

Atomic transitions 6

From the previous section

$$R_{a \rightarrow b} = \frac{\pi e^2}{3\epsilon_0 \hbar^2} |\langle \psi_b | \mathbf{r} | \psi_a \rangle|^2 \rho(\omega_0)$$

By Planck's law

$$\rho(\omega_0) = \frac{\hbar \omega_0^3}{\pi^2 c^3} \frac{1}{\exp\left(\frac{\hbar \omega_0}{kT}\right) - 1}$$

Hence

$$R_{a \rightarrow b} = \frac{e^2 \omega_0^3}{3\pi \epsilon_0 \hbar c^3} |\langle \psi_b | \mathbf{r} | \psi_a \rangle|^2 \frac{1}{\exp\left(\frac{\hbar \omega_0}{kT}\right) - 1} \quad (1)$$

Noting that

$$R_{b \rightarrow a} = R_{a \rightarrow b}$$

the spontaneous emission rate is

$$A_{b \rightarrow a} = R_{b \rightarrow a} \left[\exp\left(\frac{\hbar \omega_0}{kT}\right) - 1 \right] = \frac{e^2 \omega_0^3}{3\pi \epsilon_0 \hbar c^3} |\langle \psi_a | \mathbf{r} | \psi_b \rangle|^2$$

Verify dimensions.

$$A_{b \rightarrow a} \propto \frac{\frac{e^2}{\text{C}^2} \frac{\omega_0^3}{\text{s}^{-3}}}{\frac{\epsilon_0}{\text{C}^2 \text{J}^{-1} \text{m}^{-1}} \frac{\hbar}{\text{J s}} \frac{c^3}{\text{m}^3 \text{s}^{-3}}} \times \frac{|\langle \psi_a | \mathbf{r} | \psi_b \rangle|^2}{\text{m}^2} = \text{s}^{-1}$$

The stimulated emission rate is

$$B_{b \rightarrow a} = \frac{\pi^2 c^3}{\hbar \omega_0^3} A_{b \rightarrow a} = \frac{\pi e^2}{3\epsilon_0 \hbar^2} |\langle \psi_a | \mathbf{r} | \psi_b \rangle|^2 \quad (2)$$