16.1.1. Consider a system that has two possible single fermion states, 1 and 2, and can have anywhere from zero to two particles in it. There are therefore four possible states of this system: $|0_1, 0_2\rangle$ (the state with no particles in either single-fermion state, a state we could also write as the empty state $|0\rangle$), $|1_1, 0_2\rangle$, $|0_1, 1_2\rangle$, and $|1_1, 1_2\rangle$. (We will also choose the standard ordering of the states to be in the order 1, 2.) Any state of the system could be described as a linear combination of these four basis states, i.e.,

$$|\Psi\rangle = c_1|0_1, 0_2\rangle + c_2|1_1, 0_2\rangle + c_3|0_1, 1_2\rangle + c_4|1_1, 1_2\rangle$$

which we could also choose to write as a vector

$$|\Psi\rangle = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix}$$

(i) Construct 4×4 matrices for each of the operators $\hat{b}_1^\dagger$, $\hat{b}_1$, $\hat{b}_2^\dagger$, and $\hat{b}_2$.

(ii) Explicitly verify by matrix multiplication the anticommutation relations

$$\begin{aligned}
\hat{b}_1^\dagger \hat{b}_1 + \hat{b}_1 \hat{b}_1^\dagger &= 1 \\
\hat{b}_2^\dagger \hat{b}_2 + \hat{b}_2 \hat{b}_2^\dagger &= 1 \\
\hat{b}_1^\dagger \hat{b}_2^\dagger + \hat{b}_2^\dagger \hat{b}_1^\dagger &= 0 \\
\hat{b}_1^\dagger \hat{b}_1^\dagger + \hat{b}_1^\dagger \hat{b}_1^\dagger &= 0
\end{aligned}$$

(i) We have

$$\hat{b}_1^\dagger = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \quad \hat{b}_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

and

$$\hat{b}_2^\dagger = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad \hat{b}_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(ii) See Eigenmath demo.