9-1. Show that $\mathbf{E}$, $\mathbf{B}$, and $\mathbf{k}$ are mutually perpendicular for this plane-wave solution.

$$\mathbf{A}(\mathbf{r},t) = \mathbf{a}_\mathbf{k}(t)\exp(i\mathbf{k}\cdot\mathbf{r}) \tag{9.12}$$

For an arbitrary direction of propagation $\mathbf{k}'$, let us choose a rotation matrix $\mathbf{R}$ such that

$$\mathbf{k} = \mathbf{R}\,\mathbf{k}' = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

By the requirement that $\mathbf{a}_\mathbf{k}\cdot\mathbf{k} = 0$ let

$$\mathbf{a}_\mathbf{k} = a(t)\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

Then by equation (9.12)

$$\mathbf{A} = \mathbf{a}_\mathbf{k}\exp(i\mathbf{k}\cdot\mathbf{r}) = \begin{pmatrix} a(t)\exp(iz) \\ a(t)\exp(iz) \\ 0 \end{pmatrix}$$

By equation (9.7)

$$\mathbf{B} = \nabla\times\mathbf{A} = \begin{pmatrix} -ia(t)\exp(iz) \\ ia(t)\exp(iz) \\ 0 \end{pmatrix}$$

By equation (9.9) with $\phi = 0$

$$\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t} = -\frac{1}{c}\frac{da(t)}{dt}\begin{pmatrix} -\exp(iz) \\ -\exp(iz) \\ 0 \end{pmatrix}$$

By inspection we have $\mathbf{k}\cdot\mathbf{E} = 0$, $\mathbf{k}\cdot\mathbf{B} = 0$, and $\mathbf{E}\cdot\mathbf{B} = 0$, hence all three vectors are mutually perpendicular.