Schrodinger from propagator

Consider the free-particle propagator

$$K_0(x, t, x_a, t_a) = \left(\frac{m}{2\pi i \hbar (t - t_a)} \right)^{\frac{1}{2}} \exp \left(\frac{im(x - x_a)^2}{2\hbar(t - t_a)} \right)$$

Let $\psi(x, t)$ be a free-particle wave function. Then

$$\psi(x, t) = \int_{-\infty}^{\infty} K_0(x, t, x_a, t_a) \psi(x_a, t_a) \, dx_a \tag{1}$$

Apply $\partial/\partial t$ to both sides of (1) to obtain

$$\frac{\partial}{\partial t} \psi(x, t) = \int_{-\infty}^{\infty} \frac{\partial K_0}{\partial t} \psi(x_a, t_a) \, dx_a \tag{2}$$

Apply $\partial^2/\partial x^2$ to both sides of (1) to obtain

$$\frac{\partial^2}{\partial x^2} \psi(x, t) = \int_{-\infty}^{\infty} \frac{\partial^2 K_0}{\partial x^2} \psi(x_a, t_a) \, dx_a \tag{3}$$

By computer algebra

$$\frac{\partial K_0}{\partial t} = \frac{i\hbar}{2m} \frac{\partial^2 K_0}{\partial x^2} \tag{4}$$

Multiply both sides of (4) by $\psi(x_a, t_a)$ and integrate over x_a.

$$\underbrace{\int_{-\infty}^{\infty} \frac{\partial K_0}{\partial t} \psi(x_a, t_a) \, dx_a}_{(2)} = \frac{i\hbar}{2m} \underbrace{\int_{-\infty}^{\infty} \frac{\partial^2 K_0}{\partial x^2} \psi(x_a, t_a) \, dx_a}_{(3)} \tag{5}$$

Substitute (2) and (3) into (5) to obtain

$$\frac{\partial}{\partial t} \psi(x, t) = \frac{i\hbar}{2m} \frac{\partial^2}{\partial x^2} \psi(x, t)$$

Multiply both sides by $i\hbar$ to obtain the free-particle Schrodinger equation.

$$i\hbar \frac{\partial}{\partial t} \psi(x, t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x, t)$$