Compton scattering CERN data

See "Compton Scattering of Quasi-Real Virtual Photons at LEP," arxiv.org/abs/hep-ex/0504012.

x	y
-0.74	13380
-0.60	7720
-0.47	6360
-0.34	4600
-0.20	4310
-0.07	3700
0.06	3640
0.20	3340
0.33	3500
0.46	3010
0.60	3310
0.73	3330

The data are for the center of mass frame and have the following relationship with the differential cross section formula.

$$x = \cos \theta, \quad y = \frac{d\sigma}{d\cos \theta} = 2\pi \frac{d\sigma}{d\Omega}$$

For the high energy approximation we have

$$\langle |\mathcal{M}|^2 \rangle = 2e^4 \left(\frac{\cos \theta + 1}{2} + \frac{2}{\cos \theta + 1} \right)$$

The corresponding cross section formula is

$$\frac{d\sigma}{d\Omega} = \frac{\langle |\mathcal{M}|^2 \rangle}{64\pi^2 s} = \frac{e^4}{32\pi^2 s} \left(\frac{\cos \theta + 1}{2} + \frac{2}{\cos \theta + 1} \right), \quad s \gg m$$

Substituting $e^4 = 16\pi^2\alpha^2$ yields

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s} \left(\frac{\cos \theta + 1}{2} + \frac{2}{\cos \theta + 1} \right)$$

Multiply by 2π to obtain

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{s} \left(\frac{\cos\theta + 1}{2} + \frac{2}{\cos\theta + 1} \right)$$

To compute predicted values \hat{y} from the above formula, multiply by $(hc)^2$ to convert to SI and multiply by 10^{40} to convert square meters to picobarns.

$$\hat{y} = \frac{\pi \alpha^2}{s} \left(\frac{x+1}{2} + \frac{2}{x+1} \right) \times (hc)^2 \times 10^{40}$$

The following table shows \hat{y} for $s = (40 \,\text{GeV})^2$.

x	y	\hat{y}
-0.74	13380	12573
-0.60	7720	8358
-0.47	6360	6491
-0.34	4600	5401
-0.20	4310	4661
-0.07	3700	4204
0.06	3640	3884
0.20	3340	3643
0.33	3500	3486
0.46	3010	3375
0.60	3310	3295
0.73	3330	3248

The coefficient of determination \mathbb{R}^2 measures how well predicted values fit the data.

$$R^2 = 1 - \frac{\sum (y - \hat{y})^2}{\sum (y - \bar{y})^2} = 0.97$$

The result indicates that the model $d\sigma$ explains 97% of the variance in the data.