Quantum angular momentum 1

Definitions.

$$\mathbf{L} = \mathbf{X} \times \mathbf{P} = \begin{pmatrix} X_2P_3 - X_3P_2 \\ X_3P_1 - X_1P_3 \\ X_1P_2 - X_2P_1 \end{pmatrix}$$

$$L^2 = |\mathbf{L}|^2 = {L_1}^2 + {L_2}^2 + {L_3}^2$$

$$\mathbf{J} = \frac{1}{\hbar}\mathbf{L}$$

Let

$$X_j = x_j, \quad P_j = -i\hbar\frac{\partial}{\partial x_j}$$

Show that

1. $$[\mathbf{X}, L_3] = \begin{pmatrix} -i\hbar X_2 \\ i\hbar X_1 \\ 0 \end{pmatrix}$$

2. $$[\mathbf{P}, L_3] = \begin{pmatrix} -i\hbar P_2 \\ i\hbar P_1 \\ 0 \end{pmatrix}$$

3. $$[\mathbf{L}, L^2] = 0$$

4. $$\mathbf{J} \times \mathbf{J} = i\mathbf{J}$$

5. $$\mathbf{P} \times \mathbf{L} + \mathbf{L} \times \mathbf{P} = 2i\hbar\mathbf{P}$$

6. $$\tfrac{1}{2}\left(\mathbf{P} \times \mathbf{L} - \mathbf{L} \times \mathbf{P}\right) = \mathbf{P} \times \mathbf{L} - i\hbar\mathbf{P}$$

7. $$[L_i, P_j] = i\hbar\sum_k \epsilon_{ijk}P_k$$