3-6. Since the free-particle lagrangian is quadratic, show that

$$K(b,a) = F(t_b, t_a) \exp\left(\frac{im(x_b - x_a)^2}{2\hbar(t_b - t_a)}\right)$$

and give an argument to show that F can depend only on the difference  $F(t_b-t_a)$ .

From equation (3.51)

$$K(b,a) = \exp\left(\frac{i}{\hbar}S_{cl}(b,a)\right)F(t_b,t_a)$$

From problem (2.1)

$$S_{cl} = \frac{m}{2} \frac{(x_b - x_a)^2}{t_b - t_a}$$

Hence by substitution of  $S_{cl}$ 

$$K(b,a) = F(t_b, t_a) \exp\left(\frac{im(x_b - x_a)^2}{2\hbar(t_b - t_a)}\right)$$

The argument for why F can only depend on  $t_b - t_a$  is invariance under time translation. For any constant s we should have

$$F(t_b, t_a) = F(t_b + s, t_a + s)$$

For example, we should have

$$F(1,0) = F(3,2)$$

Hence F can only depend on the difference  $t_b - t_a$  and not the specific values of  $t_a$  and  $t_b$ .