(1.) Forward Euler: $R(z) = 1 + z$

★ Assuming h to be positive ★

$|1 - 10h| \leq 1 \Rightarrow \boxed{h \leq .2}$

Backward Euler: $R(z) = \frac{1}{1-z}$

$\left|\frac{1}{1+10h}\right| \leq 1 \Rightarrow \boxed{h \geq 0}$

Trapezoidal/Midpoint: $R(z) = \frac{\left(1 + \frac{z}{2}\right)}{\left(1 - \frac{z}{2}\right)}$

$\left|\frac{1-\frac{10h}{2}}{1+\frac{10h}{2}}\right| \leq 1 \Rightarrow \left|\frac{1-5h}{1+5h}\right| \leq 1 \Rightarrow \boxed{h \geq 0}$

RK2: $R(z) = 1 + z + z^2/2$

$\left|1 - 10h + \frac{100h^2}{2}\right| \rightarrow \left|1 - 10h + 50h^2\right| \leq 1 \Rightarrow \boxed{h \leq .2}$

RK4: $R(z) = 1 + z + z^2/2 + z^3/6 + z^4/24$

$\left|1 - 10h + 50h^2 - \frac{500}{3}h^3 + \frac{10{,}000}{24}h^4\right| \leq 1 \Rightarrow \boxed{h \leq .279}$

AB2: $R^2 - \left(1 + \frac{3z}{2}\right)R + z/2 = 0$

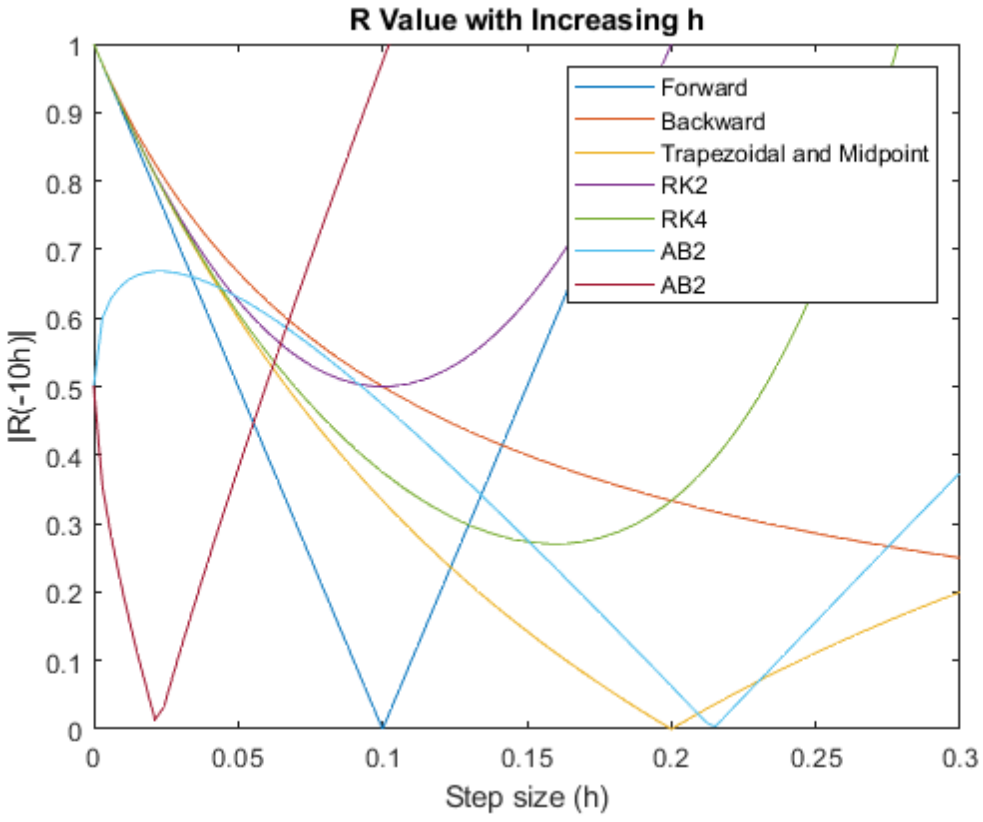
$R^2 - (1 - 15h)R - 5h = 0 \Rightarrow R = \frac{(1-15h) \pm \sqrt{(1-15h)^2 + 20h}}{2}$

