5-3. Assume f(x) is normalized, that is,

$$\int_{-\infty}^{\infty} f^*(x)f(x) \, dx = 1$$

Under this constraint, show that the state f(x) which has the highest probability of having property G is f(x) = g(x).

Consider equation (5.32).

$$P(G) = \left| \int_{-\infty}^{\infty} g^*(x) f(x) \, dx \right|^2 \tag{5.32}$$

If f(x) = g(x) then

$$P(G) = \left| \int_{-\infty}^{\infty} f^*(x) f(x) \, dx \right|^2 = 1$$

which is the highest probability.