Addition of angular momentum 3

Let Ψ be an eigenstate of J^2. Then

$$J^2\Psi = j(j+1)\hbar^2\Psi$$

The following Ψ are eigenstates of J^2 with $j = l + \frac{1}{2}$.

$$\Psi = \left(\frac{l+m+1}{2l+1}\right)^{1/2} Y_{lm}\chi_+ + \left(\frac{l-m}{2l+1}\right)^{1/2} Y_{l,m+1}\chi_- \tag{1}$$

The following Ψ are eigenstates of J^2 with $j = l - \frac{1}{2}$ and $m < l$.

$$\Psi = \left(\frac{l-m}{2l+1}\right)^{1/2} Y_{lm}\chi_+ - \left(\frac{l+m+1}{2l+1}\right)^{1/2} Y_{l,m+1}\chi_- \tag{2}$$