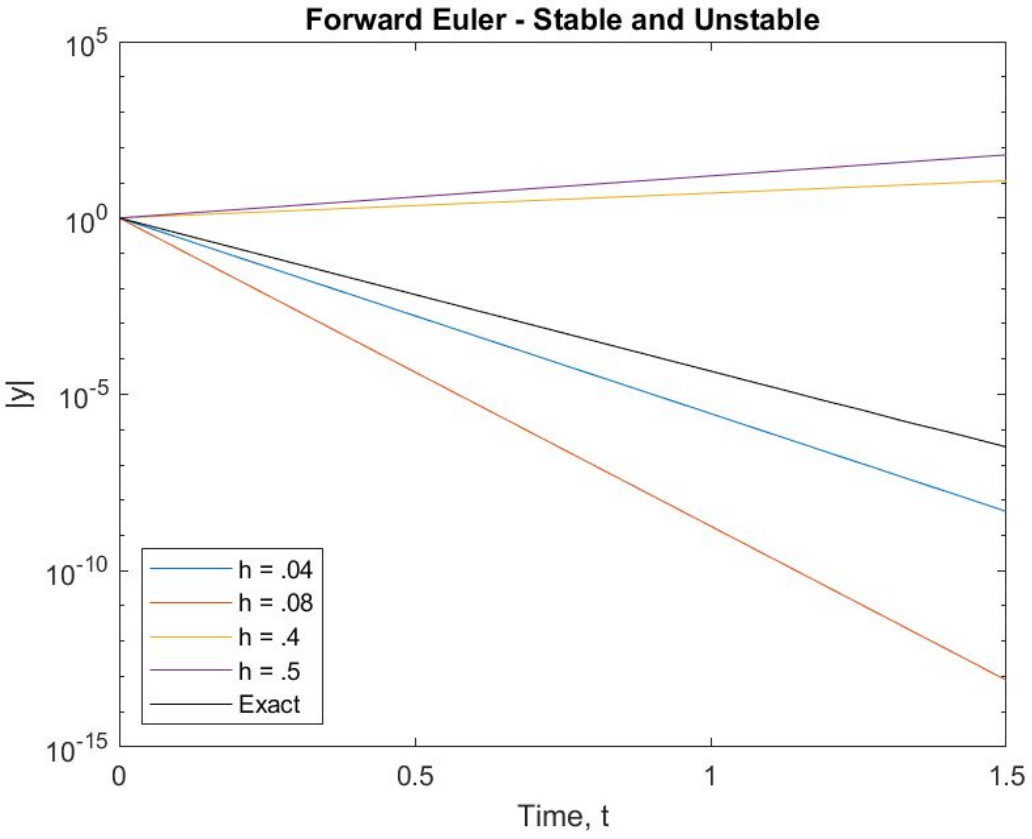
For positive root: $\boxed{h \geq 0}$

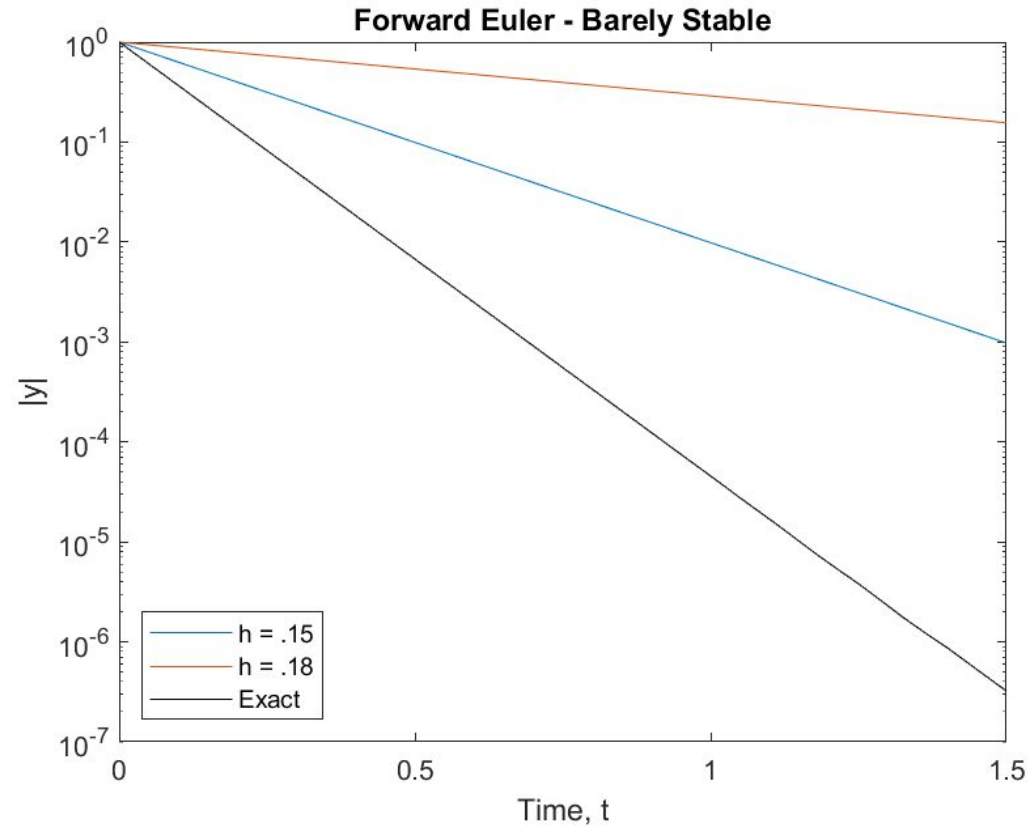
For negative root: $\boxed{h \leq .1}$

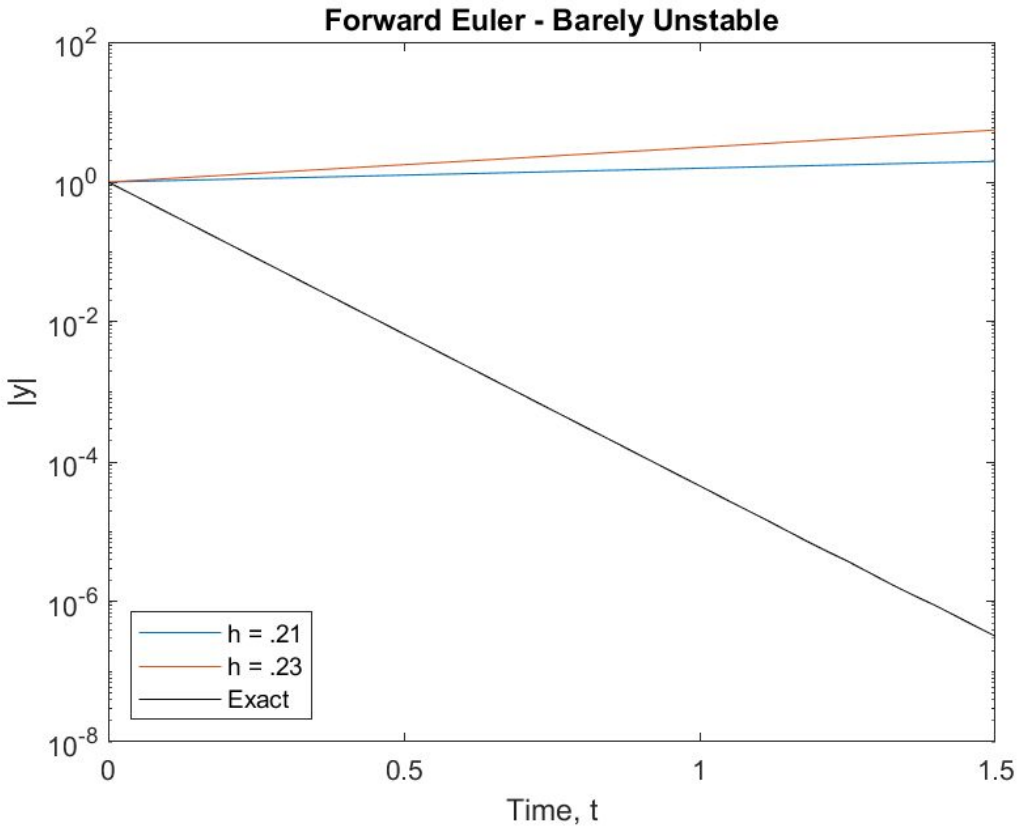
Method	Stability Range
Forward Euler	$h \leq .2$
Backward Euler	$h \geq 0$
Trapezoidal/midpoint	$h \geq 0$
RK2	$h \leq .2$
RK4	$h \leq .279$
AB2	$h \leq .1$

