This is the Dirac equation.

$$i\hbar \left(\frac{1}{c}\gamma^0 \frac{\partial}{\partial t} + \gamma^1 \frac{\partial}{\partial x} + \gamma^2 \frac{\partial}{\partial y} + \gamma^3 \frac{\partial}{\partial z} \right) \psi = mc\psi$$

The following set of gamma matrices are known as the "Dirac representation."

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \quad \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Let

$$\phi(x, y, z, t) = p_x x + p_y y + p_z z - Et$$

where

$$E = \sqrt{p_x^2 c^2 + p_y^2 c^2 + p_z^2 c^2 + m^2 c^4}$$

The four positive wave solutions to the Dirac equation are

$$\psi_1 = \begin{pmatrix} E + mc^2 \\ 0 \\ p_z c \\ p_x c + i p_y c \end{pmatrix} \exp\left(\frac{i\phi}{\hbar}\right) \quad \psi_2 = \begin{pmatrix} 0 \\ E + mc^2 \\ p_x c - i p_y c \\ -p_z c \end{pmatrix} \exp\left(\frac{i\phi}{\hbar}\right)$$

$$\psi_3 = \begin{pmatrix} p_z c \\ p_x c + i p_y c \\ E - mc^2 \\ 0 \end{pmatrix} \exp\left(\frac{i\phi}{\hbar}\right) \quad \psi_4 = \begin{pmatrix} p_x c - i p_y c \\ -p_z c \\ 0 \\ E - mc^2 \end{pmatrix} \exp\left(\frac{i\phi}{\hbar}\right)$$

The four negative wave solutions are

$$\psi_5 = \begin{pmatrix} E - mc^2 \\ 0 \\ p_z c \\ p_x c + i p_y c \end{pmatrix} \exp\left(-\frac{i\phi}{\hbar}\right) \quad \psi_6 = \begin{pmatrix} 0 \\ E - mc^2 \\ p_x c - i p_y c \\ -p_z c \end{pmatrix} \exp\left(-\frac{i\phi}{\hbar}\right)$$

$$\psi_7 = \begin{pmatrix} p_z c \\ p_x c + i p_y c \\ E + mc^2 \\ 0 \end{pmatrix} \exp\left(-\frac{i\phi}{\hbar}\right) \quad \psi_8 = \begin{pmatrix} p_x c - i p_y c \\ -p_z c \\ 0 \\ E + mc^2 \end{pmatrix} \exp\left(-\frac{i\phi}{\hbar}\right)$$

The negative wave solutions flip the sign of the mc^2 term.

The following solutions are used by quantum electrodynamics.

$$\psi_1 \quad \text{Fermion, spin up}$$
$$\psi_2 \quad \text{Fermion, spin down}$$

$$\psi_7 \quad \text{Anti-fermion, spin up}$$
$$\psi_8 \quad \text{Anti-fermion, spin down}$$