

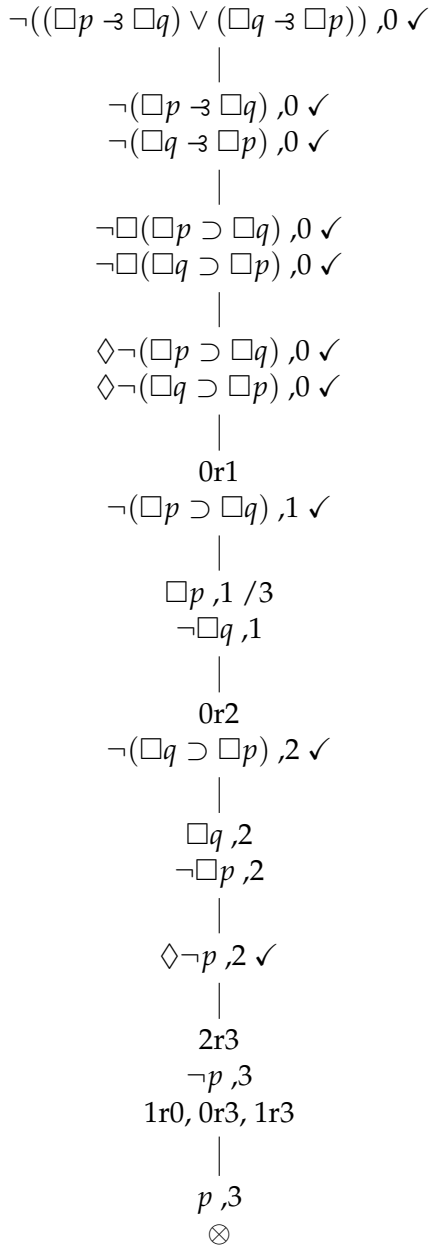
PY2010 Intermediate Logic  
UNIVERSITY OF ST. ANDREWS  
*Take Home Assessment*

ID: 200007413

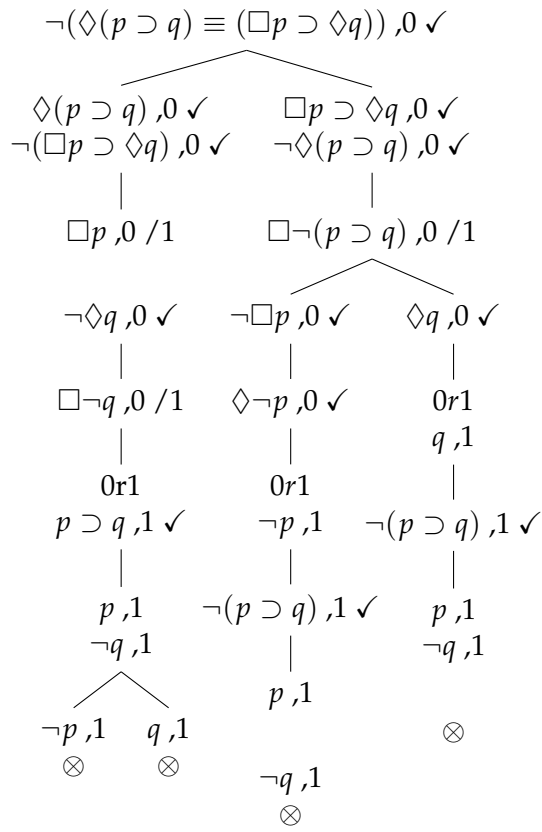
December 7, 2021

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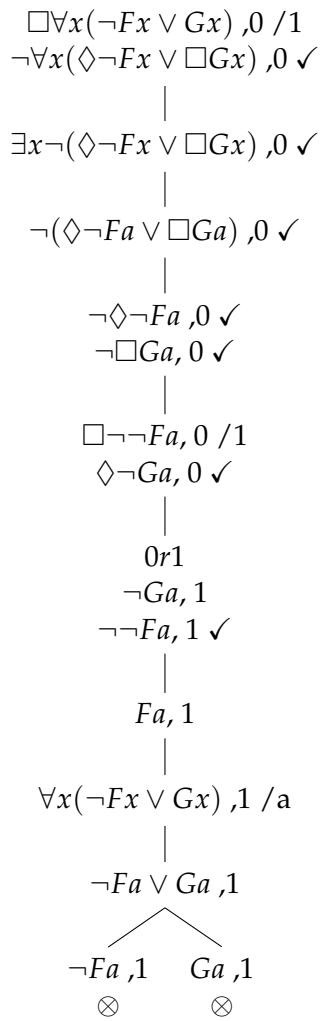
1. (a)  $\vdash_{TK\rho\sigma\tau} (\Box p \rightarrow \Box q) \vee (\Box q \rightarrow \Box p)$



(b)  $\vdash_{TK\rho} \Diamond(p \supset q) \equiv (\Box p \supset \Diamond q)$



(c)  $\Box \forall x (\neg Fx \vee Gx) \vdash_{TCK} \forall x (\Diamond \neg Fx \vee \Box Gx)$



