

The file q4.txt defines kets, operators, and a measurement function for simulating a four bit quantum computer. See [eigenmath.org/q.c](http://eigenmath.org/q.c) for the program that generates q4.txt.

Kets are unit vectors in  $\mathbb{C}^{16}$ . The dimension is 16 because a four bit quantum computer has  $2^4 = 16$  eigenstates. The following kets that represent eigenstates are defined in q4.txt.

$$\begin{aligned} |0\rangle &= |0000_2\rangle = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \\ |1\rangle &= |0001_2\rangle = (0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \\ |2\rangle &= |0010_2\rangle = (0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \\ |3\rangle &= |0011_2\rangle = (0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \\ &\vdots \\ |15\rangle &= |1111_2\rangle = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1) \end{aligned}$$

Operators are  $16 \times 16$  matrices that rotate ket vectors. The following operators and the measurement function  $M(\psi)$  are defined in q4.txt.

$H_n$	Hadamard operator on bit $n$ .
$I$	Identity matrix.
$M(\psi)$	Measurement function (not an operator).
$P_n(\phi)$	Phase shift $\phi$ on bit $n$ .
$P_{mn}(\phi)$	Controlled phase shift, $m$ is the control bit, $n$ is the target bit, $\phi$ is the phase.
$Q$	Quantum Fourier transform.
$R$	Inverse quantum Fourier transform.
$S_{mn}$	Swap bits $m$ and $n$ .
$X_n$	Pauli X (NOT) operator on bit $n$ .
$X_{mn}$	Controlled X (CNOT) operator, $m$ is the control bit, $n$ is the target bit.
$Y_n$	Pauli Y operator on bit $n$ .
$Z_n$	Pauli Z operator on bit $n$ .

Measurement function  $M(\psi)$  shows, for all  $k = 0 \dots 15$ , the probability  $P_k$  of observing eigenstate  $k$  given that the quantum computer is in state  $\psi$ .

$$\psi = \sum_{k=0}^{15} c_k |k\rangle, \quad |\psi| = 1, \quad P_k = c_k c_k^*$$

Quantum algorithms are expressed as sequences of operators applied to the initial state  $|0\rangle$ . The operator sequence should be read backwards, from right to left, although the direction makes no difference mathematically.

## Deutsch-Jozsa algorithm

Let  $f(q_0, q_1, q_2)$  be an operator ( $16 \times 16$  matrix) that operates on  $q_3$  in a manner consistent with a constant or balanced oracle. Then the Deutsch-Jozsa algorithm for identifying  $f$  is

$$\psi = H_2 H_1 H_0 f(q_0, q_1, q_2) H_3 X_3 H_2 H_1 H_0 |0\rangle$$

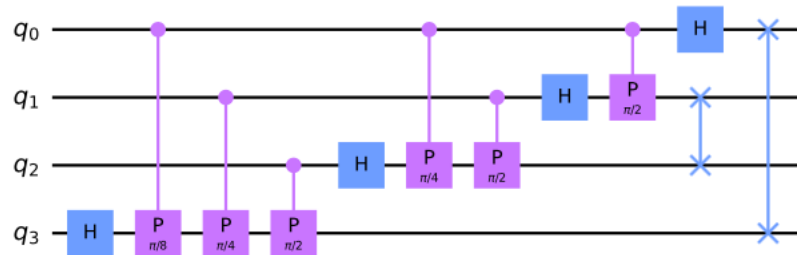
## Bernstein-Vazirani algorithm

Let  $f(q_0, q_1, q_2)$  be an operator ( $16 \times 16$  matrix) that operates on  $q_3$ . Then the Bernstein-Vazirani algorithm for identifying  $f$  is

$$\psi = H_2 H_1 H_0 f(q_0, q_1, q_2) Z_3 H_3 H_2 H_1 H_0 |0\rangle$$

## Quantum Fourier transform

The following circuit diagram shows how the QFT is implemented.<sup>1</sup>



This is how the QFT operator  $Q$  is defined in q4.txt.

```
Q = dot(
S03,          -- Swap qubits 0 and 3
S12,          -- Swap qubits 1 and 2
H0,           -- Hadamard qubit 0
P01(pi/2),    -- Controlled phase shift (0 control, 1 target)
H1,           -- Hadamard qubit 1
P12(pi/2),    -- Controlled phase shift (1 control, 2 target)
P02(pi/4),    -- Controlled phase shift (0 control, 2 target)
H2,           -- Hadamard qubit 2
P23(pi/2),    -- Controlled phase shift (2 control, 3 target)
P13(pi/4),    -- Controlled phase shift (1 control, 3 target)
P03(pi/8),    -- Controlled phase shift (0 control, 3 target)
H3)
```

The inverse QFT operator  $R$  is defined similarly except the operators appear in reverse order and the phase shifts are negated.

<sup>1</sup>[qiskit.org/textbook/ch-algorithms/quantum-fourier-transform.html](http://qiskit.org/textbook/ch-algorithms/quantum-fourier-transform.html)