

White dwarf

The radius of a white dwarf can be estimated using the electron gas model of a solid.

The total electron energy E of a spherical electron gas is

$$E = \left(\frac{3\pi^2}{2} \right)^{\frac{1}{3}} \frac{9\hbar^2 N^{\frac{5}{3}}}{20m_e R^2}$$

where R is the radius, N is the number of free electrons, and m_e is electron mass.

The gravitational energy U of a sphere with mass M and uniform density is

$$U = -\frac{3GM^2}{5R}$$

Minimize the total energy by finding R such that

$$\frac{d}{dR}(E + U) = 0$$

Hence

$$-\left(\frac{3\pi^2}{2} \right)^{\frac{1}{3}} \frac{9\hbar^2 N^{\frac{5}{3}}}{10m_e R^3} + \frac{3GM^2}{5R^2} = 0$$

Multiply both sides by R^3 .

$$-\left(\frac{3\pi^2}{2} \right)^{\frac{1}{3}} \frac{9\hbar^2 N^{\frac{5}{3}}}{10m_e} + \frac{3GM^2}{5} R = 0$$

Hence

$$R = \left(\frac{3\pi^2}{2} \right)^{\frac{1}{3}} \frac{9\hbar^2 N^{\frac{5}{3}}}{10m_e} \frac{5}{3GM^2} = \left(\frac{3\pi^2}{2} \right)^{\frac{1}{3}} \frac{3\hbar^2 N^{\frac{5}{3}}}{2m_e GM^2} \quad (1)$$

The number of free electrons is estimated to be one-half the number of nucleons.

$$N = \frac{M}{2m_p} \quad (2)$$

Substitute (2) into (1) to obtain

$$R = \frac{3\hbar^2}{8Gm_e} \left(\frac{3\pi^2}{Mm_p^5} \right)^{\frac{1}{3}} \quad (3)$$

For one solar mass $M = M_\odot = 2 \times 10^{30}$ kg we have

$$R = 7146 \text{ km}$$