Spinor boost

This vector represents an electron at rest with spin up along the z axis.

$$u^0 = \sqrt{2m}\begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

This matrix boosts a spinor in the z direction where $E^2 = p^2 + m^2$.

$$\Lambda = \frac{1}{\sqrt{2m(E+m)}}\begin{pmatrix}E+m & 0 & p & 0\\ 0 & E+m & 0 & p\\ p & 0 & E+m & 0\\ 0 & p & 0 & E+m\end{pmatrix}$$

Hence

$$u = \Lambda u^0 = \frac{1}{\sqrt{E+m}}\begin{pmatrix}E+m & 0 & p & 0\\ 0 & E+m & 0 & p\\ p & 0 & E+m & 0\\ 0 & p & 0 & E+m\end{pmatrix}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \frac{1}{\sqrt{E+m}}\begin{pmatrix}E+m\\0\\p\\0\end{pmatrix}$$

Consider the Dirac equation

$$\not{p}u = mu$$

Rewrite u as boost.

$$\not{p}\Lambda u^0 = m\Lambda u^0$$

Noting that $\gamma^0 u^0 = u^0$ we can write

$$\not{p}\Lambda u^0 = m\Lambda\gamma^0 u^0$$

Rewrite u^0 as boost.

$$\not{p}\Lambda\Lambda^{-1}u = m\Lambda\gamma^0\Lambda^{-1}u$$

Hence

$$\not{p} = m\Lambda\gamma^0\Lambda^{-1}$$

and

$$\Lambda\gamma^0 = m^{-1}\not{p}\Lambda \tag{1}$$

Consider

$$\gamma^0 u^0 - u^0 = 0$$

Apply boost to recover the Dirac equation.

$$\Lambda(\gamma^0 u^0 - u^0) = m^{-1}\not{p}\Lambda u^0 - \Lambda u^0 = m^{-1}\not{p}u - u = 0 \tag{2}$$