Hydrogen radius

Verify that

$$\langle r \rangle = \frac{3n^2 - l(l+1)}{2}\, a_0 \tag{1}$$

for $n = 1, 2, 3$ and $l = 0, \ldots, n-1$.

The Eigenmath code proceeds as follows:

Determine the probability density function $f(r)$.

$$f(r) = \int_0^{\pi} \int_0^{2\pi} |\psi_{nlm}|^2\, r^2 \sin\theta \, d\theta \, d\phi$$

Calculate the expectation value.

$$\langle r \rangle = \int_0^{\infty} r f(r) \, dr$$