

From problem 5-4 the transition amplitude is

$$\langle m|n\rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_m^*(x_b) K_V(b, a) \phi_n(x_a) dx_a dx_b \quad (1)$$

Consider equation (6.68).

$$K_V(b, a) = \sum_m \sum_n \lambda_{mn}(t_b, t_a) \phi_m(x_b) \phi_n^*(x_a) \quad (6.68)$$

Substitute (6.68) into (1).

$$\begin{aligned} \langle m|n\rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_m^*(x_b) & \left( \sum_{m'} \sum_{n'} \lambda_{m'n'}(t_b, t_a) \phi_{m'}(x_b) \phi_{n'}^*(x_a) \right) \\ & \times \phi_n(x_a) dx_a dx_b \end{aligned}$$

By distributive law

$$\langle m|n\rangle = \sum_{m'} \sum_{n'} \lambda_{m'n'}(t_b, t_a) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_m^*(x_b) \phi_{m'}(x_b) \phi_{n'}^*(x_a) \phi_n(x_a) dx_a dx_b$$

By orthogonality of eigenfunctions

$$\langle m|n\rangle = \lambda_{mn}(t_b, t_a) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_m^*(x_b) \phi_m(x_b) \phi_n^*(x_a) \phi_n(x_a) dx_a dx_b$$

By normalization of eigenfunctions

$$\langle m|n\rangle = \lambda_{mn}(t_b, t_a)$$