

PY2010 Intermediate Logic  
UNIVERSITY OF ST ANDREWS  
*Exercises Week 02*

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I hereby declare that the attached piece of written work is my own work and that I have not reproduced, without acknowledgement, the work of another.

**1**  $p \supset q, p \supset r \vdash_{NC} p \supset (q \wedge r)$

1	(1)	$p \supset q$	Ass (Premise)
2	(2)	$q \supset r$	Ass (Premise)
3	(3)	$p$	Ass ( $\supset E$ )
1,3	(4)	$q$	1,3, $\supset E$
2,3	(5)	$r$	2,3, $\supset E$
1,2,3	(6)	$q \wedge r$	4,5, $\wedge I$
1,2	(7)	$p \supset (q \wedge r)$	3,6, $\supset I$

**2**  $p \wedge q, q \supset r \vdash_{NC} p \wedge r$

1	(1)	$p \wedge q$	Ass (Premise)
2	(2)	$q \supset r$	Ass (Premise)
1	(3)	$p$	1, $\wedge E$
1	(4)	$q$	1, $\wedge E$
1,2	(5)	$r$	2,4, $\supset E$
1,2	(6)	$p \wedge r$	3,5, $\wedge I$

**3**  $\vdash_{NC} (p \supset q) \supset ((q \supset r) \supset (p \supset r))$

1	(1)	$p$	Ass ( $\supset I$ )
2	(2)	$r$	Ass ( $\supset I$ )
2	(3)	$p \supset r$	1,2, $\supset I$
4	(4)	$q$	Ass ( $\supset I$ )
2	(5)	$q \supset r$	2,4, $\supset I$
	(6)	$((q \supset r) \supset (p \supset r))$	3,5, $\supset I$
4	(7)	$p \supset q$	1,4, $\supset I$
	(8)	$(p \supset q) \supset ((q \supset r) \supset (p \supset r))$	7,8, $\supset I$

**4**  $p \wedge q \vdash_{NC} q \wedge p$

- |   |     |              |                  |
|---|-----|--------------|------------------|
| 1 | (1) | $p \wedge q$ | Ass (Premise)    |
| 1 | (2) | $p$          | 1, $\wedge E$    |
| 1 | (3) | $q$          | 1, $\wedge E$    |
| 1 | (4) | $q \wedge p$ | 2, 3, $\wedge I$ |

**5**  $p \supset (p \supset q) \vdash_{NC} p \supset q$

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|-----|-----|---------------------------|---------------------|
| 1   | (1) | $p \supset (p \supset q)$ | Ass (Premise)       |
| 2   | (2) | $p$                       | Ass ( $\supset E$ ) |
| 1,2 | (3) | $p \supset q$             | 1, 2, $\supset E$   |
| 1,2 | (4) | $q$                       | 2, 3, $\supset E$   |
| 1   | (5) | $p \supset q$             | 2, 4, $\supset I$   |