Feynman and Hibbs problem 5-6

Suppose A, B, and C are the three cartesian coordinates of momentum p_x, p_y, p_z. What is the form of function the $\chi_{a,b,c}(x,y,z)$?

From equation (5.36)

$$F_{a,b,c} = \int_{\mathbb{R}^3} \chi^*_{a,b,c}(\mathbf{x}) f(\mathbf{x})\, dx\, dy\, dz \tag{1}$$

From equation (5.6)

$$F_{a,b,c} = \phi(\mathbf{p}) = \int_{\mathbb{R}^3} \exp\left(-\frac{i\mathbf{p}\cdot\mathbf{x}}{\hbar}\right) f(\mathbf{x})\, dx\, dy\, dz \tag{2}$$

where

$$\mathbf{p} = (p_x, p_y, p_z) = (A, B, C)$$

Equate (1) and (2).

$$F_{a,b,c} = \int_{\mathbb{R}^3} \chi^*_{a,b,c}(\mathbf{x}) f(\mathbf{x})\, dx\, dy\, dz = \int_{\mathbb{R}^3} \exp\left(-\frac{i\mathbf{p}\cdot\mathbf{x}}{\hbar}\right) f(\mathbf{x})\, dx\, dy\, dz$$

Hence one solution is

$$\chi^*_{a,b,c}(\mathbf{x}) = \exp\left(-\frac{i\mathbf{p}\cdot\mathbf{x}}{\hbar}\right)$$

and

$$\chi_{a,b,c}(\mathbf{x}) = \exp\left(\frac{i\mathbf{p}\cdot\mathbf{x}}{\hbar}\right)$$