Feature index

abs(x)

Returns the absolute value or vector length of x.

```
X = (x,y,z)
abs(X)
```

$(x^2 + y^2 + z^2)^{1/2}$

adj(m)

Returns the adjunct of matrix m. Adjunct is equal to determinant times inverse.

```
A = ((a,b),(c,d))
adj(A) == det(A) inv(A)
```

1

and($a, b, \ldots$)

Returns 1 if all arguments are true (nonzero). Returns 0 otherwise.

```
and(1=1,2=2)
```

1

arccos(x)

Returns the arc cosine of x.

```
arccos(1/2)
```

$\frac{1}{3}\pi$

arccosh(x)

Returns the arc hyperbolic cosine of x.

arcsin(x)

Returns the arc sine of x.

```
arcsin(1/2)
```

$\frac{1}{6}\pi$

arcsinh(x)

Returns the arc hyperbolic sine of x.

arctan(y, x)

Returns the arc tangent of y over x. If x is omitted then $x = 1$ is used.

```
arctan(1,0)
```

$\frac{1}{2}\pi$

arctanh(x)

Returns the arc hyperbolic tangent of x.

arg(z)

Returns the angle of complex z.

```
arg(2 - 3i)
```

$-\arctan(3, 2)$

binding(s)

The result of evaluating a symbol can differ from the symbol's binding. For example, the result may be expanded. The `binding` function returns the actual binding of a symbol.

```
p = quote((x + 1)^2)
p
```

$p = x^2 + 2x + 1$

```
binding(p)
```

$(x + 1)^2$

ceiling(x)

Returns the smallest integer greater than or equal to x.

```
ceiling(1/2)
```

1

check(x)

If x is true (nonzero) then continue, else stop. Expression x can include the relational operators `=`, `==`, `<`, `<=`, `>`, `>=`. Use the `not` function to test for inequality.

```
A = exp(i pi)
B = -1
check(A == B) -- stop here if A not equal to B
```

choose(n, k)

Returns the binomial coefficient n choose k.

```
choose(52,5) -- number of poker hands
```

2598960

circexp(x)

Returns expression x with circular and hyperbolic functions converted to exponentials.

```
circexp(cos(x) + i sin(x))
```

$\exp(ix)$

clear

Clears all symbol definitions.

clock(z)

Returns complex z in polar form with base of negative 1 instead of e.

```
clock(2 - 3i)
```

$13^{1/2}\,(-1)^{-\arctan(3,2)/\pi}$

cofactor(m, i, j)

Returns the cofactor of matrix m for row i and column j.

```
A = ((a,b),(c,d))
cofactor(A,1,2) == adj(A)[2,1]
```

1

conj(z)

Returns the complex conjugate of z.

```
conj(2 - 3i)
```

$2 + 3i$

contract(a, i, j)

Returns tensor a summed over indices i and j. If i and j are omitted then 1 and 2 are used. The expression `contract(m)` computes the trace of matrix m.

```
A = ((a,b),(c,d))
contract(A)
```

$a + d$

cos(x)

Returns the cosine of x.

```
cos(pi/4)
```

$$\frac{1}{2^{1/2}}$$

cosh(x)

Returns the hyperbolic cosine of x.

```
circexp(cosh(x))
```

$\frac{1}{2}\exp(-x) + \frac{1}{2}\exp(x)$

cross(u, v)

Returns the cross product of vectors u and v.

curl(v)

Returns the curl of vector v with respect to symbols `x`, `y`, and `z`.

d$(f, x, \ldots)$

Returns the partial derivative of f with respect to x and any additional arguments.

```
d(sin(x),x)
```

$\cos(x)$

Multiderivatives are computed by extending the argument list.

```
d(sin(x),x,x)
```

$-\sin(x)$

A numeric argument n computes the nth derivative with respect to the previous symbol.

```
d(sin(x y),x,2,y,2)
```

