Rutherford scattering data

The following data is from Geiger and Marsden's 1913 paper.[1] Column y is number of scattering events for silver foil.

θ	y
150	22.2
135	27.4
120	33.0
105	47.3
75	136
60	320
45	989
37.5	1760
30	5260
22.5	20300
15	105400

This is the differential cross section for Rutherford scattering.

$$\frac{d\sigma}{d\Omega} \propto \frac{1}{(1-\cos\theta)^2}$$

Rewrite as

$$d\sigma \propto \frac{1}{(1-\cos\theta)^2}\,d\Omega$$

Substitute $d\Omega = 2\pi \sin\theta\, d\theta$ and let the proportionality absorb the 2π.

$$d\sigma \propto \frac{\sin\theta}{(1-\cos\theta)^2}\,d\theta$$

$d\sigma$ is a probability density function, not a mass function, hence must be integrated to obtain a probability. Integrate $d\sigma$ to obtain the following cumulative distribution function $F(\theta)$ where C is a normalization constant.

$$F(\theta) = \int d\sigma = \frac{C}{\cos\theta - 1}$$

Geiger and Marsden give the aperture of the detector as 15°. Hence the probability of scattering into the detector at angle θ is

$$\Pr(\theta) = F(\theta + 7.5^\circ) - F(\theta - 7.5^\circ)$$

It turns out that $F(\theta)$ cannot be normalized analytically because $F(0)$ is undefined. To obtain predicted values $\hat{y}_k$ use regression to fit the unnormalized data. Let $C = 1$ and let

$$x_k = F(\theta_k + 7.5^\circ) - F(\theta_k - 7.5^\circ)$$

[1] www.chemteam.info/Chem-History/GeigerMarsden-1913/GeigerMarsden-1913.html

Then the best fit for the regression model $\hat{y} = \beta x$ is

$$\beta = \frac{\mathbf{x} \cdot \mathbf{y}}{\mathbf{x} \cdot \mathbf{x}} = 1008.66$$

The following table shows the predicted values.

θ	y	$\hat{y}$
150	22.2	38.2
135	27.4	64.5
120	33.0	103
105	47.3	163
75	136	474
60	320	944
45	989	2303
37.5	1760	4095
30	5260	8370
22.5	20300	22073
15	105400	104650

The coefficient of determination R^2 measures how well predicted values fit the data.

$$R^2 = 1 - \frac{\sum(y - \hat{y})^2}{\sum(y - \bar{y})^2} = 0.998$$

The result indicates that $d\sigma$ explains 99.8% of the variance in the data.