

6-21. Consider the special case that the perturbing potential V has no matrix elements except between the two states 1 and 2; and further, suppose these states are degenerate, that is, suppose $E_1 = E_2$. Let $V_{12} = V_{21} = v$ and let V_{11} , V_{22} , and all other V_{mn} be zero. Show that

$$\lambda_{11} = 1 - \frac{v^2 T^2}{2\hbar^2} + \frac{v^4 T^4}{24\hbar^4} - \dots = \cos \frac{vT}{\hbar} \quad (6.81)$$

$$\lambda_{12} = -i \frac{vT}{\hbar} + i \frac{v^3 T^3}{6\hbar^3} - \dots = -i \sin \frac{vT}{\hbar} \quad (6.82)$$

Consider equation (6.75).

$$\begin{aligned} \lambda_{mn}(t_b, t_a) = & \delta_{mn} \exp \left(-\frac{i}{\hbar} E_m(t_b - t_a) \right) \\ & - \frac{i}{\hbar} \int_{t_a}^{t_b} \exp \left(-\frac{i}{\hbar} E_m(t_b - t_a) \right) \sum_j V_{mj}(t_c) \lambda_{jn}(t_c, t_a) dt_c \end{aligned} \quad (6.75)$$

Let $E = E_1 = E_2$ and $T = t_b - t_a$. Then by (6.75) we have

$$\begin{aligned} \lambda_{11}(t_b, t_a) &= \exp \left(-\frac{iET}{\hbar} \right) - \frac{i}{\hbar} \int_{t_a}^{t_b} \exp \left(-\frac{i}{\hbar} E(t_b - t_c) \right) v(t_c) \lambda_{21}(t_c, t_a) dt_c \\ \lambda_{12}(t_b, t_a) &= -\frac{i}{\hbar} \int_{t_a}^{t_b} \exp \left(-\frac{i}{\hbar} E(t_b - t_c) \right) v(t_c) \lambda_{21}(t_c, t_a) dt_c \\ \lambda_{21}(t_b, t_a) &= -\frac{i}{\hbar} \int_{t_a}^{t_b} \exp \left(-\frac{i}{\hbar} E(t_b - t_c) \right) v(t_c) \lambda_{12}(t_c, t_a) dt_c \\ \lambda_{22}(t_b, t_a) &= \exp \left(-\frac{iET}{\hbar} \right) - \frac{i}{\hbar} \int_{t_a}^{t_b} \exp \left(-\frac{i}{\hbar} E(t_b - t_c) \right) v(t_c) \lambda_{12}(t_c, t_a) dt_c \end{aligned}$$