On firm effects

The opinions expressed in this paper are those of the authors alone and do not reflect the views of the Internal Revenue Service or the US Treasury Department. This work is a component of a larger project on income risk in the United States, conducted through the SOI Joint Statistical Research Program.

Introduction – The AKM model and estimator

• Abowd, Kramarz and Margolis (1999, AKM) proposed the model (net of covariates for simplicity):

$$\underbrace{Y_{it}}_{\text{log-earnings}} = \underbrace{\alpha_i}_{\text{worker}} + \underbrace{\psi_{j(i,t)}}_{\text{firm}} + \underbrace{\varepsilon_{it}}_{\text{error}}.$$

• This allows documenting the contributions of worker heterogeneity, firm heterogeneity, and sorting patterns to earnings dispersion.

• AKM proposed a least-squares fixed-effects estimator for $\alpha_1, ..., \alpha_N$ and $\psi_1, ...\psi_J$. AKM estimates can be used to decompose the log-earnings variance.

• The AKM estimator is widely used in labor, and elsewhere (e.g., teachers, hospitals, banks...)

Introduction – Influential findings based on AKM

- Several influential substantive findings are based on AKM:

– Firms matter for inequality. Card *et al.* (2018) conclude that dispersion of firm premia explains 20% of earnings dispersion.

– Sorting estimates are often small, but their increase contributes to wage inequality (Card *et al.*, 2013, Song *et al.*, 2018).

- AKM estimates are also used to guide theories of the labor market:

– The dispersion in firm premia raises the question: "why are similar workers paid differently?" (Mortensen, 2003).

– Motivates study of sorting mechanisms (e.g., Postel-Vinay and Robin, 2002, Eeckhout and Kircher, 2011, Hagedorn *et al.*, 2017).

Outline

- Introduction
- The AKM estimator and limited mobility bias
- Methods for bias reduction
- Evidence on limited mobility bias in the US
- Evidence on limited mobility bias in other countries

The AKM estimator and limited mobility bias

The model

- The AKM model is

$$Y_{it} = X_{it}'\beta + \alpha_i + \psi_{j(i,t)} + \varepsilon_{it},$$

under the exogeneity assumption

$$\mathbb{E}(\varepsilon_{it} \,|\, X, j, \alpha, \psi) = 0.$$

- In this paper we focus on parameter estimation in this model. Yet:

– Exogenous mobility and the absence of state dependence can be at odds with dynamic models of workers and firms (e.g., wage posting, sequential bargaining).

– The additivity assumption rules out interactions between worker effects α_i and firm effects $\psi_{j(i,t)}$ that may be economically relevant.

– Relax later, Bonhomme, Lamadon and Manresa (19).

The model (cont.)

• Treating α and ψ as parameter vectors, the AKM model is a linear regression model. β can be estimated using OLS after partialling out worker and firm indicators. We thus now re-define: $Y_{it} - X_{it}'\beta \mapsto Y_{it}$.

• Stacking the NT observations, we have

$$Y = A\gamma + \varepsilon,$$

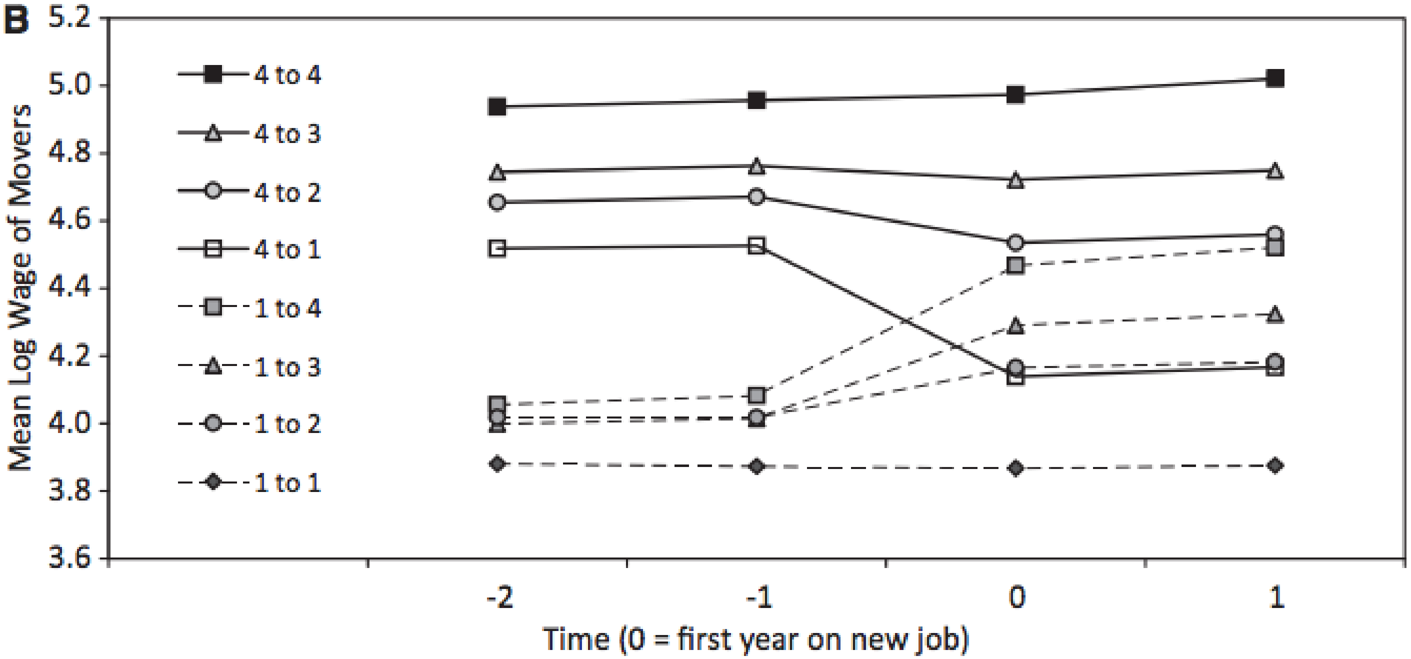
where $\gamma = (\alpha_1, ..., \alpha_N, \psi_1, ..., \psi_J)'$, and $A = [A_W \, A_F]$ depends on worker-firm link indicators.

• Assume that $A'A$ is non-singular. This requires imposing one normalization, and working with a connected component of the firm-worker graph (Abowd *et al.*, 2002). 1

Fixed effect in many countries

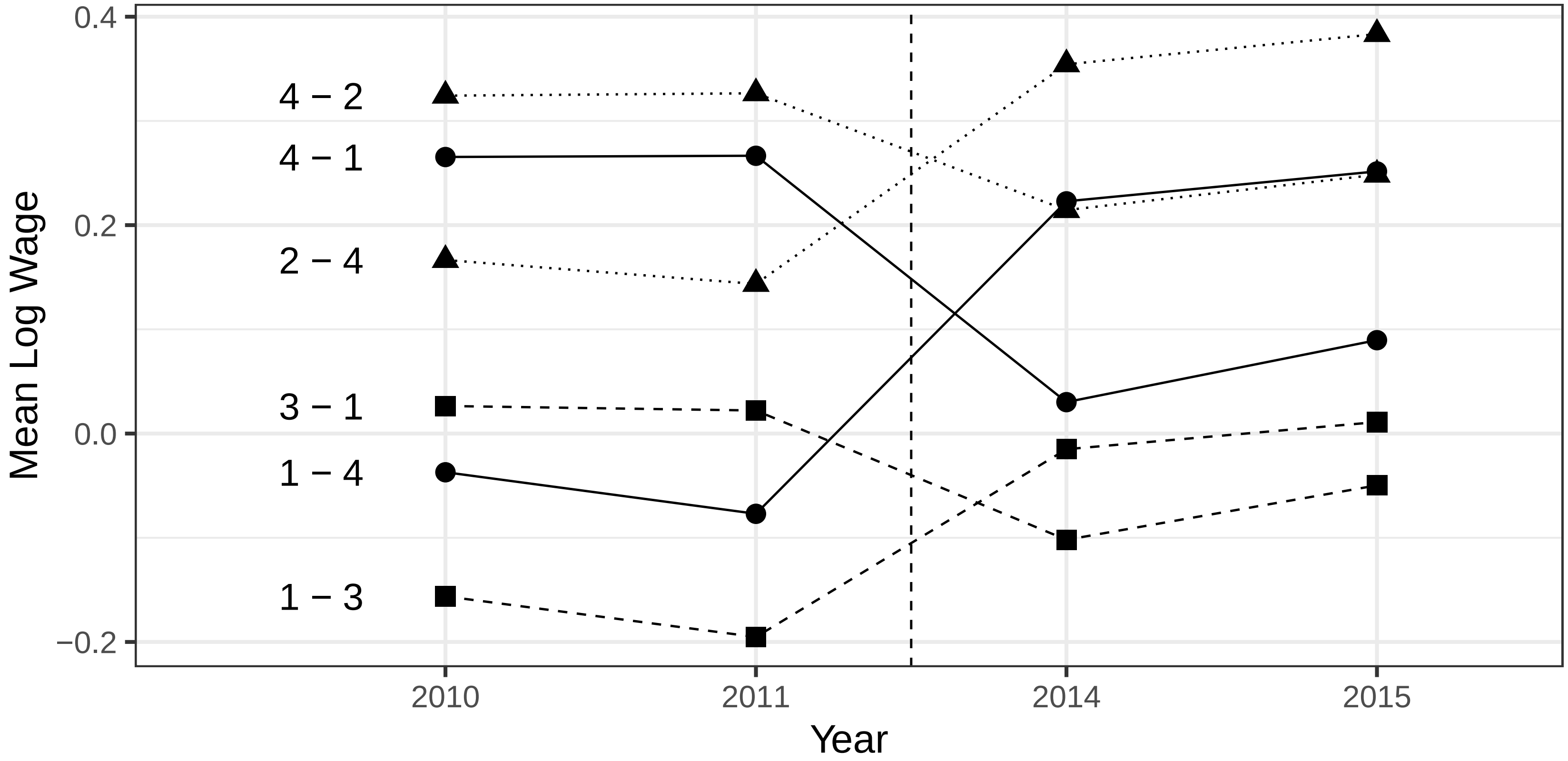
	firms	workers	stayers	movers	var(y)	btw firm	fe firm	fe cov
AT6thresh	206	3,396	2,645	1,123	0.187	45.5%	18.7%	2.4%
(6 years, threshold at mean(earnings)*.325)	140	3,240	2,536	1,055	0.182	43.7%	15.5%	4.3%
AT6full	118	2,641	2,392	355	0.235	48.1%	27.9%	-5.9%
(6 years, full-year)	62	2,395	2,192	297	0.223	45.4%	20.9%	-0.6%
AT3thresh	117	2,845	2,504	387	0.183	43.7%	19.7%	-2.7%
(3 years, threshold at mean(earnings)*.325)	68	2,604	2,309	336	0.178	41.8%	15.0%	0.7%
IT6thresh	92	1,111	855	379	0.167	46.1%	23.1%	-0.6%
(6 years, threshold at mean(earnings)*.325)	61	1,034	800	346	0.168	44.8%	19.3%	2.3%
IT6full	35	645	669	100	0.189	46.2%	25.8%	-7.8%
(6 years, full-year)	17	546	572	79	0.193	44.1%	17.9%	-1.0%
IT3thresh	54	864	733	148	0.176	44.9%	24.1%	-4.2%
(3 years, threshold at mean(earnings)*.325)	30	755	648	121	0.181	43.5%	18.5%	0.7%
SW6thresh	63	1,921	1,504	608	0.164	31.6%	14.6%	-4.1%
(6 years, threshold at mean(earnings)*.325)	52	1,850	1,440	596	0.164	30.9%	11.6%	-1.6%
SW6full	50	1,644	1,388	343	0.199	31.4%	15.9%	-6.1%
(6 years, full-year)	35	1,525	1,283	325	0.198	30.2%	10.4%	-1.4%
SW3thresh	42	1,497	1,285	237	0.161	31.3%	23.6%	-14.3%
(3 years, threshold at mean(earnings)*.325)	29	1,377	1,179	221	0.161	30.2%	15.5%	-7.0%
NO6thresh	114	1,286	913	556	0.239	47.2%	24.4%	-3.9%
(6 years, threshold at mean(earnings)*.325)	78	1,199	849	519	0.236	45.8%	19.2%	0.4%
US-national	2,568	55,464	45,519	14,888	0.414	39.6%	12.2%	0.5%
(6 years, 15000 dollars threshold)	1,689	52,484	43,124	13,968	0.416	38.8%	9.5%	2.9%
US-national-select 2010 2012	1,241	36,826	33,125	4,252	0.436	38.2%	16.3%	-6.0%
(3 years, 15000 dollars threshold)	670	33,031	29,863	3,645	0.440	37.6%	10.4%	-0.4%

Event study in CHK



Event study for the US

- We compute wage changes around worker moves across firms



Fixed-effects and limited mobility bias

• The AKM estimator of worker and firm effects is the least-squares estimator

$$\widehat{\gamma} = (A'A)^{-1}A'Y.$$

• We are interested in variance components, which can be written as quadratic forms in γ: $V_Q = \gamma' Q \gamma$, for some matrix Q. The AKM estimator of V_Q is

$$\widehat{V}_Q^{\mathsf{FE}} = \widehat{\gamma}' Q \widehat{\gamma}.$$

• The AKM estimator is biased, since

$$\mathbb{E}[\widehat{V}_Q^{\mathsf{FE}} \mid A] = V_Q + \underbrace{\mathbb{E}[\varepsilon' A(A'A)^{-1}Q(A'A)^{-1}A'\varepsilon \mid A]}_{\text{Bias due to noise}}.$$

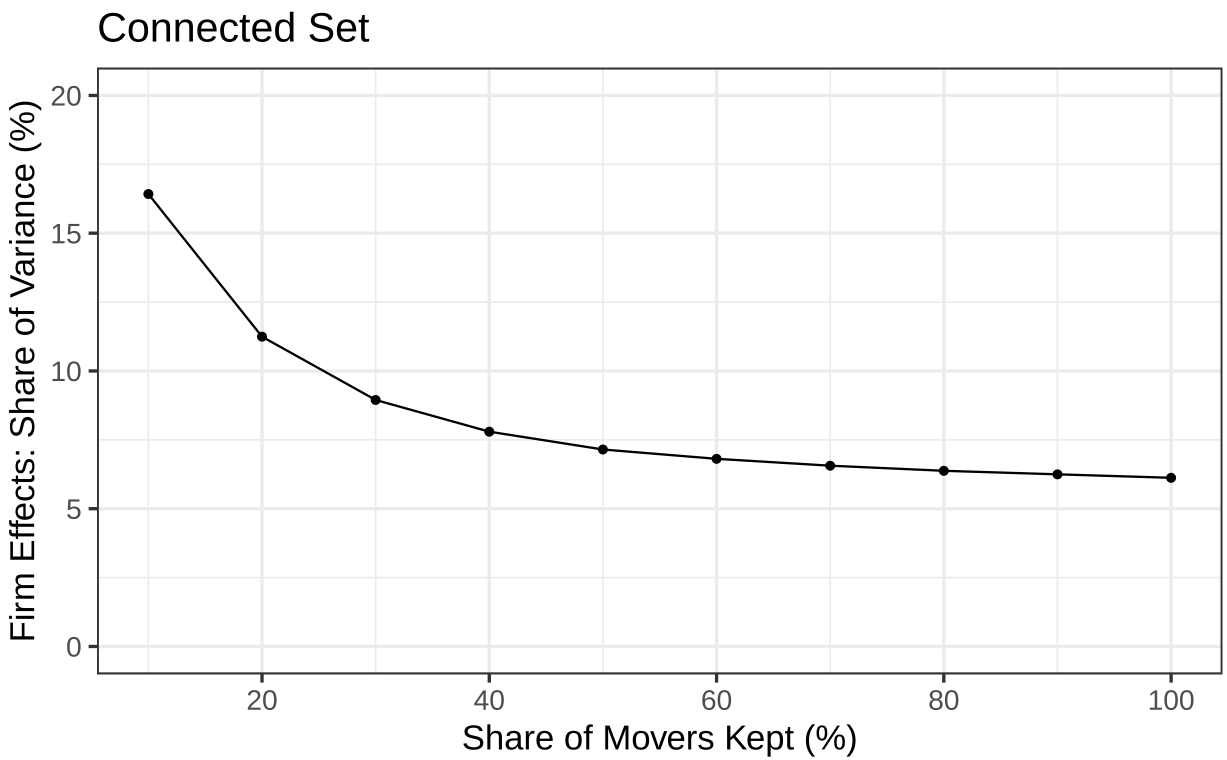
Intuition on the bias

• Intuitively: the bias arises from an insufficient number of job movers in the firm.