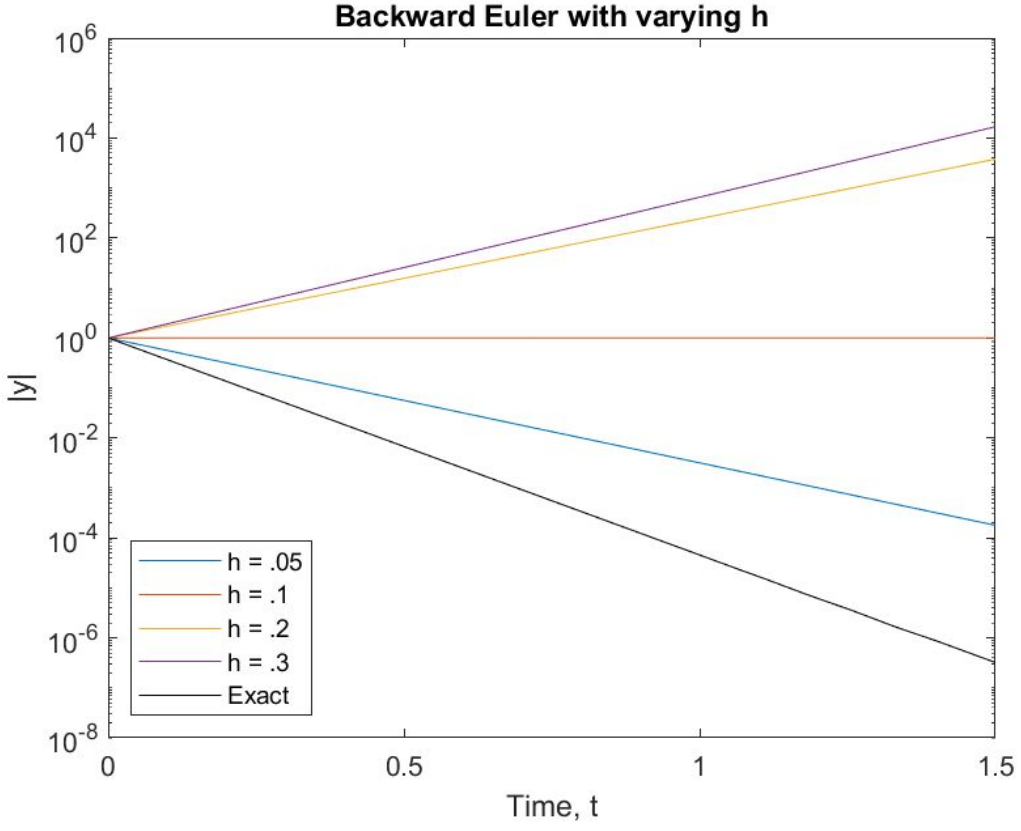
```
clc; clear all; close all;

h = linspace(0,.3,100);
z = -10.*h;

r = {
    1+z,                                            ... % fwd
    1./(1-z),                                       ... % bwd
    (1+z./2)./(1-z./2),                             ... % trap and mid
    1+z+(z.^2)./2,                                  ... % rk2
    1+z+(z.^2)./2+(z.^3)./6+(z.^4)./24,             ... % rk4
    ((1-15.*h)+sqrt((1-15.^h).^2+20.*h))./2,        ... % ab2
    ((1-15.*h)-sqrt((1-15.^h).^2+20.*h))./2         ... % ab2
};


figure(1)
for i = 1:length(r)
    x = h;
    y = abs(r{i});
    plot(x,y)
    xlim([0 0.3])
    ylim([0 1])
    hold on
end
legend('Forward','Backward','Trapezoidal and Midpoint','RK2','RK4','AB2','AB2')
title('R Value with Increasing h')
xlabel('Step size (h)')
ylabel('|R(-10h)|')
```

