Dirac from boost

Let u_0 be the spinor for a spin-up electron at rest.

$$u_0 = \begin{pmatrix} \sqrt{2m} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Matrix A boosts u_0 in the z direction with $E = \sqrt{p_z^2 + m^2}$.

$$A = \frac{1}{\sqrt{2m(E+m)}} \begin{pmatrix} E+m & 0 & p_z & 0 \\ 0 & E+m & 0 & -p_z \\ p_z & 0 & E+m & 0 \\ 0 & -p_z & 0 & E+m \end{pmatrix}$$

Let u be the boosted spinor.

$$\begin{aligned} u &= Au_0 \\ &= \frac{1}{\sqrt{2m(E+m)}} \begin{pmatrix} E+m & 0 & p_z & 0 \\ 0 & E+m & 0 & -p_z \\ p_z & 0 & E+m & 0 \\ 0 & -p_z & 0 & E+m \end{pmatrix} \begin{pmatrix} \sqrt{2m} \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ &= \frac{1}{\sqrt{E+m}} \begin{pmatrix} E+m \\ 0 \\ p_z \\ 0 \end{pmatrix} \end{aligned}$$

Spinor u is normalized such that

$$u^\dagger u = 2E$$

We will now derive $\not{p}$ from A and the Dirac equation

$$\not{p}u = mu$$

Substitute Au_0 for u on the right-hand side.

$$\not{p}u = mAu_0$$

By the identity $\gamma^0 u_0 = u_0$, substitute $\gamma^0 u_0$ for u_0.

$$\not{p}u = mA\gamma^0 u_0$$

Substitute $A^{-1}u$ for u_0.

$$\not{p}u = mA\gamma^0 A^{-1} u$$

Cancel u to obtain

$$\not{p} = mA\gamma^0 A^{-1} \tag{1}$$

To recover the Dirac equation from boost A, start with the identity

$$\gamma^0 u_0 = u_0$$

Boost both sides of the equation.

$$A\gamma^0 u_0 = Au_0$$

By equation (1) substitute $(\not{p}/m)A$ for $A\gamma^0$.

$$\frac{\not{p}}{m} Au_0 = Au_0$$

Substitute u for Au_0.

$$\frac{\not{p}}{m} u = u$$

Multiply both sides by m to obtain the Dirac equation

$$\not{p} u = mu \tag{2}$$

Eigenmath script