CS4102 - P1

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Submission Note

Throughout this submission real values are displayed. For ease of display, values are rounded to 5dp. However, all calculations were done in python without rounding intermediate values.

1 Object manipulation

(a)

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & \delta_x \\ 0 & 1 & 0 & \delta_y \\ 0 & 0 & 1 & \delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{S} = \begin{bmatrix} \sigma_x & 0 & 0 & 0 \\ 0 & \sigma_y & 0 & 0 \\ 0 & 0 & \sigma_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R_x} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x & 0 \\ 0 & \sin\theta_x & \cos\theta_x & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{R_y} = \begin{bmatrix} \cos\theta_y & 0 & \sin\theta_y & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta_y & 0 & \cos\theta_y & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{R_z} = \begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 & 0 \\ \sin\theta_z & \cos\theta_z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b)

$$\mathbf{M}_{\text{model}} = \mathbf{R_z R_y R_x S T} =$$

$$\begin{bmatrix} \cos\theta_z & -\sin\theta_z & 0 & 0 \\ \sin\theta_z & \cos\theta_z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_y & 0 & \sin\theta_y & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta_y & 0 & \cos\theta_y & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x & 0 \\ 0 & \sin\theta_x & \cos\theta_x & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \sigma_x & 0 & 0 & 0 \\ 0 & \sigma_y & 0 & 0 \\ 0 & 0 & \sigma_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & \delta_x \\ 0 & 1 & 0 & \delta_y \\ 0 & 0 & 1 & \delta_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(c)

$$\begin{bmatrix} \cos 0.1 & -\sin 0.1 & 0 & 0 \\ \sin\theta_z & \cos 0.1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 0.2 & 0 & \sin 0.2 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 0.2 & 0 & \cos 0.2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 0.2 & -\sin 0.2 & 0 \\ 0 & \sin 0.2 & \cos 0.2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1.2 & 0 & 0 & 0 \\ 0 & 1.2 & 0 & 0 \\ 0 & 0 & 1.2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1.17020 & -0.07029 & 0.21357 & 0.28657 \\ 0.11741 & 1.17493 & -0.17824 & 1.99339 \\ -0.23840 & 0.23365 & 0.96053 & 2.38836 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(d)

$$\mathbf{V}' = \mathbf{M}_{\text{model}} \cdot \mathbf{V} =$$

$$\begin{bmatrix} -1.34284 & 3.33797 & 3.19740 & -1.48341 & -2.62426 & 2.05655 & 1.91598 & -2.76483 & -2.23227 & 2.44855 \\ 0.04892 & 0.51857 & 2.86843 & 2.39878 & 1.11835 & 1.58800 & 3.93786 & 3.46821 & 1.34934 & 1.81899 \\ 5.51311 & 4.55950 & 5.02680 & 5.98041 & -0.25007 & -1.20369 & -0.73638 & 0.21722 & 1.78781 & 0.83420 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

2 Geometry

(a) $P1' = \begin{bmatrix}-1.34284 & 0.04892 & 5.51311\end{bmatrix}^T$

$P2' = \begin{bmatrix}3.33797 & 0.51857 & 4.55950\end{bmatrix}^T$

$P3' = \begin{bmatrix}3.19740 & 2.86843 & 5.02680\end{bmatrix}^T$

$P4' = \begin{bmatrix}-1.48341 & 2.39878 & 5.98041\end{bmatrix}^T$

$P5' = \begin{bmatrix}-2.62426 & 1.11835 & -0.25007\end{bmatrix}^T$

$P6' = \begin{bmatrix}2.05655 & 1.58800 & -1.20369\end{bmatrix}^T$

$P7' = \begin{bmatrix}1.91598 & 3.93786 & -0.73638\end{bmatrix}^T$

$P8' = \begin{bmatrix}-2.76483 & 3.46821 & 0.21722\end{bmatrix}^T$

$P9' = \begin{bmatrix}-2.23227 & 1.34934 & 1.78781\end{bmatrix}^T$

$P10' = \begin{bmatrix}2.44855 & 1.81899 & 0.83420\end{bmatrix}^T$

(b) $v_1 = P2' - P1' = \begin{bmatrix}4.68081 & 0.46965 & -0.95361\end{bmatrix}^T$

$v_2 = P4' - P1' = \begin{bmatrix}-0.14057 & 2.34986 & 0.46730\end{bmatrix}^T$

$v_3 = P5' - P1' = \begin{bmatrix}-1.28142 & 1.06943 & -5.76318\end{bmatrix}^T$

(c) Two vectors are orthogonal if they multiply to get 0.

$v_1 \cdot v_2 = \begin{bmatrix}4.68081 & 0.46965 & -0.95361\end{bmatrix}^T \cdot \begin{bmatrix}-0.14057 & 2.34986 & 0.46730\end{bmatrix}^T = -9.36593889367488e^{-16}$

$v_1 \cdot v_3 = \begin{bmatrix}4.68081 & 0.46965 & -0.95361\end{bmatrix}^T \cdot \begin{bmatrix}-1.28142 & 1.06943 & -5.76318\end{bmatrix}^T = -1.896281173374806e^{-15}$

$v_2 \cdot v_3 = \begin{bmatrix}-0.14057 & 2.34986 & 0.46730\end{bmatrix}^T \cdot \begin{bmatrix}-1.28142 & 1.06943 & -5.76318\end{bmatrix}^T = -2.7824883025005647e^{-15}$

These values are so small, that given any reasonable level of precision in graphics (e.g. we have been using 5 decimal places), they round to 0.00000, and so each vector is orthogonal to each of the other vectors.

