

## Static spherical metric

Let  $g_{\mu\nu}$  be the following metric.

$$g_{\mu\nu} = \begin{pmatrix} -e^{2\Phi} & 0 & 0 & 0 \\ 0 & e^{2\Lambda} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{pmatrix}$$

Verify that

$$G_{\mu\nu} = \begin{pmatrix} G_{tt} & 0 & 0 & 0 \\ 0 & G_{rr} & 0 & 0 \\ 0 & 0 & G_{\theta\theta} & 0 \\ 0 & 0 & 0 & G_{\phi\phi} \end{pmatrix}$$

where

$$G_{tt} = \frac{e^{2\Phi}}{r^2} \frac{d}{dr} (r (1 - e^{-2\Lambda}))$$

$$G_{rr} = \frac{1 - e^{2\Lambda}}{r^2} + \frac{2\Phi'}{r}$$

$$G_{\theta\theta} = r^2 e^{-2\Lambda} \left( \Phi'' + (\Phi')^2 + \frac{\Phi'}{r} - \Phi' \Lambda' - \frac{\Lambda'}{r} \right)$$

$$G_{\phi\phi} = G_{\theta\theta} \sin^2 \theta$$

Symbols  $\Phi$  and  $\Lambda$  are unspecified functions of  $r$ . See “A First Course in General Relativity” p. 255.