

I. FEYNMAN RULES

$$\begin{aligned}
S_{int} &= \int_x -g f^{abc} A^{a\alpha}(x) A^{b\beta}(x) \partial_\alpha A_\beta^c(x) \\
&= \int_x -g f^{abc} g_{\alpha\beta} g_{\gamma\delta} A^{a\gamma}(x) A^{b\alpha}(x) \partial^\delta A^{c\beta}(x) \\
&= -g f^{abc} \int_x \int_{k_1, k_2, k_3} g_{\alpha\beta} g_{\gamma\delta} e^{-ik_1 x} A^{a\gamma}(k_1) e^{-ik_2 x} A^{b\alpha}(k_2) e^{-ik_3 x} k_3^\delta A^{c\beta}(k_3)
\end{aligned}$$

$$\begin{aligned}
& i(-g) f^{def} g^{\delta\mu} g^{\nu\rho} \frac{\delta}{\delta A^{d\mu}} \frac{\delta}{\delta A^{e\nu}} \left(\frac{\delta}{\delta A^{f\rho}} \partial_\delta \right) \left\{ \int_x \int_{k_1, k_2, k_3} e^{-ik_1 x} A^{a\gamma}(k_1) e^{-ik_2 x} A^{b\alpha}(k_2) e^{-ik_3 x} A^{c\beta}(k_3) \right\} \\
&= -ig f^{def} g^{\delta\mu} g^{\nu\rho} \left\{ -ik_{1\delta} (\delta_f^a \delta_\rho^\gamma) (\delta_e^b \delta_\nu^\alpha) (\delta_d^c \delta_\mu^\beta) - ik_{1\delta} (\delta_f^a \delta_\rho^\gamma) (\delta_e^c \delta_\nu^\beta) (\delta_d^b \delta_\mu^\alpha) \right. \\
&\quad - ik_{2\delta} (\delta_f^b \delta_\rho^\alpha) (\delta_e^a \delta_\nu^\gamma) (\delta_d^c \delta_\mu^\beta) - ik_{2\delta} (\delta_f^b \delta_\rho^\alpha) (\delta_e^c \delta_\nu^\beta) (\delta_d^a \delta_\mu^\gamma) \\
&\quad \left. - ik_{3\delta} (\delta_f^c \delta_\rho^\beta) (\delta_e^a \delta_\nu^\gamma) (\delta_d^b \delta_\mu^\alpha) - ik_{3\delta} (\delta_f^c \delta_\rho^\beta) (\delta_e^b \delta_\nu^\alpha) (\delta_d^a \delta_\mu^\gamma) \right\} \int_x e^{-i(k_1+k_2+k_3)x} \\
&= -ig \{ f^{abc} (-ik_{1\delta}) g^{\delta\beta} g^{\alpha\gamma} + f^{acb} (-ik_{1\delta}) g^{\delta\alpha} g^{\beta\gamma} \\
&\quad f^{bac} (-ik_{2\delta}) g^{\delta\beta} g^{\gamma\alpha} + f^{bca} (-ik_{2\delta}) g^{\delta\gamma} g^{\beta\alpha} \\
&\quad f^{cab} (-ik_{3\delta}) g^{\delta\alpha} g^{\gamma\beta} + f^{cba} (-ik_{3\delta}) g^{\delta\gamma} g^{\alpha\beta} \} \delta^4(k_1 + k_2 + k_3) \\
&= -ig \{ f^{abc} (-ik_{1\delta}) g^{\delta\beta} g^{\alpha\gamma} - f^{abc} (-ik_{1\delta}) g^{\delta\alpha} g^{\beta\gamma} \\
&\quad - f^{abc} (-ik_{2\delta}) g^{\delta\beta} g^{\gamma\alpha} + f^{abc} (-ik_{2\delta}) g^{\delta\gamma} g^{\beta\alpha} \\
&\quad f^{abc} (-ik_{3\delta}) g^{\delta\alpha} g^{\gamma\beta} - f^{abc} (-ik_{3\delta}) g^{\delta\gamma} g^{\alpha\beta} \} \delta^4(k_1 + k_2 + k_3) \\
&= -ig f^{abc} \{ (-ik_1^\beta) g^{\alpha\gamma} - (-ik_1^\alpha) g^{\beta\gamma} \\
&\quad - (-ik_2^\beta) g^{\gamma\alpha} + (-ik_2^\gamma) g^{\beta\alpha} \\
&\quad (-ik_3^\alpha) g^{\gamma\beta} - (-ik_3^\gamma) g^{\alpha\beta} \} \delta^4(k_1 + k_2 + k_3) \\
&= g f^{abc} \{ (-k_1^\beta) g^{\alpha\gamma} - (-k_1^\alpha) g^{\beta\gamma} \\
&\quad - (-k_2^\beta) g^{\gamma\alpha} + (-k_2^\gamma) g^{\beta\alpha} \\
&\quad (-k_3^\alpha) g^{\gamma\beta} - (-k_3^\gamma) g^{\alpha\beta} \} \delta^4(k_1 + k_2 + k_3) \\
&= g f^{abc} \{ (k_2 - k_1)^\beta g^{\alpha\gamma} + (k_1 - k_3)^\alpha g^{\beta\gamma} + (k_3 - k_2)^\gamma g^{\alpha\beta} \} \delta^4(k_1 + k_2 + k_3)
\end{aligned}$$

