

This is the Rodrigues formula.

$$P_\ell(x) = \frac{1}{2^\ell \ell!} \left( \frac{d}{dx} \right)^\ell (x^2 - 1)^\ell$$

Binomial formula

$$(x^2 - 1)^\ell = \sum_{k=0}^{\ell} \binom{\ell}{k} x^{2(\ell-k)} (-1)^k$$

Hence

$$\left( \frac{d}{dx} \right)^\ell (x^2 - 1)^\ell = \sum_{k=0}^{\ell} c_k x^{2(\ell-k)-\ell}$$