ECON40003 Econometrics III

Final Exam

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Question 1

(a) asd

(b)

Question 2

(a) (i) $E[\bar{X}_n] = 1$; $V[\bar{X}_n] = \frac{0.5}{n}$; $\text{plim}_{n\to\infty}\bar{X}_n = 1$, the last by Kolmogorov's WLLN.

(ii) It is not usually valid to claim that $E[1/X] = 1/E[X]$. Instead we use Jensen's inequality which states that $E[g(X)] \geq g(E[X])$ when g is convex, which it is for $g = 1/x$. Hence we can say that $E[Z] \geq 1$. Also, by the continuous mapping theorem $\text{plim}_{n\to\infty}Z_n = 1$. So Z approaches X from above as n tends to infinity.

(iii) First we estimate the limiting distribution of $\sqrt{n}(\bar{X}_n - 1)$. We appeal to the Lindeberg-Levy Central Limit Theorem, where $E[X_i] = 1, Var[X_i] = \sigma^2 = 0.5/n$. Since we are dealing with a sampling distribution where the variance is already divided by n, we do not scale by $\sqrt{n}$ in our stabilising transformation, giving $T = \frac{\bar{X}-\mu}{\sigma} = \frac{\bar{X}-1}{\sigma/\sqrt{n}} = \frac{\sqrt{n}(\bar{X}-1)}{0.5} \xrightarrow{d} N(0,1)$.

Next, we apply the same logic to our transformation Z_n. We seek the limiting distribution of $\sqrt{n}(1/\bar{X}_n - 1)$. In this case, the variance