

Translation

Let K be coordinate translation operators.

$$K_1 = \frac{1}{\hbar}P_1, \quad K_2 = \frac{1}{\hbar}P_2, \quad K_3 = \frac{1}{\hbar}P_3$$

Let U be the unitary transformation

$$U = 1 - i\epsilon K_3 - \frac{1}{2}\epsilon^2 K_3^2$$

Show that to order ϵ^2

$$U^{-1}L_1U = L_1 - \epsilon P_2, \quad U^{-1}L_2U = L_2 + \epsilon P_1, \quad U^{-1}L_3U = L_3$$

where L are the angular momentum operators

$$L_1 = X_2P_3 - X_3P_2, \quad L_2 = X_3P_1 - X_1P_3, \quad L_3 = X_1P_2 - X_2P_1$$