l'Hôpital's rule

Find the following limit.

$$y = \lim_{n\to\infty}\left(1-\frac{\alpha}{n^2}\right)^n$$

Rewrite as

$$y = \lim_{n\to\infty}\exp\left[n\log\left(1-\frac{\alpha}{n^2}\right)\right]$$

By composition limit law rewrite as exponential of limit.

$$y = \exp\left[\lim_{n\to\infty} n\log\left(1-\frac{\alpha}{n^2}\right)\right]$$

Rewrite again as

$$y = \lim_{n\to\infty}\exp\left[\frac{\log\left(1-\frac{\alpha}{n^2}\right)}{\frac{1}{n}}\right]$$

Both numerator and denominator have limit zero hence apply l'Hôpital's rule.

$$y = \exp\left[\lim_{n\to\infty}\frac{\frac{d}{dn}\log\left(1-\frac{\alpha}{n^2}\right)}{\frac{d}{dn}\frac{1}{n}}\right] = \exp\left(\lim_{n\to\infty}\frac{\frac{2\alpha}{n^3-\alpha n}}{-\frac{1}{n^2}}\right)$$

Hence

$$y = \exp\left(\lim_{n\to\infty}\frac{2\alpha}{\frac{\alpha}{n}-n}\right) = \exp(0) = 1$$