Let $\phi$ be the field

$$\phi(x,y,z,t) = p_x x + p_y y + p_z z - Et$$

where

$$E = \sqrt{p_x^2 c^2 + p_y^2 c^2 + p_z^2 c^2 + m^2 c^4}$$

The following solutions to the Dirac equation are used to model fermions.

$$\underset{\text{fermion spin up}}{\psi_1 = \begin{pmatrix} E/c + mc \\ 0 \\ p_z \\ p_x + ip_y \end{pmatrix} \exp\left(\frac{i\phi}{\hbar}\right)} \qquad \underset{\text{fermion spin down}}{\psi_2 = \begin{pmatrix} 0 \\ E/c + mc \\ p_x - ip_y \\ -p_z \end{pmatrix} \exp\left(\frac{i\phi}{\hbar}\right)}$$

$$\underset{\text{anti-fermion spin up}}{\psi_7 = \begin{pmatrix} p_z \\ p_x + ip_y \\ E/c + mc \\ 0 \end{pmatrix} \exp\left(-\frac{i\phi}{\hbar}\right)} \qquad \underset{\text{anti-fermion spin down}}{\psi_8 = \begin{pmatrix} p_x - ip_y \\ -p_z \\ 0 \\ E/c + mc \end{pmatrix} \exp\left(-\frac{i\phi}{\hbar}\right)}$$

A spinor is the vector part of $\psi$. The following spinors are used for scattering calculations. Symbol $u$ indicates a fermion and symbol $v$ indicates an anti-fermion.

$$\underset{\text{fermion spin up}}{u_1 = \begin{pmatrix} E_1/c + mc \\ 0 \\ p_{1z} \\ p_{1x} + ip_{1y} \end{pmatrix}} \qquad \underset{\text{fermion spin down}}{u_2 = \begin{pmatrix} 0 \\ E_1/c + mc \\ p_{1x} - ip_{1y} \\ -p_{1z} \end{pmatrix}} \qquad \underset{\text{anti-fermion spin up}}{v_1 = \begin{pmatrix} p_{2z} \\ p_{2x} + ip_{2y} \\ E_2/c + mc \\ 0 \end{pmatrix}} \qquad \underset{\text{anti-fermion spin down}}{v_2 = \begin{pmatrix} p_{2x} - ip_{2y} \\ -p_{2z} \\ 0 \\ E_2/c + mc \end{pmatrix}}$$

These are the associated momentum vectors.

$$p_1 = \begin{pmatrix} E_1/c \\ p_{1x} \\ p_{1y} \\ p_{1z} \end{pmatrix} \qquad p_2 = \begin{pmatrix} E_2/c \\ p_{2x} \\ p_{2y} \\ p_{2z} \end{pmatrix}$$

Spinors are solutions to the following momentum-space equations with $\not{p} = p \cdot (\gamma^0, \gamma^1, \gamma^2, \gamma^3)$.

$$\not{p}u = mcu \qquad \not{p}v = -mcv$$

Up and down spinors have the following "completeness property."

$$u_{11}\bar{u}_{11} + u_{12}\bar{u}_{12} = (E_1/c + mc)(\not{p}_1 + mc) \qquad v_{21}\bar{v}_{21} + v_{22}\bar{v}_{22} = (E_2/c + mc)(\not{p}_2 - mc)$$

The adjoint of a spinor is $\bar{u} = u^\dagger \gamma^0$. The adjoint is a row vector hence $u\bar{u}$ is an outer product.