ASSESSED LOGIC EXERCISES

Due on Monday of Week 5 (11 Oct.)

Directions: Respond to all of the following exercises. Write your answers by hand, as clearly as possible. Submit your exercises to MMS before 17:00. Marking is done blindly so please do not write your name anywhere in the document. On the first page of your coursework you should include: your matriculation number, your tutor's name, and the following statement:

> 'I hereby declare that the attached piece of written work is my own work and that I have not reproduced, without acknowledgement, the work of another'.

(1) Explain why the following are true in the semantics of classical propositional logic.

(a) $q \vee \neg q \vDash_{\mathsf{C}} ((p \supset q) \supset p) \supset p)$

(b) $\neg\neg\neg(p \wedge q) \vDash_{\mathsf{C}} \neg p \vee \neg q$

(2) Give a proof of each of the following using natural deduction.

(a) $q \supset r \vdash_{\mathsf{NC}} (p \wedge q) \supset r$

(b) $(\neg p \wedge \neg q) \vee (p \wedge q) \vdash_{\mathsf{NC}} p \equiv q$
HINT: use the definition $p \equiv q := (p \supset q) \wedge (q \subset p)$

(3) Check the following using tableaux for classical propositional logic. Give a countermodel if the argument is invalid. (Make sure you explain *why* the interpretation you give 'works' as a countermodel)

(a) $(p \vee (\neg p \wedge (p \vee q))) \vdash_{\mathsf{TC}} p \vee q$

(b) $\neg(p \supset (q \vee r)) \vdash_{\mathsf{TC}} \neg((q \vee r) \supset p)$

(4) Explain why the following are true in the semantics of the various modal logics.

(a) $\Diamond p \vDash_{\mathsf{K}} \neg \Box \neg p$

(b) $\Box \Box \Box p \vDash_{\mathsf{TK}\rho\tau} \Box p$

(5) Check the following using tableaux for the various modal logics. Give a countermodel if the argument is invalid. (Make sure you explain *why* the interpretation you give 'works' as a countermodel)

(a) $\Diamond p, \Box q \vdash_{\mathsf{NK}} \Diamond(p \wedge q)$

(b) $p \dashv\!3\, r, q \dashv\!3\, r \vdash_{\mathsf{TK}\rho\tau} p \dashv\!3\, q$
HINT: use the definition $A \dashv\!3\, B := \Box(A \supset B)$