– The AKM variance of firm effects tends to be overstated.

– The covariance between worker and firm effects tends to be negatively biased, since worker effects and firm effects enter the model additively.

Preliminary evidence on limited mobility bias in the US
An exercise in the spirit of Andrews et al. (2008, 2012)



Notes: We select firms with more than 15 movers. We remove a given share of movers and re-compute the AKM estimator of the variance of firm effects. Details on the US data to come in a few slides.

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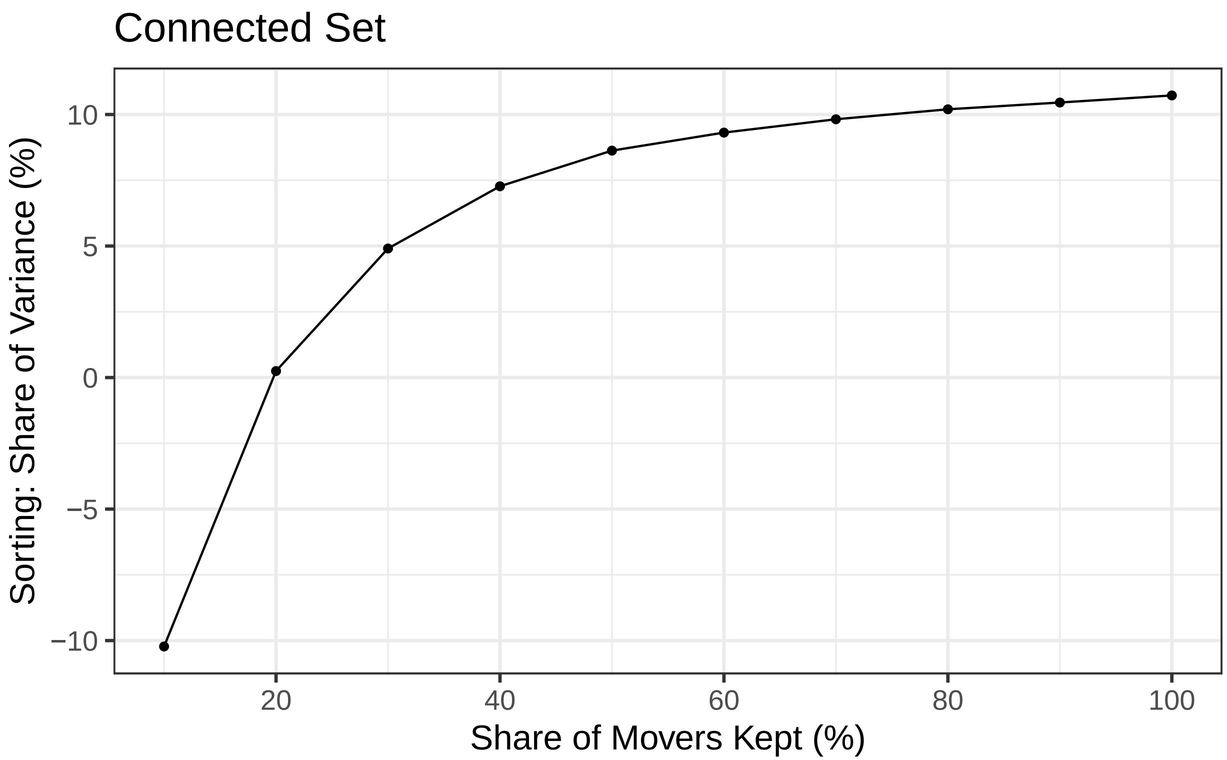

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Methods for bias correction

Fixed-effects bias correction

• Andrews *et al.* (08) note that the bias term is

$$\mathbb{E}[\varepsilon' A(A'A)^{-1}Q(A'A)^{-1}A'\varepsilon \,|\, A] = \text{Tr}\left(A(A'A)^{-1}Q(A'A)^{-1}A'\Omega(A)\right),$$

where $\Omega(A) = \text{Var}(\varepsilon \,|\, A)$.

• They propose an estimator of the bias in the homoskedastic case where $\Omega(A) = \sigma^2 I$. They construct: $\widehat{\sigma}^2\, \text{Tr}\left((A'A)^{-1}Q\right)$, for some $\widehat{\sigma}^2$.

• Gaure (14) proposes an heteroskedastic and clustered correction.

• Kline *et al.* (20) propose an heteroskedastic generalization based on the jackknife, maintaining independence across workers.

– They characterize the asymptotic properties of the resulting bias-reduced estimator of V_Q.

Random-effects approach

• Biases in AKM estimation reflect overfit. An alternative is to model the distribution of α and ψ given A, as in Woodcock (2008, 2015).

• Given mean and variance specifications $\mu(A) = \mathbb{E}(\gamma \mid A)$ and $\Sigma(A) = \mathrm{Var}(\gamma \mid A)$, a direct estimator of $\mathbb{E}(V_Q \mid A)$ is

$$\widehat{V}_Q^{\mathsf{RE}} = \mathrm{Tr}\left(\mathbb{E}_{\mu,\Sigma}(\gamma\gamma' \mid A)Q\right) = \mathrm{Tr}(\Sigma(A)Q) + \mu(A)'Q\mu(A).$$

• Alternatively, a posterior estimator is given by

$$\widehat{V}_Q^{\mathsf{P}} = \mathbb{E}_{\mu,\Sigma,\Omega}(\gamma' Q\gamma \mid Y, A),$$

where the prior means and variances are given by $\mu(A)$ and $\Sigma(A)$, and ε has mean 0 and variance $\Omega(A)$.

Random-effects estimation

• To estimate the parameters in $\mu(A)$ and $\Sigma(A)$ we rely on the moment restrictions

$$\mathbb{E}(Y \mid A) = A\mu(A), \quad \mathsf{Var}(Y \mid A) = A\Sigma(A)A' + \Omega(A).$$

• Interestingly, a fully flexible random-effects approach coincides with bias-reduced fixed-effects, since mean and variance restrictions imply

$$\mathbb{E}_{\mu,\Sigma}(\gamma\gamma' \mid A) = (A'A)^{-1}A'\mathbb{E}(YY' \mid A)A(A'A)^{-1} - (A'A)^{-1}A'\Omega(A)A(A'A)^{-1},$$

so

$$\widehat{V}_Q^{\mathsf{RE}} = \mathsf{Tr}\left(\mathbb{E}_{\mu,\Sigma}(\gamma\gamma' \mid A)Q\right) = \mathbb{E}(\widehat{V}_Q^{\mathsf{FE}} \mid A) - \underbrace{\mathsf{Tr}\left(A(A'A)^{-1}Q(A'A)^{-1}A'\Omega(A)\right)}_{\text{Bias reduction}}.$$

Random-effects specifications

1) Independent random-effects (Woodcock, 2008, 2015) is obtained when $\mu(A) = \mu$ and $\Sigma(A) = \Sigma$ do not depend on A, and Σ is diagonal.

– Posterior estimators enjoy robustness properties under misspecification. However, this "no-sorting" prior may be too informative.

2) We implement a correlated random-effects estimator where $\mu(A)$ and $\Sigma(A)$ depend on the worker-firm link indicators.

– We rely on the grouping strategy of Bonhomme *et al.* (2019), and impose that the means and variances of worker and firm effects are independent of link indicators conditional on firm groups.

– Key additional assumption: we assume that firm effects before and after the move are uncorrolated conditional on the groups.

Correlated random-effects with discrete types (CRE)

• Let $k_{it} = k(j(i,t))$ denote the group of the firm where i is employed in t. Following BLM we will group firms based on empirical distribution of wages.

• A being the matrix of worker and firm indicators, we specify $\mu(A)$ and $\Sigma(A)$ as follows:

$$\begin{aligned}
&\mathbb{E}[\alpha_i \,|\, A] = \mathbb{E}[\alpha_i \,|\, k_{i1}, ..., k_{iT}] = \mu_W(k_{i1}, ..., k_{iT}),\\
&\mathbb{E}[\psi_j \,|\, A] = \mathbb{E}[\psi_j \,|\, k_j] = \mu_F(k_j),\\
&\mathrm{Var}[\alpha_i \,|\, A] = \mathrm{Var}[\alpha_i \,|\, k_{i1}, ..., k_{iT}] = \sigma^2_W(k_{i1}, ..., k_{iT}),\\
&\mathrm{Var}[\psi_j \,|\, A] = \mathrm{Var}[\psi_j \,|\, k_j] = \sigma^2_F(k_j),\\
&\mathrm{Cov}[\psi_j, \psi_{j'} \,|\, A] = 0, \quad j \neq j'
\end{aligned}$$

• Importantly we allow for sorting within groups

$$Cov(\alpha_i, \psi_{j(i,t)}, A) \neq 0, \qquad \text{and} \qquad Cov(\alpha_i, \alpha_{i'} | j(i,t) = j(i',t'), A) \neq 0$$

Correlated random-effects with discrete types (cont.)

• Let's look at the implications for 2 movers leaving from same firm $j(i,1) = j(i',1) = j$ and moving to different firms j' and j''.

$$
\begin{aligned}
\text{Cov}[Y_{1i}, Y_{1i'}] &= \text{Cov}[\alpha_i + \psi_j,\, \alpha_{i'} + \psi_j] \\
&= \sigma^2_{\alpha\alpha'} + \sigma^2_{\psi} + 2\sigma^2_{\alpha\psi} \\
\text{Cov}[Y_{1i}, Y_{2i'}] &= \text{Cov}[\alpha_i + \psi_j,\, \alpha_{i'} + \psi_{j''}] \\
&= \sigma^2_{\alpha\alpha'} + \sigma^2_{\alpha\psi} \\
\text{Cov}[Y_{2i}, Y_{2i'}] &= \text{Cov}[\alpha_i + \psi_{j'},\, \alpha_{i'} + \psi_{j''}] \\
&= \sigma^2_{\alpha\alpha'}
\end{aligned}
$$

• In practice we combine all moments to extract within variances and co-variances.

Random-effects vs fixed-effects

• Although leaving worker and firm effects unrestricted is conceptually appealing, the large number of parameters causes overfit.

• Random-effects approaches induce automatic bias reduction. When based on parsimonious specifications, they allow making inference about the full sample, not only connected components.

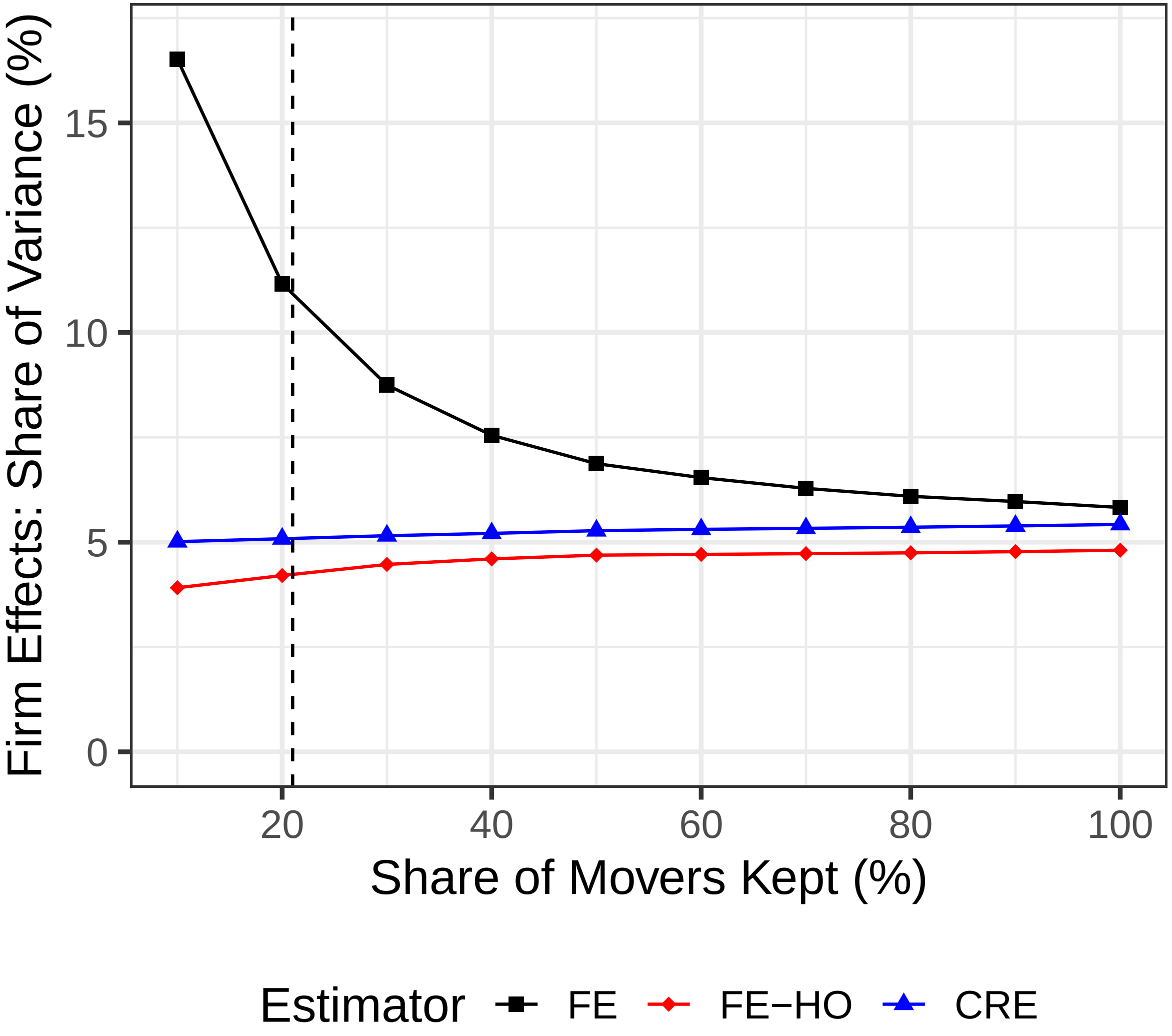
• Random-effects may be misspecified. Modeling worker and firm effects conditional on worker-firm link indicators is challenging.

• Posterior estimators provide simple specification checks.