(d) First, we get three different vectors between the points

$v1' = P1' - P2' = \begin{bmatrix}-1.34284 & 0.04892 & 5.51311\end{bmatrix}^T - \begin{bmatrix}3.33797 & 0.51857 & 4.55950\end{bmatrix}^T$

$= \begin{bmatrix}-4.68081757 & -0.4696483 & 0.95361279\end{bmatrix}$

$v2' = P1' - P3' = \begin{bmatrix}-1.34284 & 0.04892 & 5.51311\end{bmatrix}^T - \begin{bmatrix}3.19740 & 2.86843 & 5.02680\end{bmatrix}^T$

$= \begin{bmatrix}-4.54024699 & -2.81951398 & 0.48631078\end{bmatrix}$

$v3' = P1' - P4' = \begin{bmatrix}-1.34284 & 0.04892 & 5.51311\end{bmatrix}^T - \begin{bmatrix}-1.48341 & 2.39878 & 5.98041\end{bmatrix}^T$

$= \begin{bmatrix}0.14057058 & -2.34986569 & -0.46730201\end{bmatrix}$

The normal to the plane between two of the vectors (including three of the points)

$n = v1' \times v2' = \begin{bmatrix}-4.68081757 & -0.4696483 & 0.95361279\end{bmatrix} \times \begin{bmatrix}-4.54024699 & -2.81951398 & 0.48631078\end{bmatrix}$

$= \begin{bmatrix}2.46032956 & -2.05330556 & 11.06531133\end{bmatrix}$

Then, projecting the third vector including the fourth point, we get a value which, under any reasonable level of accuracy (in our case 5dp.) rounds to zero, and thus is orthonormal to the normal, meaning it projects onto the same plane created from the other three points.

$n \cdot v3' = \begin{bmatrix}2.46032956 & -2.05330556 & 11.06531133\end{bmatrix} \cdot \begin{bmatrix}0.14057058 & -2.34986569 & -0.46730201\end{bmatrix}$

$= -8.705161717594623e^{-15} \approx 0.00000$

3 Camera Positioning

(a)

$$\mathbf{T} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & d_c\\ 0 & 0 & 0 & 1\end{bmatrix} \mathbf{R_x} = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \cos\theta_c & -\sin\theta_x & 0\\ 0 & \sin\theta_c & \cos\theta_c & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \mathbf{R_y} = \begin{bmatrix}\cos\phi_c & 0 & \sin\phi_c & 0\\ 0 & 1 & 0 & 0\\ -\sin\phi_c & 0 & \cos\phi_c & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

(b)

$$\mathbf{M}_{\text{view}} = \mathbf{R_y R_x T} = \begin{bmatrix} 0.98007 & 0.04915 & 0.19249 & 0.57748 \\ 0.00000 & 0.96891 & -0.24740 & -0.74221 \\ -0.19867 & 0.24247 & 0.94960 & 2.84880 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(c)

$$\mathbf{M}_{\text{model\_view}} = \mathbf{M}_{\text{view}}\mathbf{M}_{\text{model}} = \begin{bmatrix} 0.98007 & 0.04915 & 0.19249 & 0.57748 \\ 0 & 0.96891 & -0.24740 & -0.74221 \\ -0.19867 & 0.24247 & 0.94960 & 2.84880 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(d)

$$\mathbf{V}'' = \mathbf{M}_{\text{model\_view}}\mathbf{V} =$$

$$\begin{bmatrix} 0.32504 & 4.75208 & 4.81976 & 0.39273 & -1.98764 & 2.43939 & 2.50707 & -1.91996 & -1.19983 & 3.22721 \\ -2.05878 & -1.36781 & 0.79340 & 0.10242 & 0.40324 & 1.09421 & 3.25541 & 2.56444 & 0.12287 & 0.81384 \\ 8.36268 & 6.64107 & 7.68253 & 9.40413 & 3.40386 & 1.68225 & 2.72370 & 4.44531 & 5.31716 & 3.59555 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

4 In-Place Manipulation

$$R = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R}_{\text{in\_place}} = \mathbf{M}_{\text{model\_view}}\mathbf{R}\mathbf{M}^{\mathbf{-1}}_{\text{model\_view}} =$$

$$\begin{bmatrix} 0.98007 & 0.04915 & 0.19249 & 0.57748 \\ 0 & 0.96891 & -0.24740 & -0.74221 \\ -0.19867 & 0.24247 & 0.94960 & 2.84880 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.76858 & 0.11996 & -0.29889 & 0.49667 \\ 0.02350 & 0.75042 & 0.36162 & -2.48677 \\ 0.38545 & -0.41034 & 0.82647 & -4.88159 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1.85063 & 0.13277 & -0.3308 & 0.5497 \\ 0.13277 & 1.02072 & -0.05163 & 0.0858 \\ -0.3308 & -0.05163 & 1.12864 & -0.21377 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5 Advanced Tasks

(a)

(b)