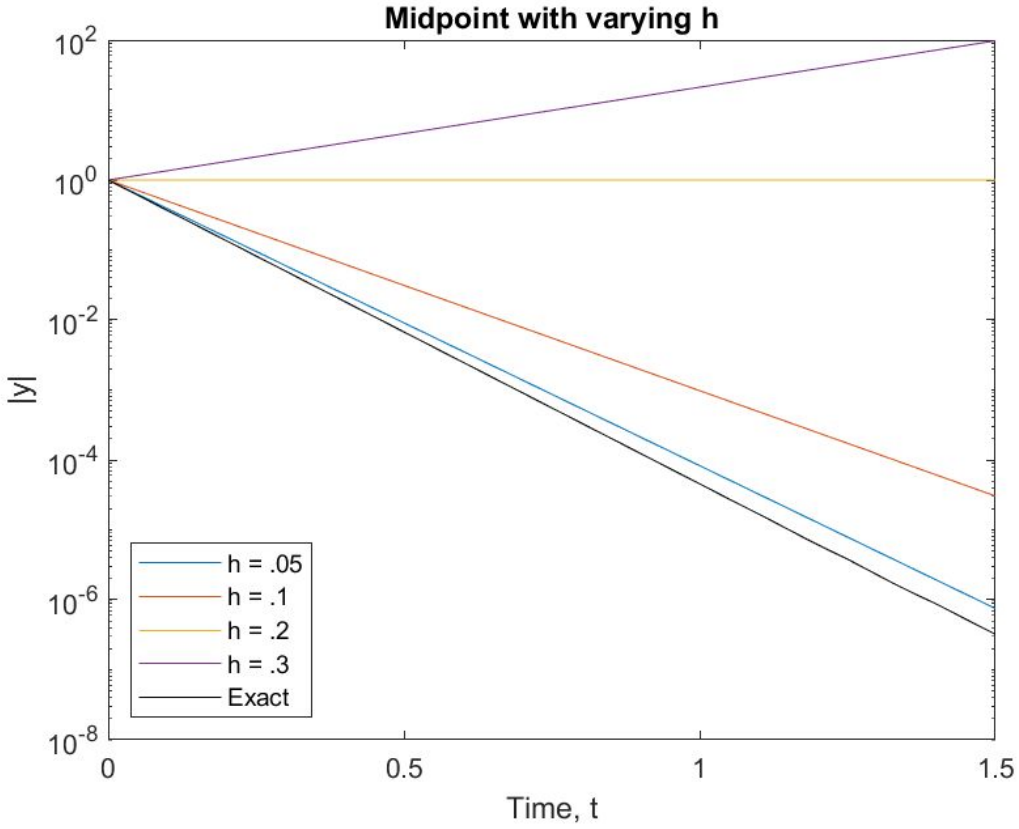


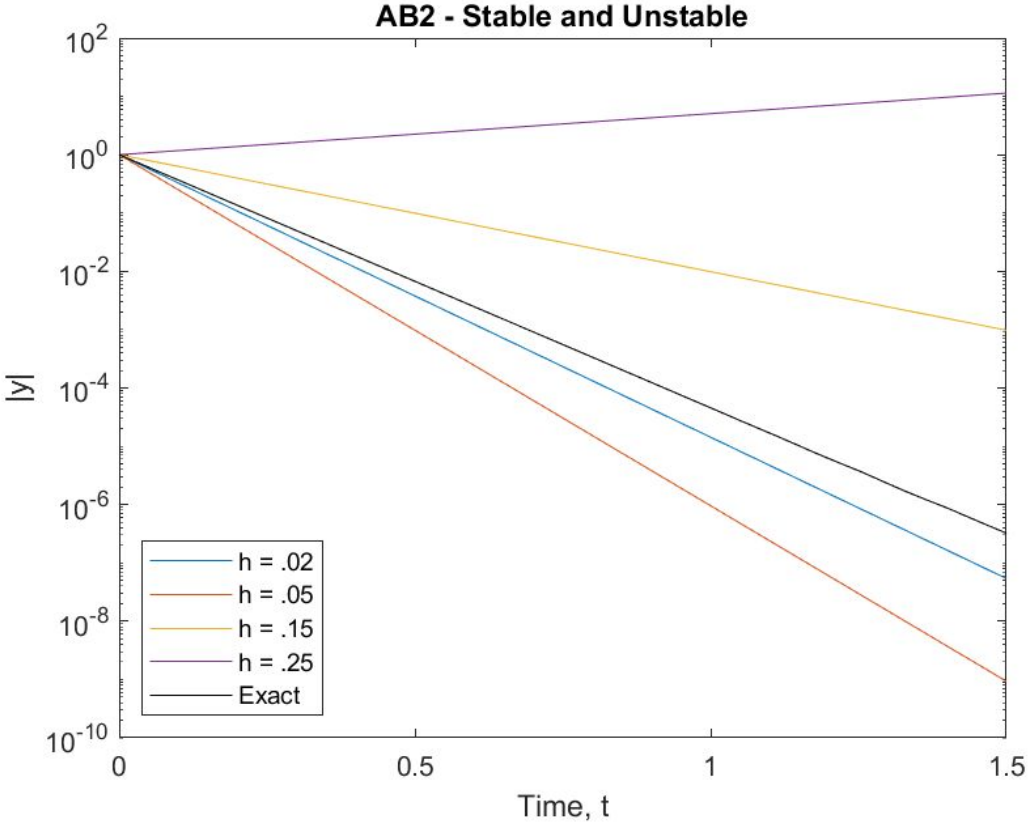

Forward Euler - Stable and Unstable
10^{5}
10^{0}
10^{-5}
10^{-10}
10^{-15}
|y|
h = .04
h = .08
h = .4
h = .5
Exact
0
0.5
1
1.5
Time, t