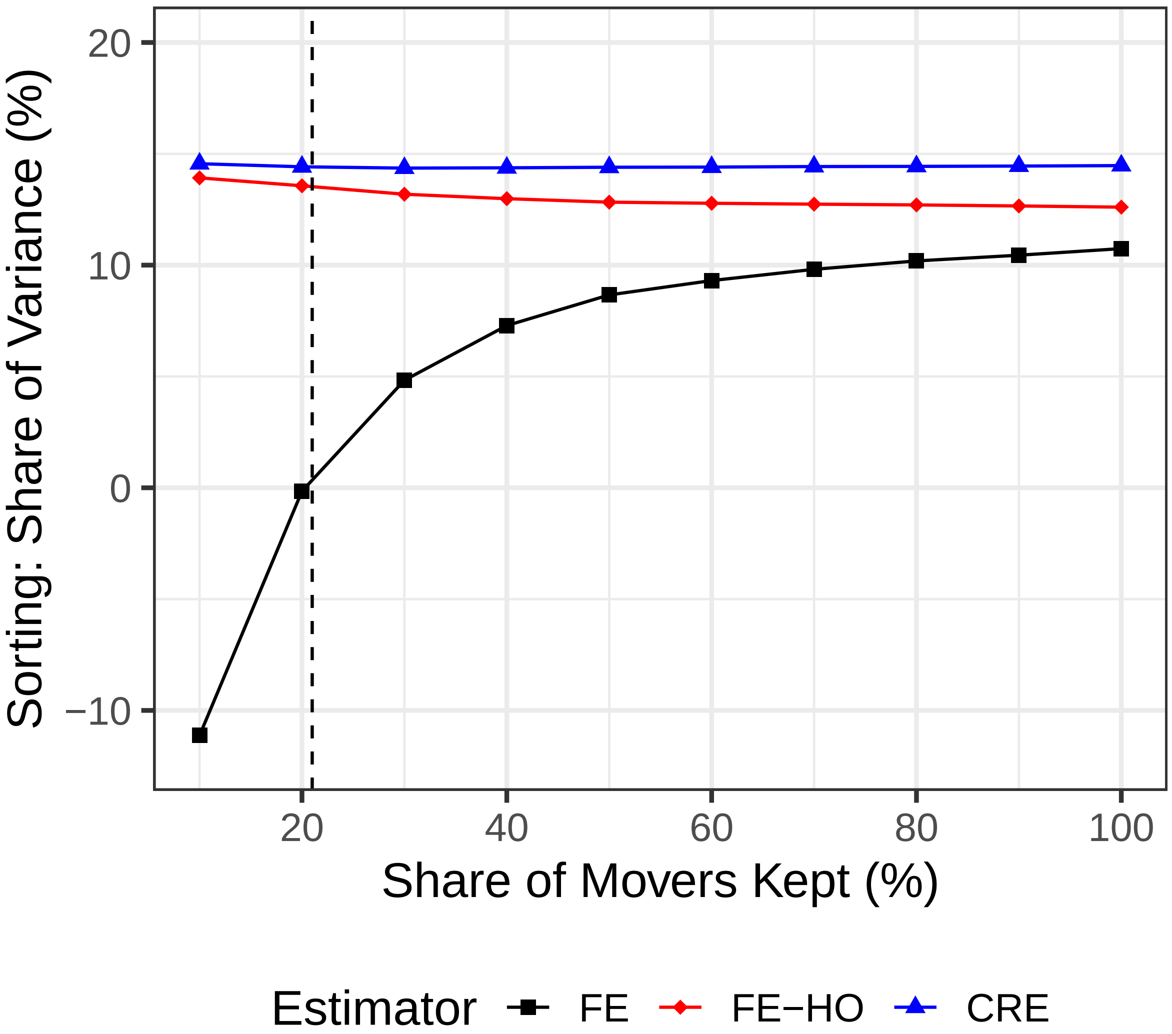
Computational considerations

- AKM boils down to inverting the sparse matrix of worker mobility
  - We use multigrid solver, but iterative solvers can do the job.
- Andrews et al. (08) in addition requires the computation of $Tr[(A'A)^{-1}Q]$
  - S. Gaure (14) proposes an approximation based on sampling.
- Kline et al. (20) in addition requires computation of leave-out connected set and leverages $P_i = A_i(A'A)^{-1}A_i'$ to construct jackknife etimates.
  - propose using a similar approximation, but requires many draws
- CRE requires a classification and computing simple covariances.
  - Classification based on clustering scales extremely well.

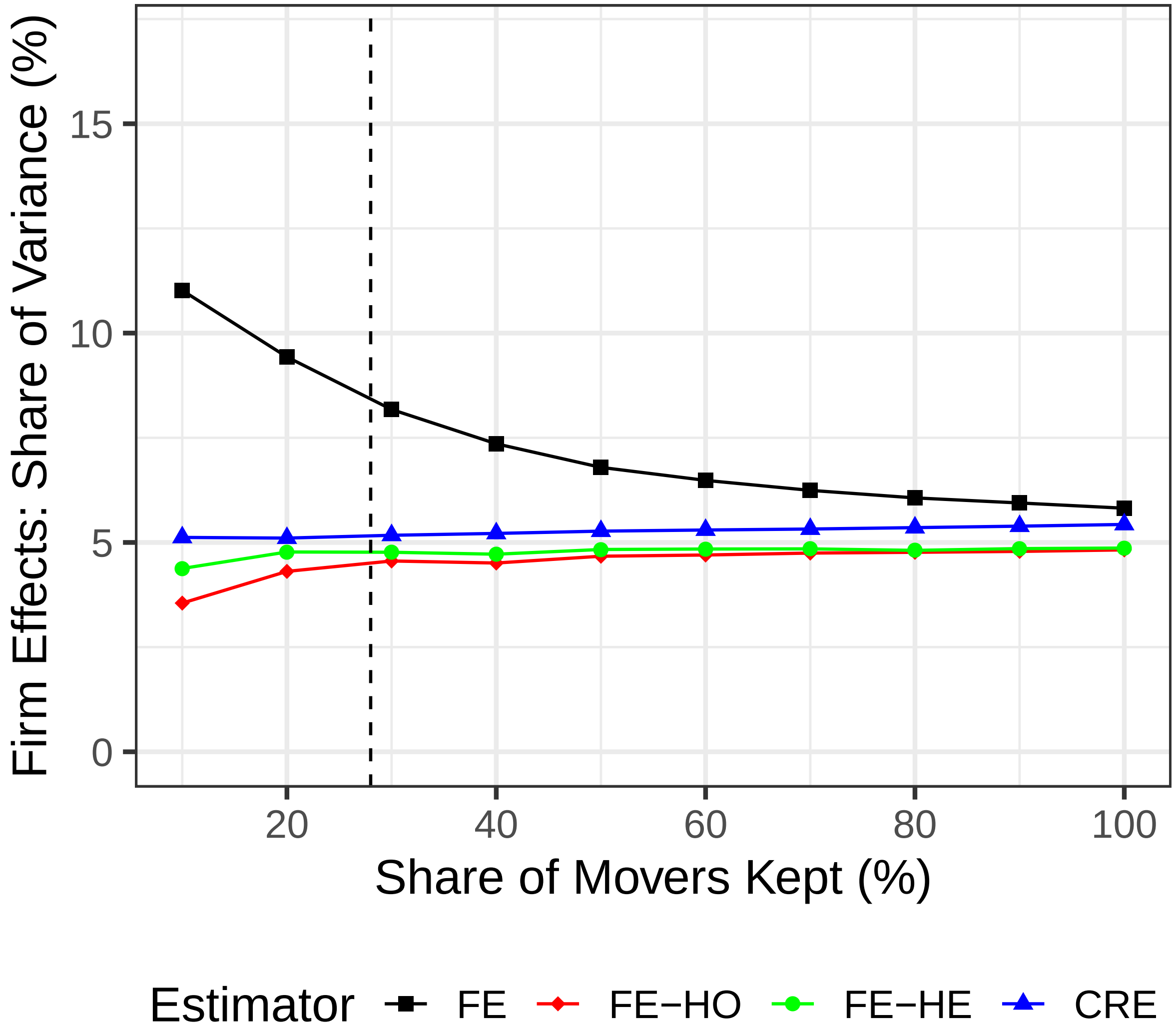
Preliminary evidence on bias correction in the US (cont.)



Preliminary evidence on bias correction in the US (cont.)



Preliminary evidence on bias correction on leave-out set



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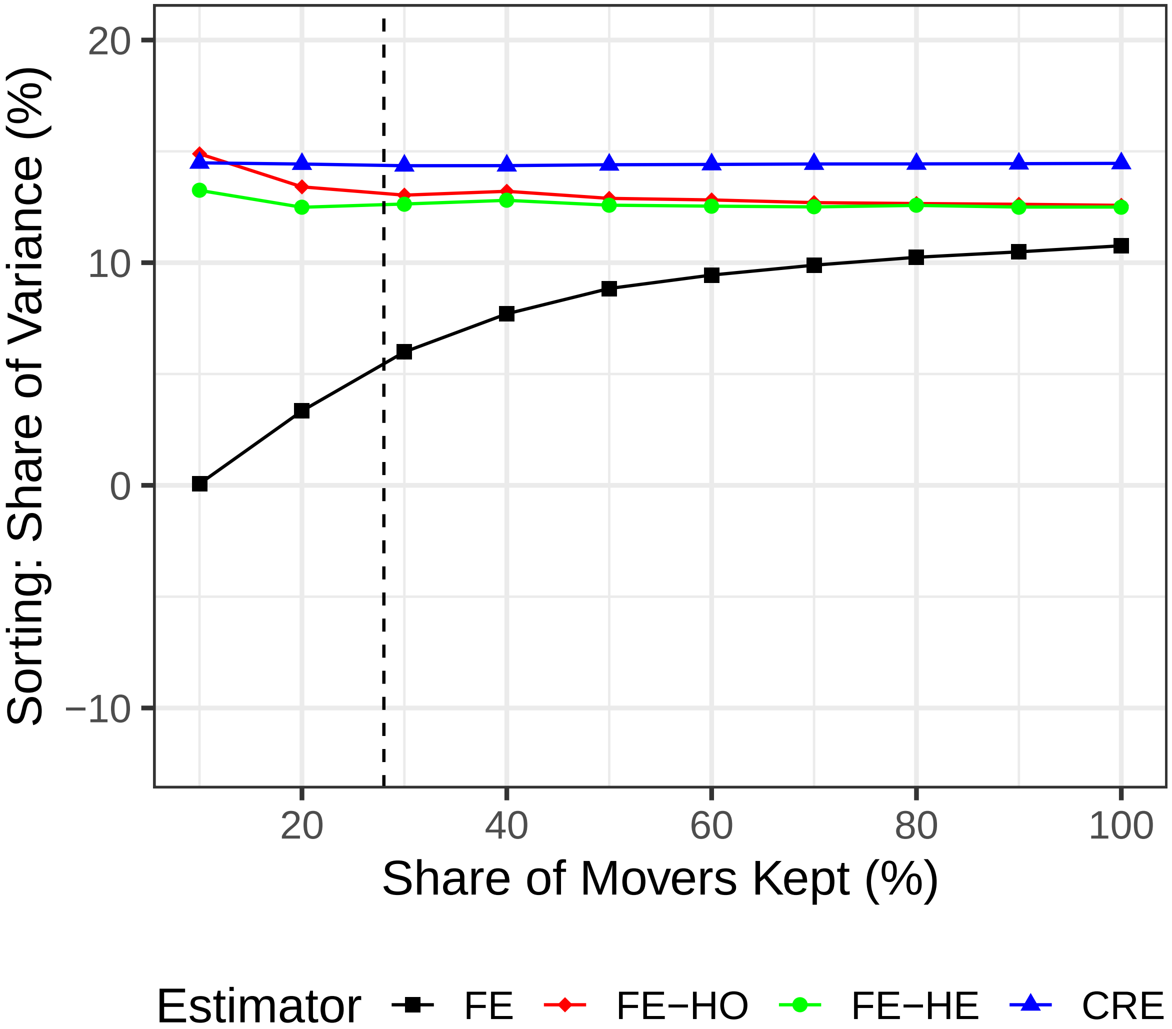
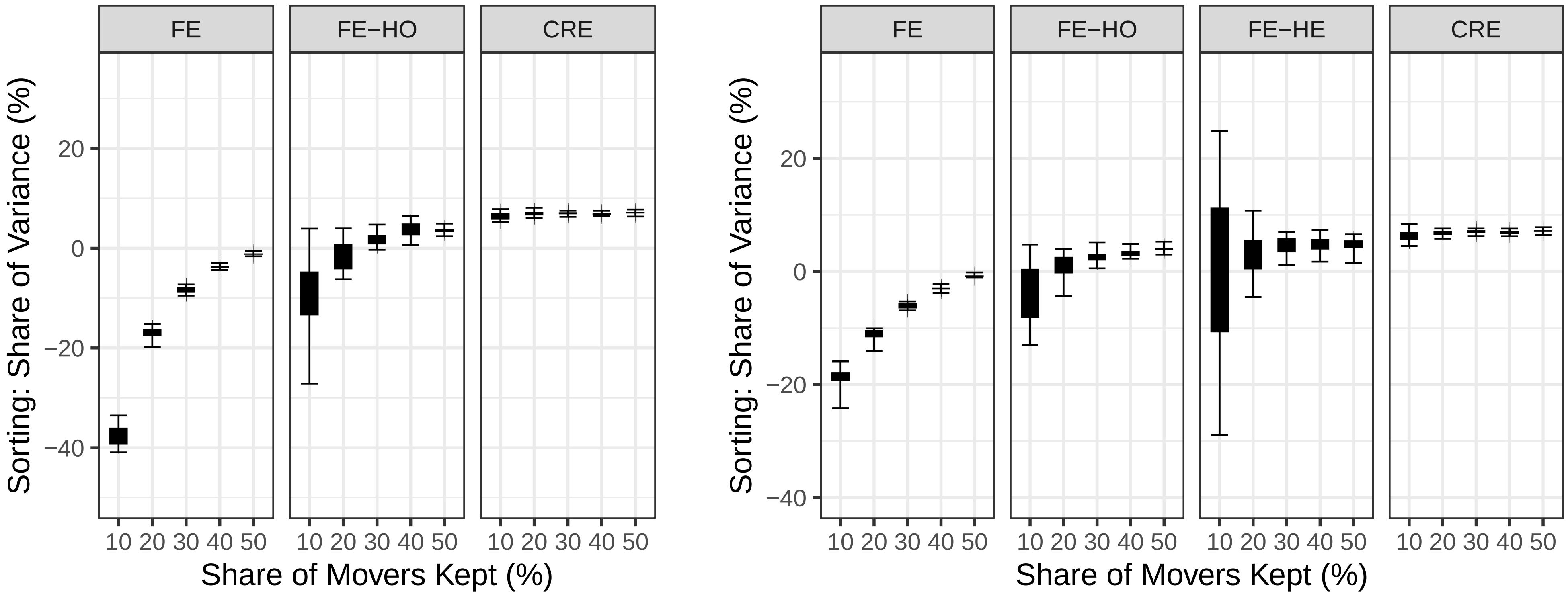
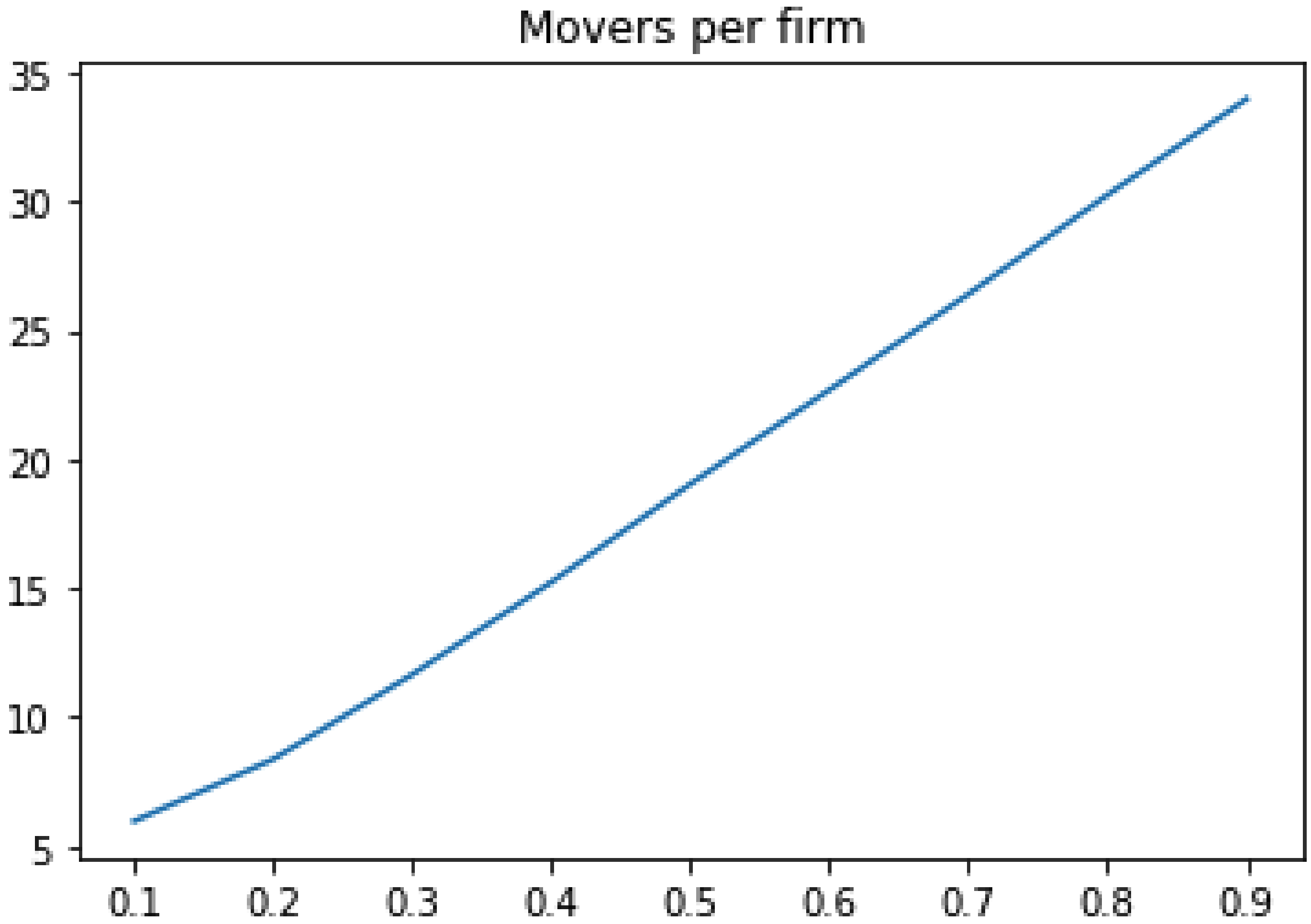


Variability of the estimators



Number of mover per firms in attrition exercice



A note on reported moments

• At this point we have only relied the fact that the ε_{it} are uncorrelated across individuals.

