Harmonic oscillator coherent state

The coherent state minimizes $\Delta x \Delta p$.

This is the coherent state for a quantum harmonic oscillator.

$$\begin{aligned}\psi_{n,r,\theta}(x) = \frac{1}{\sqrt{2^n n!}} \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} H_n\left(\sqrt{\frac{m\omega}{\hbar}}\left(x - \langle x\rangle\right)\right) \\ \times \exp\left[-\frac{m\omega}{2\hbar}\left(x - \langle x\rangle\right)^2 + \frac{i}{\hbar}\langle p\rangle\left(x - \frac{\langle x\rangle}{2}\right) - i\left(n + \frac{1}{2}\right)\omega t\right]\end{aligned}$$

Parameters r and θ are polar coordinates in phase space and

$$\langle x\rangle = \sqrt{\frac{2\hbar}{m\omega}}\, r\cos(\omega t - \theta), \quad \langle p\rangle = -\sqrt{2m\hbar\omega}\, r\sin(\omega t - \theta)$$

Exercises

1. Verify

$$i\hbar\frac{d}{dt}\psi_1 = \hat{H}\psi_1$$

2. Verify

$$\int_{-\infty}^{\infty} \psi_1^* \psi_1 \, dx = 1$$

3. Verify for ψ_0

$$\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2} = \sqrt{\frac{\hbar}{2m\omega}}$$

and

$$\Delta p = \sqrt{\langle p^2\rangle - \langle p\rangle^2} = \sqrt{\frac{m\hbar\omega}{2}}$$

Hence

$$\Delta x \Delta p = \frac{\hbar}{2}$$