

The file q4.txt defines kets, operators, and a measurement function for simulating a four qbit quantum computer. See eigenmath.org/q.c for the program that generates q4.txt.

Kets are unit vectors in \mathbb{C}^{16} . The dimension is 16 because four qbits have $2^4 = 16$ basis states. Qbit numbering is $|q_3q_2q_1q_0\rangle$. The following basis kets are defined in q4.txt.

$$\begin{aligned} |0\rangle &= |0000_2\rangle = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \\ |1\rangle &= |0001_2\rangle = (0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \\ |2\rangle &= |0010_2\rangle = (0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \\ |3\rangle &= |0011_2\rangle = (0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \\ &\vdots \\ |15\rangle &= |1111_2\rangle = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1) \end{aligned}$$

Operators are 16×16 matrices that rotate ket vectors. (A ket always has unit length.) The following operators are defined in q4.txt.

C_{mn}	Controlled not (CNOT) operator, m is the control bit, n is the target bit.
H_n	Hadamard operator on bit n .
I	Identity matrix.
$P_{mn}(\phi)$	Controlled phase shift, m is the control bit, n is the target bit, ϕ is the phase.
Q	Quantum Fourier transform.
R	Inverse quantum Fourier transform.
S_{mn}	Swap bits m and n .
X_n	Pauli X (NOT) operator on bit n .
Y_n	Pauli Y operator on bit n .
Z_n	Pauli Z operator on bit n .

Function M measures the final state by drawing a graph of the probability for each of 16 states.

$$M(\psi)$$

Quantum algorithms are expressed as sequences of operators applied to the initial state $|0\rangle$. The operator sequence should be read backwards, from right to left, although the direction makes no difference mathematically.

Deutsch-Jozsa algorithm

Let $f(q_0, q_1, q_2)$ be an operator (16×16 matrix) that operates on q_3 in a manner consistent with a constant or balanced oracle. Then the Deutsch-Jozsa algorithm for identifying f is

$$\psi = H_2 H_1 H_0 f(q_0, q_1, q_2) H_3 X_3 H_2 H_1 H_0 |0\rangle$$

Bernstein-Vazirani algorithm

Let $f(q_0, q_1, q_2)$ be an operator (16×16 matrix) that operates on q_3 . Then the Bernstein-Vazirani algorithm for identifying f is

$$\psi = H_2 H_1 H_0 f(q_0, q_1, q_2) Z_3 H_3 H_2 H_1 H_0 |0\rangle$$