$x^2y^2\sin(xy) - 4xy\cos(xy) - 2\sin(xy)$

Argument f can be a tensor of any rank. Argument x can be a vector. When x is a vector the result is the gradient of f.

```
F = (f(),g(),h())
X = (x,y,z)
d(F,X)
```

$$\begin{bmatrix} \mathrm{d}(f(),x) & \mathrm{d}(f(),y) & \mathrm{d}(f(),z) \\ \mathrm{d}(g(),x) & \mathrm{d}(g(),y) & \mathrm{d}(g(),z) \\ \mathrm{d}(h(),x) & \mathrm{d}(h(),y) & \mathrm{d}(h(),z) \end{bmatrix}$$

Symbol `d` can be used as a variable name. Doing so does not conflict with function `d`.

Symbol `d` can be redefined as a different function. The function `derivative`, a synonym for `d`, can be used to obtain a partial derivative.

defint(f, x, a, b)

Returns the definite integral of f with respect to x evaluated from a to b. The argument list can be extended for multiple integrals as shown in the following example.

```
f = (1 + cos(theta)^2) sin(theta)
-- integrate over theta then over phi
defint(f, theta, 0, pi, phi, 0, 2 pi)
```

$\frac{16}{3}\pi$

denominator(x)

Returns the denominator of expression x.

```
denominator(a/b)
```

b

det(m)

Returns the determinant of matrix m.

```
A = ((a,b),(c,d))
det(A)
```

$ad - bc$

dim(a, n)

Returns the dimension of the nth index of tensor a. Index numbering starts with 1.

```
A = ((1,2),(3,4),(5,6))
dim(A,1)
```

3

div(v)

Returns the divergence of vector v with respect to symbols `x`, `y`, and `z`.

do($a, b, \ldots$)

Evaluates each argument from left to right. Returns the result of the final argument.

```
do(A=1,B=2,A+B)
```

3

dot($a, b, \ldots$)

Returns the dot product of vectors, matrices, and tensors. Also known as the matrix product. Arguments are evaluated from right to left. The following example solves for X in $AX = B$.

```
A = ((1,2),(3,4))
B = (5,6)
X = dot(inv(A),B)
X
```

$$\begin{bmatrix} -4 \\ \frac{9}{2} \end{bmatrix}$$

eigenvec(m)

Returns eigenvectors for matrix m. Matrix m is required to be numerical, real, and symmetric. The return value is a matrix with each column an eigenvector. Eigenvalues are obtained as shown.

```
A = ((1,2,3),(2,6,4),(3,4,5))
Q = eigenvec(A)
D = dot(transpose(Q),A,Q) -- eigenvalues on the diagonal of D
dot(Q,D,transpose(Q))
```

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 6 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

eval($f, x, a, y, b, \ldots$)

Returns f evaluated with x replaced by a, y replaced by b, etc. All arguments can be expressions.

```
f = sqrt(x^2 + y^2)
eval(f,x,3,y,4)
```

5

In the following example, `eval` is used to replace `x` with `cos(theta)`.

```
-- associated legendre of cos theta
P(l,m,x) = test(m < 0, (-1)^m (l + m)! / (l - m)! P(l,-m),
           1 / (2^l l!) sin(theta)^m *
           eval(d((x^2 - 1)^l, x, l + m), x, cos(theta)))

P(2,-1)
```

$-\frac{1}{2}\cos(\theta)\sin(\theta)$

exp(x)

Returns the exponential of x.

```
exp(i pi)
```

-1

expcos(z)

Returns the cosine of z in exponential form.

```
expcos(z)
```

$\frac{1}{2}\exp(iz) + \frac{1}{2}\exp(-iz)$

expcosh(z)

Returns the hyperbolic cosine of z in exponential form.

```
expcosh(z)
```

$\frac{1}{2}\exp(-z) + \frac{1}{2}\exp(z)$

expsin(z)

Returns the sine of z in exponential form.

```
expsin(z)
```

$-\frac{1}{2}i\exp(iz) + \frac{1}{2}i\exp(-iz)$

expsinh(z)

