Exercise 6.2. Show that if the two normalization conditions of Eqs. 6.4 are satisfied, then the state vector of Eq. 6.5 is automatically normalized as well. In other words, show that for this product state, normalizing the overall state vector does not put any additional constraints on the α's and β's.

These are the normalization conditions.

$$\alpha_u^*\alpha_u + \alpha_d^*\alpha_d = 1 \tag{6.4a}$$

$$\beta_u^*\beta_u + \beta_d^*\beta_d = 1 \tag{6.4b}$$

Consider equation (6.5).

$$\alpha_u\beta_u|uu\rangle + \alpha_u\beta_d|ud\rangle + \alpha_d\beta_u|du\rangle + \alpha_d\beta_d|dd\rangle \tag{6.5}$$

In order for this state to be normalized we must have $P = 1$ where

$$P = |\alpha_u\beta_u|^2 + |\alpha_u\beta_d|^2 + |\alpha_d\beta_u|^2 + |\alpha_d\beta_d|^2$$

Rewrite as

$$P = \alpha_u^*\alpha_u\,\beta_u^*\beta_u + \alpha_u^*\alpha_u\,\beta_d^*\beta_d + \alpha_d^*\alpha_d\,\beta_u^*\beta_u + \alpha_d^*\alpha_d\,\beta_d^*\beta_d$$

By the distributive law

$$P = \alpha_u^*\alpha_u(\beta_u^*\beta_u + \beta_d^*\beta_d) + \alpha_d^*\alpha_d(\beta_u^*\beta_u + \beta_d^*\beta_d) \tag{1}$$

Substitute (6.4b) into (1) to obtain

$$P = \alpha_u^*\alpha_u + \alpha_d^*\alpha_d$$

By (6.4a) we have $P = 1$ hence the state vector in (6.5) is normalized.