• In all methods, recovering $Var(\alpha_i)$ requires additional assumption on the panel structure ε_{it}

– For stayers for instance, in the presence of strong persistence, hard to separate ε from α_i.

– $Var(\varepsilon)$ independent of move is a possibility.

• At this stage, we will focus on $Var(\psi)$ and $Cov(\alpha, \psi)$

Evidence on limited mobility bias in the US

Data and sample: United States

- Population tax records (W-2, filed by firms):

– Data on annual earnings and worker-firm links, on 2001-2015.

– In each year, we select the highest-paying firm. Earnings include taxable salaries, bonuses, etc.

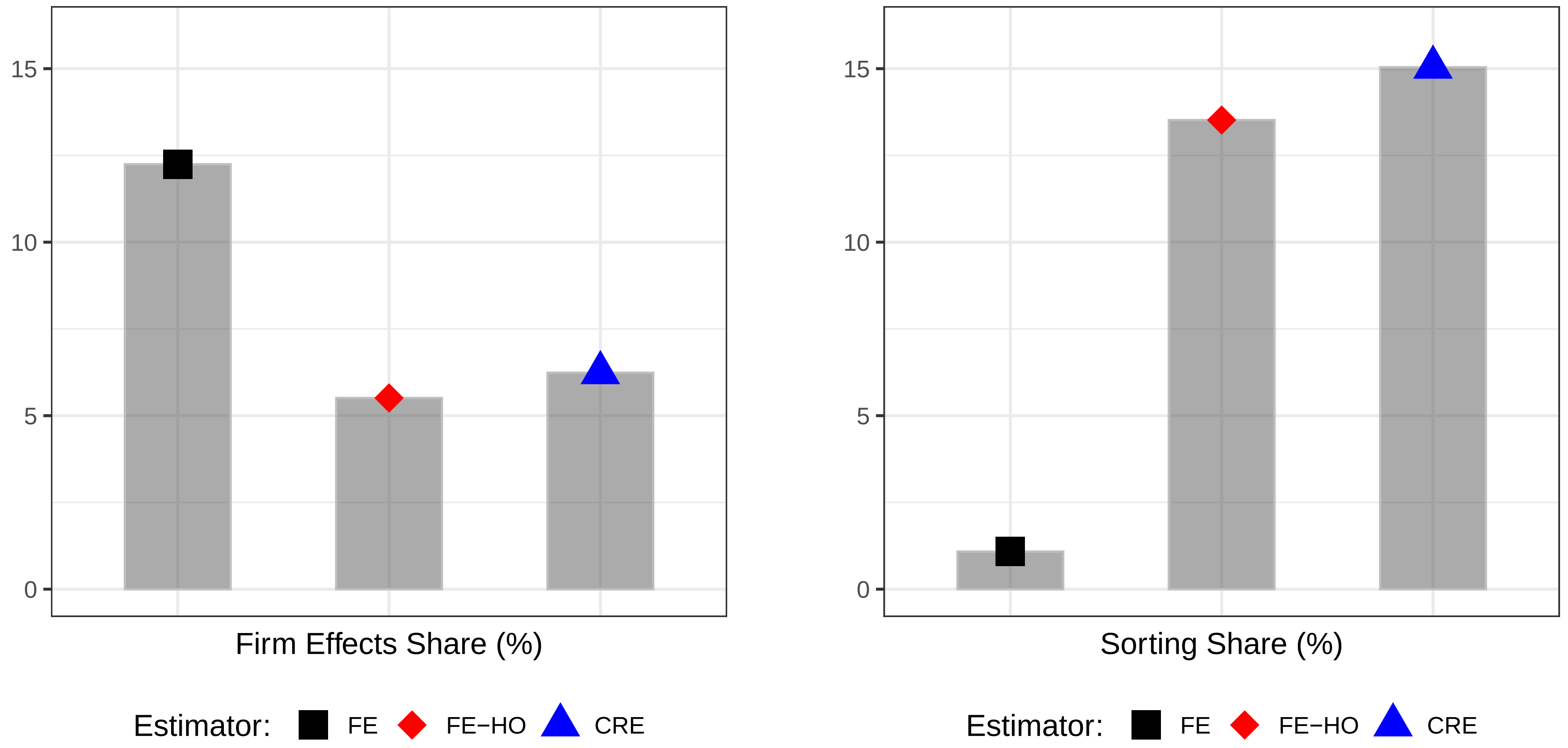
- Estimation samples:

– Full sample: Aged 25-60, with earnings greater than full-time minimum-wage equivalent. Private sector firms only, no self-employed, no contractors.

– We compute spell averages of log-wages (unweighted) and construct event-study type data (before/after move spell earnings)

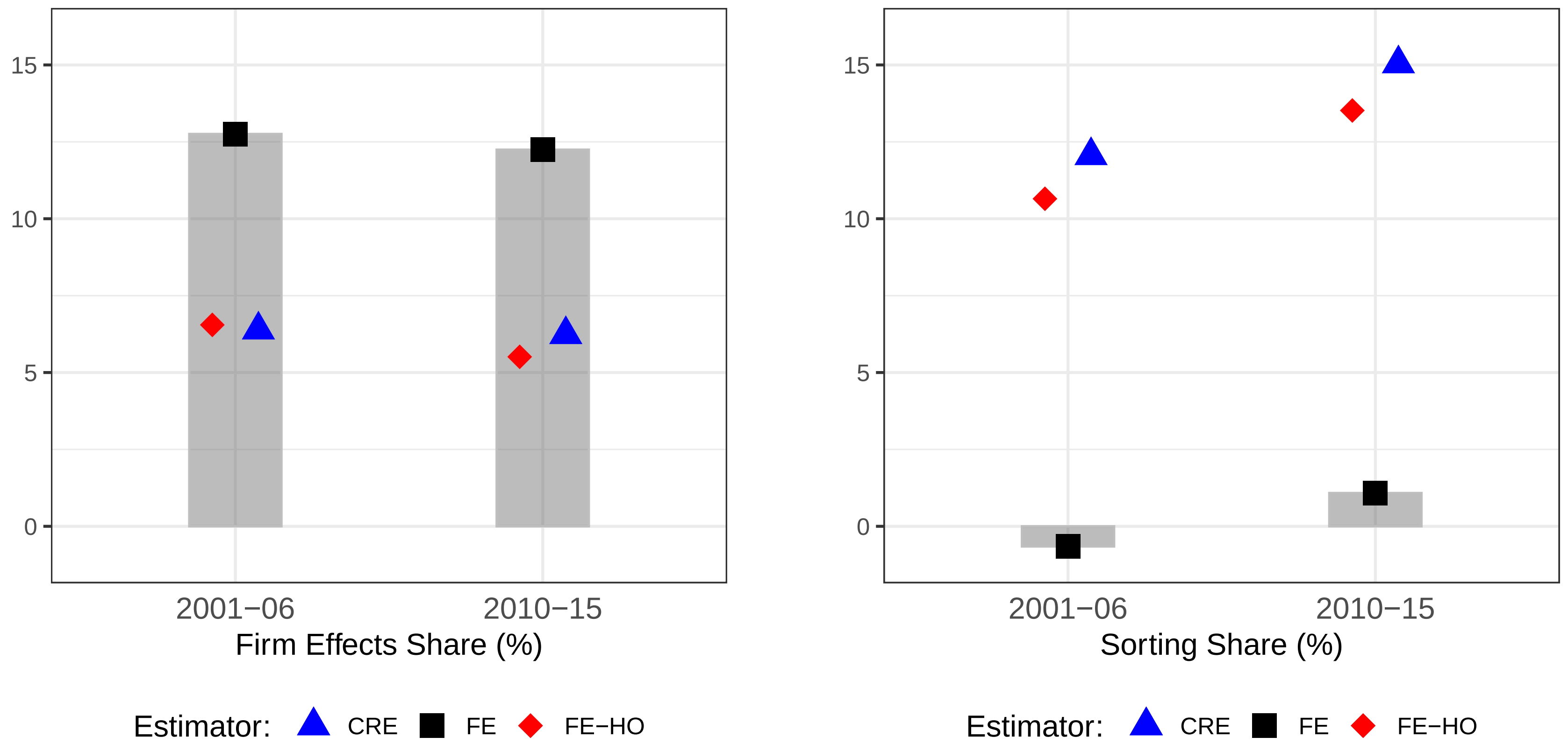
– We compute regression residuals on a cubic in age and time dummies ($R^2 = 3\%$).

Fixed-effects versus correlated random-effects estimates



Notes: FE is the AKM estimator. FE-HO is the bias-corrected estimator of Andrews et al. (2008). CRE is the correlated random-effects estimator allowing for within-group firm heterogeneity.

Earnings inequality over time: 2001-2006 versus 2010-2015



Notes: Over this period, the variance of log-earnings increased from 0.39 in 2001-2006 to 0.41 in 2010-2015.

Evidence on limited mobility bias in other countries

Data and sample: Other countries

- Sweden: Administrative records, 1997-2007. Universe of workers and firms. Yearly earnings and months worked in a given firm.

- Austria: Administrative records, 2000-2016. Universe of workers and firms. Yearly earnings and months worked in a given firm.

- Italy: Veneto Worker Histories, 1975-2001. Workers who ever worked in Treviso and Vincenza (provinces of Veneto). Workers are followed across other provinces of Veneto.

- Norway: Administrative records, 2011-2013. Ongoing: administrative records from Spain and Germany.

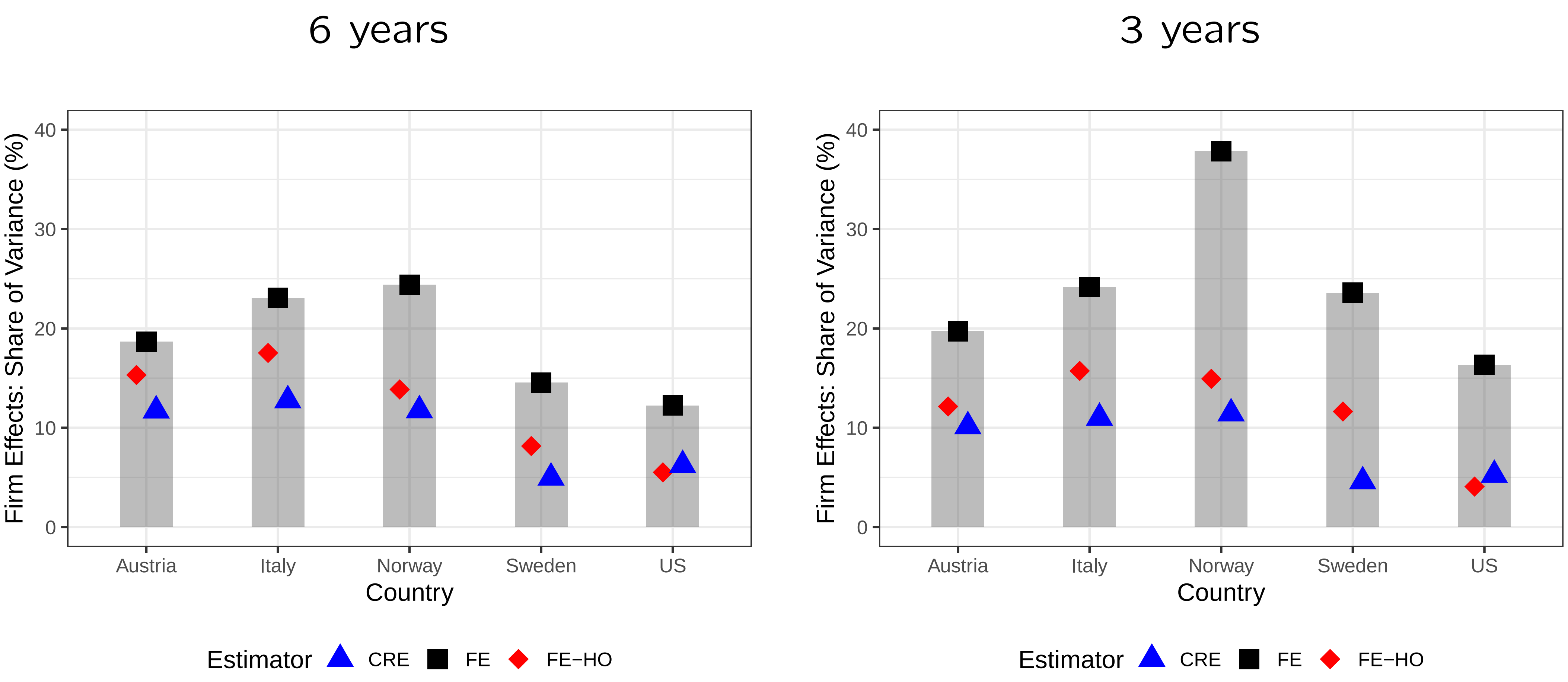
- Comparable sample selection and employment definition as for the US.