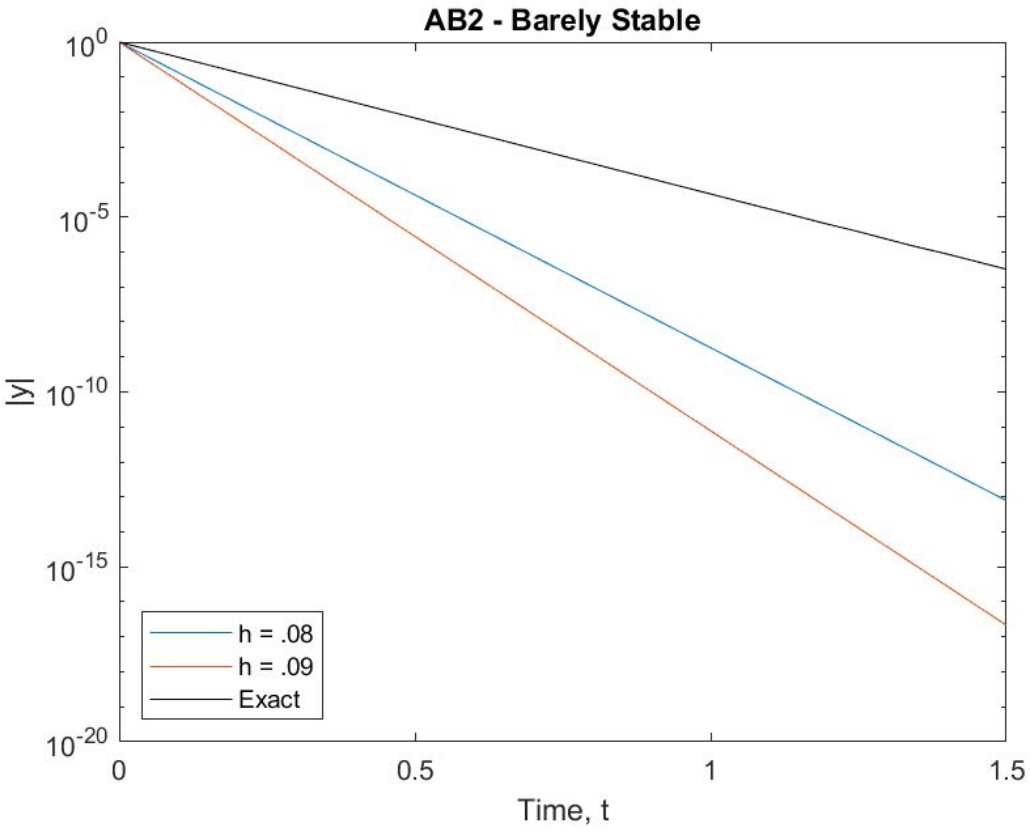

Forward Euler - Barely Stable
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
|y|
h = .15
h = .18
Exact
0
0.5
1
1.5
Time, t

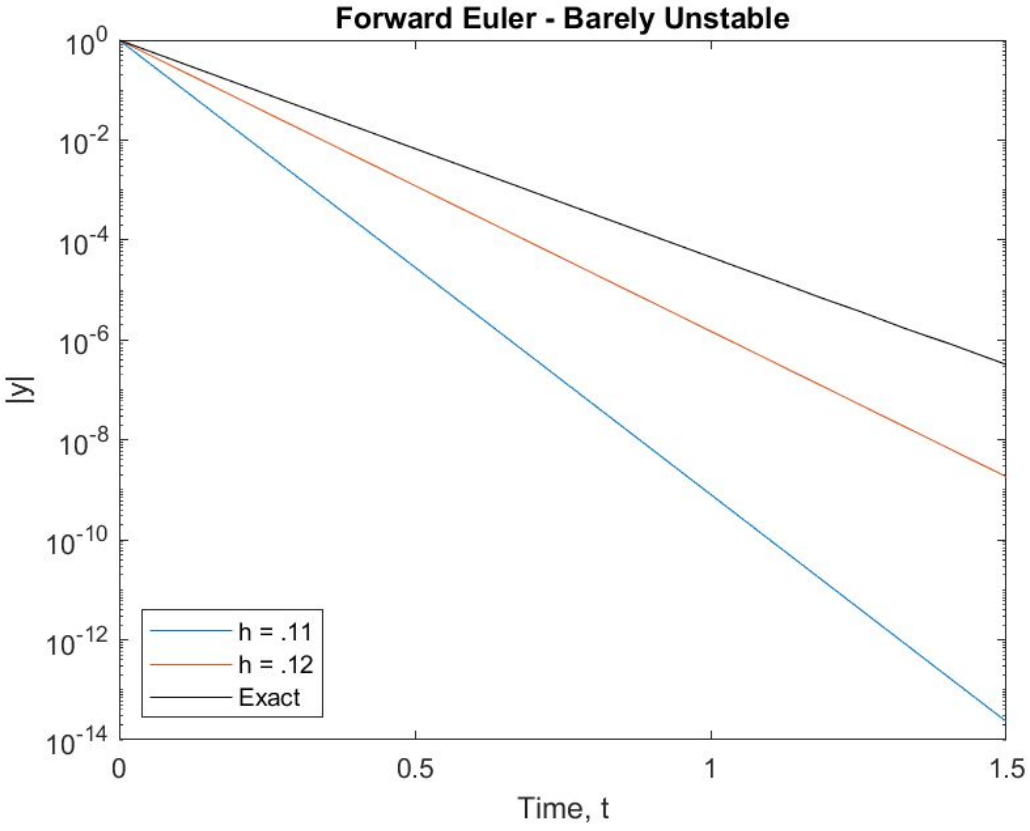

Forward Euler - Barely Unstable
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
|y|
h = .21
h = .23
Exact
0
0.5
1
1.5
Time, t

