

Addition of angular momentum 3

Let Ψ be an eigenstate of J^2 . Then for total angular momentum quantum number j

$$J^2\Psi = j(j+1)\hbar^2\Psi$$

The following Ψ are eigenstates with $j = l + \frac{1}{2}$.

$$\begin{aligned}\Psi &= Y_{l,l}\chi_+ \\ \Psi &= Y_{l,-l}\chi_-\end{aligned}$$

For $m < l$ the following linear combinations are eigenstates.

$$\begin{aligned}\Psi &= \left(\frac{l+m+1}{2l+1}\right)^{1/2} Y_{lm}\chi_+ + \left(\frac{l-m}{2l+1}\right)^{1/2} Y_{l,m+1}\chi_- & j = l + \frac{1}{2} \\ \Psi &= \left(\frac{l-m}{2l+1}\right)^{1/2} Y_{lm}\chi_+ - \left(\frac{l+m+1}{2l+1}\right)^{1/2} Y_{l,m+1}\chi_- & j = l - \frac{1}{2}\end{aligned}$$

Eigenmath code