CS5010 - P1

Uncertainty

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Question 1

1. $p_0 = 0.032$

2. See Tables 1.1, 1.2 and 1.3 for CFPs.

Table 1.1: CFP for $P(A)$

A	$P(A)$
0	0.2
1	0.8

Table 1.2: CFP for $P(B|A)$

A	$P(B=0|A)$	$P(B=1|A)$
0	0.3	0.7
1	0.2	0.8

Table 1.3: CFP for $P(C|A,B)$

A	B	$P(C=0|A,B)$	$P(C=1|A,B)$
0	0	0.6	0.4
0	1	0.6	0.4
1	0	0.8	0.2
1	1	0.8	0.2

As A has no parents, there are no conditional dependencies on it, so the CFP is just it's marginal which is calculated using the sum rule on A values (see. Table 1.1).

B has one parent A, and so it's CFP is $P(B|A)$. This can be calculated using the conditional probabiltiy definition:

$$P(B|A) = {}^{P(A,B)}/_{P(A)}$$

We can easily calculate the joint distribution $P(A, B)$ using the sum rule (see. Table 1.4), and then divide by the values in Table 1.1 to calculate the conditional probability (see. Table 1.2).

Table 1.4: Probability distribution $P(A, B)$

A	B	$P(A, B)$
0	0	0.06
0	1	0.14
1	0	0.16
1	1	0.64

Finally, C has two conditional parents A and B and so it's CFP is $P(C|A, B)$ which is calculated using the same conditional probability definition

$$P(C|A, B) = {}^{P(A,B,C)}/_{P(A,B)}$$

We have the full joint probability from the question, and have already calculated $P(A, B)$ (see. Table 1.4), so we can easily calculate this conditional probability (see. Table 1.3).

3. a. The definition of marginal independence is the same as independence. Therefore, if B are marginally independent, then:

$$P(A, B) = P(A)P(B)$$

We already have $P(A)$ from Table 1.1, and we can easily select and normalize to get $P(A, B)$ and $P(B)$ (see. Tables 1.4 and 1.5). Finally, we can multiply the values from the individual probability distributions to get distribution $P(A)P(B)$. This can be compared, for each value of A and B, to the joint probability. If they are all equal, then the variables are marginally independent. However, as Table 1.6 shows, the values differ, so the statement is false.

Table 1.5: Probability distribution $P(B)$

B	$P(B)$
0	0.22
1	0.78

Table 1.6: Probability distributions $P(A)P(B)$ and $P(A, B)$ showing a difference, meaning A and B are not marginally independent.

A	B	$P(A)P(B)$	$P(A, B)$
0	0	$0.2 \times 0.22 = 0.044$	0.06
0	1	$0.2 \times 0.78 = 0.156$	0.14
1	0	$0.8 \times 0.22 = 0.176$	0.16
1	1	$0.8 \times 0.78 = 0.624$	0.64

b. If B,C are conditional independent given A, then the impact of B and C on eachother is fully mediated by A which would mean:

$$P(B|A, C) = P(B|A) \wedge P(C|A, B) = P(C|A)$$

We have $P(B|A)$ from Table 1.2 and $P(C|A, B)$ from Table 1.3, and we can calculate $P(C|A)$ and $P(B|A, C)$

Table 1.7: Joint probability distribution $P(A, C)$

A	C	$P(A, C)$
0	0	0.12
0	1	0.08
1	0	0.64
1	1	0.16

Table 1.8: Conditional probability distribution $P(B|A, C)$

A	C	$P(B = 0|A, C)$	$P(B = 1|A, C)$
0	0	0.3	0.7
0	1	0.3	0.7
1	0	0.2	0.8
1	1	0.2	0.8

By selecting and normalizing both conditional probabilities for the value of B, this gives:

Table 1.9: Conditional probability distribution $P(B|A, C)$, selected and normalized for B, with the probability distribution for $P(B|A)$, selected and normalized for B, showing the first conditional independence condition is met.

B	$P(B|A, C)$	$P(B|A)$
0	$0.3+0.3+0.2+0.2/4 = 0.25$	$0.3+0.2/2 = 0.25$
1	$0.7+0.7+0.8+0.8/4 = 0.75$	$0.7+0.8/2 = 0.75$

We can then use the same process for $P(C|A, B)$ (see Table 1.3), and $P(C|A)$:

Table 1.10: Conditional probability distribution $P(C|A)$

A	$P(C=0|A)$	$P(C=1|A)$
0	0.6	0.4
1	0.8	0.2

Table 1.11: Conditional probability distribution $P(C|A,B)$, selected and normalized for C, with the probability distribution for $P(C|A)$, selected and normalized for C, showing the second conditional independence condition is met.

C	$P(C|A,B)$	$P(C|A)$
0	${}^{0.6+0.6+0.8+0.8}/_{4} = 0.7$	${}^{0.6+0.8}/_{2} = 0.7$
1	${}^{0.4+0.4+0.2+0.2}/_{4} = 0.3$	${}^{0.4+0.2}/_{2} = 0.3$

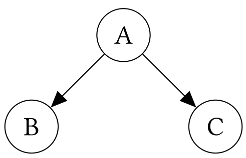
Therefore, Tables 1.9 and 1.11 show that the conditional relationship between B and C is fully mediated through A, and so the given statement is true.

c. Yes it can be simplified.

