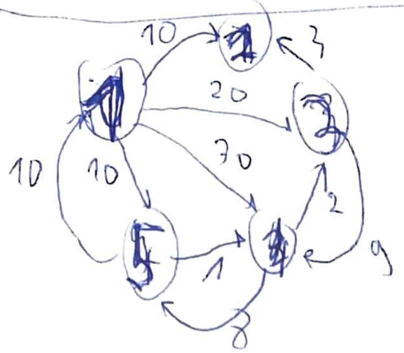
Floyd - Warshall algorithm



D_i := cost matrix, P_i := previous matrices

$$D_0 = \begin{pmatrix} 0 & 10 & 20 & 30 & 10 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & 3 & 0 & 9 & \infty \\ \infty & \infty & 2 & 0 & 8 \\ 10 & \infty & \infty & 1 & 0 \end{pmatrix}$$

$$P_0 = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 3 & 0 \\ 0 & 0 & 4 & 0 & 4 \\ 5 & 0 & 0 & 5 & 0 \end{pmatrix}$$

$k=1$ → using vertex 1 as intermediate ~~value~~ vertex

$$D_1 = \begin{pmatrix} 0 & 10 & 20 & 30 & 10 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & 3 & 0 & 9 & \infty \\ \infty & \infty & 2 & 0 & 8 \\ 10 & \boxed{20} & \boxed{30} & 1 & 0 \end{pmatrix} \quad P_1 = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 3 & 0 \\ 0 & 0 & 4 & 0 & 4 \\ 5 & 1 & 1 & 5 & 0 \end{pmatrix}$$

$k=2$ → using vertex 2 as intermediate vertex.

$$D_2 = \begin{pmatrix} 0 & 10 & 20 & 30 & 10 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & 3 & 0 & 9 & \infty \\ \infty & \infty & 2 & 0 & 8 \\ 10 & 20 & 30 & 1 & 0 \end{pmatrix} \quad P_2 = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 3 & 0 \\ 0 & 0 & 4 & 0 & 4 \\ 5 & 1 & 1 & 5 & 0 \end{pmatrix}$$

$k=3$ → using vertex 3 as intermediate vertex.

$$D_3 = \begin{pmatrix} 0 & 10 & 20 & \boxed{29} & 10 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & 3 & 0 & 9 & \infty \\ \infty & \boxed{5} & 2 & 0 & 8 \\ 10 & 20 & 30 & 1 & 0 \end{pmatrix} \quad P_3 = \begin{pmatrix} 0 & 1 & 1 & \boxed{3} & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 3 & 0 \\ 0 & \boxed{3} & 4 & 0 & 4 \\ 5 & 1 & 1 & 5 & 0 \end{pmatrix}$$

$k=4 \rightarrow$ using the vertex 4 as an intermediate vertex

$$D_4 = \begin{pmatrix} 0 & 10 & 20 & 19 & 10 \\ \infty & 0 & \infty & \infty & \infty \\ \infty & 3 & 0 & 9 & \boxed{14} \\ \infty & 5 & 2 & 0 & 8 \\ 10 & 6 & 3 & 1 & 0 \end{pmatrix} \qquad P_4 = \begin{pmatrix} 0 & 1 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 3 & \boxed{4} \\ 0 & 3 & 4 & 0 & 4 \\ 5 & 3 & 4 & 5 & 0 \end{pmatrix}$$

$k=5 \rightarrow$ using the vertex 5 as an intermediate vertex

$$D_5 = \begin{pmatrix} 0 & 10 & 13 & 11 & 10 \\ \infty & 0 & \infty & \infty & \infty \\ 27 & 3 & 0 & 9 & 14 \\ 18 & 5 & 2 & 0 & 8 \\ 10 & 6 & 3 & 1 & 0 \end{pmatrix} \qquad P_5 = \begin{pmatrix} 0 & 1 & 4 & 5 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 5 & 3 & 0 & 3 & 4 \\ 5 & 3 & 4 & 0 & 4 \\ 5 & 3 & 4 & 5 & 0 \end{pmatrix} \;\; |1,4|=5 \longrightarrow \text{stop}$$

