Linear algebra

The `dot` function is used to multiply vectors, matrices, and tensors. For example, let

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

The product AX is computed as follows.

```
A = ((a11,a12),(a21,a22))
X = (x1,x2)
dot(A,X)
```

$$\begin{bmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{bmatrix}$$

The following example shows how to use `dot` and `inv` to solve for vector X in $AX = B$.

```
A = ((3,7),(1,-9))
B = (16,-22)
X = dot(inv(A),B)
X
```

$$X = \begin{bmatrix} -\frac{5}{17} \\ \frac{41}{17} \end{bmatrix}$$

The `dot` function can have more than two arguments. For example, `dot(A,B,C)` can be used for the dot product of three tensors.

Square brackets are used for component access. Index numbering starts with 1.

```
A = ((a,b),(c,d))
A[1,2] = -A[1,1]
A
```

$$\begin{bmatrix} a & -a \\ c & d \end{bmatrix}$$

The following example demonstrates the relation $A^{-1} = (\det A)^{-1} \operatorname{adj} A$.

```
A = ((a,b),(c,d))
inv(A) == adj(A) / det(A)
```

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Sometimes a calculation will be simpler if it can be reorganized to use `adj` instead of `inv`. The main idea is to try to prevent the determinant from appearing as a divisor. For example, suppose for matrices A and B you want to show that

$$A - B^{-1} = 0$$

Depending on the complexity of $\det B$, the software may not be able to find a simplification that yields zero. Should that occur, the following alternative formulation can be tried.

$$A \det B - \operatorname{adj} B = 0$$