The file q4.txt defines kets, operators, and a measurement function for simulating a four bit quantum computer. See eigenmath.org/q.c for the program that generates q4.txt.

Kets are unit vectors in $\mathbb{C}^{16}$. The dimension is 16 because four quantum bits have $2^4 = 16$ basis states. Quantum bit numbering is $|q_3q_2q_1q_0\rangle$. The following basis kets are defined in q4.txt.

$$
\begin{aligned}
|0\rangle &= |0000_2\rangle = (1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0) \\
|1\rangle &= |0001_2\rangle = (0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0) \\
|2\rangle &= |0010_2\rangle = (0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0) \\
|3\rangle &= |0011_2\rangle = (0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0) \\
&\vdots \\
|15\rangle &= |1111_2\rangle = (0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1)
\end{aligned}
$$

Operators are 16×16 matrices that rotate ket vectors. (A ket always has unit length.) The following operators are defined in q4.txt.

H_n	Hadamard operator on bit n.
I	Identity matrix.
$P_{mn}(\phi)$	Controlled phase shift, m is the control bit, n is the target bit, ϕ is the phase.
Q	Quantum Fourier transform.
R	Inverse quantum Fourier transform.
S_{mn}	Swap bits m and n.
X_n	Pauli X (NOT) operator on bit n.
X_{mn}	Controlled X (CNOT) operator, m is the control bit, n is the target bit.
Y_n	Pauli Y operator on bit n.
Z_n	Pauli Z operator on bit n.

Function M (measurement function) shows the probability of observing each of the 16 basis states given that the system is in state ψ.

$$M(\psi)$$

Quantum algorithms are expressed as sequences of operators applied to the initial state $|0\rangle$. The operator sequence should be read backwards, from right to left, although the direction makes no difference mathematically.

Deutsch-Jozsa algorithm

Let $f(q_0, q_1, q_2)$ be an operator (16×16 matrix) that operates on q_3 in a manner consistent with a constant or balanced oracle. Then the Deutsch-Jozsa algorithm for identifying f is

$$\psi = H_2\ H_1\ H_0\ f(q_0, q_1, q_2)\ H_3\ X_3\ H_2\ H_1\ H_0\ |0\rangle$$

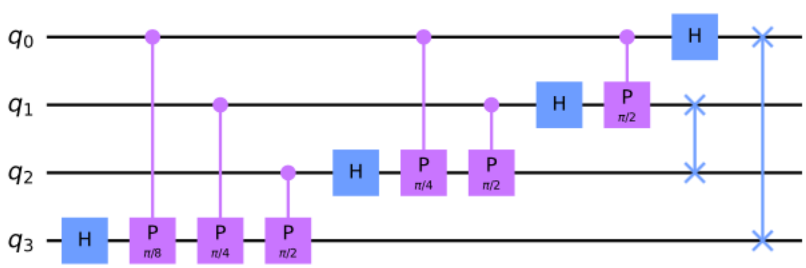
Bernstein-Vazirani algorithm

Let $f(q_0, q_1, q_2)$ be an operator (16×16 matrix) that operates on q_3. Then the Bernstein-Vazirani algorithm for identifying f is

$$\psi = H_2\ H_1\ H_0\ f(q_0, q_1, q_2)\ Z_3\ H_3\ H_2\ H_1\ H_0\ |0\rangle$$

Quantum Fourier transform

The following circuit diagram[1] shows how to implement the QFT.



This is how the QFT operator Q is defined in q4.txt.

```
Q = dot(
S03,
S12,
H0,
P01(pi/2),
H1,
P12(pi/2),
P02(pi/4),
H2,
P23(pi/2),
P13(pi/4),
P03(pi/8),
H3)
```

The inverse QFT operator R is defined similarly except the operators appear in reverse order and the phase shifts are negated.

[1] qiskit.org/textbook/ch-algorithms/quantum-fourier-transform.html