

Consider a light wave propagating in the  $z$  direction. For simplicity let the light be linearly polarized with electric field vector  $\mathbf{E}$  pointing in the  $x$  direction.

$$\mathbf{E}(t, x, y, z) = \begin{pmatrix} E_x \cos(kz - \omega t) \\ 0 \\ 0 \end{pmatrix}$$

Symbol  $\omega$  is angular frequency and  $k$  is the wave number  $k = \omega/c$ .

The corresponding wave function is

$$\psi = A \left| n - \frac{1}{2} \right\rangle + B \left| n + \frac{1}{2} \right\rangle$$

where  $n$  is the number of photons per unit volume and

$$A = \exp \left( -i \left( n - \frac{1}{2} \right) \omega t \right) \\ B = \exp \left( -i \left( n + \frac{1}{2} \right) \omega t \right)$$

The electric field operator is

$$\hat{\mathcal{E}} = C\hat{a} + C^*\hat{a}^\dagger$$

where  $\hat{a}$  and  $\hat{a}^\dagger$  are the lowering and raising operators such that

$$a \left| n + \frac{1}{2} \right\rangle = \sqrt{n} \left| n - \frac{1}{2} \right\rangle \\ a^\dagger \left| n - \frac{1}{2} \right\rangle = \sqrt{n} \left| n + \frac{1}{2} \right\rangle$$

The quantity  $C$  is

$$C = \sqrt{\frac{\hbar\omega}{2V\varepsilon_0}} \exp(ikz)$$

where  $V$  is a unit volume.

Apply electric field operator  $\hat{\mathcal{E}}$  to wave function  $\psi$ .

$$\begin{aligned} \hat{\mathcal{E}}\psi &= C\hat{a}\psi + C^*\hat{a}^\dagger\psi \\ &= CA\sqrt{n-1} \left| n - \frac{3}{2} \right\rangle + CB\sqrt{n} \left| n - \frac{1}{2} \right\rangle + C^*A\sqrt{n} \left| n + \frac{1}{2} \right\rangle + C^*B\sqrt{n+1} \left| n + \frac{3}{2} \right\rangle \end{aligned}$$

The observed electric field is the eigenvalue  $\mathcal{E}$  such that  $\hat{\mathcal{E}}\psi = \mathcal{E}\psi$ .

$$\begin{aligned} \mathcal{E} &= \psi^\dagger \hat{\mathcal{E}} \psi \\ &= \left\langle n - \frac{1}{2} \right| A^* C B \sqrt{n} \left| n - \frac{1}{2} \right\rangle + \left\langle n + \frac{1}{2} \right| B^* C^* A \sqrt{n} \left| n + \frac{1}{2} \right\rangle \\ &= \sqrt{n} C \exp(-i\omega t) + \sqrt{n} C^* \exp(i\omega t) \\ &= \sqrt{\frac{2n\hbar\omega}{V\varepsilon_0}} \cos(kz - \omega t) \end{aligned}$$

Identifying  $\mathcal{E}$  as the first component of  $\mathbf{E}$  we have  $\mathcal{E} = E_x \cos(kz - \omega t)$ . Hence the electric field amplitude  $E_x$  is proportional to the square root of photon density.

$$E_x = \sqrt{\frac{2n\hbar\omega}{V\varepsilon_0}}$$