For a band such that $-1 \leq f(z) \leq 1$ there are N allowed energy states. For $\beta = -1.5$ we have $f(0) = 1 + \beta = -0.5$ and $-1 \leq f(|z|) \leq -0.5$. Hence there are $N/4$ positive energy states and $N/4$ negative energy states for a total of $N/2$ allowed energy states in the first band.

The allowed negative energies are E such that $E < 0$ and

$$\cos\left(\frac{2\pi n}{N}\right) = \cos\left(\frac{\sqrt{-2mE}}{\hbar}a\right) + \frac{m\alpha}{\hbar\sqrt{-2mE}}\sin\left(\frac{\sqrt{-2mE}}{\hbar}a\right), \quad n = 0, 1, 2, \ldots, N-1$$

Equivalently

$$\cos\left(\frac{2\pi n}{N}\right) = \cos(\kappa a) + \frac{m\alpha}{\hbar^2\kappa}\sin(\kappa a)$$

For $\beta = m\alpha a/\hbar^2$ we have

$$\cos\left(\frac{2\pi n}{N}\right) = \cos(\kappa a) + \beta\frac{\sin(\kappa a)}{\kappa a}$$

Note: $\alpha < 0$