Returns the hyperbolic sine of z in exponential form.

```
expsinh(z)
```

$-\frac{1}{2}\exp(-z) + \frac{1}{2}\exp(z)$

exptan(z)

Returns the tangent of z in exponential form.

```
exptan(z)
```

$$\frac{i}{\exp(2iz)+1} - \frac{i\exp(2iz)}{\exp(2iz)+1}$$

exptanh(z)

Returns the hyperbolic tangent of z in exponential form.

```
exptanh(z)
```

$$-\frac{1}{\exp(2z)+1} + \frac{\exp(2z)}{\exp(2z)+1}$$

factorial(n)

Returns the factorial of n. The expression `n!` can also be used.

```
20!
```

2432902008176640000

float(x)

Returns expression x with rational numbers and integers converted to floating point values. The symbol `pi` and the natural number are also converted.

```
float(212^17)
```

3.52947×10^{39}

floor(x)

Returns the largest integer less than or equal to x.

```
floor(1/2)
```

0

for($i, j, k, a, b, \ldots$)

For i equals j through k evaluate a, b, etc.

```
for(k,1,3,A=k,print(A))
```

$A = 1$
$A = 2$
$A = 3$

Note: The original value of i is restored after `for` completes. If symbol `i` is used for index variable i then the imaginary unit is overridden in the scope of `for`.

grad(f)

Returns the gradient `d(f,(x,y,z))`.

```
grad(f())
```

$$\begin{bmatrix} \mathrm{d}(f(), x) \\ \mathrm{d}(f(), y) \\ \mathrm{d}(f(), z) \end{bmatrix}$$

hadamard$(a, b, \ldots)$

Returns the Hadamard (element-wise) product.

```
X = (a,b,c)
hadamard(X,X)
```

$$\begin{bmatrix} a^2 \\ b^2 \\ c^2 \end{bmatrix}$$

i

Symbol `i` is initialized to the imaginary unit $\sqrt{-1}$.

```
exp(i pi)
```

-1

Note: It is ok to clear or redefine `i` and use the symbol for something else.

imag(z)

Returns the imaginary part of complex z.

```
imag(2 - 3i)
```

-3

infixform(x)

Converts expression x to a string and returns the result.

```
p = (x + 1)^2
infixform(p)
```

```
x^2 + 2 x + 1
```

inner$(a, b, \ldots)$

Returns the inner product of vectors, matrices, and tensors. Also known as the matrix product.

```
A = ((a,b),(c,d))
B = (x,y)
inner(A,B)
```

$$\begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

Note: `inner` and `dot` are the same function.

integral(f, x)

Returns the integral of f with respect to x.

```
integral(x^2,x)
```

$\frac{1}{3}x^3$

inv(m)

Returns the inverse of matrix m.

```
A = ((1,2),(3,4))
inv(A)
```

$$\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

j

Set `j=sqrt(-1)` to use `j` for the imaginary unit instead of `i`.

```
j = sqrt(-1)
1/sqrt(-1)
```

$-j$

kronecker($a, b, \ldots$)

Returns the Kronecker product of vectors and matrices.

```
A = ((1,2),(3,4))
B = ((a,b),(c,d))
kronecker(A,B)
```

$$\begin{bmatrix} a & b & 2a & 2b \\ c & d & 2c & 2d \\ 3a & 3b & 4a & 4b \\ 3c & 3d & 3c & 4d \end{bmatrix}$$

last

The result of the previous calculation is stored in `last`.

```
212^17
```

3529471145760275132301897342055866171392

```
last^(1/17)
```

212

Symbol `last` is an implied argument when a function has no argument list.

```
212^17
```

3529471145760275132301897342055866171392

```
float
```

3.52947×10^{39}

log(x)

Returns the natural logarithm of x.

```
log(x^y)
```

$y \log(x)$

mag(z)

