

PY2010 Intermediate Logic
UNIVERSITY OF ST. ANDREWS
Exercises Week 04

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I hereby declare that the attached piece of written work is my own work and that I have not reproduced, without acknowledgement, the work of another.

1. Show the following in semantics for the relevant logic

(a) $\Box A \models_{K\rho} \Diamond A$

Show: For all $\langle W, R, v \rangle$ w/ R reflexive, if $v_w(\Box A) = 1$ then $v_w(\Diamond A) = 1$.

Assume for an arbitrary $\langle w, R, v \rangle$ with reflexive R , $v_w(\Box A) = 1$.

$v_w(\Box A) = 1$ iff at ALL x such that wRx , $v_x(A) = 1$ [\Box semantics], so $v_w(A) = 1$ [ρ rule]

Therefore, by definition, $v_w(\Diamond A) = 1$ [\Diamond semantics]. QED

(b) $\Diamond\Diamond A \models_{K\rho\sigma\tau} \Diamond A$

Show: For all $\langle W, R, v \rangle$ w/ R reflexive, symmetrical, and transitive, if $v_w(\Diamond\Diamond A) = 1$, then $v_w(\Diamond A) = 1$.

Assume for an arbitrary $\langle w, R, v \rangle$ with R reflexive, symmetrical, and transitive, $v_w(\Diamond\Diamond A) = 1$.

So, there's an x such that wRx and $v_x(\Diamond A) = 1$ [\Diamond rule]. Also, we can deduce xRw and thus $v_w(\Diamond A) = 1$ [σ rule]. QED

2. Show the following in $K\rho$ using tableaux

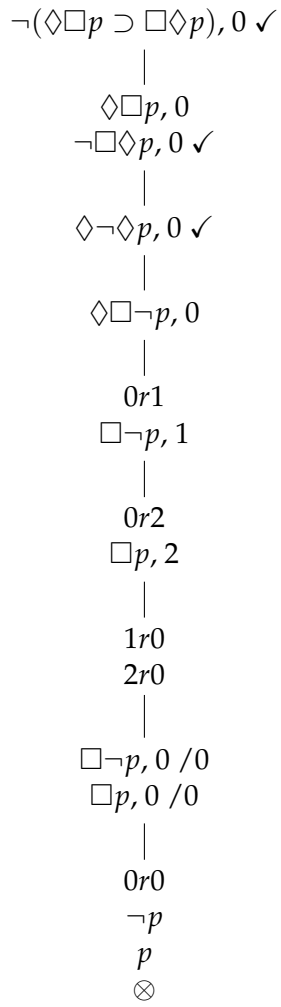
(a) $\vdash (\Diamond\neg p \vee \Diamond\neg q) \vee \Diamond(p \vee q)$

$$\begin{array}{c}
\neg((\Diamond\neg p \vee \Diamond\neg q) \vee \Diamond(p \vee q)), 0 \checkmark \\
| \\
\neg(\Diamond\neg p \vee \Diamond\neg q), 0 \checkmark \\
\neg\Diamond(p \vee q), 0 \checkmark \\
| \\
\Box\neg(p \vee q), 0 / 0 \\
| \\
\neg\Diamond\neg p, 0 \checkmark \\
\neg\Diamond\neg q, 0 \checkmark \\
| \\
\Box\neg\neg p, 0 \checkmark \\
\Box\neg\neg q, 0 \checkmark \\
| \\
\Box p, 0 / 0 \\
\Box q, 0 / 0 \\
| \\
0r0 \\
p, 0 \\
q, 0 \\
| \\
\neg(p \vee q), 0 \\
| \\
\neg p, 0 \\
\neg q, 0 \\
\otimes
\end{array}$$

$$\begin{array}{c}
\text{(b)} \vdash \Box p \supset \Diamond p \\
\neg(\Box p \supset \Diamond p), 0 \checkmark \\
| \\
\Box p, 0 / 0 \\
\neg\Diamond p, 0 \checkmark \\
| \\
\Box\neg p, 0 / 0 \\
| \\
0r0 \\
p, 0 \\
\neg p, 0
\end{array}$$

3. Do the following hold in $K\Box\tau$ using tableaux?

$$\text{(a)} \vdash \Diamond\Box p \supset \Box\Diamond p$$



(b) $\Box(p \supset q) \vdash \Box(\Box p \supset \Box q)$