$$\begin{aligned}
& i(-g)f^{def}g_{\delta\mu}g_{\nu\rho}\frac{\delta}{\delta J_\mu^d(x)}\frac{\delta}{\delta J_\nu^e(x)}\left(\frac{\delta}{\delta J_\rho^f(x)}\partial_\delta\right)\left\{\int_{x_1,y_1}J_\alpha^a(x_1)\Delta_{\alpha\beta}^{ab}(x_1-y_1)J_\alpha^b(y_1)\times\right. \\
& \quad \left.\int_{x_2,y_2}J_\alpha^a(x_2)\Delta_{\alpha\beta}^{ab}(x_2-y_2)J_\alpha^b(y_2)\int_{x_3,y_3}J_\alpha^a(x_3)\Delta_{\alpha\beta}^{ab}(x_3-y_3)J_\alpha^b(y_3)\right\} \\
& = -igf^{def}g^{\delta\mu}g^{\nu\rho}\left\{-ik_{1\delta}\Delta_{\rho\beta}^{fb}(x-y_1)J_\alpha^b(y_1)(\delta_e^b\delta_\nu^\alpha)(\delta_d^c\delta_\mu^\beta) - ik_{1\delta}(\delta_f^a\delta_\rho^\gamma)(\delta_e^c\delta_\nu^\beta)(\delta_d^b\delta_\mu^\alpha)\right. \\
& = -igf^{def}g^{\delta\mu}g^{\nu\rho}\left\{-ik_{1\delta}(\delta_f^a\delta_\rho^\gamma)(\delta_e^b\delta_\nu^\alpha)(\delta_d^c\delta_\mu^\beta) - ik_{1\delta}(\delta_f^a\delta_\rho^\gamma)(\delta_e^c\delta_\nu^\beta)(\delta_d^b\delta_\mu^\alpha)\right. \\
& \quad - ik_{2\delta}(\delta_f^b\delta_\rho^\alpha)(\delta_e^a\delta_\nu^\gamma)(\delta_d^c\delta_\mu^\beta) - ik_{2\delta}(\delta_f^b\delta_\rho^\alpha)(\delta_e^c\delta_\nu^\beta)(\delta_d^a\delta_\mu^\gamma) \\
& \quad \left.- ik_{3\delta}(\delta_f^c\delta_\rho^\beta)(\delta_e^a\delta_\nu^\gamma)(\delta_d^b\delta_\mu^\alpha) - ik_{3\delta}(\delta_f^c\delta_\rho^\beta)(\delta_e^b\delta_\nu^\alpha)(\delta_d^a\delta_\mu^\gamma)\right\}\int_x e^{-i(k_1+k_2+k_3)x} \\
& = -ig\{f^{abc}(-ik_{1\delta})g^{\delta\beta}g^{\alpha\gamma} + f^{acb}(-ik_{1\delta})g^{\delta\alpha}g^{\beta\gamma} \\
& \quad f^{bac}(-ik_{2\delta})g^{\delta\beta}g^{\gamma\alpha} + f^{bca}(-ik_{2\delta})g^{\delta\gamma}g^{\beta\alpha} \\
& \quad f^{cab}(-ik_{3\delta})g^{\delta\alpha}g^{\gamma\beta} + f^{cba}(-ik_{3\delta})g^{\delta\gamma}g^{\alpha\beta}\}\delta^4(k_1+k_2+k_3) \\
& = -ig\{f^{abc}(-ik_{1\delta})g^{\delta\beta}g^{\alpha\gamma} - f^{abc}(-ik_{1\delta})g^{\delta\alpha}g^{\beta\gamma} \\
& \quad - f^{abc}(-ik_{2\delta})g^{\delta\beta}g^{\gamma\alpha} + f^{abc}(-ik_{2\delta})g^{\delta\gamma}g^{\beta\alpha} \\
& \quad f^{abc}(-ik_{3\delta})g^{\delta\alpha}g^{\gamma\beta} - f^{abc}(-ik_{3\delta})g^{\delta\gamma}g^{\alpha\beta}\}\delta^4(k_1+k_2+k_3) \\
& = -igf^{abc}\{(-ik_1^\beta)g^{\alpha\gamma} - (-ik_1^\alpha)g^{\beta\gamma} \\
& \quad - (-ik_2^\beta)g^{\gamma\alpha} + (-ik_2^\gamma)g^{\beta\alpha} \\
& \quad (-ik_3^\alpha)g^{\gamma\beta} - (-ik_3^\gamma)g^{\alpha\beta}\}\delta^4(k_1+k_2+k_3) \\
& = gf^{abc}\{(-k_1^\beta)g^{\alpha\gamma} - (-k_1^\alpha)g^{\beta\gamma} \\
& \quad - (-k_2^\beta)g^{\gamma\alpha} + (-k_2^\gamma)g^{\beta\alpha} \\
& \quad (-k_3^\alpha)g^{\gamma\beta} - (-k_3^\gamma)g^{\alpha\beta}\}\delta^4(k_1+k_2+k_3) \\
& = gf^{abc}\{(k_2-k_1)^\beta g^{\alpha\gamma} + (k_1-k_3)^\alpha g^{\beta\gamma} + (k_3-k_2)^\gamma g^{\alpha\beta}\}\delta^4(k_1+k_2+k_3)
\end{aligned}$$

