

Exercise 9.2. Prove Eq. 9.10 by expanding each side and comparing the results.

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We are given

$$[\mathbf{P}^2, \mathbf{X}] = \mathbf{P}[\mathbf{P}, \mathbf{X}] + [\mathbf{P}, \mathbf{X}]\mathbf{P} \quad (9.10)$$

We have

$$[\mathbf{P}^2, \mathbf{X}] = \mathbf{P}\mathbf{P}\mathbf{X} - \mathbf{X}\mathbf{P}\mathbf{P} \quad (1)$$

$$\mathbf{P}[\mathbf{P}, \mathbf{X}] = \mathbf{P}\mathbf{P}\mathbf{X} - \mathbf{P}\mathbf{X}\mathbf{P} \quad (2)$$

$$[\mathbf{P}, \mathbf{X}]\mathbf{P} = \mathbf{P}\mathbf{X}\mathbf{P} - \mathbf{X}\mathbf{P}\mathbf{P} \quad (3)$$

Substitute (1), (2), and (3) into (9.10) to obtain

$$\mathbf{P}\mathbf{P}\mathbf{X} - \mathbf{X}\mathbf{P}\mathbf{P} = \mathbf{P}\mathbf{P}\mathbf{X} - \mathbf{P}\mathbf{X}\mathbf{P} + \mathbf{P}\mathbf{X}\mathbf{P} - \mathbf{X}\mathbf{P}\mathbf{P}$$

The  $\mathbf{P}\mathbf{X}\mathbf{P}$  cancel and (9.10) is proved.