Rotating wave approximation

Let $\Psi(\mathbf{r},t)$ be the following wave function for a two state system.

$$\Psi(\mathbf{r},t)=\psi_a(\mathbf{r})c_a(t)\exp(-\tfrac{i}{\hbar}E_at)+\psi_b(\mathbf{r})c_b(t)\exp(-\tfrac{i}{\hbar}E_bt)$$

Let $\hat{H}(\mathbf{r},t)$ be the Hamiltonian

$$\hat{H}(\mathbf{r},t)=\hat{H}_0(\mathbf{r})+\hat{H}_1(\mathbf{r},t)$$

where

$$\hat{H}_0\psi_a=E_a\psi_a,\quad \hat{H}_0\psi_b=E_b\psi_b,\quad \hat{H}_0\Psi=(E_a+E_b)\Psi$$

It was shown that the Schrödinger equation requires

$$\frac{d}{dt}c_a(t)=-\frac{i}{\hbar}\langle\psi_a|\hat{H}_1|\psi_b\rangle\exp(-i\omega_0t)c_b(t),\quad \frac{d}{dt}c_b(t)=-\frac{i}{\hbar}\langle\psi_b|\hat{H}_1|\psi_a\rangle\exp(i\omega_0t)c_a(t) \tag{1}$$

where

$$\omega_0=\frac{E_b-E_a}{\hbar}$$

Let $\hat{H}_1(\mathbf{r},t)$ be the perturbation

$$\hat{H}_1(\mathbf{r},t)=\hat{V}(\mathbf{r})\cos(\omega t)$$

Then

$$\langle\psi_a|\hat{H}_1|\psi_b\rangle=\langle\psi_a|\hat{V}|\psi_b\rangle\left[\tfrac{1}{2}\exp(i\omega t)+\tfrac{1}{2}\exp(-i\omega t)\right]$$

The rotating wave approximation discards the second term and asserts

$$\langle\psi_a|\hat{H}_1|\psi_b\rangle=\tfrac{1}{2}\langle\psi_a|\hat{V}|\psi_b\rangle\exp(i\omega t) \tag{2}$$

Substitute equation (2) into (1) to obtain

$$\frac{d}{dt}c_a(t)=-\frac{i}{2\hbar}\langle\psi_a|\hat{V}|\psi_b\rangle\exp\big(i(\omega-\omega_0)t\big)c_b(t) \tag{3}$$

and

$$\frac{d}{dt}c_b(t)=-\frac{i}{2\hbar}\langle\psi_b|\hat{V}|\psi_a\rangle\exp\big(i(\omega_0-\omega)t\big)c_a(t) \tag{4}$$

Solve for $c_b(t)$ with initial conditions $c_a(0)=1$ and $c_b(0)=0$.

$$c_b(t)=-\frac{i}{2\hbar\omega_r}\langle\psi_b|\hat{V}|\psi_a\rangle\sin(\omega_rt)\exp\left(\tfrac{i}{2}(\omega_0-\omega)t\right)$$

Symbol ω_r is the Rabi flopping frequency

$$\omega_r=\frac{1}{2}\sqrt{(\omega_0-\omega)^2+\frac{|\langle\psi_b|\hat{V}|\psi_a\rangle|^2}{\hbar^2}}$$

Use equation (1) to solve for $c_a(t)$.

$$c_a(t)=\left[\cos(\omega_rt)+\frac{i(\omega_0-\omega)}{2\omega_r}\sin(\omega_rt)\right]\exp\left(-\tfrac{i}{2}(\omega_0-\omega)t\right)$$