

Dirac from boost

This is a Dirac spinor that represents an electron at rest with spin up along the z axis.

$$u_0 = \sqrt{2m} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

This matrix boosts a spinor in the z direction where $E^2 = p^2 + m^2$.

$$\Lambda = \frac{1}{\sqrt{2m(E+m)}} \begin{pmatrix} E+m & 0 & p & 0 \\ 0 & E+m & 0 & p \\ p & 0 & E+m & 0 \\ 0 & p & 0 & E+m \end{pmatrix}$$

Hence

$$u = \Lambda u_0 = \frac{1}{\sqrt{E+m}} \begin{pmatrix} E+m & 0 & p & 0 \\ 0 & E+m & 0 & p \\ p & 0 & E+m & 0 \\ 0 & p & 0 & E+m \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{E+m}} \begin{pmatrix} E+m \\ 0 \\ p \\ 0 \end{pmatrix}$$

This is the Dirac equation in spinor form.

$$\not{p}u = mu$$

Substitute Λu_0 for u .

$$\not{p}\Lambda u_0 = m\Lambda u_0$$

By the identity $\gamma^0 u_0 = u_0$ substitute $\gamma^0 u_0$ for u_0 on the right hand side.

$$\not{p}\Lambda u_0 = m\Lambda \gamma^0 u_0$$

Substitute $\Lambda^{-1}u$ for u_0 .

$$\not{p}\Lambda \Lambda^{-1}u = m\Lambda \gamma^0 \Lambda^{-1}u$$

Hence

$$\not{p} = m\Lambda \gamma^0 \Lambda^{-1}$$

and

$$\Lambda \gamma^0 = m^{-1} \not{p} \Lambda \tag{1}$$

Boost $\gamma^0 u_0 = u_0$ to recover the Dirac equation.

$$\begin{aligned} \Lambda \gamma^0 u_0 &= \Lambda u_0 \\ m^{-1} \not{p} u &= u \\ \not{p} u &= mu \end{aligned} \tag{2}$$