Harmonic oscillator coherent state

The coherent state minimizes $\Delta x \Delta p$.

This is the coherent state for a quantum harmonic oscillator.

$$\psi = \left(\frac{m\omega}{\pi\hbar}\right) \exp\left(-\frac{m\omega}{2\hbar}\Big(x - \langle x\rangle\Big)^2 + \frac{i}{\hbar}\langle p\rangle\Big(x - \tfrac{1}{2}\langle x\rangle\Big) - \frac{i}{2}\omega t\right)$$

where

$$\langle x\rangle = \sqrt{\frac{2\hbar}{m\omega}}\,\alpha_0 \cos(\omega t - \phi), \quad \langle p\rangle = -\sqrt{2m\hbar\omega}\,\alpha_0 \sin(\omega t - \phi)$$

Exercises

1. Verify that

$$i\hbar\frac{d}{dt}\psi = \hat{H}\psi$$

2. Verify that

$$\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2} = \sqrt{\frac{\hbar}{2m\omega}}$$

and

$$\Delta p = \sqrt{\langle p^2\rangle - \langle p\rangle^2} = \sqrt{\frac{m\hbar\omega}{2}}$$

Hence

$$\Delta x \Delta p = \frac{\hbar}{2}$$