

Consider the following equation from p. 216.

$$Q_\alpha = \frac{1}{\sqrt{2}}(Q_\alpha^c - iQ_\alpha^s)$$

It follows that

$$Q_\alpha^* Q_\alpha = \frac{1}{2}(Q_\alpha^c)^2 + \frac{1}{2}(Q_\alpha^s)^2 \quad (1)$$

Substitute (1) into (8.78).

$$L = \frac{1}{4} \sum_{\alpha=0}^{N-1} \left((\dot{Q}_\alpha^c)^2 + (\dot{Q}_\alpha^s)^2 - \omega_\alpha^2 (Q_\alpha^c)^2 - \omega_\alpha^2 (Q_\alpha^s)^2 \right) \quad (2)$$

Consider equation (2.7).

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{Q}} = \frac{\partial L}{\partial Q} \quad (2.7)$$

Substitute (2) into (2.7) to obtain the following equations of motion.

$$\ddot{Q}_\alpha^c(t) = -\omega_\alpha^2 Q_\alpha^c(t) \quad \ddot{Q}_\alpha^s(t) = -\omega_\alpha^2 Q_\alpha^s(t) \quad (3)$$

From equation (8.58) and the associated text on p. 210, the unnormalized ground state eigenfunction corresponding to (3) is

$$\phi_0(x_\alpha) = \exp \left(-\frac{\omega_\alpha x_\alpha^2}{2\hbar} \right)$$

Then by equation (8.62)

$$\Phi_0 = \prod_{\alpha=0}^{N-1} \phi_0(Q_\alpha^c) \phi_0(Q_\alpha^s) = \exp \left(-\frac{1}{2\hbar} \sum_{\alpha=0}^{N-1} \omega_\alpha (Q_\alpha^c + Q_\alpha^s) \right)$$