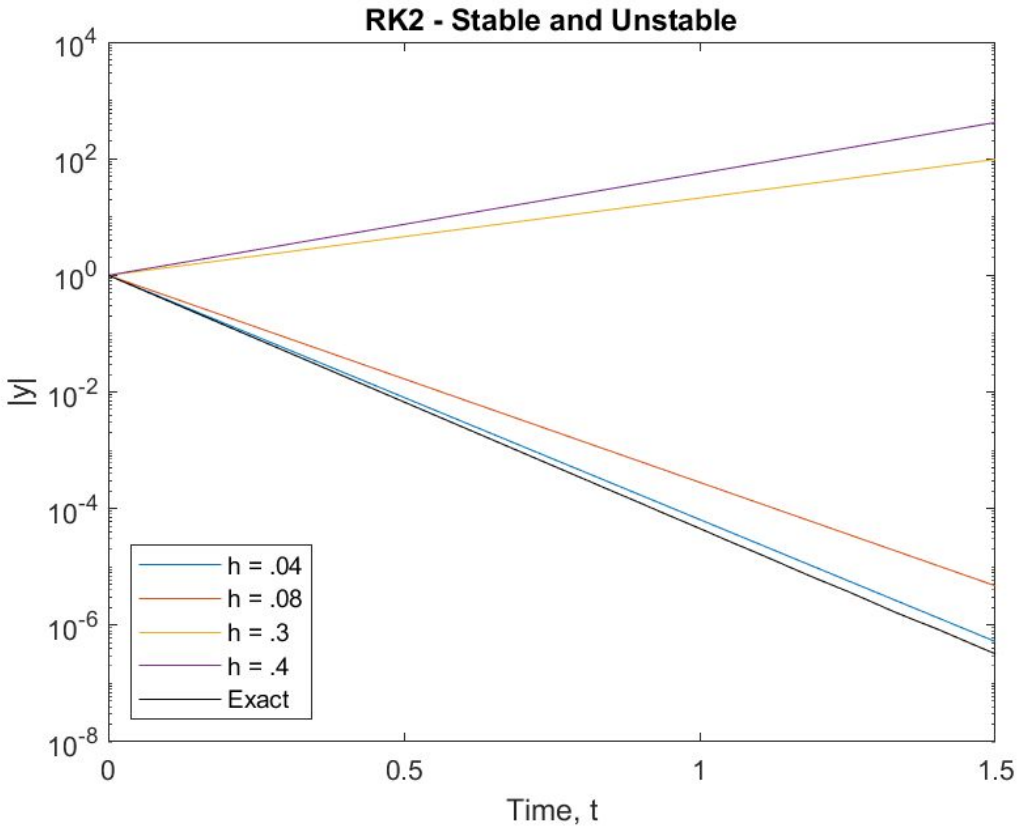

Backward Euler with varying h
10^{6}
10^{4}
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
|y|
h = .05
h = .1
h = .2
h = .3
Exact
0
0.5
1
1.5
Time, t

