

Angular separation

Let P_1 and P_2 be two points in polar coordinates such that

$$\begin{aligned} P_1 &= r_1 \hat{\mathbf{e}}_r + \theta_1 \hat{\mathbf{e}}_\theta + \phi_1 \hat{\mathbf{e}}_\phi \\ P_2 &= r_2 \hat{\mathbf{e}}_r + \theta_2 \hat{\mathbf{e}}_\theta + \phi_2 \hat{\mathbf{e}}_\phi \end{aligned}$$

The angular separation θ_{12} between P_1 and P_2 is

$$\begin{aligned} \cos \theta_{12} &= \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \cos(\phi_1 - \phi_2) \\ &= \frac{r_1^2 + r_2^2 - r_{12}^2}{2r_1 r_2} \\ &= \frac{x_1 x_2 + y_1 y_2 + z_1 z_2}{r_1 r_2} \end{aligned}$$

where

$$r_{12}^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$$

and

$$\begin{array}{lll} x_1 = r_1 \sin \theta_1 \cos \phi_1 & y_1 = r_1 \sin \theta_1 \sin \phi_1 & z_1 = r_1 \cos \theta_1 \\ x_2 = r_2 \sin \theta_2 \cos \phi_2 & y_2 = r_2 \sin \theta_2 \sin \phi_2 & z_2 = r_2 \cos \theta_2 \end{array}$$