Returns the magnitude of complex z. Function `mag` treats undefined symbols as real while `abs` does not.

```
mag(x + i y)
```

$(x^2 + y^2)^{1/2}$

minor(m, i, j)

Returns the minor of matrix m for row i and column j.

```
A = ((1,2,3),(4,5,6),(7,8,9))
minor(A,1,1) == det(minormatrix(A,1,1))
```

1

minormatrix(m, i, j)

Returns a copy of matrix m with row i and column j removed.

```
A = ((1,2,3),(4,5,6),(7,8,9))
minormatrix(A,1,1)
```

$$\begin{bmatrix} 5 & 6 \\ 8 & 9 \end{bmatrix}$$

noexpand(x)

Evaluates expression x without expanding products of sums.

```
noexpand((x + 1)^2 / (x + 1))
```

$x + 1$

not(x)

Returns 0 if x is true (nonzero). Returns 1 otherwise.

```
not(1=1)
```

0

nroots(p, x)

Returns the approximate roots of polynomials with real or complex coefficients. Multiple roots are returned as a vector.

```
p = x^5 - 1
nroots(p,x)
```

$$\begin{bmatrix} 1 \\ -0.809017 + 0.587785\,i \\ -0.809017 - 0.587785\,i \\ 0.309017 + 0.951057\,i \\ 0.309017 - 0.951057\,i \end{bmatrix}$$

numerator(x)

Returns the numerator of expression x.

```
numerator(a/b)
```

a

or$(a, b, \ldots)$

Returns 1 if at least one argument is true (nonzero). Returns 0 otherwise.

```
or(1=1,2=2)
```

1

outer$(a, b, \ldots)$

Returns the outer product of vectors, matrices, and tensors.

```
A = (a,b,c)
B = (x,y,z)
outer(A,B)
```

$$\begin{bmatrix} ax & ay & az \\ bx & by & bz \\ cx & cy & cz \end{bmatrix}$$

pi

Symbol for π.

```
exp(i pi)
```

-1

polar(z)

Returns complex z in polar form.

```
polar(x - i y)
```

$(x^2 + y^2)^{1/2} \exp(-i \arctan(y, x))$

power

Use ^ to raise something to a power. Use parentheses for negative powers.

```
x^(-2)
```

$$\frac{1}{x^2}$$

print($a, b, \ldots$)

Evaluate expressions and print the results. Useful for printing from inside a `for` loop.

```
for(j,1,3,print(j))
```

$j = 1$
$j = 2$
$j = 3$

product(i, j, k, f)

For i equals j through k evaluate f. Returns the product of all f.

```
product(j,1,3,x + j)
```

$x^3 + 6x^2 + 11x + 6$

The original value of i is restored after `product` completes. If symbol `i` is used for index variable i then the imaginary unit is overridden in the scope of `product`.

product(y)

Returns the product of components of y.

```
y = (1,2,3,4)
product(y)
```

24

quote(x)

Returns expression x without evaluating it first.

```
quote((x + 1)^2)
```

$(x + 1)^2$

rank(a)

Returns the number of indices that tensor a has.

```
A = ((a,b),(c,d))
rank(A)
```

2

rationalize(x)

Returns expression x with everything over a common denominator.

```
rationalize(1/a + 1/b + 1/2)
```

$$\frac{2a + ab + 2b}{2ab}$$

Note: `rationalize` returns an unexpanded expression. If the result is assigned to a symbol, evaluating the symbol will expand the result. Use `binding` to retrieve the unexpanded expression.

```
f = rationalize(1/a + 1/b + 1/2)
binding(f)
```

$$\frac{2a + ab + 2b}{2ab}$$

real(z)

Returns the real part of complex z.

```
real(2 - 3i)
```

2

rect(z)

Returns complex z in rectangular form.

```
rect(exp(i x))
```

$\cos(x) + i\sin(x)$

roots(p, x)