B and C are conditionally independent given A, meaning the impact of A and B together on C is fully explained by A so:

$$P(C|A,B) = P(C|A)$$

The former is the CFP for C from the original diagram, and the latter is the CFP for C when removing the relationship from B to C, and so the diagrams are equivalent in this case:



We cannot simplify any further, as A and B are not marginally independent, and so there must be a conditional relationship between them. We can check if A and

C are marginally independent using the same method as previously explained (see. Table 1.12) to find that they are not marginally independent, so there must also be a conditional relationship between A and C.

Table 1.12: Probability distribution for $P(A,C)$ and $P(A)P(C)$ showing A and C are not marginally independent.

A	C	$P(A)P(C)$	$P(A,C)$
0	0	0.152	0.12
0	1	0.048	0.08
1	0	0.608	0.64
1	1	0.192	0.16

Question 2

1. - V = has been Vaccinated against Flu-19
   - F = has Flu-19
   - S = has Flu-19 Symptoms
   - T_A = Test result of rapid Antigen test
   - T_P = Test result of PCR test

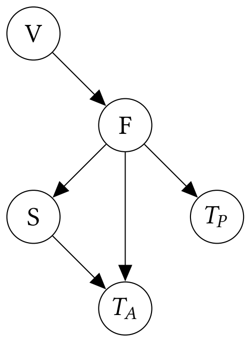


Table 2.1: CFP for $P(V)$

V	$P(V))$
0	0.1
1	0.9

Table 2.2: CFP for $P(F|V)$

V	$P(F=0|V)$	$P(F=1|V)$
0	0.78	0.22
1	0.95	0.05

Table 2.3: CFP for $P(S|F)$

F	$P(S=0|F)$	$P(S=1|F)$
0	1	0
1	$1/3$	$2/3$

Table 2.4: CFP for $P(T_P|F)$

F	$P(T_P=0|F)$	$P(T_P=1|F)$
0	0.992	0.008
1	0.018	0.982

Table 2.5: CFP for $P(T_A|S,F)$

S F	$P(T_A=0|S,F)$	$P(T_A=1|S,F)$	
0	0	0.9	0.1
0	1	0.42	0.58
1	0	0.92	0.08
1	1	0.2	0.8

2. There are chain patterns

$$V \to F \to S \to T_A$$

$$V \to F \to T_A$$

$$V \rightarrow F \rightarrow T_P$$

The fork patterns are

$$S \leftarrow F \rightarrow T_A$$

$$S \leftarrow F \rightarrow T_P$$

$$T_A \leftarrow F \rightarrow T_P$$

TODO: DISCUSS CONDITIONAL RELATIONSHIPS

Question 3

Card prediction

1. - C = card picked
   - F_1 = first face color
   - F_2 = second face color

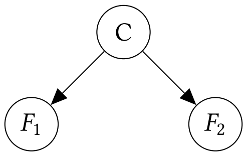


Table 3.1: CFP for $P(C)$

C	$P(C))$
1	$1/3$
2	$1/3$
3	$1/3$

Table 3.2: CFP for $P(F_1|C)$

C	$P(F_1 = w|C))$	$P(F_1 = b|C))$
1	0.5	0.5
2	0	1
3	1	0

Table 3.3: CFP for $P(F_2|C)$

C	$P(F_2 = w|C))$	$P(F_2 = b|C))$
1	0.5	0.5
2	0	1
3	1	0

2. Query: $P(F_2|F_1)$. Nuisance: $\{C\}$

$P(F_2|F_1 = b)$

$$\begin{aligned}
&= \alpha P(F_2, F_1 = b) && : \text{conditional probability}\\
&= \alpha \sum_{c'} P(C = c', F_2, F_1 = b) && : \text{sum rule}\\
&= \alpha \sum_{c'} P(C = c')P(F_1 = b|C = c')P(F_2|C = c') && : \text{factor property}\\
\text{for} F_2 = w &= \alpha \times ((1/3 \times 0.5 \times 0.5) + (1/3 \times 1 \times 0) + (1/3 \times 0 \times 1))\\
&= \alpha \times 1/12\\
\text{for} F_2 = b &= \alpha \times ((1/3 \times 0.5 \times 0.5) + (1/3 \times 1 \times 1) + (1/3 \times 0 \times 0))\\
&= \alpha \times 5/12\\
\alpha &= 1/_{1/12+5/12} = 2
\end{aligned}$$

Therefore: $P(F_2 = w|F_1 = b) = 2 \times 1/12 = 1/6$

Question 4

A mysterious urn

1. - 10 coins
   - 2 mints: A bias = 0.7, B bias = 0.3
   - method: (randomly pick a coin, (toss x 6), then return) x 20
   - query: x = number of A bias coins / 10

   Variables:

   - x = number of A bias coins
   - c_i = probability of picking an A bias coin for each pick i
   - y_j = probability of getting heads for each toss j

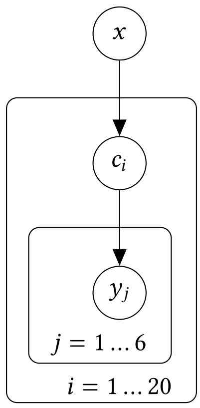


Figure 4.1: DAG for the mysterious urn problem

TODO: CFPs

2. TODO: enum-all tree