To extend (7.33) to three dimensions, change derivative to gradient, hence

$$\langle \nabla_k F \rangle = -\frac{i}{\hbar} \langle F \nabla_k S \rangle \tag{1}$$

where

$$\nabla_k = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$$

and

$$\mathbf{i} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{j} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{k} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Let

$$F = x_k + y_k + z_k$$

Then

$$\nabla_k F = \frac{\partial F}{\partial x_k}\mathbf{i} + \frac{\partial F}{\partial y_k}\mathbf{j} + \frac{\partial F}{\partial z_k}\mathbf{k}$$
$$= \mathbf{i} + \mathbf{j} + \mathbf{k}$$

Hence

$$\langle \nabla_k F \rangle = \langle 1 \rangle \mathbf{i} + \langle 1 \rangle \mathbf{j} + \langle 1 \rangle \mathbf{k} \tag{2}$$

By equation (7.43)

$$\left\langle (x_k + y_k + z_k)\frac{\partial S}{\partial x_k} \right\rangle \mathbf{i} = \left\langle x_k\frac{\partial S}{\partial x_k} \right\rangle \mathbf{i}$$

$$\left\langle (x_k + y_k + z_k)\frac{\partial S}{\partial y_k} \right\rangle \mathbf{j} = \left\langle y_k\frac{\partial S}{\partial y_k} \right\rangle \mathbf{j}$$

$$\left\langle (x_k + y_k + z_k)\frac{\partial S}{\partial z_k} \right\rangle \mathbf{k} = \left\langle z_k\frac{\partial S}{\partial z_k} \right\rangle \mathbf{k}$$

Hence

$$\langle F \nabla_k S \rangle = \left\langle x_k\frac{\partial S}{\partial x_k} \right\rangle \mathbf{i} + \left\langle y_k\frac{\partial S}{\partial y_k} \right\rangle \mathbf{j} + \left\langle z_k\frac{\partial S}{\partial z_k} \right\rangle \mathbf{k} \tag{3}$$

By equations (1), (2), and (3)

$$\langle 1 \rangle \mathbf{i} = -\frac{i}{\hbar} \left\langle x_k\frac{\partial S}{\partial x_k} \right\rangle \mathbf{i}$$

$$\langle 1 \rangle \mathbf{j} = -\frac{i}{\hbar} \left\langle y_k\frac{\partial S}{\partial y_k} \right\rangle \mathbf{j}$$

$$\langle 1 \rangle \mathbf{k} = -\frac{i}{\hbar} \left\langle z_k\frac{\partial S}{\partial z_k} \right\rangle \mathbf{k}$$

Then by the same arguments in the book for equation (7.49) we have

$$\left\langle (x_{k+1} - x_k)^2 \right\rangle \mathbf{i} = -\frac{\hbar\epsilon}{im} \langle 1 \rangle \mathbf{i}$$

$$\left\langle (y_{k+1} - y_k)^2 \right\rangle \mathbf{j} = -\frac{\hbar\epsilon}{im} \langle 1 \rangle \mathbf{j}$$

$$\left\langle (z_{k+1} - z_k)^2 \right\rangle \mathbf{k} = -\frac{\hbar\epsilon}{im} \langle 1 \rangle \mathbf{k}$$

Hence (7.50) is shown to be true.

The action for a particle in three dimensions is

$$S = \int_{t_a}^{t_b} \left(\frac{m}{2} \left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2 \right) - V(x, y, z) \right) dt$$

Hence

$$\nabla_k S = \frac{\partial S}{\partial x_k} \mathbf{i} + \frac{\partial S}{\partial y_k} \mathbf{j} + \frac{\partial S}{\partial z_k} \mathbf{k}$$

where

$$\frac{\partial S}{\partial x_k} = -m \left(\frac{x_{k+1} - x_k}{\epsilon} - \frac{x_k - x_{k-1}}{\epsilon} \right) - \epsilon \frac{\partial V}{\partial x} \bigg|_{x_k, y_k, z_k}$$

$$\frac{\partial S}{\partial y_k} = -m \left(\frac{y_{k+1} - y_k}{\epsilon} - \frac{y_k - y_{k-1}}{\epsilon} \right) - \epsilon \frac{\partial V}{\partial y} \bigg|_{x_k, y_k, z_k}$$

$$\frac{\partial S}{\partial z_k} = -m \left(\frac{z_{k+1} - z_k}{\epsilon} - \frac{z_k - z_{k-1}}{\epsilon} \right) - \epsilon \frac{\partial V}{\partial z} \bigg|_{x_k, y_k, z_k}$$

FIXME show (7.51)