# CS3050 - Exercise - 2

## gp87

### September 26, 2022

(a) $(P \land Q) \to (Q \land P)$

| | | | |
|---|---|---|---|
| 1 | (1) | $P \land Q$ | ASSUMPTION ($\land E$) |
| 1 | (2) | $P$ | $1, \land E1$ |
| 1 | (3) | $Q$ | $1, \land E2$ |
| 1 | (4) | $Q \land P$ | $3, 2, \land I$ |
| | (5) | $(P \land Q) \to (Q \land P)$ | $1, 4, \to I$ |

(b) $(P \to (Q \to (P \land Q)))$

| | | | |
|---|---|---|---|
| 1 | (1) | $P \land Q$ | ASSUMPTION ($\land E$) |
| 1 | (2) | $P$ | $1, \land E1$ |
| 1 | (3) | $Q$ | $1, \land E2$ |
| | (4) | $Q \to (P \land Q)$ | $3, 1 \to I$ |
| | (5) | $P \to (Q \to (P \land Q))$ | $2, 4 \to I$ |

(c) $(P \to ((\neg P) \to Q))$

| | | | |
|---|---|---|---|
| 1 | (1) | $Q \land \neg Q$ | ASSUMPTION ($\land E$) |
| 1 | (2) | $Q$ | $1, \land E1$ |
| 1 | (3) | $\neg Q$ | $1, \land E2$ |
| 1 | (4) | $\bot$ | $2, 3, \neg E$ |
| 2 | (5) | $P$ | ASSUMPTION ($\neg I$) |
| 1 | (6) | $\neg P$ | $4, 5, \neg I$ |
| | (7) | $(\neg P) \to Q$ | $6, 2, \to I$ |
| | (8) | $P \to ((\neg P) \to Q)$ | $5, 7, \to I$ |

(d) $(P \lor Q) \to (((P \to R) \land (Q \to R)) \to R)$

| | | | |
|---|---|---|---|
| 1 | (1) | $R$ | ASSUMPTION ($\to I$) |
| 2 | (2) | $P$ | ASSUMPTION ($\to I$) |
| 3 | (3) | $Q$ | ASSUMPTION ($\to I$) |
| 1 | (4) | $P \to R$ | $1, 2 \to I$ |
| 1 | (5) | $Q \to R$ | $1, 3 \to I$ |
| 1 | (6) | $(P \to R) \land (Q \to R)$ | $4, 5 \land I$ |