Returns the rational roots of a polynomial. Multiple roots are returned as a vector.

```
p = (x + 1) (x - 2)
roots(p,x)
```

$$\begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

If no roots are found then `nil` is returned. A `nil` result is not printed so the following example uses `infixform` to print `nil` as a string.

```
p = x^2 + 1
infixform(roots(p,x))
```

nil

rotate($u, s, k, \ldots$)

Rotates vector u and returns the result. Vector u is required to have 2^n elements where n is an integer from 1 to 15. Arguments $s, k, \ldots$ are a sequence of rotation codes where s is an upper case letter and k is a qubit number from 0 to $n-1$. Rotations are evaluated from left to right. See the section on quantum computing for a list of rotation codes.

```
psi = (1,0,0,0)
rotate(psi,H,0)
```

$$\begin{bmatrix} \frac{1}{2^{1/2}} \\ \frac{1}{2^{1/2}} \\ 0 \\ 0 \end{bmatrix}$$

run(x)

Run script x where x evaluates to a filename string. Useful for importing function libraries.

```
run("/Users/heisenberg/EVA2.txt")
```

simplify(x)

Returns expression x in a simpler form.

```
simplify(sin(x)^2 + cos(x)^2)
```

1

sin(x)

Returns the sine of x.

```
sin(pi/4)
```

$$\frac{1}{2^{1/2}}$$

sinh(x)

Returns the hyperbolic sine of x.

```
circexp(sinh(x))
```

$-\frac{1}{2}\exp(-x) + \frac{1}{2}\exp(x)$

sqrt(x)

Returns the square root of x.

```
sqrt(10!)
```

$720\ 7^{1/2}$

stop

In a script, it does what it says.

sum(i, j, k, f)

For i equals j through k evaluate f. Returns the sum of all f.

```
sum(j,1,5,x^j)
```

$x^5 + x^4 + x^3 + x^2 + x$

The original value of i is restored after `sum` completes. If symbol `i` is used for index variable i then the imaginary unit is overridden in the scope of `sum`.

sum(y)

Returns the sum of components of y.

```
y = (1,2,3,4)
sum(y)
```

10

tan(x)

Returns the tangent of x.

```
simplify(tan(x) - sin(x)/cos(x))
```

0

tanh(x)

Returns the hyperbolic tangent of x.

```
circexp(tanh(x))
```

$$-\frac{1}{\exp(2x)+1} + \frac{\exp(2x)}{\exp(2x)+1}$$

test$(a, b, c, d, \ldots)$

If argument a is true (nonzero) then b is returned, else if c is true then d is returned, etc. If the number of arguments is odd then the final argument is returned if all else fails. Expressions can include the relational operators `=`, `==`, `<`, `<=`, `>`, `>=`. Use the `not` function to test for inequality. (The equality operator `==` is available for contexts in which `=` is the assignment operator.)

```
A = 1
B = 1
test(A=B,"yes","no")
```

yes

trace

Set `trace=1` in a script to print the script as it is evaluated. Useful for debugging.

```
trace = 1
```

Note: The `contract` function is used to obtain the trace of a matrix.

transpose(a, i, j)

Returns the transpose of tensor a with respect to indices i and j. If i and j are omitted then 1 and 2 are used. Hence a matrix can be transposed with a single argument.

```
A = ((a,b),(c,d))
transpose(A)
```

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

Note: The argument list can be extended for multiple transpose operations. Arguments are evaluated from left to right. For example, `transpose(A,1,2,2,3)` is equivalent to `transpose(transpose(A,1,2),2,3)`

tty

Set `tty=1` to show results in string format. Set `tty=0` to turn off. Can be useful when displayed results exceed window size.

```
tty = 1
(x + 1)^2
```

```
x^2 + 2 x + 1
```

unit(n)

Returns an n by n identity matrix.

```
unit(3)
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

zero($i, j, \ldots$)

Returns a null tensor with dimensions i, j, etc. Useful for creating a tensor and then setting component values.

```
A = zero(3,3)
for(k,1,3,A[k,k]=k)
A
```

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$