

2.1. Solve the Klein-Gordon equation.

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This is the Klein-Gordon equation.

$$\left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} + \frac{m^2 c^2}{\hbar^2} \right) \psi = 0$$

One solution is

$$\psi = \exp \left( -\frac{i}{\hbar} (Et - p_x x - p_y y - p_z z) \right) \quad (1)$$

where

$$E = \sqrt{m^2 c^4 + p_x^2 c^2 + p_y^2 c^2 + p_z^2 c^2}$$

Let us inspect dimensions. We have

$$Et \propto \text{joule second}$$

Hence

$$\frac{Et}{\hbar} \propto \frac{\text{joule second}}{\text{joule second}} = 1$$

We have

$$p_x x \propto \frac{\text{kilogram meter}}{\text{second}} \times \text{meter} = \text{joule second}$$

Hence

$$\frac{p_x x}{\hbar} \propto \frac{\text{joule second}}{\text{joule second}} = 1$$