(14.2) *A demonstration that the photon has spin-1, with only two spin polarizations.*

A photon γ propagates with momentum $q^\mu = (|\mathbf{q}|, 0, 0, |\mathbf{q}|)$. Working with a basis where the two transverse photon polarizations are $\epsilon^\mu_{\lambda=1}(q) = (0, 1, 0, 0)$ and $\epsilon^\mu_{\lambda=2}(q) = (0, 0, 1, 0)$, it may be shown, using Noether's theorem, that the operator $\hat{S}^z$, whose eigenvalue is the z-component spin angular momentum of the photon, obeys the commutation relation

$$\left[\hat{S}^z, \hat{a}^\dagger_{\mathbf{q}\lambda}\right] = i\epsilon^{\mu=1*}_\lambda(q)\hat{a}^\dagger_{\mathbf{q}\lambda=2} - i\epsilon^{\mu=2*}_\lambda(q)\hat{a}^\dagger_{\mathbf{q}\lambda=1} \tag{14.36}$$

(i) Define creation operators for the circular polarizations via

$$\begin{aligned} \hat{b}^\dagger_{\mathbf{q}R} &= -\frac{1}{\sqrt{2}}\left(\hat{a}^\dagger_{\mathbf{q}1} + i\hat{a}^\dagger_{\mathbf{q}2}\right) \\ \hat{b}^\dagger_{\mathbf{q}L} &= \frac{1}{\sqrt{2}}\left(\hat{a}^\dagger_{\mathbf{q}1} - i\hat{a}^\dagger_{\mathbf{q}2}\right) \end{aligned} \tag{14.37}$$

Show that

$$\begin{aligned} \left[\hat{S}^z, \hat{b}^\dagger_{\mathbf{q}R}\right] &= \hat{b}^\dagger_{\mathbf{q}R} \\ \left[\hat{S}^z, \hat{b}^\dagger_{\mathbf{q}L}\right] &= -\hat{b}^\dagger_{\mathbf{q}L} \end{aligned} \tag{14.38}$$