PY2010 Intermediate Logic
UNIVERSITY OF ST. ANDREWS
Exercises Week 04

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I hereby declare that the attached piece of written work is my own work and that I have not reproduced, without acknowledgement, the work of another.

1. Show the following in semantics for the relevant logic

   (a) $\Box A \models_{K\rho} \Diamond A$
   Show: For all $\langle W, R, v\rangle$ w/ R reflexive, if $v_w(\Box A) = 1$ then $v_w(\Diamond A) = 1$.
   Assume for an arbitrary $\langle w, R, v\rangle$ with reflexive R, $v_w(\Box A) = 1$.
   $v_w(\Box A) = 1$ iff at ALL x such that wRx, $v_x(A) = 1$ [$\Box$ semantics], so $v_w(A) = 1$ [ρ rule]
   Therefore, by definition, $v_w(\Diamond A) = 1$ [$\Diamond$ semantics]. QED

   (b) $\Diamond\Diamond A \models_{K\rho\sigma\tau} \Diamond A$
   Show: For all $\langle W, R, v\rangle$ w/ R reflective, symmetrical, and transitive, if $v_w(\Diamond\Diamond A) = 1$, then $v_w(\Diamond A) = 1$.
   Assume for an arbitrary $\langle w, R, v\rangle$ with R reflective, symmetrical, and transitive, $v_w(\Diamond\Diamond A) = 1$.
   So, there's an x such that wRx and $v_x(\Diamond A) = 1$ [$\Diamond$ rule]. Also, we can deduce xRw and thus $v_w(\Diamond A) = 1$ [σ rule]. QED

2. Show the following in $K\rho$ using tableaux

   (a) $\vdash (\Diamond\neg p \vee \Diamond\neg q) \vee \Diamond(p \vee q)$

$$\neg((\Diamond\neg p \vee \Diamond\neg q) \vee \Diamond(p \vee q)), 0 \checkmark$$

$$|$$

$$\neg(\Diamond\neg p \vee \Diamond\neg q), 0 \checkmark$$
$$\neg\Diamond(p \vee q), 0 \checkmark$$

$$|$$

$$\Box\neg(p \vee q), 0 \ /0$$

$$|$$

$$\neg\Diamond\neg p, 0 \checkmark$$
$$\neg\Diamond\neg q, 0 \checkmark$$

$$|$$

$$\Box\neg\neg p, 0 \checkmark$$
$$\Box\neg\neg q, 0 \checkmark$$

$$|$$

$$\Box p, 0 \ /0$$
$$\Box q, 0 \ /0$$

$$|$$

$$0r0$$
$$p, 0$$
$$q, 0$$

$$|$$

$$\neg(p \vee q), 0$$

$$|$$

$$\neg p, 0$$
$$\neg q, 0$$
$$\otimes$$

(b) $\vdash \Box p \supset \Diamond p$

$$\neg(\Box p \supset \Diamond p), 0 \checkmark$$

$$|$$

$$\Box p, 0 \ /0$$
$$\neg\Diamond p, 0 \checkmark$$

$$|$$

$$\Box\neg p, 0 \ /0$$

$$|$$

$$0r0$$
$$p, 0$$
$$\neg p, 0$$

3. Do the following hold in $K\rho\tau$ using tableaux?

   (a) $\vdash \Diamond\Box p \supset \Box\Diamond p$

$$\neg(\Diamond\Box p \supset \Box\Diamond p), 0 \checkmark$$

$$|$$

$$\Diamond\Box p, 0$$
$$\neg\Box\Diamond p, 0 \checkmark$$

$$|$$

$$\Diamond\neg\Diamond p, 0 \checkmark$$

$$|$$

$$\Diamond\Box\neg p, 0$$

$$|$$

$$0r1$$
$$\Box\neg p, 1$$

$$|$$

$$0r2$$
$$\Box p, 2$$

$$|$$

$$1r0$$
$$2r0$$

$$|$$

$$\Box\neg p, 0\ /0$$
$$\Box p, 0\ /0$$

$$|$$

$$0r0$$
$$\neg p$$
$$p$$
$$\otimes$$

(b) $\Box(p \supset q) \vdash \Box(\Box p \supset \Box q)$

$$\Box(p \supset q), 0 \ /2$$
$$\neg\Box(\Box p \supset \Box q), 0 \ \checkmark$$

$$\Diamond\neg(\Box p \supset \Box q), 0 \ \checkmark$$

$$0r1$$
$$\neg(\Box p \supset \Box q), 1 \ /1$$

$$\Box p, 1 \ /2$$
$$\neg\Box q, 1 \ \checkmark$$

$$\Diamond\neg q, 1 \ \checkmark$$

$$1r2$$
$$\neg q, 2$$
$$p, 2$$

$$0r2$$
$$p \supset q, 2$$

$$\begin{array}{cc} \neg p, 2 & q, 2 \\ \otimes & \otimes \end{array}$$