

4-4. Show that

$$\frac{\partial^2}{\partial x^2}x = x \frac{\partial^2}{\partial x^2} + 2 \frac{\partial}{\partial x} \quad (4.22)$$

and therefore for the H in equation (4.15)

$$Hx - xH = -\frac{\hbar^2}{m} \frac{\partial}{\partial x}$$

We have

$$\frac{\partial}{\partial x}(x\psi) = \psi + x \frac{\partial}{\partial x}\psi$$

Differentiate a second time.

$$\begin{aligned} \frac{\partial^2}{\partial x^2}(x\psi) &= \frac{\partial}{\partial x} \left(\psi + x \frac{\partial}{\partial x}\psi \right) \\ &= \frac{\partial}{\partial x}\psi + \frac{\partial}{\partial x}\psi + x \frac{\partial^2}{\partial x^2}\psi \\ &= 2 \frac{\partial}{\partial x}\psi + x \frac{\partial^2}{\partial x^2}\psi \end{aligned}$$

This is equation (4.15).

$$H = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x, t)$$

Solve for $\partial^2/\partial x^2$.

$$\frac{\partial^2}{\partial x^2} = -\frac{2m}{\hbar^2}(H - V(x, t)) \quad (1)$$

By equation (4.22) above

$$\frac{\partial^2}{\partial x^2}x - x \frac{\partial^2}{\partial x^2} = 2 \frac{\partial}{\partial x} \quad (2)$$

Substitute (1) into (2).

$$-\frac{2m}{\hbar^2}(H - V(x, t))x + x \frac{2m}{\hbar^2}(H - V(x, t)) = 2 \frac{\partial}{\partial x}$$

The two V terms cancel. Multiply both sides by $-\hbar^2/2m$.

$$Hx - xH = -\frac{\hbar^2}{m} \frac{\partial}{\partial x}$$