

In[65]:= **Diff** = {{2 / (τ₁ * x₁), 0, 0}, {0, 2 / (τ₂ * x₂), 0}, {0, 0, 2 / (τ₃ * x₃)}}

Out[65]= $\left\{ \left\{ \frac{2}{x_1 \tau_1}, 0, 0 \right\}, \left\{ 0, \frac{2}{x_2 \tau_2}, 0 \right\}, \left\{ 0, 0, \frac{2}{x_3 \tau_3} \right\} \right\}$

In[66]:= **MatrixForm**[Diff]

Out[66]//MatrixForm=

$$\begin{pmatrix} \frac{2}{x_1 \tau_1} & 0 & 0 \\ 0 & \frac{2}{x_2 \tau_2} & 0 \\ 0 & 0 & \frac{2}{x_3 \tau_3} \end{pmatrix}$$

In[67]:= **M** = {{1 / τ₁, 0, 0}, {-1 / τ₂, 1 / τ₂, 0}, {-1 / τ₃, 0, 1 / τ₃}}

Out[67]= $\left\{ \left\{ \frac{1}{\tau_1}, 0, 0 \right\}, \left\{ -\frac{1}{\tau_2}, \frac{1}{\tau_2}, 0 \right\}, \left\{ -\frac{1}{\tau_3}, 0, \frac{1}{\tau_3} \right\} \right\}$

In[68]:= **MatrixForm**[M]

Out[68]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\tau_1} & 0 & 0 \\ -\frac{1}{\tau_2} & \frac{1}{\tau_2} & 0 \\ -\frac{1}{\tau_3} & 0 & \frac{1}{\tau_3} \end{pmatrix}$$

In[69]:= **n** = {{η₁₁, η₁₂, η₁₃}, {η₁₂, η₂₂, η₂₃}, {η₁₃, η₂₃, η₃₃}}

Out[69]= {{η₁₁, η₁₂, η₁₃}, {η₁₂, η₂₂, η₂₃}, {η₁₃, η₂₃, η₃₃}}

In[70]:= **MatrixForm**[n]

Out[70]//MatrixForm=

$$\begin{pmatrix} \eta_{11} & \eta_{12} & \eta_{13} \\ \eta_{12} & \eta_{22} & \eta_{23} \\ \eta_{13} & \eta_{23} & \eta_{33} \end{pmatrix}$$

In[71]:= **M.n + Transpose**[M.n]

Out[71]= $\left\{ \left\{ \frac{2 \eta_{11}}{\tau_1}, \frac{\eta_{12}}{\tau_1} - \frac{\eta_{11}}{\tau_2} + \frac{\eta_{12}}{\tau_2}, \frac{\eta_{13}}{\tau_1} - \frac{\eta_{11}}{\tau_3} + \frac{\eta_{13}}{\tau_3} \right\}, \right.$
 $\left. \left\{ \frac{\eta_{12}}{\tau_1} - \frac{\eta_{11}}{\tau_2} + \frac{\eta_{12}}{\tau_2}, -\frac{2 \eta_{12}}{\tau_2} + \frac{2 \eta_{22}}{\tau_2}, -\frac{\eta_{13}}{\tau_2} + \frac{\eta_{23}}{\tau_2} - \frac{\eta_{12}}{\tau_3} + \frac{\eta_{23}}{\tau_3} \right\}, \right.$
 $\left. \left\{ \frac{\eta_{13}}{\tau_1} - \frac{\eta_{11}}{\tau_3} + \frac{\eta_{13}}{\tau_3}, -\frac{\eta_{13}}{\tau_2} + \frac{\eta_{23}}{\tau_2} - \frac{\eta_{12}}{\tau_3} + \frac{\eta_{23}}{\tau_3}, -\frac{2 \eta_{13}}{\tau_3} + \frac{2 \eta_{33}}{\tau_3} \right\} \right\}$

In[72]:= **M.n + Transpose[M.n] == Diff**

$$\text{Out[72]} = \left\{ \left\{ \frac{2 \eta_{11}}{\tau_1}, \frac{\eta_{12}}{\tau_1} - \frac{\eta_{11}}{\tau_2} + \frac{\eta_{12}}{\tau_2}, \frac{\eta_{13}}{\tau_1} - \frac{\eta_{11}}{\tau_3} + \frac{\eta_{13}}{\tau_3} \right\}, \right. \\ \left\{ \frac{\eta_{12}}{\tau_1} - \frac{\eta_{11}}{\tau_2} + \frac{\eta_{12}}{\tau_2}, -\frac{2 \eta_{12}}{\tau_2} + \frac{2 \eta_{22}}{\tau_2}, -\frac{\eta_{13}}{\tau_2} + \frac{\eta_{23}}{\tau_2} - \frac{\eta_{12}}{\tau_3} + \frac{\eta_{23}}{\tau_3} \right\}, \\ \left. \left\{ \frac{\eta_{13}}{\tau_1} - \frac{\eta_{11}}{\tau_3} + \frac{\eta_{13}}{\tau_3}, -\frac{\eta_{13}}{\tau_2} + \frac{\eta_{23}}{\tau_2} - \frac{\eta_{12}}{\tau_3} + \frac{\eta_{23}}{\tau_3}, -\frac{2 \eta_{13}}{\tau_3} + \frac{2 \eta_{33}}{\tau_3} \right\} \right\} == \\ \left\{ \left\{ \frac{2}{x_1 \tau_1}, 0, 0 \right\}, \left\{ 0, \frac{2}{x_2 \tau_2}, 0 \right\}, \left\{ 0, 0, \frac{2}{x_3 \tau_3} \right\} \right\}$$

