

Galilean transformation

Let U be the transformation

$$U = 1 - i\epsilon G_1 - \frac{1}{2}\epsilon^2 G_1^2$$

where

$$G_1 = \frac{m}{\hbar} X_1$$

Show that for $\epsilon^4 = \epsilon^3 = 0$ we have

$$U^{-1}P_1U = P_1 - \epsilon m, \quad U^{-1}P_2U = P_2, \quad U^{-1}P_3U = P_3 \quad (1)$$

and

$$U^{-1}HU = H - \epsilon P_1 + \frac{1}{2}\epsilon^2 m \quad (2)$$

where

$$H = \frac{1}{2m} (P_1^2 + P_2^2 + P_3^2)$$