Sample Characteristics

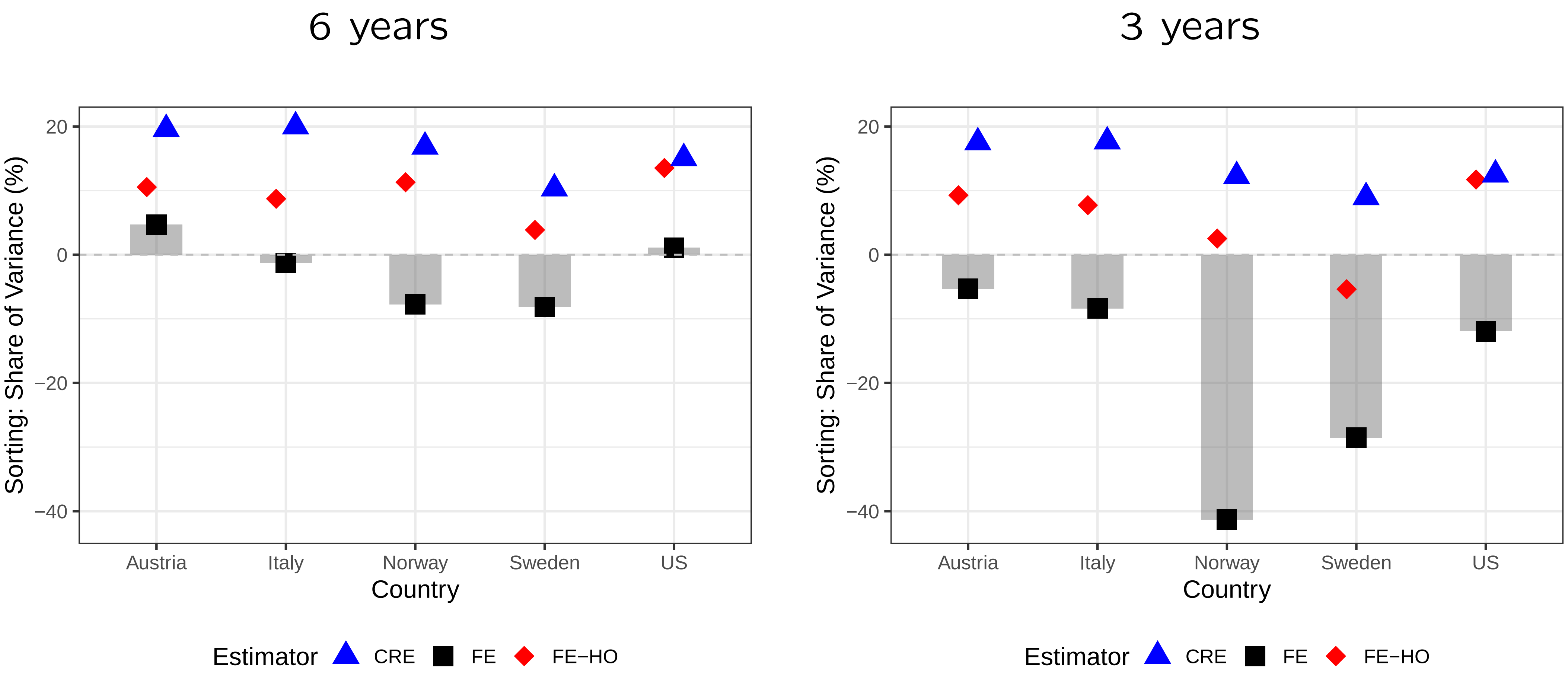
	Austria			Italy			Norway			Sweden			US		
Set:															
Baseline Years	2010-2015			1996-2001			2009-2014			2000-2005			2010-2015		
Full Set	✓	×	×	✓	×	×	✓	×	×	✓	×	×	✓	×	×
Connected Set	×	✓	×	×	✓	×	×	✓	×	×	✓	×	×	✓	×
Leave-one-out Set	×	×	✓	×	×	✓	×	×	✓	×	×	✓	×	×	✓
Sample Counts (1,000):															
Unique Firms	446	206	140	198	92	61	233	114	78	136	63	52	7,565	2,568	1,689
(Share of Full Set)	(100%)	(46%)	(31%)	(100%)	(47%)	(31%)	(100%)	(49%)	(34%)	(100%)	(46%)	(38%)	(100%)	(34%)	(22%)
Unique Workers	3,582	3,396	3,240	1,188	1,111	1,034	1,379	1,286	1,199	1,979	1,921	1,850	59,621	55,464	52,484
(Share of Full Set)	(100%)	(95%)	(90%)	(100%)	(94%)	(87%)	(100%)	(93%)	(87%)	(100%)	(97%)	(93%)	(100%)	(93%)	(88%)
Distribution of Moves:															
Moves per Firm	2	5	8	2	4	6	2	5	7	4	10	11	2	6	8
Worker-weighted quantiles:															
10th Quantile	4	4	5	3	3	4	3	3	4	4	5	6	3	4	5
50th Quantile	52	51	56	22	22	25	26	26	29	77	77	82	56	58	67
90th Quantile	605	605	629	313	311	326	397	399	420	2,354	2,352	2,484	4,214	4,304	4,676
Log Earnings Distrib.:															
Variance	0.195	0.187	0.182	0.169	0.167	0.168	0.241	0.239	0.236	0.164	0.164	0.164	0.413	0.414	0.416
Between-firm Share	43%	46%	44%	46%	46%	45%	47%	47%	46%	31%	32%	31%	40%	40%	39%

Notes: This table presents characteristics of the connected set and the leave-one-out set for employer-employee administrative data from 5 countries.

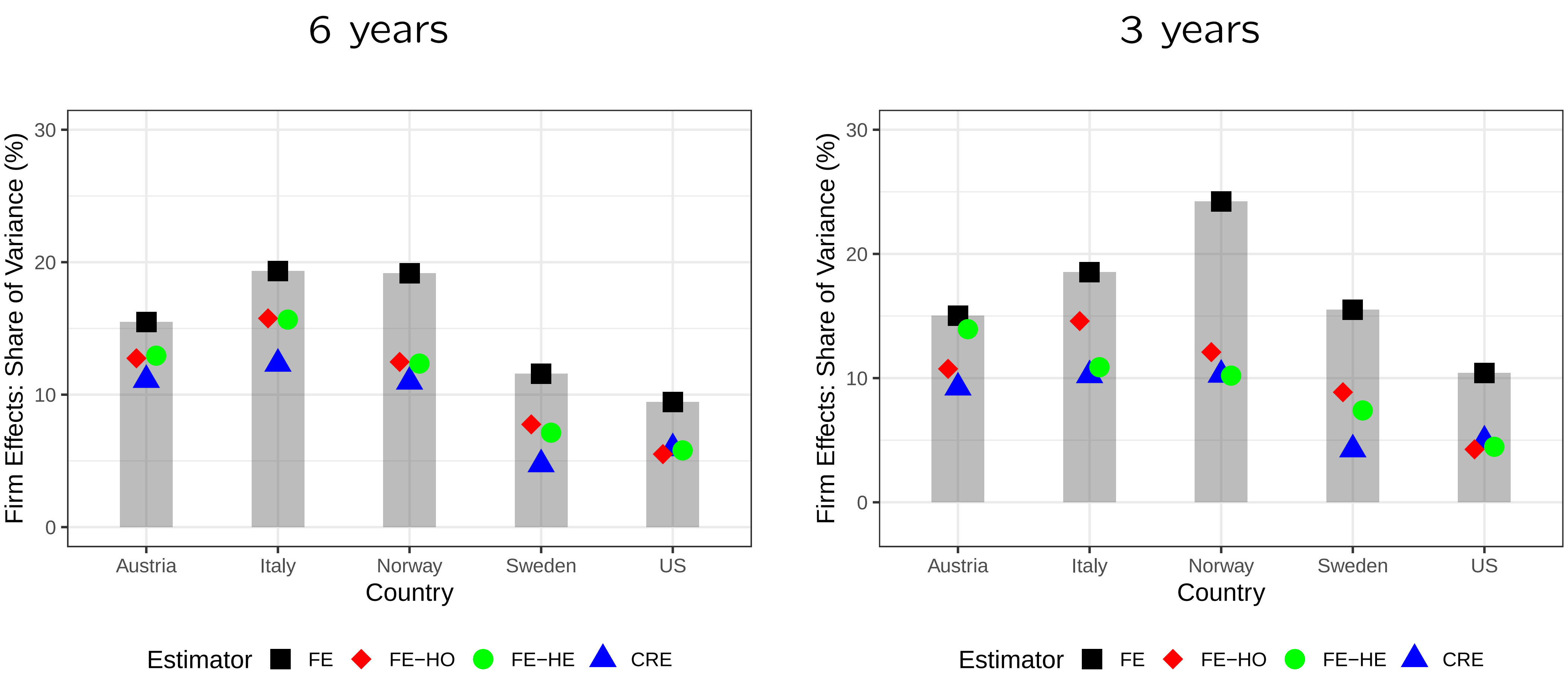
Firm effects estimates (connected set) 6 and 3 years



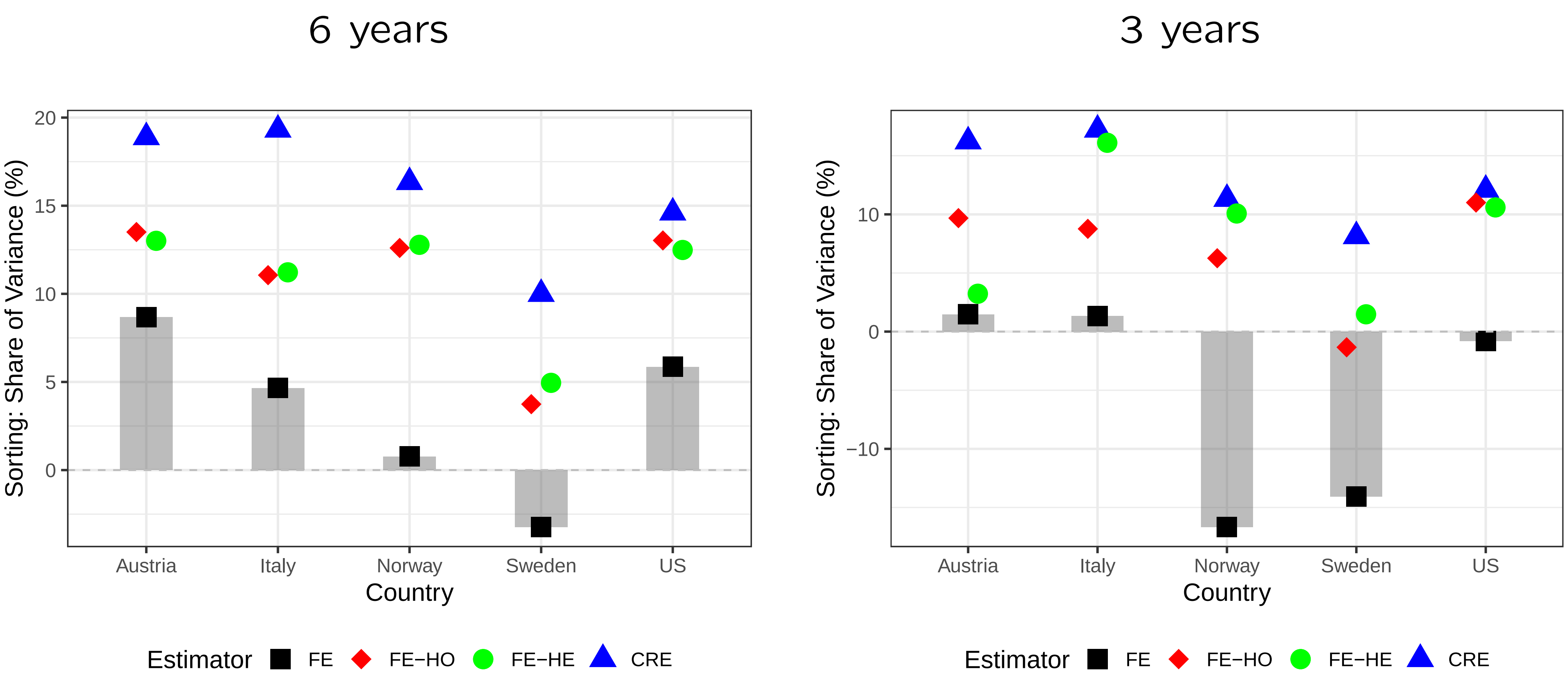
Sorting estimates (connected set) 6 and 3 years



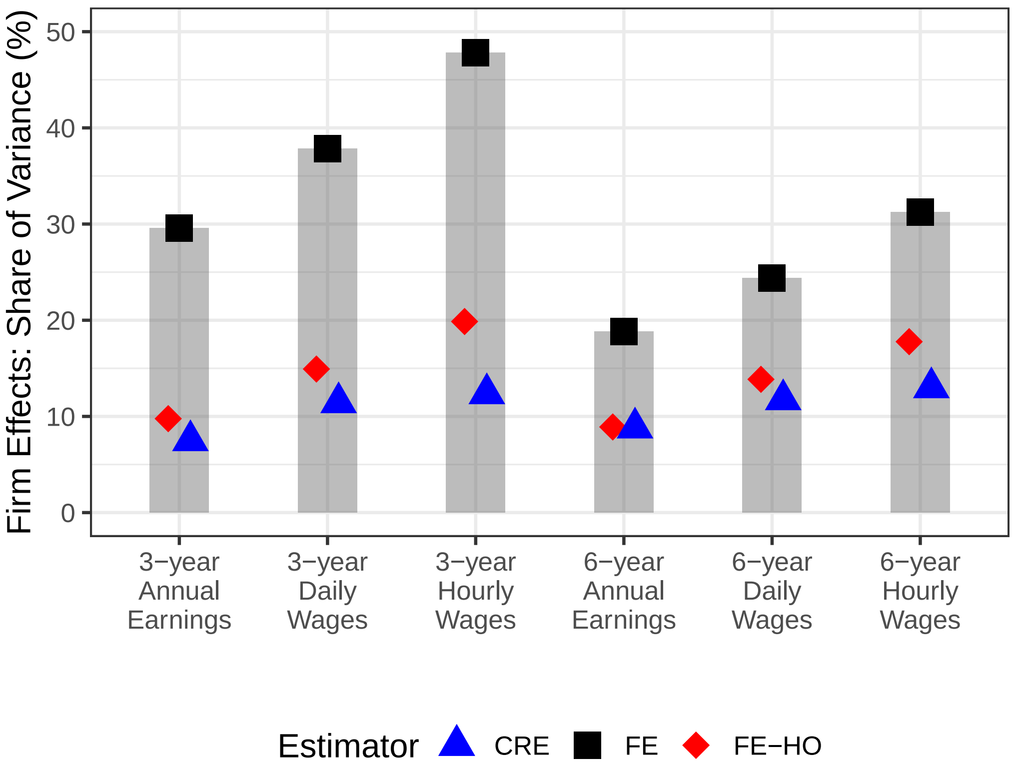
Firm effects estimates (leaveout set) 6 and 3 years

