Bohr model

By an argument that is no longer relevant the Bohr model for hydrogen energy levels is

$$E_n = -\frac{\alpha^2 mc^2}{2n^2}$$

By the kinetic energy relation

$$v^2 = -\frac{2E_n}{m}$$

velocity v reduces to

$$v = \frac{\alpha c}{n}$$

The Bohr model quantizes orbital angular momentum as

$$mvr_n = n\hbar$$

Hence the radius is

$$r_n = \frac{n\hbar}{mv} = \frac{n^2\hbar}{\alpha mc}$$

For $n = 1$ and $m = m_e$ we have (r_1 is exactly the Bohr radius a_0)

$$E_1 = -13.6057\,\text{eV}, \quad r_1 = 5.29177 \times 10^{-11}\,\text{meter} = a_0$$

For reduced electron mass

$$m = \frac{m_e m_p}{m_e + m_p}$$

the result is

$$E_1 = -13.5983\,\text{eV}, \quad r_1 = 5.29465 \times 10^{-11}\,\text{meter}$$

The model can be made more convoluted by the substitution

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c}$$

leading to

$$E_n = -\frac{me^4}{2(4\pi\varepsilon_0)^2\hbar^2 n^2}$$

and

$$r_n = \frac{4\pi\varepsilon_0\hbar^2 n^2}{me^2}$$

Some authors use $4\pi\varepsilon_0 = 1$. Wikipedia uses

$$k_e = \frac{1}{4\pi\varepsilon_0}$$