PY2010 Intermediate Logic
UNIVERSITY OF ST. ANDREWS
Exercises wk. 01

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1 Well Formed Formulae

1. $\neg(p \wedge \neg q) \supset (p \supset \neg q)$

   This is a wff (well formed formula). This is because p, and q are atomic formulas and thus are wffs. Which means the negation of q ($\neg q$) is still a wff. Connecting the wff to $(p \wedge \neg q)$ and $(p \supset \neg q)$ then are also wffs, which means the negation $\neg(p \wedge \neg q)$ is also a wff and so is the final connective of two wffs to form $\neg(p \wedge \neg q) \supset (p \supset \neg q)$.

2. $p \wedge \neg p \supset p$

   This is not a wff because, despite p, and therefore $\neg q$ being wffs, the rule for connecting wffs require there to be a single connective joining two wffs per set of brackets. As nothing else is a wff, this use of connectives cannot form a wff.

3. $A \supset (B \supset A)$

   We do not know whether this is a wff, as we don't know if A and B are wffs. Assuming they are, then it would be a wff, as joining two wffs with $\supset$ forms another wff. However, if either A or B is not a wff, then the entire thing cannot be a wff.

2 Semantic Validity

1. $p \wedge \neg p \vDash_c q$

   Show: For all interpretations of v, if $v(p \wedge \neg p) = 1$, then $v(q) = 1$.

   Assuming $v(p \wedge \neg p) = 1$, $v(p) = 1)$ and $v(\neg p) = 1$.

   $v(\neg p) = 1$ means that $v(p) = 0$ from the negation rule.

   As we already have $v(p) = 1)$, the value of $v(p)$ cannot be both, since v is a function.

   So, there's no v which satisfies $v(p \wedge \neg p) = 1$, without satisfying $v(p \wedge \neg p) = 1$".

   QED.

2. $p \vee q \vDash_c (p \supset q) \vee (\neg p \supset \neg q)$

   Show: For all interpretations of v, if $v(p \vee q) = 1$, then $v((p \supset q) \vee (\neg p \supset \neg q)) = 1$

   Assuming the premise $v(p \vee q) = 1$, by the rule $v(A \vee B) = 1$ iff $v(A) = 1$ or $v(B) = 1$, either $v(p) = 1$ or $v(q) = 1$.

   For $v((p \supset q) \vee (\neg p \supset \neg q)) = 1$, either $v(p \supset q) = 1$, or $v(\neg p \supset \neg q) = 1$.

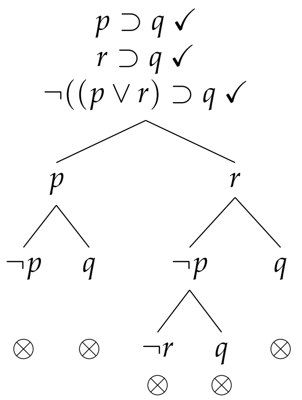
   For $v(\neg p \supset \neg q) = 1$, then either $v(\neg p) = 0$ or $v(\neg q) = 1$, and $v(p) = 1$, then necessarily $v(\neg p) = 0$ and so $v(\neg p \supset \neg q) = 1$, and so the conclusion $v((p \supset q) \vee (\neg p \supset \neg q)) = 1$.

   For $v(p \supset q) = 1$, then either $v(p) = 0$ or $v(q) = 1$, which is satisfied by the other premise case and so $v(p \supset q) = 1$, and so the conclusion $v((p \supset q) \vee (\neg p \supset \neg q)) = 1$.

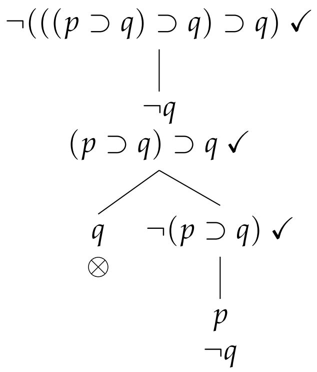
   QED

3 Tableau

1. $p \supset q, r \supset q \vdash_{tc} (p \vee r) \supset q$



2. $\vdash_{tc} \neg(((p \supset q) \supset q) \supset q$



   This is invalid. As there are no premises, the antecedent is always satisfied. However, the conclusion can never be satisfied. Starting with the atomic values of $v(p) = 1$ and $v(q) = 0$, which can be derived from $v(\neg q) = 1$ using the $\neg$ rule. $v(p \supset q) = 1$ iff $v(p) = 0$ or $v(q) = 1$, which is false, and so $v(p \supset q) = 0$, which means that $v((p \supset q) \supset q) = 1$ because of the same $\supset$ rule as the antecedent is false. That means that $v(((p \supset q) \supset q) \supset q) = 0$ for the same rule again, as the antecedent is true, as well as already establishing that $v(q) = 0$."

3. $\neg(p \supset q) \wedge \neg(p \supset r)$ ✓

$\neg(\neg q \vee \neg r)$ ✓

q

r

$\neg(p \supset q)$ ✓

$\neg(p \supset r)$ ✓

$\neg q \quad p$

$\otimes \quad p \quad \neg r$

$\otimes$

This is an invalid argument. Starting with the values $v(p) = 1$, $v(q) = 1$, and $v(r) = 1$, we can use the rule that $v(p \supset q) = 1$ iff $v(p) = 0$ or $v(q) = 1$ to determine that $v(p \supset q) = 0$. We can do the same with $v(p \supset r) = 1$ iff $v(p) = 0$ or $v(r) = 1$ to get $v(p \supset r) = 0$. Then, using the rule $v(\neg A) = 1$ iff $v(A) = 0$, we can get $v(\neg(p \supset q)) = 1$ and $v(\neg(p \supset r)) = 1$ and finally, as $v(A \wedge B = 1$ iff $v(A) = 1$ and $v(B) = 1$, we can determine $v(\neg(p \supset q) \wedge \neg(p \supset r)) = 1$ from the previous values thus satisfying the premises.

We can see that because $v(q) = 1$, and $v(r) = 1$, the negation rule must mean that $v(\neg q) = 0$, and $v(\neg r) = 0$. This means that $v(\neg q \vee \neg r) = 0$ also because $v(A \vee B) = 1$ iff $v(A) = 1$ or $v(B) = 1$.

Therefore, there is an counter-model for how the premises are satisfied, whilst the conclusion is not and so it is invalid.