(2.4) Using $\hat{a}|0\rangle = 0$ and eqns 2.9 and 2.10 together with $\langle x|\hat{p}|\psi\rangle = -i\hbar \frac{d}{dx}\langle x|\psi\rangle$, show that

$$0 = \left(x + \frac{\hbar}{m\omega}\frac{d}{dx}\right)\langle x|0\rangle \tag{2.69}$$

and hence

$$\langle x|0\rangle = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \exp\left(-\frac{m\omega x^2}{2\hbar}\right) \tag{2.70}$$

$$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + \frac{i}{m\omega}\hat{p}\right) \tag{2.9}$$

$$\hat{a}^\dagger = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} - \frac{i}{m\omega}\hat{p}\right) \tag{2.10}$$

We have

$$\left(x + \frac{\hbar}{m\omega}\frac{d}{dx}\right)\langle x|0\rangle = \langle x|\left(\hat{x} + \frac{i}{m\omega}\hat{p}\right)|0\rangle = \langle x|\hat{a}|0\rangle = 0$$

Let

$$f = \exp\left(-\frac{m\omega x^2}{2\hbar}\right)$$

Then

$$\frac{d}{dx}f = -\frac{m\omega}{\hbar}x\exp\left(-\frac{m\omega x^2}{2\hbar}\right)$$

It follows that

$$\frac{\hbar}{m\omega}\frac{d}{dx}f = -xf$$

Hence (2.70) is a solution to differential equation (2.69).