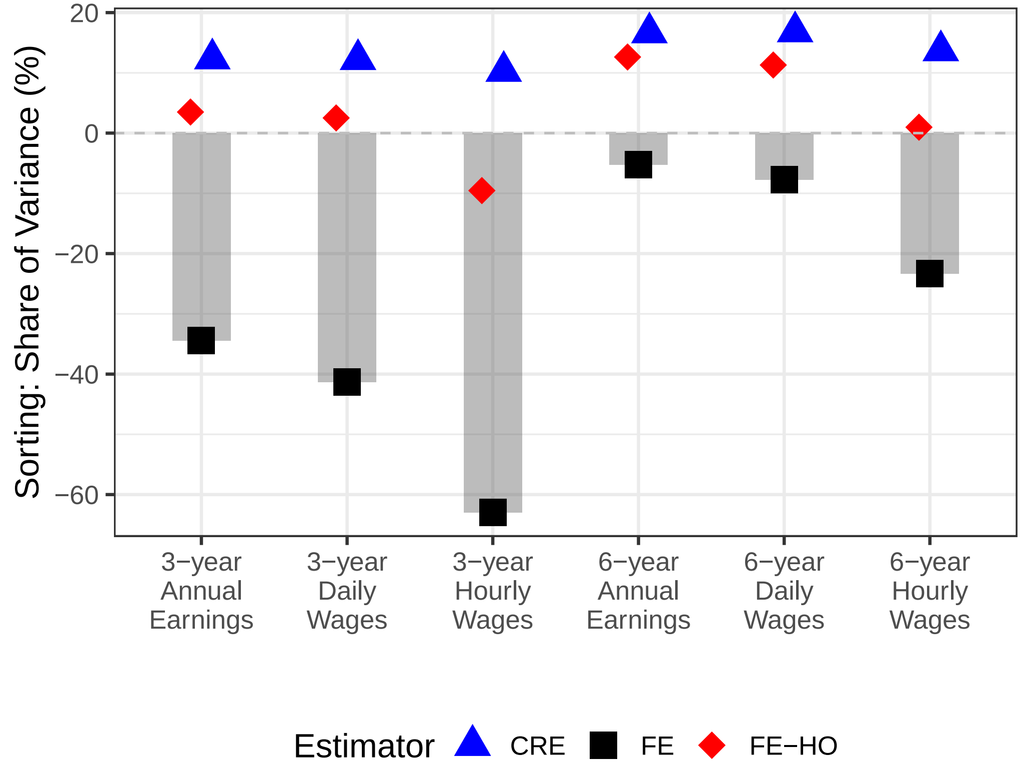


Sorting estimates (leaveout set) 6 and 3 years



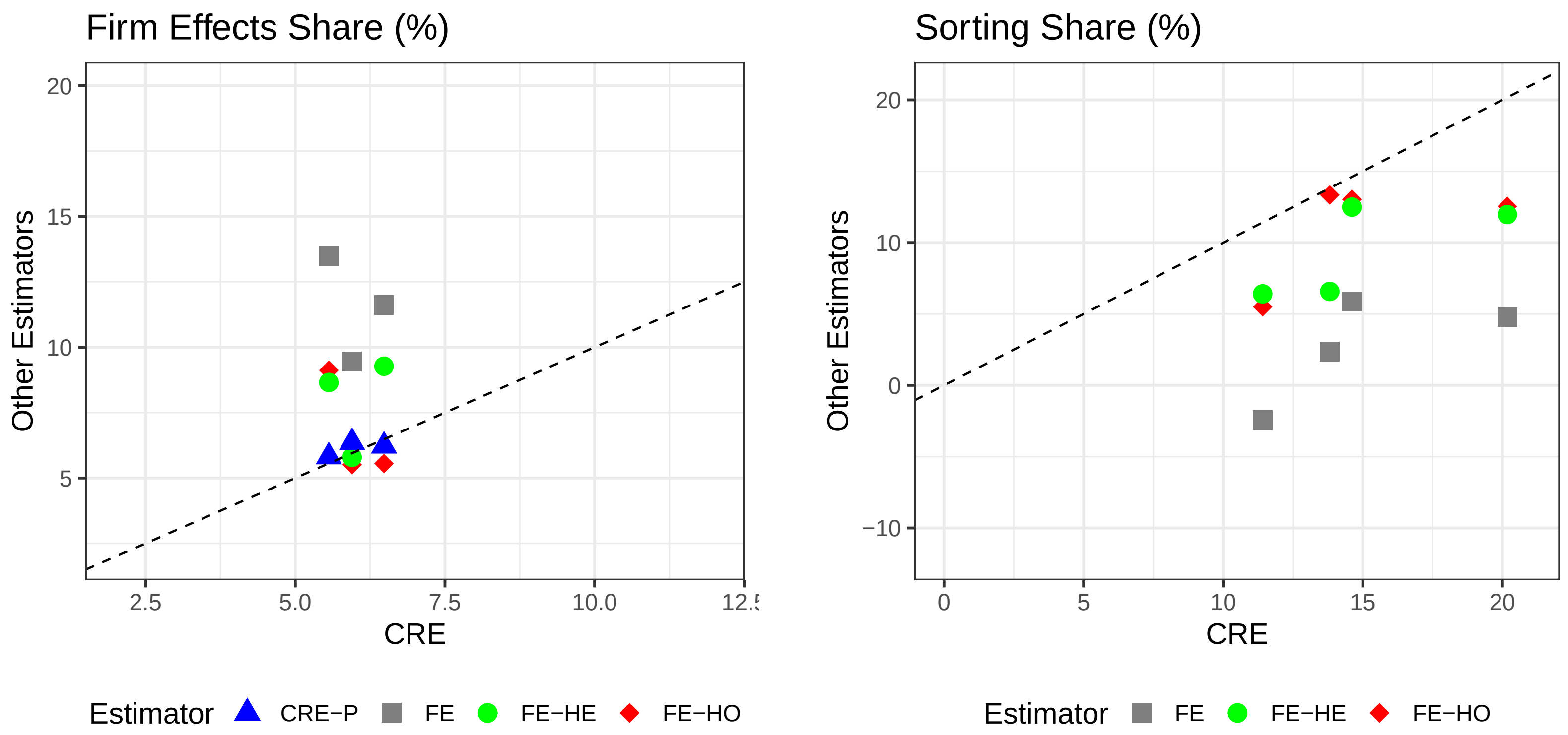
Using Wages in Norway





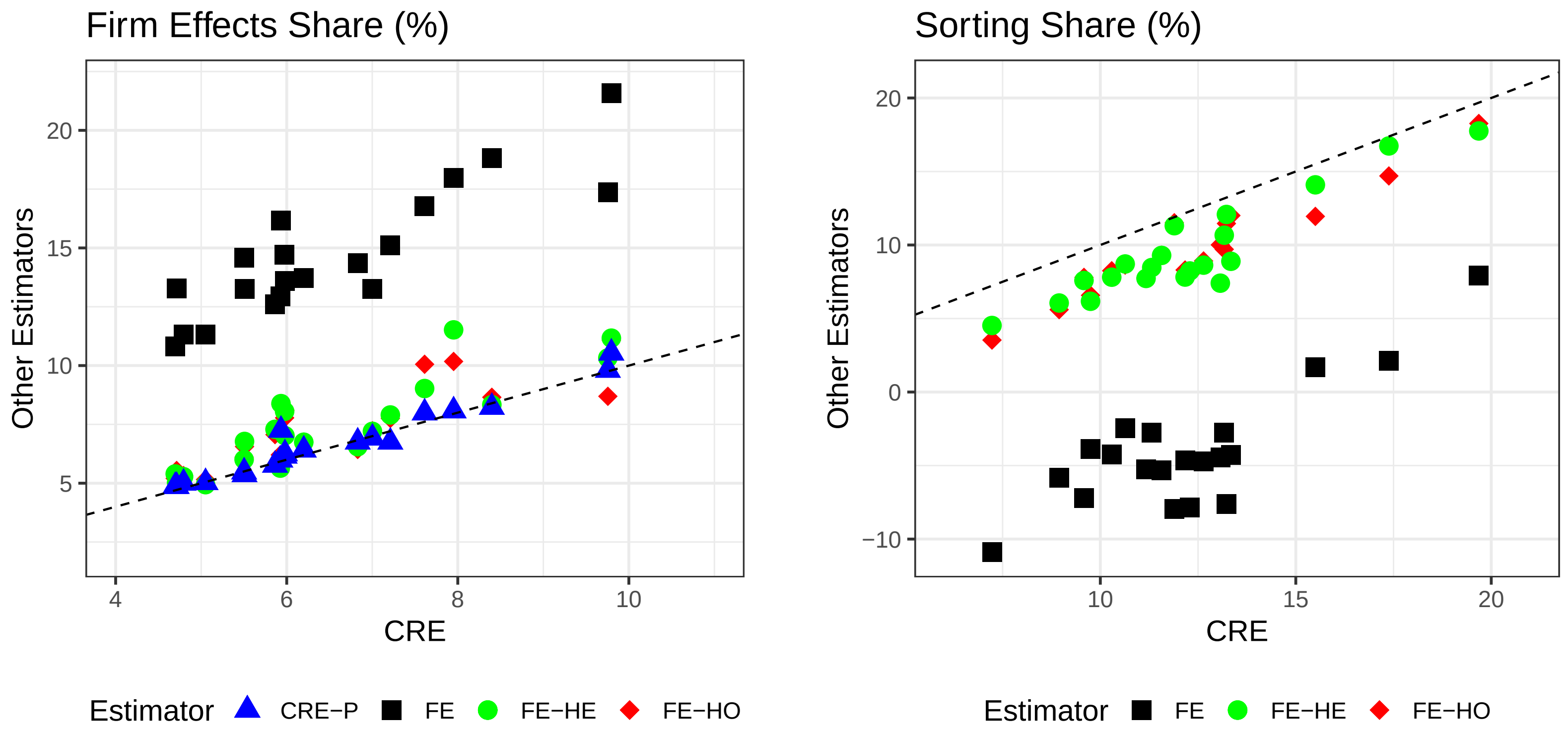
Robustness

CRE and Posterior (leave-one-out set)



Notes: AKM estimator in grey. FE-HO is the bias-corrected estimator of Andrews et al. (2008). FE-HE is the bias-corrected estimator of Kline et al. (2019). CRE-P is the posterior correlated random-effects estimator allowing for within-group firm heterogeneity. Baseline CRE on x-axis.

CRE vs Posterior and Other Estimators: Small U.S. States



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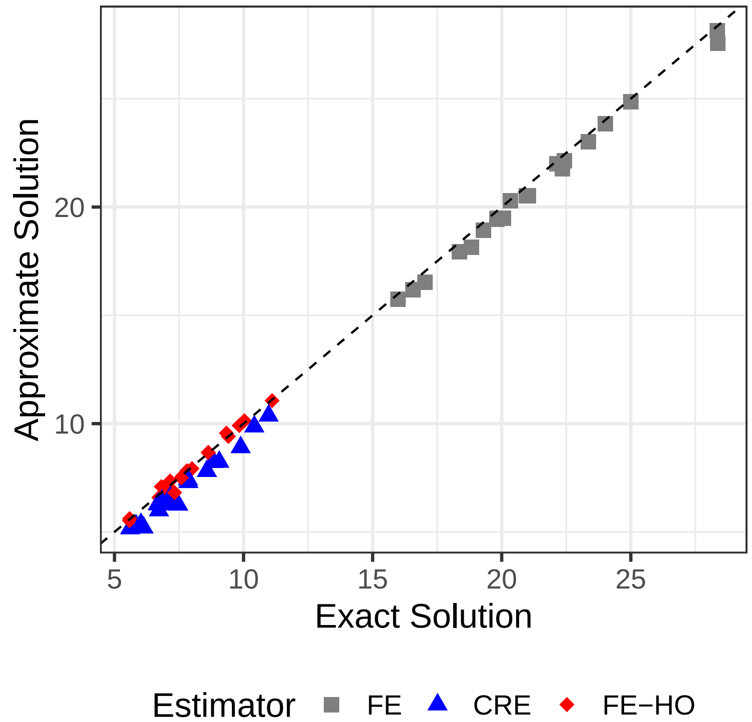
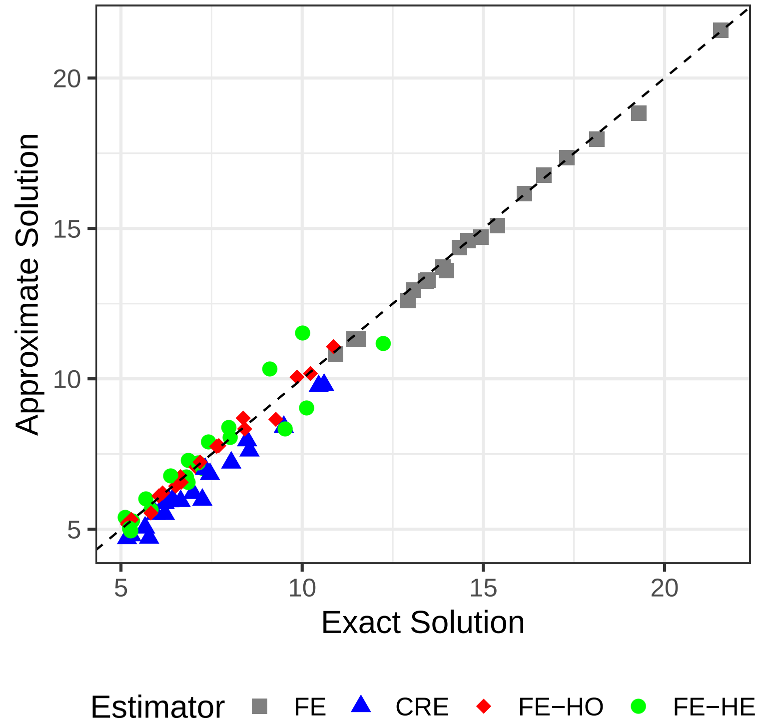
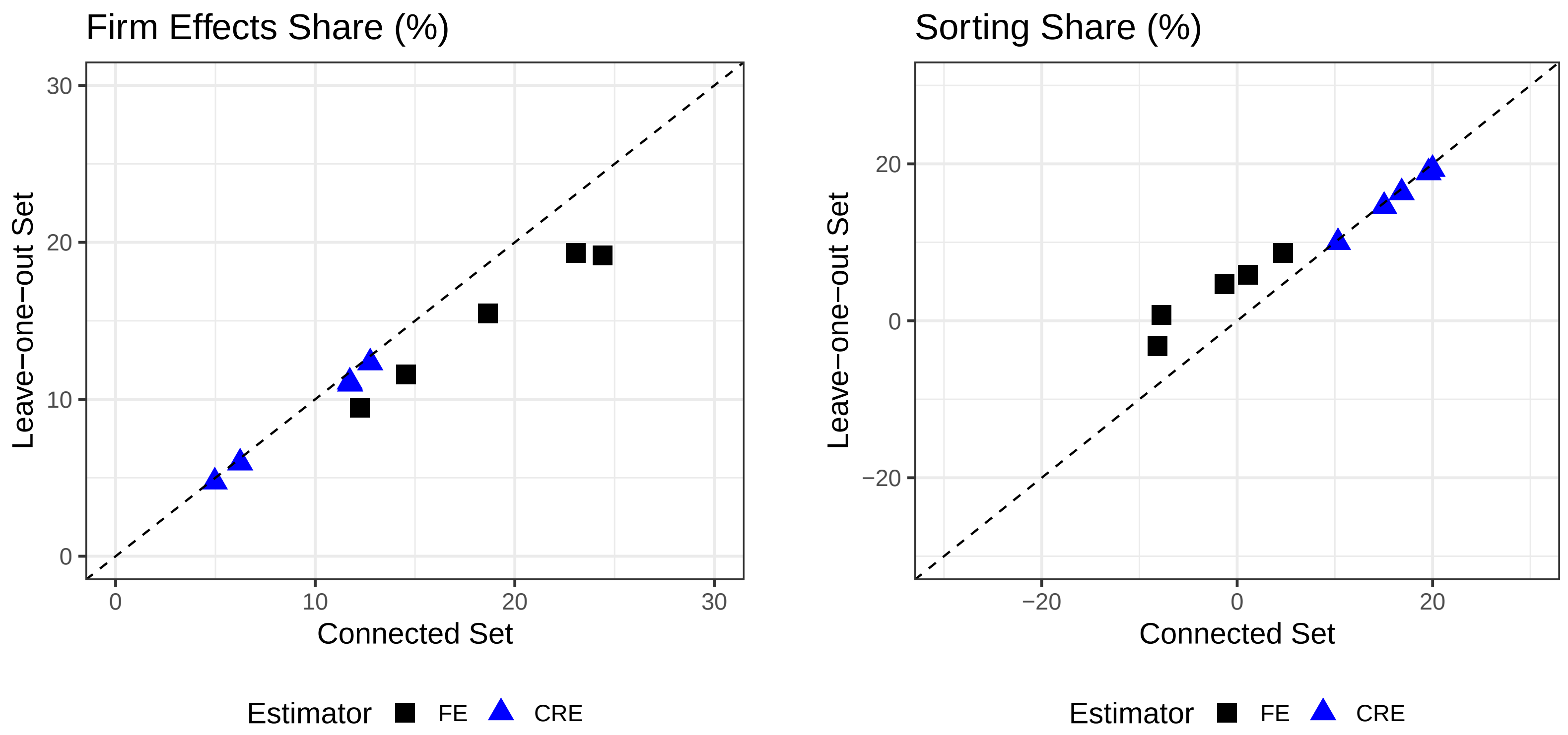
Exact vs Approximate Solutions on Small U.S. States





Notes: AKM estimator in grey. FE-HO is the bias-corrected estimator of Andrews et al. (2008). FE-HE is the bias-corrected estimator of Kline et al. (2019). CRE is the correlated random-effects estimator allowing for within-group firm heterogeneity.

