4-4. Show that

$$\frac{\partial^2}{\partial x^2}x = x\frac{\partial^2}{\partial x^2} + 2\frac{\partial}{\partial x} \tag{4.22}$$

and therefore for the H in equation (4.15)

$$Hx - xH = -\frac{\hbar^2}{m} \frac{\partial}{\partial x}$$

We have

$$\frac{\partial}{\partial x}(x\psi) = \psi + x \frac{\partial}{\partial x}\psi$$

Differentiate a second time.

$$\frac{\partial^2}{\partial x^2}(x\psi) = \frac{\partial}{\partial x} \left(\psi + x \frac{\partial}{\partial x} \psi \right)$$
$$= \frac{\partial}{\partial x} \psi + \frac{\partial}{\partial x} \psi + x \frac{\partial^2}{\partial x^2} \psi$$
$$= 2 \frac{\partial}{\partial x} \psi + x \frac{\partial^2}{\partial x^2} \psi$$

This is equation (4.15).

$$H = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x,t)$$

Solve for $\partial^2/\partial x^2$.

$$\frac{\partial^2}{\partial x^2} = -\frac{2m}{\hbar^2} (H - V(x, t)) \tag{1}$$

By equation (4.22) above

$$\frac{\partial^2}{\partial x^2}x - x\frac{\partial^2}{\partial x^2} = 2\frac{\partial}{\partial x} \tag{2}$$

Substitute (1) into (2).

$$-\frac{2m}{\hbar^2}(H - V(x,t))x + x\frac{2m}{\hbar^2}(H - V(x,t)) = 2\frac{\partial}{\partial x}$$

The two V terms cancel. Multiply both sides by $-\hbar^2/2m$.

$$Hx - xH = -\frac{\hbar^2}{m} \frac{\partial}{\partial x}$$