda = 2\pi$, $c = 1$, $h = .1$, $\sigma = .5$

$1 - \sigma + \sigma\cos B$

$1 - .5 + .5\cos .1 = .9975$

Fou: $|g| = \sqrt{|1 - \sigma + \sigma\cos B|^2 + (\sigma \sin B)^2}$ $\quad \sigma^2 \sin^2 B = .00249$

$= \sqrt{(.9975)^2 + \sigma^2 \sin^2 B}$

$= \sqrt{.995 + .00249}$

$|g| = .99875^N \Rightarrow N = \frac{\ln .5}{\ln .99875} = \boxed{555}$

$.99875^N = .5$

First order upwind requires 555 steps before the amplitude is $\frac{1}{2}$ of the exact solution.

L-W: $|G| = 1 - \sigma^2 + \sigma^2 \cos B + i\sigma \sin B$

$|g| = \sqrt{(1 - \sigma^2 + \sigma^2 \cos B)^2 + (\sigma \sin B)^2}$

$|g| = .999997$

$.999997^N = .5 \rightarrow \frac{\ln .5}{\ln .999997} \Rightarrow \boxed{N = 296,247}$

Lax-Wendroff requires 296247 steps before the amplitude is $\frac{1}{2}$ of the exact solution.

2d.) FOU: $|g| = \sqrt{(1-\sigma+\sigma\cos\beta)^2 + (\sigma\sin\beta)^2}$

LW: $|g| = \sqrt{(1-\sigma^2+\sigma^2\cos\beta)^2 + (\sigma\sin\beta)^2}$

$|g| = .9999972602\ldots$

$.9999972602 = \sqrt{(1-.5+.5\cos h)^2 + (.5\sin h)^2}$

$.9999945204 = (.5+.5\cos(h))^2 + (.5\sin(h))^2$

$\boxed{h = .0043}$

$2\pi/h$

This h will require 1462 points in each dimension.

In 3D, our number of points becomes N^3. Lax-Wendroff becomes advantagious as you add dimensions because the number of required points is significantly lower.