Matrix mechanics 3

Let J_1, J_2, and J_3 be rotation matrices.

$$J_1 = \frac{1}{\hbar}L_1, \quad J_2 = \frac{1}{\hbar}L_2, \quad J_3 = \frac{1}{\hbar}L_3$$

Let U be the unitary transformation

$$U = 1 - i\epsilon J_3 - \tfrac{1}{2}\epsilon^2 {J_3}^2$$

1. Show that to order ϵ^2

$$\begin{aligned}
&U^{-1}X_1U = \left(1 - \tfrac{1}{2}\epsilon^2\right)X_1 - \epsilon X_2 \\
&U^{-1}X_2U = \left(1 - \tfrac{1}{2}\epsilon^2\right)X_2 + \epsilon X_1 \\
&U^{-1}X_3U = X_3
\end{aligned}$$

2. Show that to order ϵ^2

$$\begin{aligned}
&U^{-1}P_1U = \left(1 - \tfrac{1}{2}\epsilon^2\right)P_1 - \epsilon P_2 \\
&U^{-1}P_2U = \left(1 - \tfrac{1}{2}\epsilon^2\right)P_2 + \epsilon P_1 \\
&U^{-1}P_3U = P_3
\end{aligned}$$

3. Show that to order ϵ^2

$$\begin{aligned}
&U^{-1}L_1U = \left(1 - \tfrac{1}{2}\epsilon^2\right)L_1 - \epsilon L_2 \\
&U^{-1}L_2U = \left(1 - \tfrac{1}{2}\epsilon^2\right)L_2 + \epsilon L_1 \\
&U^{-1}L_3U = L_3
\end{aligned}$$

4. Show that to order ϵ^2

$$U^{-1}HU = H$$

where

$$H = \frac{1}{2m}\left({P_1}^2 + {P_2}^2 + {P_3}^2\right)$$