

## Arc length

Let  $g(t)$  be a parametric function that draws a curve in  $\mathbb{R}^n$ . The arc length from  $g(a)$  to  $g(b)$  is given by

$$\int_a^b |g'(t)| dt$$

where  $|g'(t)|$  is the length of the tangent vector at  $g(t)$ .

Example 1. Find the length of the curve  $y = x^2$  from  $x = 0$  to  $x = 1$ .

```
g = (t,t^2)
defint(abs(d(g,t)),t,0,1)
```

$$\frac{1}{2} 5^{1/2} - \frac{1}{4} \log(2) + \frac{1}{4} \log(2 \cdot 5^{1/2} + 4)$$

```
float
```

```
1.47894
```

As expected, the result is greater than  $\sqrt{2} \approx 1.414$ , the length of a straight line from  $(0, 0)$  to  $(1, 1)$ .

The following script does a discrete computation of the arc length by dividing the curve into 100 pieces.

```
g(t) = (t,t^2)
h(k) = abs(g(k/100.0) - g((k-1)/100.0))
sum(k,1,100,h(k))
```

```
1.47894
```

As expected, the discrete result matches the analytic result.

Example 2. Find the length of the curve  $y = x^{3/2}$  from the origin to  $x = \frac{4}{3}$ .

```
g = (t,t^(3/2))
defint(abs(d(g,t)),t,0,4/3)
```

$$\frac{56}{27}$$