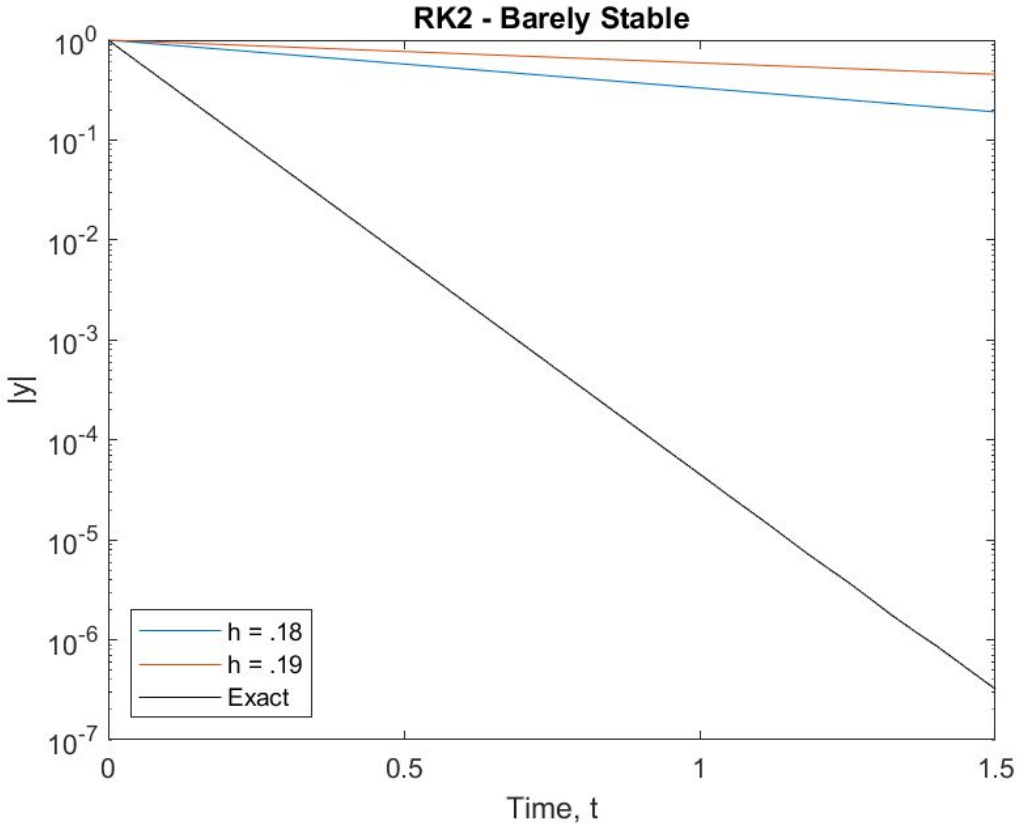

Midpoint with varying h
|y|
Time, t
h = .05
h = .1
h = .2
h = .3
Exact

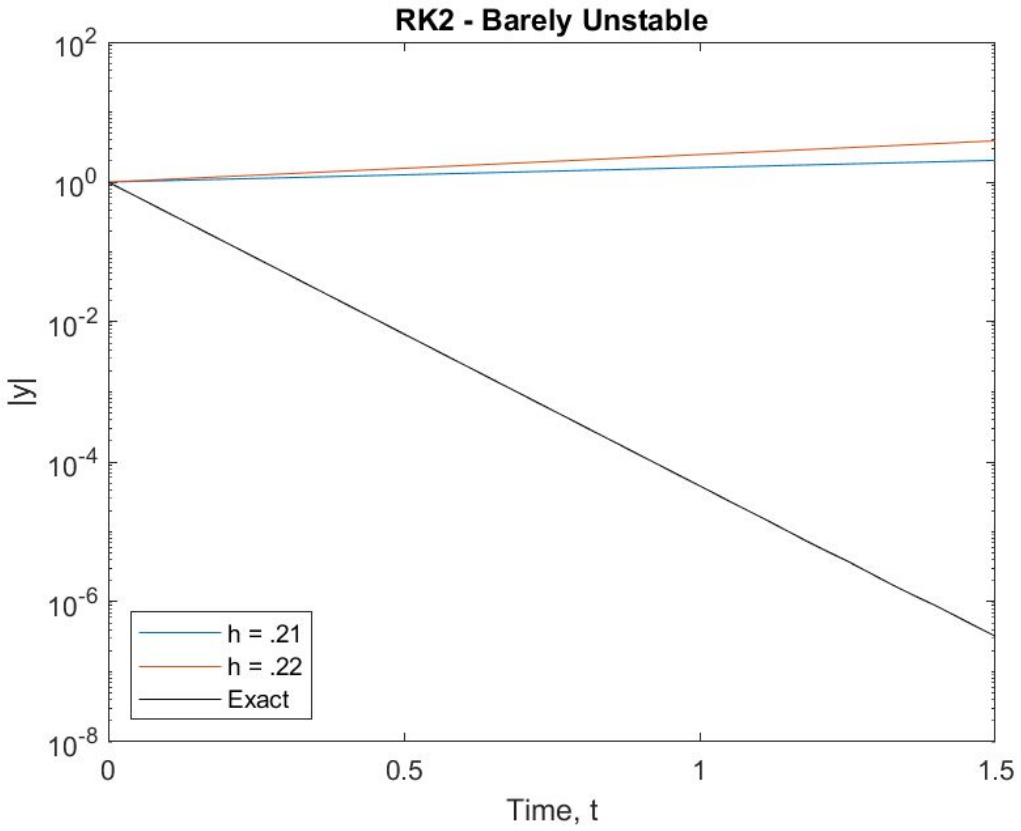

AB2 - Stable and Unstable
|y|
Time, t
h = .02
h = .05
h = .15
h = .25
Exact

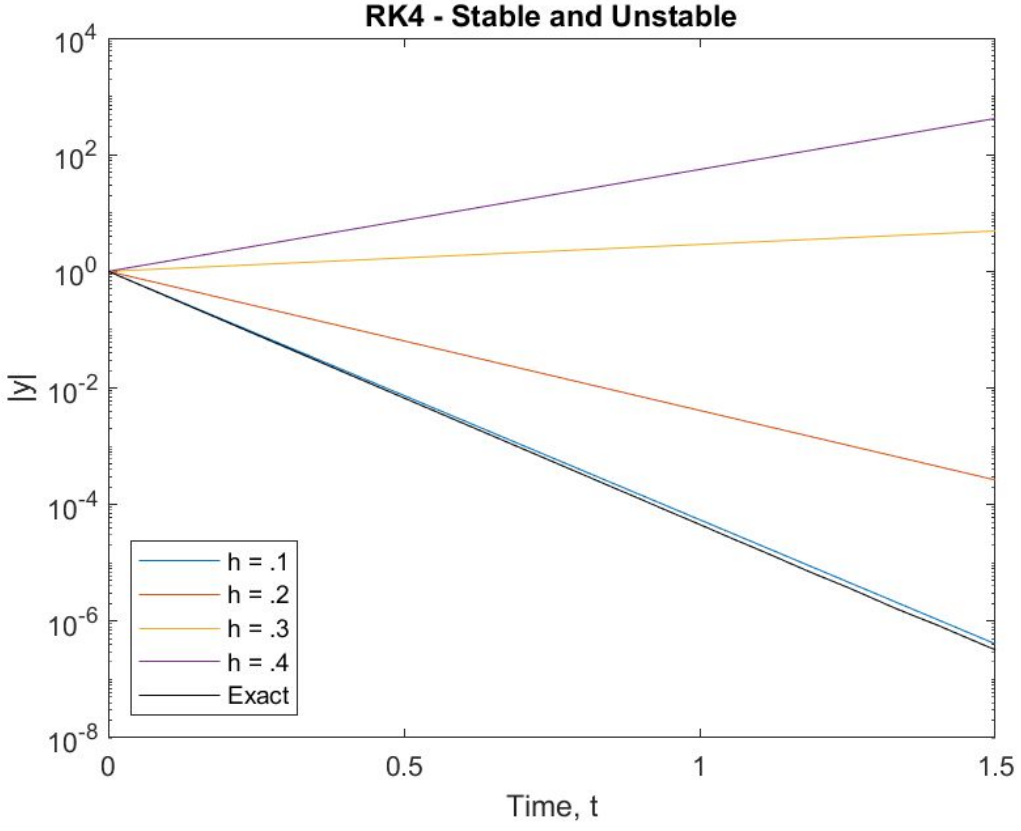

AB2 - Barely Stable
|y|
Time, t
h = .08
h = .09
Exact

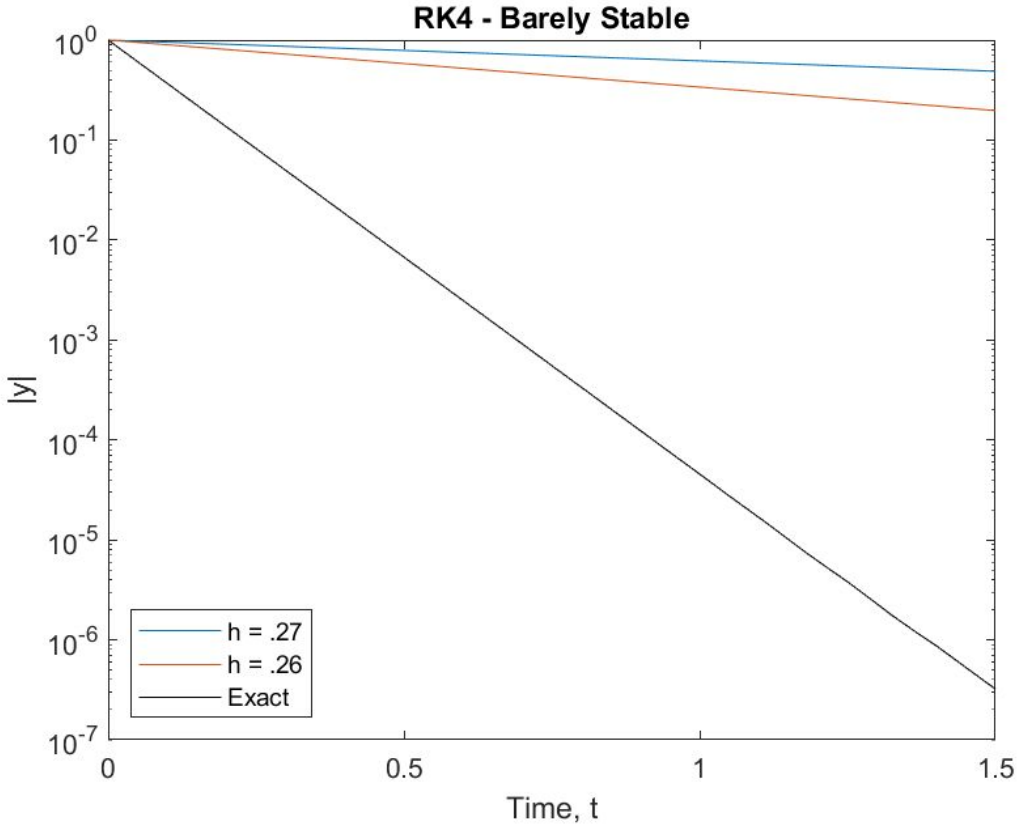

Forward Euler - Barely Unstable
|y|
Time, t
h = .11