2. (a)  $(p \wedge q) \vee r, \neg r \vdash_{NC} p$
- |     |      |                       |                     |
|-----|------|-----------------------|---------------------|
| 1   | (1)  | $(p \wedge q) \vee r$ | Ass(premise)        |
| 2   | (2)  | $\neg r$              | Ass(premise)        |
| 3   | (3)  | $(p \wedge q)$        | Ass( $\vee E$ )     |
| 3   | (4)  | $p$                   | 3, $\wedge E$       |
| 5   | (5)  | $r$                   | Ass( $\vee E$ )     |
| 2,5 | (6)  | $\perp$               | 2,5, $\neg E$       |
| 7   | (7)  | $\neg p$              | Ass( $\neg I$ )     |
| 2,5 | (8)  | $\neg \neg p$         | 6,7, $\neg I$       |
| 2,5 | (9)  | $p$                   | 8, DN               |
| 1,2 | (10) | $p$                   | 1,3,4,5,9, $\vee E$ |

- (b)  $\exists x(Fx \vee Gx) \vdash_{NC} \exists xFx \vee \exists xGx$

1	(1)	$\exists x(Fx \vee Gx)$	Ass(premise)
2	(2)	$Fx \vee Gx$	Ass( $\exists E$ )
3	(3)	$Fx$	Ass( $\vee E$ )
3	(4)	$\exists Fx$	3, $\exists I$
3	(5)	$\exists Fx \vee \exists Gx$	4, $\vee I$
6	(6)	$Gx$	Ass( $\vee E$ )
6	(7)	$\exists Gx$	6, $\exists I$
6	(8)	$\exists Fx \vee \exists Gx$	7, $\vee I$
2	(9)	$\exists Fx \vee \exists Gx$	2,3,5,6,8, $\vee E$
1	(10)	$\exists Fx \vee \exists Gx$	1,2,9, $\exists E$

(c)  $\neg p \wedge \neg q \vdash_{NI} (p \vee q)$

1	(1)	$\neg p \wedge \neg q$	Ass(premise)
1	(2)	$\neg p$	1, $\wedge E$
1	(3)	$\neg q$	1, $\wedge E$
4	(4)	$p \vee q$	Ass( $\neg I$ )
5	(5)	$p$	Ass( $\vee E$ )
1,5	(6)	$\perp$	2,5, $\neg E$
7	(7)	$q$	Ass( $\vee E$ )
1,7	(8)	$\perp$	3,7, $\neg E$
1,4	(9)	$\perp$	4,5,6,7,8 $\vee E$
1	(10)	$\neg (p \vee q)$	4,9 $\neg I$

3. (a)  $p \vee \neg p \vdash \neg(q \wedge \neg q)$  - check in both  $K_3$  and  $LP$

i.  $p \vee \neg p \vdash_{K_3} \neg(q \wedge \neg q)$

$p \vee \neg p, + \checkmark$
$\neg(q \wedge \neg q), - \checkmark$
$\neg q \vee \neg \neg q, - \checkmark$
$\neg q, -$
$\neg \neg q, - \checkmark$
$q, -$
$p, +$ $\neg p, +$

Counter Model:

$$v(p) = 1, v(q) = n$$

$$D \in \{1\}$$

As  $v(p) = 1$ , then the semantics for  $\neg$  mean that  $v(\neg p) = 0$ .

Then, the semantics of  $\vee$  mean that  $v(p \vee \neg p) = 1$  because the maximum of  $v(p) = 1$  and  $v(\neg p) = 0$  is 1. Therefore, as  $1 \in D$ , we have a case where the premises are true.

Next, with  $v(q) = n$ , then the semantics for  $\neg$  mean that  $v(\neg q) = n$ .

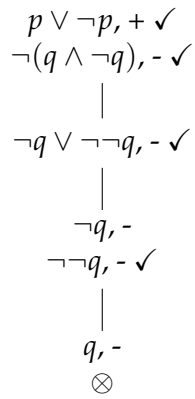
Then, the semantics for  $\wedge$  means  $v(q \wedge \neg q)$  is the minimum of  $v(q)$  and  $v(\neg q)$  which are both  $n$  and so  $v(q \wedge \neg q) = n$ .

Then, the semantics for  $\neg$  mean that  $v(\neg(q \wedge \neg q)) = n$  as  $v(q \wedge \neg q) = n$ .

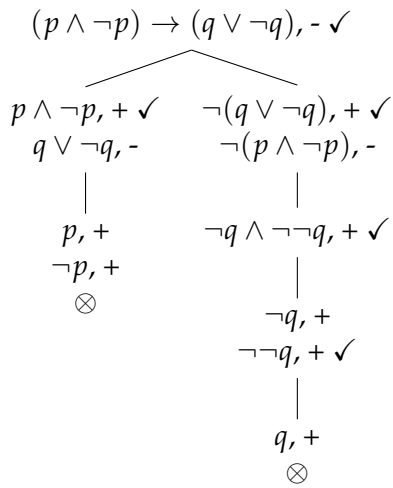
Therefore, we have a scenario where the premises are all satisfied, but the conclusion has value  $n$  and  $n \notin D$  meaning it is not satisfied. Therefore,  $p \vee \neg p \not\vdash_{K_3} \neg(q \wedge \neg q)$

QED.