$$\begin{aligned}
& \frac{\delta}{\delta A^{d\mu}}\frac{\delta}{\delta A^{e\nu}}\left(\frac{\delta}{\delta A^{f\rho}}\partial^\delta\right)\left\{gf^{abc}\int_x\int_{k_1,k_2,k_3}g_{\alpha\beta}g_{\gamma\delta}e^{-ik_1x}A^{a\gamma}(k_1)e^{-ik_2x}A^{b\alpha}(k_2)e^{-ik_3x}A^{c\beta}(k_3)\right\} \\
& = gf^{abc}g_{\alpha\beta}g_{\gamma\delta}\left\{-ik_1^\delta(\delta_f^a\delta_\rho^\gamma)(\delta_e^b\delta_\nu^\alpha)(\delta_d^c\delta_\mu^\beta) - ik_1^\delta(\delta_f^a\delta_\rho^\gamma)(\delta_e^c\delta_\nu^\beta)(\delta_d^b\delta_\mu^\alpha)\right. \\
& \quad - ik_2^\delta(\delta_f^b\delta_\rho^\alpha)(\delta_e^a\delta_\nu^\gamma)(\delta_d^c\delta_\mu^\beta) - ik_2^\delta(\delta_f^b\delta_\rho^\alpha)(\delta_e^c\delta_\nu^\beta)(\delta_d^a\delta_\mu^\gamma) \\
& \quad \left.- ik_3^\delta(\delta_f^c\delta_\rho^\beta)(\delta_e^a\delta_\nu^\gamma)(\delta_d^b\delta_\mu^\alpha) - ik_3^\delta(\delta_f^c\delta_\rho^\beta)(\delta_e^b\delta_\nu^\alpha)(\delta_d^a\delta_\mu^\gamma)\right\} \\
& = gf^{fed}(-ik_1^\delta g_{\nu\mu}g_{\rho\delta} - ik_1^\delta g_{\nu\mu}g_{\rho\delta})
\end{aligned}$$

$$\begin{aligned}
S_{int} &= \int_x g^2 f^{abe} f^{cde} A^{a\mu}(x) A^{b\nu}(x) A_\mu^c(x) A_\nu^d(x) \\
&= g^2 f^{abe} f^{cde} \int_x \int_{k_1} e^{ik_1 x} A^{a\mu}(k_1) \int_{k_2} e^{ik_2 x} A^{b\nu}(k_2) \int_{k_3} e^{ik_3 x} A_\mu^c(k_3) \int_{k_4} e^{ik_4 x} A_\nu^d(k_4) \\
&= g^2 f^{abe} f^{cde} \int_x e^{ik_1 x} e^{ik_2 x} e^{ik_3 x} e^{ik_4 x} \int_{k_1, k_2, k_3, k_4} A^{a\mu}(k_1) A^{b\nu}(k_2) A_\mu^c(k_3) A_\nu^d(k_4) \\
&= g^2 f^{abe} f^{cde} \delta^4(k_1 + k_2 + k_3 + k_4) \int_{k_1, k_2, k_3, k_4} A^{a\mu}(k_1) A^{b\nu}(k_2) A_\mu^c(k_3) A_\nu^d(k_4)
\end{aligned}$$

$$\frac{\delta}{\delta A^{a\mu}} \frac{\delta}{\delta A^{b\nu}} \frac{\delta}{\delta A_\mu^c} \frac{\delta}{\delta A_\nu^c}$$

$$a1a4a2a3 + a$$

$$E81 : \frac{25}{2}$$

$$E61(\%o14) - \frac{512\,i\,g4B1}{9\,M^6}(\%o15)E41(\%o16)\frac{448\,i\,g4B1}{9\,M^4}(\%o17)E21(\%o18) - \frac{64\,i\,g4B1}{3\,M^2}(\%o19)E01(\%o20),$$