h = .12
Exact

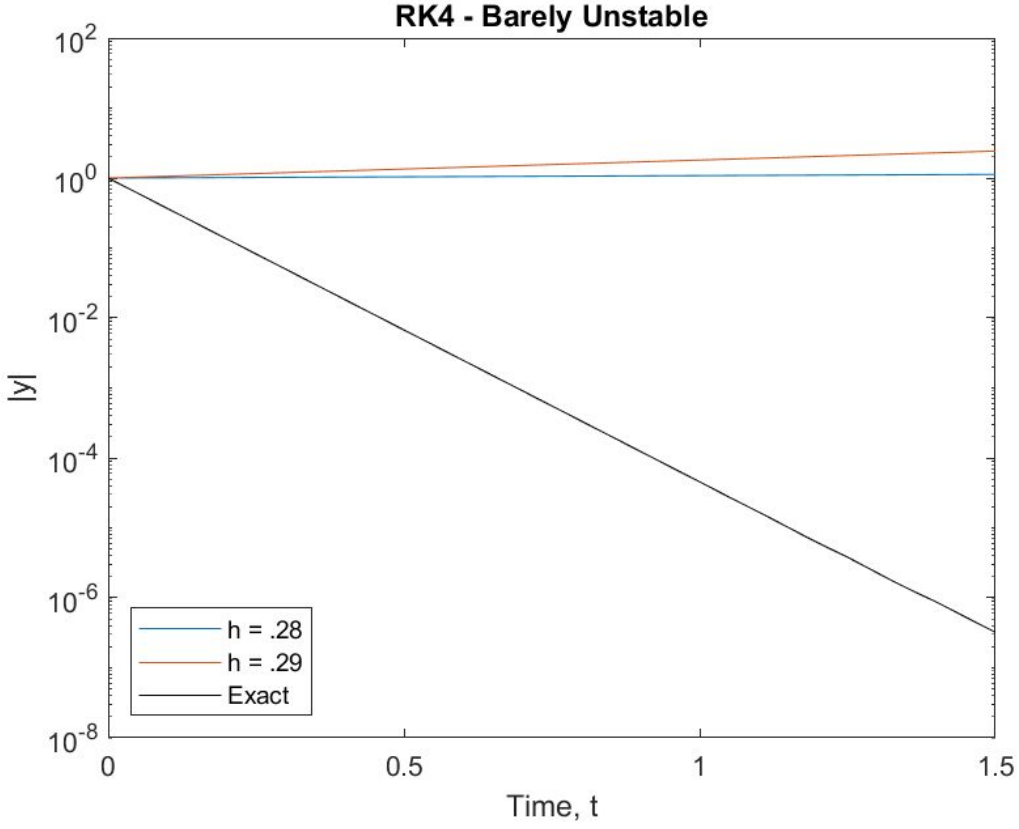

RK2 - Stable and Unstable
|y|
Time, t
h = .04
h = .08
h = .3
h = .4
Exact


RK2 - Barely Stable
|y|
Time, t
h = .18
h = .19
Exact


RK2 - Barely Unstable
|y|
Time, t
h = .21
h = .22
Exact


RK4 - Stable and Unstable
|y|
Time, t
h = .1
h = .2
h = .3
h = .4
Exact


RK4 - Barely Stable
|y|
Time, t
h = .27
h = .26
Exact


RK4 - Barely Unstable
|y|
Time, t
h = .28
h = .29
Exact