Matrix mechanics 3

From "Lectures on Quantum Mechanics" by Weinberg.

Let $\mathbf{R}$ be the Runge-Lenz vector

$$\mathbf{R} = -\frac{Ze^2\mathbf{x}}{r} + \frac{1}{2m}(\mathbf{p} \times \mathbf{L} - \mathbf{L} \times \mathbf{p}) \tag{4.8.1}$$

where $\mathbf{L}$ is the orbital angular momentum vector

$$\mathbf{L} = \mathbf{x} \times \mathbf{p}$$

Verify that

$$\mathbf{R}^2 = Z^2e^4 + \frac{2}{m}H(\mathbf{L}^2 + \hbar^2) \tag{4.8.7}$$

where H is the Hamiltonian

$$H = \frac{\mathbf{p}^2}{2m} - \frac{Ze^2}{r}$$

Eigenmath script