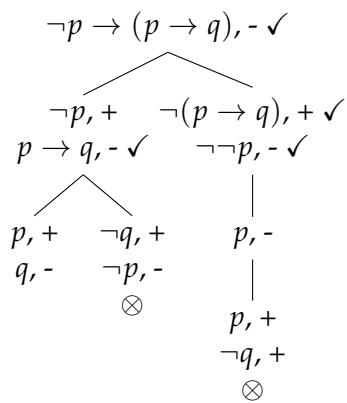
ii.  $p \vee \neg p \vdash_{LP} \neg(q \wedge \neg q)$



(b)  $\vdash_{L3} (p \wedge \neg p) \rightarrow (q \vee \neg q)$



(c)  $\vdash_{RM3} \neg p \rightarrow (p \rightarrow q)$



Counter Model:

$$v(q) = 0, v(p) = b$$

$$D \in \{1, b\}$$

As there are no premises, they are automatically satisfied.

Then, because  $v(p) = b$ , the semantics for  $\neg$  mean that  $v(\neg p) = b$ .

Then, with  $v(p) = b$ , and  $v(q) = 0$ , the semantics for  $\rightarrow$  mean that  $v(p \rightarrow q) = 0$ .

Then, the semantics of  $\rightarrow$  again mean that  $v(\neg p \rightarrow (p \rightarrow q)) = 0$  because  $v(\neg p) = b$  and  $v(p \rightarrow q) = 0$ .

Therefore, as  $0 \notin D$ , we have a scenario where all the premises are satisfied, but the conclusion is not, and so  $\not\models_{RM3} \neg p \rightarrow (p \rightarrow q)$ . QED.

4. n/a

5. (a)  $\neg p \models_{L_{\mathcal{K}}} p \rightarrow q$

SHOW: for an arbitrary  $v$ , when  $v(\neg p) = 1$ ,  $v(p \rightarrow q) = 1$

PROOF: Assume for some arbitrary  $v$ ,  $v(\neg p) = 1$ .

Then, by the semantics of  $\neg$ ,  $v(p) = 0$ .

Since the values in  $L_{\mathcal{K}}$  are linearly ordered, we know that either (i)  $v(p) > v(q)$ , or (ii)  $v(p) \leq v(q)$ .

However, as  $v(p) = 0$ , it cannot be that (i)  $v(p) > v(q)$ , as there are no values smaller than 0 and so  $v(p)$  cannot be strictly greater than  $v(q)$ . Therefore  $v(p) \leq v(q)$ .

The semantics of  $\rightarrow$  mean that when  $v(p) \leq v(q)$ ,  $v(p \rightarrow q) = 1$ . QED.

(b)  $\models_G \neg(q \vee \neg q) \rightarrow p$

SHOW: for any arbitrary  $v$ ,  $v(\neg(q \vee \neg q) \rightarrow p) = 1$

PROOF: Since the semantic values in  $G$  are linearly ordered, We know that for any  $v$ .  $0 \leq v(q) \leq 1$ .

The semantics for  $\neg$  give us two options, either: (i)  $v(q) = 0$  giving  $v(\neg q) = 1$ ; or (ii)  $v(q) > 0$  giving  $v(\neg q) = 0$ .

i. With  $v(q) = 0$  and  $v(\neg q) = 1$ , we know by the semantics of  $\vee$  that  $v(q \vee \neg q) = 1$

From this, the semantics of  $\neg$  mean that  $v(\neg(q \vee \neg q)) = 0$  because  $0 > 1$  and so  $v(q \vee \neg q) > 0$ .

ii. With  $v(q) > 0$  and  $v(\neg q) = 0$ , we know that  $v(\neg q) \leq v(q)$  because there are no potential values of  $v(q)$  smaller than 0. So, by the semantics of  $\vee$ ,  $v(q \vee \neg q) > 0$ .

From this, the semantics of  $\neg$  mean that  $v(\neg(q \vee \neg q)) = 0$ .

Therefore, from both potential values for  $v(p)$  with (i) and (ii), we get  $v(\neg(q \vee \neg q)) = 0$ .

We also know that  $0 \leq v(p) \leq 1$ . No matter what the value of  $v(p)$  is,  $v(\neg(q \vee \neg q)) = 0$  and so  $v(\neg(q \vee \neg q)) \leq v(p)$  as there are no values smaller than 0 on the linear scale.

Therefore, by the semantics of  $\rightarrow$ ,  $v(\neg(q \vee \neg q) \rightarrow p) = 1$  for all  $v$ . QED.