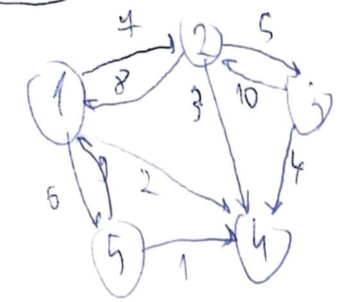
$D_5(2,1) = \infty \Rightarrow$ no walk

The minimum cost walk from 1 to 4 has cost $D_5(1,4) = 11$, and is obtained by going backwards using P_5:

$$(1,4) \xrightarrow{P(1,4)=5} (1,5) \xrightarrow{P(1,5)=1} (1,1)$$

Walk: $1 \xrightarrow{10} 5 \xrightarrow{1} 4$

Floyd-Warshall 2



D_i = cost matrix, P_i = previous matrix

$$D_0 = \begin{pmatrix} 0 & 7 & \infty & 2 & 6 \\ 8 & 0 & 5 & 3 & \infty \\ \infty & 10 & 0 & 4 & \infty \\ \infty & \infty & \infty & 0 & \infty \\ 9 & \infty & \infty & 2 & 0 \end{pmatrix} \quad P_0 = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 2 & 0 & 2 & 2 & 0 \\ 0 & 3 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 5 & 0 \end{pmatrix}$$

$k=1 \rightarrow$ using the vertex 1 as the intermediate vertex

$$D_1 = \begin{pmatrix} 0 & 7 & \infty & 2 & 6 \\ 8 & 0 & 5 & 3 & \boxed{14} \\ 0 & 10 & 0 & 4 & \infty \\ \infty & \infty & \infty & 0 & \infty \\ 9 & \boxed{16} & \infty & 2 & 0 \end{pmatrix} \quad P_1 = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 2 & 0 & 2 & 2 & \boxed{1} \\ 0 & 3 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 5 & \boxed{1} & 0 & 5 & 0 \end{pmatrix}$$

$k=2 \rightarrow$ using the vertex 2 as the intermediate vertex

$$D_2 = \begin{pmatrix} 0 & 7 & \boxed{12} & 2 & 6 \\ 8 & 0 & 5 & 3 & 14 \\ \boxed{18} & 10 & 0 & 4 & \boxed{24} \\ \infty & \infty & \infty & 0 & \infty \\ 9 & 16 & \boxed{21} & 2 & 0 \end{pmatrix} \quad P_2 = \begin{pmatrix} 0 & 1 & \boxed{2} & 1 & 1 \\ 2 & 0 & 2 & 2 & 1 \\ \boxed{2} & 3 & 0 & 3 & \boxed{1} \\ 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & \boxed{2} & 5 & 0 \end{pmatrix}$$

$k=3 \rightarrow$ using the vertex 3 as the intermediate vertex

$$D_3 = \begin{pmatrix} 0 & 7 & 12 & 2 & 6 \\ 8 & 0 & 5 & 3 & 14 \\ 18 & 10 & 0 & 4 & 24 \\ \infty & \infty & \infty & 0 & \infty \\ 9 & 16 & 21 & 2 & 0 \end{pmatrix} \quad P_3 = \begin{pmatrix} 0 & 1 & 2 & 1 & 1 \\ 2 & 0 & 2 & 2 & 1 \\ 2 & 3 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & 2 & 5 & 1 \end{pmatrix}$$

$k=4 \to$ using vertex 4 as intermediate vertex

$$D_4 = \begin{pmatrix} 0 & 7 & 12 & 2 & 6 \\ 8 & 0 & 5 & 3 & 14 \\ 18 & 10 & 0 & 4 & 24 \\ \infty & \infty & \infty & 0 & \infty \\ 9 & 16 & 21 & 2 & 0 \end{pmatrix} \quad P_4 = \begin{pmatrix} 0 & 1 & 2 & 1 & 1 \\ 2 & 0 & 2 & 2 & 1 \\ 2 & 3 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & 2 & 5 & 1 \end{pmatrix}$$

$k=5 \to$ using vertex 5 as intermediate vertex

$$D_5 = \begin{pmatrix} 0 & 7 & 12 & 2 & 6 \\ 8 & 0 & 5 & 3 & 14 \\ 18 & 10 & 0 & 4 & 24 \\ \infty & \infty & \infty & 0 & \infty \\ 9 & 16 & 21 & 2 & 0 \end{pmatrix} \quad P_5 = \begin{pmatrix} 0 & 1 & 2 & 1 & 1 \\ 2 & 0 & 2 & 2 & 1 \\ 2 & 3 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 5 & 1 & 2 & 5 & 1 \end{pmatrix} \quad |V|=5 \Rightarrow \text{stop}$$

$D_5[4,1]=\infty \Rightarrow$ no walk from 4 to 1

The minimum cost walk from 1 to 5 is $D_5[1,5] = 6$

$[1,5] \xrightarrow{P_5[1,5]=1} 1$

$1 \xrightarrow{6} 5$