

5-5. Assume that the function  $f(x, y, z, \dots)$  can be represented by

$$f(x, y, z, \dots) = \sum_a \sum_b \sum_c \cdots F'_{a,b,c,\dots} \chi_{a,b,c,\dots}(x, y, z, \dots) \quad (5.38)$$

By substituting this relation into equation (5.36), and using the orthogonal properties of  $\chi$  as defined by equation (5.35), show that  $F'_{a,b,c,\dots} = F_{a,b,c,\dots}$ .

Take (5.38) and prime the indices.

$$f(x, y, z, \dots) = \sum_{a'} \sum_{b'} \sum_{c'} \cdots F'_{a',b',c',\dots} \chi_{a',b',c',\dots}(x, y, z, \dots) \quad (1)$$

This is equation (5.36).

$$F_{a,b,c,\dots} = \int_{-\infty}^{\infty} \chi_{a,b,c,\dots}^*(x) f(x) dx \quad (5.36)$$

Substitute the right-hand side of (1) for  $f(x)$  in (5.36).

$$\begin{aligned} F_{a,b,c,\dots} &= \int_{\mathbb{R}^n} \chi_{a,b,c,\dots}^*(x, y, z, \dots) \\ &\quad \times \left( \sum_{a'} \sum_{b'} \sum_{c'} \cdots F'_{a',b',c',\dots} \chi_{a',b',c',\dots}(x, y, z, \dots) \right) dx dy dz \cdots \end{aligned}$$

Combine factors.

$$\begin{aligned} F_{a,b,c,\dots} &= \int_{\mathbb{R}^n} \sum_{a'} \sum_{b'} \sum_{c'} \cdots \\ &\quad \chi_{a,b,c,\dots}^*(x, y, z, \dots) F'_{a',b',c',\dots} \chi_{a',b',c',\dots}(x, y, z, \dots) dx dy dz \cdots \end{aligned}$$

Interchange the order of integration and summation.

$$\begin{aligned} F_{a,b,c,\dots} &= \sum_{a'} \sum_{b'} \sum_{c'} \cdots \int_{\mathbb{R}^n} \\ &\quad \chi_{a,b,c,\dots}^*(x, y, z, \dots) F'_{a',b',c',\dots} \chi_{a',b',c',\dots}(x, y, z, \dots) dx dy dz \cdots \end{aligned}$$

Factor out  $F'$ .

$$\begin{aligned} F_{a,b,c,\dots} &= \sum_{a'} \sum_{b'} \sum_{c'} \cdots F'_{a',b',c',\dots} \\ &\quad \times \left( \int_{\mathbb{R}^n} \chi_{a,b,c,\dots}^*(x, y, z, \dots) \chi_{a',b',c',\dots}(x, y, z, \dots) dx dy dz \cdots \right) \end{aligned}$$

By equation (5.35) the integral becomes a product of delta functions.

$$F_{a,b,c,\dots} = \sum_{a'} \sum_{b'} \sum_{c'} \cdots F'_{a',b',c',\dots} \delta(a - a') \delta(b - b') \delta(c - c') \cdots$$

Hence for  $F_{a,b,c,\dots} \neq 0$  we must have  $a = a'$ ,  $b = b'$ , etc. Therefore

$$F_{a,b,c,\dots} = F'_{a',b',c',\dots} = F'_{a,b,c,\dots}$$