(1.5) For a three-dimensional elastic medium, the potential energy is

$$V = \frac{\mathcal{T}}{2} \int d^3x \, (\nabla \psi)^2 \tag{1.46}$$

and the kinetic energy is

$$T = \frac{\rho}{2} \int d^3x \, \left(\frac{\partial \psi}{\partial t} \right)^2 \tag{1.47}$$

Use these results, and the functional derivative approach, to show that ψ obeys the wave equation

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

where v is the velocity of the wave.

By equation (1.19)

$$\frac{\delta V}{\delta \psi} = -\mathcal{T} \nabla^2 \psi \qquad \frac{\delta T}{\delta \psi} = -\rho \frac{\partial^2 \psi}{\partial t^2}$$

By equation (1.22)

$$\mathcal{T} \nabla^2 \psi = \rho \frac{\partial^2 \psi}{\partial t^2}$$

Then for $v^2 = \mathcal{T}/\rho$ we have

$$v^2 \nabla^2 \psi = \frac{\partial^2 \psi}{\partial t^2}$$