Problem 15.6.2. Using the results of Prob. 15.6.1, find an expression for the expectation value of the "position" ξ_λ for the coherent state in Eq. (15.94) in terms of $\bar{n}$, ω_λ and time t.

The expectation $\langle \hat{\xi}_\lambda \rangle$ for the coherent state $|\Psi_{\lambda\bar{n}}\rangle$ is

$$\langle \hat{\xi}_\lambda \rangle = \langle \Psi_{\lambda\bar{n}} | \hat{\xi}_\lambda | \Psi_{\lambda\bar{n}} \rangle$$

The operator $\hat{\xi}_\lambda$ is given by equation (15.78).

$$\hat{\xi}_\lambda \equiv \frac{1}{\sqrt{2}} \left(\hat{a}_\lambda + \hat{a}_\lambda^\dagger \right) \tag{15.78}$$

Hence

$$\langle \hat{\xi}_\lambda \rangle = \frac{1}{\sqrt{2}} \langle \Psi_{\lambda\bar{n}} | \left(\hat{a}_\lambda + \hat{a}_\lambda^\dagger \right) | \Psi_{\lambda\bar{n}} \rangle$$

Expand the right-hand side.

$$\langle \hat{\xi}_\lambda \rangle = \frac{1}{\sqrt{2}} \langle \Psi_{\lambda\bar{n}} | \hat{a}_\lambda | \Psi_{\lambda\bar{n}} \rangle + \frac{1}{\sqrt{2}} \langle \Psi_{\lambda\bar{n}} | \hat{a}_\lambda^\dagger | \Psi_{\lambda\bar{n}} \rangle \tag{1}$$

From problem 15.6.1 we have

$$\hat{a}_\lambda | \Psi_{\lambda\bar{n}} \rangle = \sqrt{\bar{n}} \exp(-i\omega_\lambda t) | \Psi_{\lambda\bar{n}} \rangle \tag{2}$$

It follows that

$$\langle \Psi_{\lambda\bar{n}} | \hat{a}_\lambda^\dagger = \left(\hat{a}_\lambda | \Psi_{\lambda\bar{n}} \rangle \right)^\dagger = \sqrt{\bar{n}} \exp(i\omega_\lambda t) \langle \Psi_{\lambda\bar{n}} | \tag{3}$$

Substitute (2) and (3) into (1) to obtain

$$\langle \hat{\xi}_\lambda \rangle = \sqrt{\frac{\bar{n}}{2}} \exp(-i\omega_\lambda t) \langle \Psi_{\lambda\bar{n}} | \Psi_{\lambda\bar{n}} \rangle + \sqrt{\frac{\bar{n}}{2}} \exp(i\omega_\lambda t) \langle \Psi_{\lambda\bar{n}} | \Psi_{\lambda\bar{n}} \rangle$$

Noting that $\langle \Psi_{\lambda\bar{n}} | \Psi_{\lambda\bar{n}} \rangle = 1$ we have

$$\langle \hat{\xi}_\lambda \rangle = \sqrt{\frac{\bar{n}}{2}} \exp(-i\omega_\lambda t) + \sqrt{\frac{\bar{n}}{2}} \exp(i\omega_\lambda t)$$

Finally, by the identity

$$2\cos(\omega_\lambda t) = \exp(-i\omega_\lambda t) + \exp(i\omega_\lambda t)$$

we have

$$\langle \hat{\xi}_\lambda \rangle = \sqrt{2\bar{n}} \cos(\omega_\lambda t)$$