In[73]:= **Solve[%72, {x₁, x₂, x₃, η₁₁, η₁₂, η₁₃, η₂₂, η₂₃, η₃₃, τ₁, τ₂, τ₃}]**

Solve: Equations may not give solutions for all "solve" variables.

$$\text{Out[73]} = \left\{ \left\{ x_1 \rightarrow \frac{1}{\eta_{11}}, \eta_{22} \rightarrow \frac{1 + x_2 \eta_{12}}{x_2}, \eta_{23} \rightarrow \frac{2 \eta_{11} - \eta_{12} - \eta_{13}}{-2 + \frac{\eta_{11}}{\eta_{12}} + \frac{\eta_{11}}{\eta_{13}}}, \eta_{33} \rightarrow \frac{1 + x_3 \eta_{13}}{x_3}, \right. \right. \\ \left. \tau_2 \rightarrow \frac{\eta_{11} \tau_1 - \eta_{12} \tau_1}{\eta_{12}}, \tau_3 \rightarrow \frac{\eta_{11} \tau_1 - \eta_{13} \tau_1}{\eta_{13}} \right\}, \left\{ x_1 \rightarrow \frac{1}{\eta_{11}}, \eta_{12} \rightarrow \eta_{13}, \eta_{22} \rightarrow \frac{1 + x_2 \eta_{13}}{x_2}, \right. \\ \left. \eta_{23} \rightarrow \eta_{13}, \eta_{33} \rightarrow \frac{1 + x_3 \eta_{13}}{x_3}, \tau_2 \rightarrow \frac{\eta_{11} \tau_1 - \eta_{13} \tau_1}{\eta_{13}}, \tau_3 \rightarrow \frac{\eta_{11} \tau_1 - \eta_{13} \tau_1}{\eta_{13}} \right\} \}$$

$$\text{In[81]} := \text{Solve} \left[x_1 == \frac{1}{\eta_{11}} \&\& \eta_{22} == \frac{1 + x_2 \eta_{12}}{x_2} \&\& \eta_{23} == \frac{2 \eta_{11} - \eta_{12} - \eta_{13}}{-2 + \frac{\eta_{11}}{\eta_{12}} + \frac{\eta_{11}}{\eta_{13}}} \&\& \eta_{33} == \frac{1 + x_3 \eta_{13}}{x_3} \&\& \right. \\ \left. \eta_{33} == \frac{1 + x_3 \eta_{13}}{x_3} \&\& \tau_2 == \frac{\eta_{11} \tau_1 - \eta_{12} \tau_1}{\eta_{12}} \&\& \tau_3 == \frac{\eta_{11} \tau_1 - \eta_{13} \tau_1}{\eta_{13}}, \{ \eta_{11}, \eta_{12}, \eta_{13}, \eta_{22}, \eta_{23}, \eta_{33} \} \right]$$

$$\text{In[87]} := \left\{ \left\{ \eta_{11} \rightarrow \frac{1}{x_1}, \eta_{12} \rightarrow \frac{\tau_1}{x_1 (\tau_1 + \tau_2)}, \eta_{13} \rightarrow \frac{\tau_1}{x_1 (\tau_1 + \tau_3)}, \eta_{22} \rightarrow \frac{x_1 \tau_1 + x_2 \tau_1 + x_1 \tau_2}{x_1 x_2 (\tau_1 + \tau_2)}, \right. \right. \\ \left. \eta_{23} \rightarrow \frac{\tau_1 (\tau_1 \tau_2 + \tau_1 \tau_3 + 2 \tau_2 \tau_3)}{x_1 (\tau_2 + \tau_3) (\tau_1^2 + \tau_1 \tau_2 + \tau_1 \tau_3 + \tau_2 \tau_3)}, \eta_{33} \rightarrow \frac{x_1 \tau_1 + x_3 \tau_1 + x_1 \tau_3}{x_1 x_3 (\tau_1 + \tau_3)} \right\} \}$$

In[89]:= **Simplify[%87]**

$$\text{Out[89]} = \left\{ \left\{ \eta_{11} \rightarrow \frac{1}{x_1}, \eta_{12} \rightarrow \frac{\tau_1}{x_1 (\tau_1 + \tau_2)}, \eta_{13} \rightarrow \frac{\tau_1}{x_1 (\tau_1 + \tau_3)}, \right. \right. \\ \left. \eta_{22} \rightarrow \frac{1}{x_2} + \frac{\tau_1}{x_1 (\tau_1 + \tau_2)}, \eta_{23} \rightarrow \frac{\tau_1 (2 \tau_2 \tau_3 + \tau_1 (\tau_2 + \tau_3))}{x_1 (\tau_1 + \tau_2) (\tau_1 + \tau_3) (\tau_2 + \tau_3)}, \eta_{33} \rightarrow \frac{1}{x_3} + \frac{\tau_1}{x_1 (\tau_1 + \tau_3)} \right\} \}$$