Probability current

Let $\mathbf{J}$ be the probability current

$$\mathbf{J} = \frac{i\hbar}{2m} \left(\Psi \nabla \Psi^* - \Psi^* \nabla \Psi\right)$$

Show that

$$\nabla \cdot \mathbf{J} = -\frac{\partial}{\partial t} |\Psi|^2$$

Recall that

$$\nabla \cdot \nabla = \nabla^2$$

By the product rule for divergence we have

$$\nabla \cdot \left(\Psi \nabla \Psi^*\right) = \nabla \Psi \cdot \nabla \Psi^* + \Psi \nabla^2 \Psi^*$$

Hence

$$\nabla \cdot \mathbf{J} = \frac{i\hbar}{2m} \left(\Psi \nabla^2 \Psi^* - \Psi^* \nabla^2 \Psi\right) \tag{1}$$

By the Schrödinger equation

$$\frac{\partial \Psi}{\partial t} = \frac{i\hbar}{2m} \nabla^2 \Psi - \frac{i}{\hbar} V \Psi$$

we have

$$\frac{\partial}{\partial t} |\Psi|^2 = \Psi^* \frac{\partial \Psi}{\partial t} + \Psi \frac{\partial \Psi^*}{\partial t} = \frac{i\hbar}{2m} \left(\Psi^* \nabla^2 \Psi - \Psi \nabla^2 \Psi^*\right) \tag{2}$$

Hence by (1) and (2)

$$\nabla \cdot \mathbf{J} = -\frac{\partial}{\partial t} |\Psi|^2$$