Connected vs Leave-one-out Set: 5 Countries



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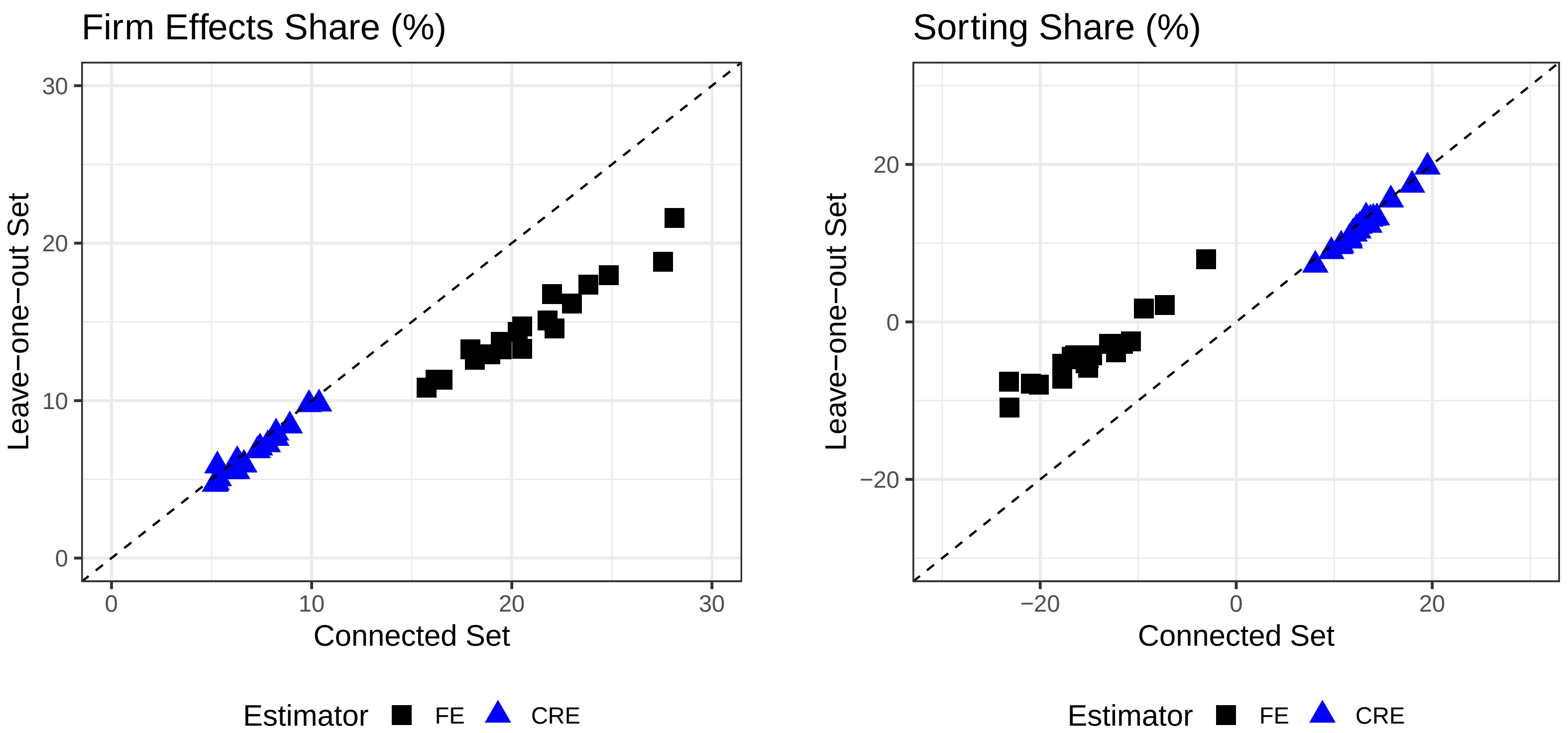
Connected vs Leave-one-out Set: 20 Small U.S. States



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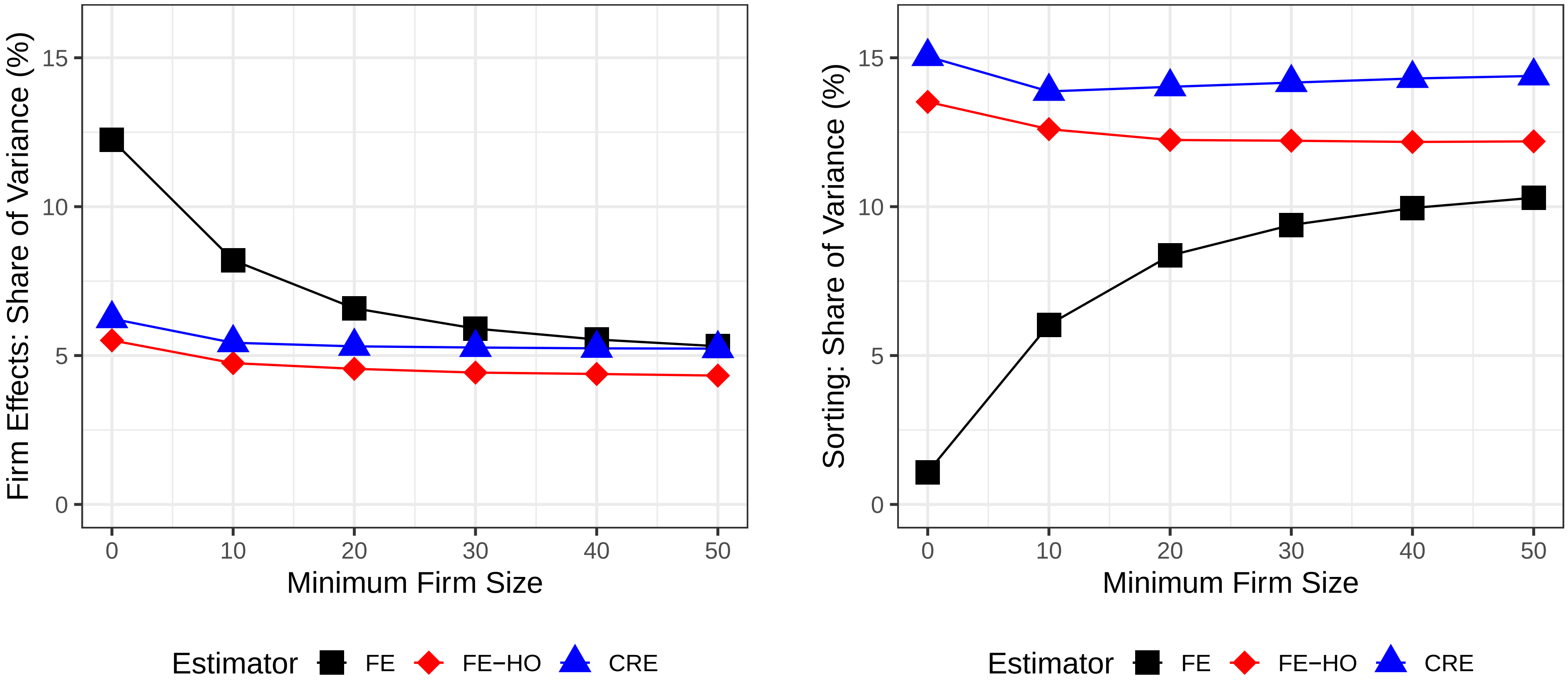
Firm Size Restrictions in the US (Connected Set)



Summary

Conclusion

• Research question: How much should we trust estimates of firm effects and worker sorting?

• Our results suggest: not so much...

• The previous literature that relies on AKM estimates suggests the coexistence of weak sorting and large firm effects.

• Our results based on several recently proposed methods suggest a different picture: relatively small firm effects and strong sorting patterns.

$\Rightarrow$ How to sctructuraly interpret such patterns? (Lamadon, Mogstad and Setzler, 2019)

Going forward

• On the methodological side, random-effects estimators may provide valuable alternatives to fixed-effects estimators, which are prominent in matched data settings.

• Random-effects specifications are more parsimonious, and can be used to extend the AKM model in important directions:

– To allow for worker-firm interactions, or for firm shocks, worker productivity processes and other dynamic patterns.

Additional slides

Estimates of a nonlinear model in the US, over time

Notes: Estimates of an interacted regression model using the estimator of Bonhomme et al. (2019) with $K = 10$ groups.

Correlated random-effects with discrete types (cont.)

• We estimate the parameters using minimum distance based on the previous mean and variance restrictions (with some weights).

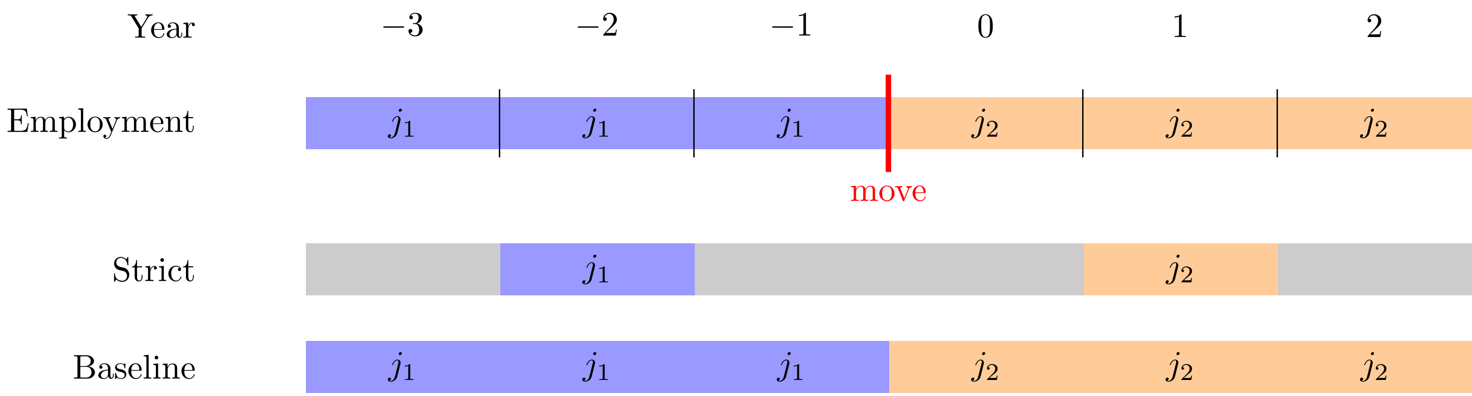
• To estimate the partition $\{k_j, j = 1, ..., J\}$, we follow Bonhomme *et al.* (2019) and cluster firms together based on the wage distribution functions, evaluated on a grid. For computation we use Lloyds' kmeans algorithm with a large number of starting values.

• Consistency of kmeans is not straightforward to establish in this context, due to the presence of within-k firm heterogeneity. In single-agent panel data Bonhomme *et al.* (2017) show consistency as K tends to infinity with the sample size.

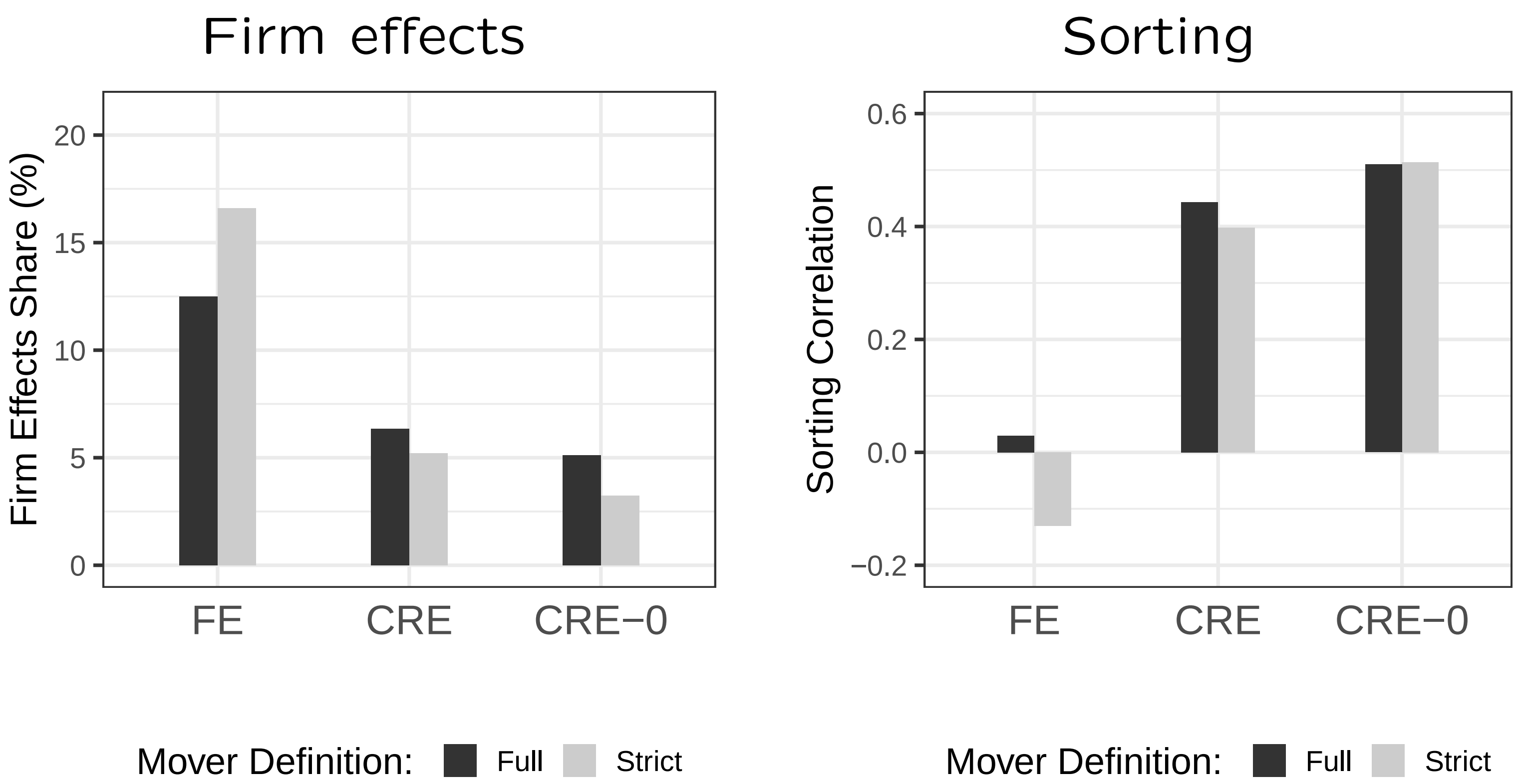
Descriptive statistics on all countries - short panel

	Austria	Italy	Norway	Sweden	US (state mean)	US (national)
Years Included	2012-2014	1999-2001	2011-2013	2001-2003	2010-2015	2010-2015
Unique Firms (1,000)						
Full Sample	217	82	78	58	33	3,871
Connected Set	28	7	19	14	13	2,106
Leave-one-out Set	11	2	10	6	8	
Unique Stayers (1,000)						
Full Sample	2,555	635	487	1,330	248	45,657
Connected Set	1,667	280	313	935	209	41,965
Leave-one-out Set	1,375	181	249	781	191	
Unique Movers (1,000)						
Full Sample	78	51	76	65	47	13,964
Connected Set	60	26	66	59	45	13,499
Leave-one-out Set	38	16	49	48	39	
Mean Movers per Firm						
Full Sample	3	3	4	6	5	9
Connected Set	4	6	5	8	6	11
Leave-one-out Set	6	13	8	16	9	

