School of Computer Science, University of St Andrews, CS3050, 2022-23

CS3050: Practical 1 – Proofs

Assignment: P1 (Practical 1)

Deadline: 14th October 2022 at 9:00 pm

Weighting: 45% of coursework weight

Please note that MMS is the definitive source for deadline and credit details. You are expected to have read and understood all the information in this specification and any accompanying documents at least a week before the deadline. You must contact the lecturers regarding any queries well in advance of the deadline.

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1   Objective

This practical is aimed at developing and writing proofs in various systems.

Competencies Addressed

- Develop your skills in writing logical statements and proofs.

- Develop your skills in writing detailed proofs with justifications.

- Deepen your understanding of the important proof systems of equational logic and natural deduction.

2   Practical Requirements

You are required to answer the questions below. You may write your answers in any word-processing software (we recommend LaTeX), or write them by hand and scan them. In either case, please return your answers as a single PDF. If you handwrite, please make sure the pages are legible, the right way up and in the right order. Your submission should be your own work. Please cite any sources you consulted.

For all answers, you must show working and reasoning for full credit. Partial credit will be available for attempts that do not succeed in achieving a complete proof, as long as some understanding is demonstrated.

3 Questions

1. Assume the domain of discourse is the set of natural numbers $\mathbb{N}$ (where $0 \in \mathbb{N}$). We have predicates *odd*, *even*, $<$ and $>$. We have function symbols $+$, $\times$.

   Determine the truth values of the following formulae under this interpretation:

   (a) $\exists y.\forall x.(y < x)$

   (b) $\exists x.\forall y.(y < x)$

   (c) $\forall y.\forall x.(y = x \vee y > x)$

   (d) $\forall x.(odd(x) \vee odd(x + 2))$

2. For the following propositions, perform truth-tables tests and state in each case whether the formula is tautologous, contingent, or contradictory:

   (a) $P \rightarrow P$

   (b) $P \rightarrow \neg P$

   (c) $\neg(P \rightarrow P)$

   (d) $(P \vee \neg Q) \wedge \neg(\neg P \rightarrow \neg Q)$

   (e) $(P \wedge Q \rightarrow R) \rightarrow (P \rightarrow R) \rightarrow (Q \rightarrow R)$

   (f) $(P \vee Q \rightarrow R) \Leftrightarrow ((P \rightarrow R) \wedge (Q \rightarrow R))$

   (g) $(P \rightarrow Q) \wedge (R \rightarrow S) \rightarrow (P \vee R \rightarrow Q \vee S)$

3. Prove the following using the logical equivalences from Lecture PropLogic-1b (slides 21 & 22 of that lecture):

   $\neg((P \wedge Q) \wedge((P \vee(R \wedge P)) \wedge(\neg Q \wedge \neg P)) \wedge(\neg\neg P \vee(Q \vee \neg Q))) \Leftrightarrow T$

4. Prove the following using the rules of natural deduction:

   (a) Give a proof of transitivity of implication, from assumptions $P \rightarrow Q$ and $Q \rightarrow R$. In other words, prove: $P \rightarrow Q, Q \rightarrow R \vdash P \rightarrow R$

   (b) $\vdash ((P \vee Q) \rightarrow R) \rightarrow ((P \rightarrow R) \wedge (Q \rightarrow R))$

   (c) $\vdash (P \rightarrow (Q \rightarrow R)) \rightarrow ((P \wedge Q) \rightarrow R)$

   (d) $P \vee Q \vdash \neg(\neg P \wedge \neg Q)$

5. Prove the following using the rules of natural deduction:

   (a) $\forall x.(P(x) \rightarrow Q) \vdash ((\exists x.P(x)) \rightarrow Q)$

   (b) $((\exists x.P(x)) \rightarrow Q) \vdash \forall x.(P(x) \rightarrow Q)$

4 Marking

Marks will be awarded for correct proofs, naturally, but some of the marks will be reserved for demonstrating understanding of proof rules and techniques, and for being clear and explicit. If we cannot understand the justification for the steps in your proof, you will not get full credit. See also the standard mark descriptors in the School Student Handbook:

http://info.cs.st-andrews.ac.uk/student-handbook/learning-teaching/
feedback.html#Mark_Descriptors

Each question from 1 to 5 will be assigned a mark on the 20 point scale, and these will be averaged to produce your mark for the practical.

Mark range	Typical attributes of work gaining a mark in this range
0	no attempt
1–3	answer containing little or no content relevant to the question
4–6	answer including some relevant information from lectures or textbooks, but no substantial steps towards a solution of the main problem
7–10	answer includes a reasonable attempt to solve the problem which is nevertheless far from solving any non-routine parts of it
11–13	answer makes significant progress towards a solution in a reasonable way, but falls short of reaching it. Could include a solution which is superficially complete, but severely lacking in clarity, precision, and/or justification
14–16	an answer that comes close to solving the problem, but may contain some minor errors, or significant failings in clarity, precision, and/or justification
17–18	an answer that clearly and precisely solves the problem, with at most trivial errors in minor details, or minor failing in presentation, justification, and precision
19–20	a clear, mathematically precise answer, very well expressed, which completely solves the problem

5 Policy Guidelines

Lateness: The standard penalty for late submission applies (Scheme B: 1 mark per 8 hour period, or part thereof):
http://info.cs.st-andrews.ac.uk/student-handbook/learning-teaching/
assessment.html#lateness-penalties

Good Academic Practice: The University policy on Good Academic Practice applies:
https://www.st-andrews.ac.uk/students/rules/academicpractice/