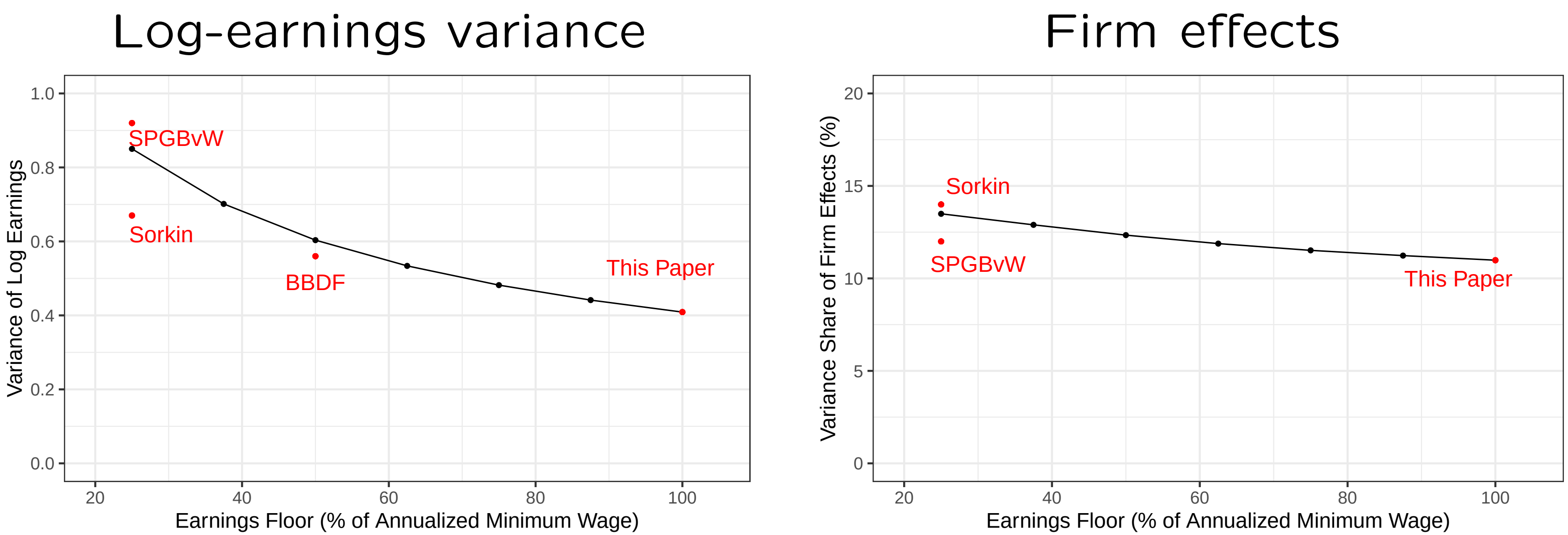
Definitions of mobility



Robustness to movers' definition in the US

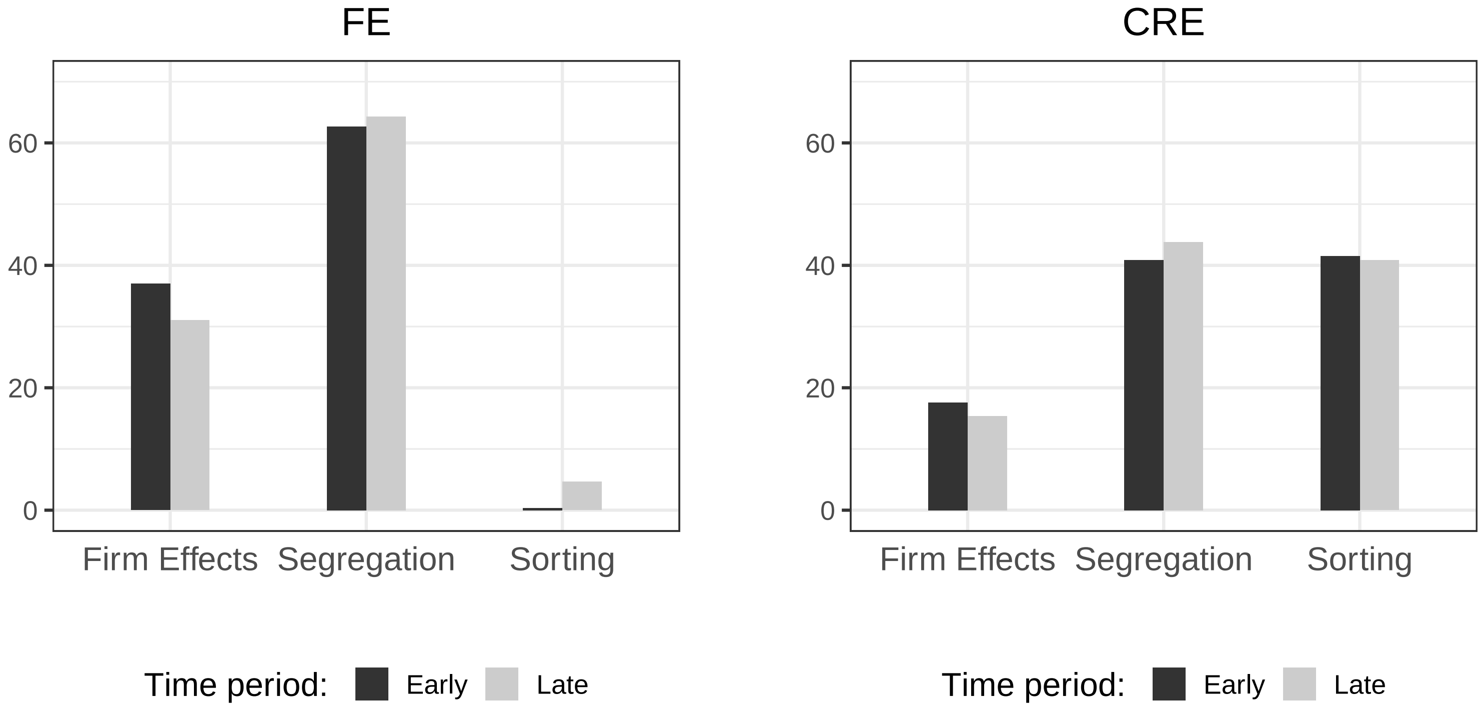


Effects of earnings floors



Notes: SPGBvW is Song et al. (2018), BBDF is Barth et al. (2016), Sorkin is Sorkin (2017).

Inequality between firms over time



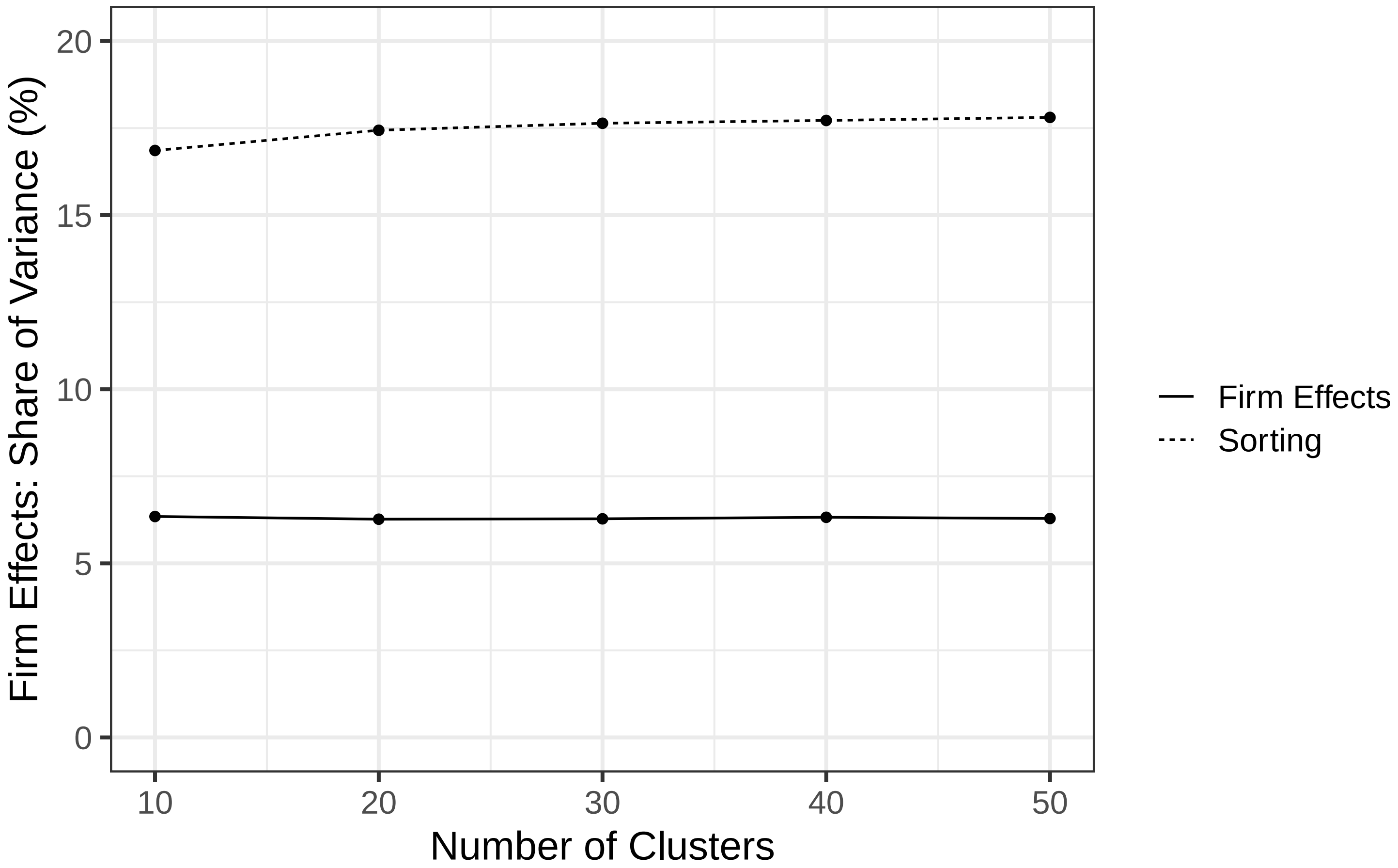
Notes: *"Segregation" is* $\mathrm{Var}(\mathbb{E}(\alpha_i \,|\, j(i,t)))$, *"Sorting" is* $2\,\mathrm{Cov}(\alpha_i, \psi_{j(i,t)})$.

Comparison to bias-reduced fixed-effects: sample sizes

	Population		Average of 21 Small States	
	Full	**Connected**	**Connected**	**Leave-one-out**
Sample Counts (thousands)				
Unique Workers	59,621	55,464	254	229
Movers	13,964	13,499	45	39
Stayers	45,657	41,965	209	191
Unique Firms	3,871	2,106	13	8

Notes: Numbers in thousands.

Robustness to number of firm groups



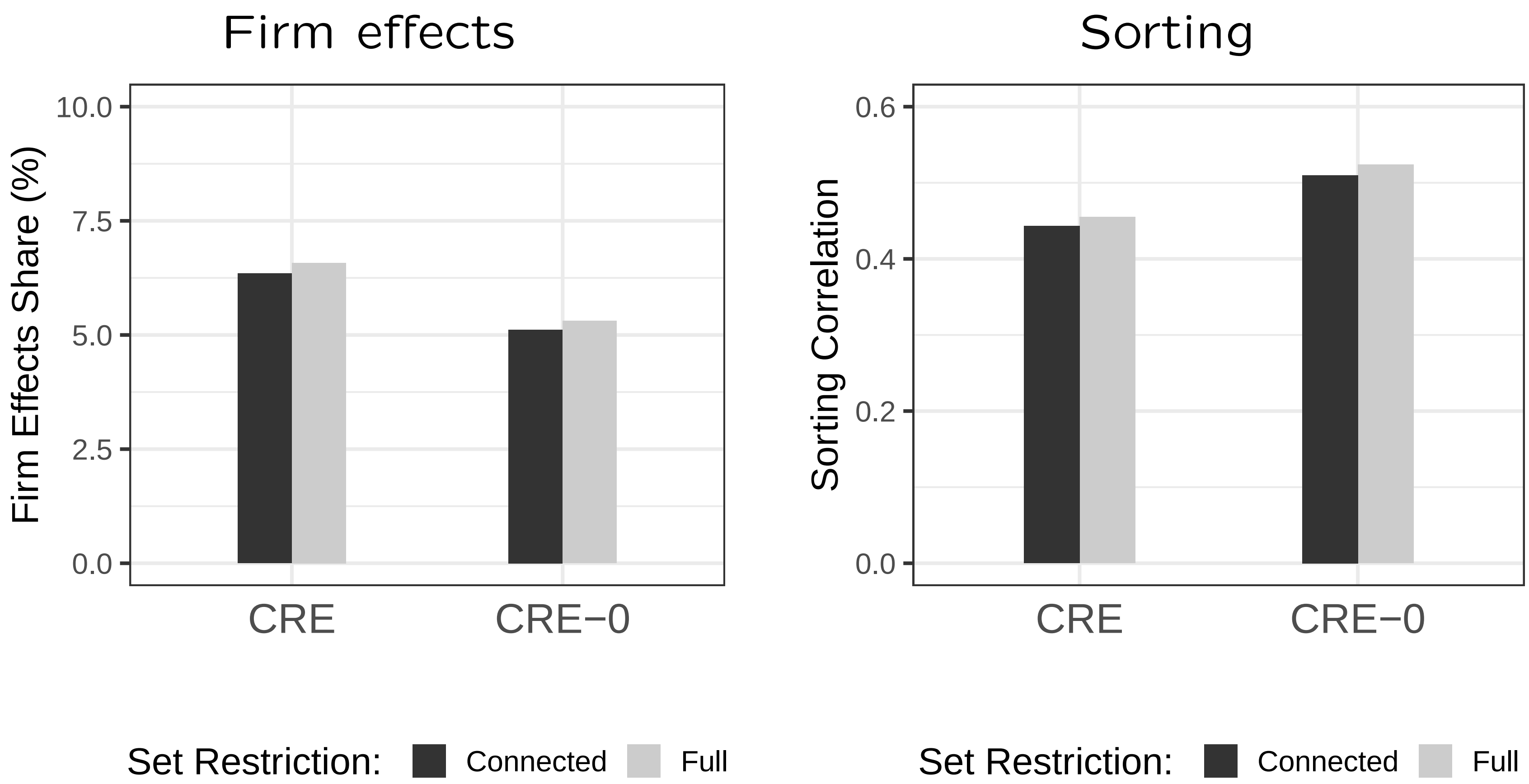
Notes: Groups are estimated using kmeans clustering. Correlated random-effects estimator.

Descriptive statistics on the $K = 10$ estimated groups in the US

Cluster:	1	2	3	4	5	6	7	8	9	10
Counts:										
Workers:	3,542,185	6,947,467	9,206,624	7,653,576	6,851,333	6,758,441	8,552,783	6,911,364	5,643,129	3,766,032
Firms:	329,395	344,266	323,765	329,782	189,937	267,642	206,847	194,766	111,210	75,335
Mean Size:	11	21	30	24	38	26	44	37	54	55
Fraction firms $\geq$ 10:	22%	39%	49%	45%	55%	45%	61%	46%	46%	32%
Fraction firms $\geq$ 50:	3%	6%	9%	8%	12%	9%	15%	11%	13%	9%
Wage Distribution:										
Mean:	-0.734	-0.568	-0.417	-0.310	-0.171	-0.146	0.014	0.196	0.438	0.839
Variance:	0.116	0.151	0.209	0.184	0.334	0.203	0.298	0.315	0.329	0.465
Skewness:	1.925	1.441	1.279	1.122	1.013	1.003	0.811	0.760	0.542	0.698
Kurtosis:	6.680	4.530	3.549	3.874	2.020	3.869	2.334	2.806	2.647	2.693

Notes: Groups are estimated using kmeans clustering.

Correlated random-effects estimates: largest connected component versus full sample



Estimates of firm effects variances in previous studies

Study	Country	Sample	$Var(y)$	$Var(\psi)$	share
Gruetter Lalive 2009	Austria	1990-1997	0.224	0.060	26.6%
Lopes de Melo 2018⋆	Brasil	1995-2005	0.746	0.180	24.12%
Alvarez etal 2018	Brasil	1988-1992	0.750	0.160	21.3%
Alvarez etal 2018	Brasil	2008-2012	0.470	0.070	14.9%
Engbom Moser 2018	Brasil	1996-2000	0.690	0.160	23.2%
Bagger 2014	Denmark	1985-2003	0.097	0.014	14.4%
Abowd etal 2003⋆	France	1976-1996	0.354	0.218	61.4%
Goux Maurin 1999⋆	France	1990-1992	0.181	0.023	12.9%
Goux Maurin 1999⋆	France	1993-1995	0.151	0.030	19.6%
Abowd etal 2002	France	1976-1987 ($\neq$ 1981, 1983)	0.269	0.081	30.1%
Abowd etal 1999a	France	1976-1987 ($\neq$ 1981, 1983)	0.269	0.234	87.0%
Card 2013	Germany	Universe, 1985-1991	0.137	0.025	18.2%
Card 2013	Germany	Universe, 2002-2009	0.249	0.053	21.3%
Andrews etal 2008⋆	Germany	LIAB 1993-1997, Bias Corr.	0.056	0.012	21.5%
Andrews etal 2008⋆	Germany	LIAB 1993-1997, Not Corr.	0.057	0.013	23.5%
Iranzo 2008	Italy	Manufacturing, 1981-1997	0.332	0.120	36.1%
Kline etal 2018	Italy (BL, RO)	1999-2001, AKM	0.146	0.061	41.4%
Kline etal 2018	Italy (BL, RO)	1999-2001, Homosk. Corr.	0.146	0.054	36.7%
Kline etal 2018	Italy (BL, RO)	1999-2001, Leave-out	0.146	0.044	30.2%
Card etal 2018⋆	Portugal	2005-2009	0.275	0.062	22.8%
Song etal 2018	USA	1980-1986	0.708	0.084	11.9%
Song etal 2018	USA	2007-2013	0.924	0.081	8.8%
Woodcock 2015	USA	LEHD, 1990-1999	0.410	0.080	19.5%
Sorkin 2017	USA	LEHD 2000-2008	0.700	0.098	14.0%
Abowd etal 2003⋆	USA	LEHD 1990-2000	0.719	0.130	18.1%
Abowd etal 2002	USA, WA	1984-1993	0.278	0.053	19.2%

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