(5.2) Show that Poisson brackets anticommute

$$\{A, B\}_{\mathrm{PB}} = -\{B, A\}_{\mathrm{PB}} \tag{5.59}$$

and also satisfy the Jacobi identity

$$\{\{A, B\}_{\mathrm{PB}}, C\}_{\mathrm{PB}} + \{\{C, A\}_{\mathrm{PB}}, B\}_{\mathrm{PB}} + \{\{B, C\}_{\mathrm{PB}}, A\}_{\mathrm{PB}} = 0 \tag{5.60}$$

and show that quantum mechanical commutators also have the same properties.

From equation (5.11)

$$\{A, B\}_{\mathrm{PB}} = \frac{\partial A}{\partial q_i}\frac{\partial B}{\partial p_i} - \frac{\partial A}{\partial p_i}\frac{\partial B}{\partial q_i} \tag{5.11}$$

Hence

$$\begin{aligned}
&\{\{A, B\}_{\mathrm{PB}}, C\}_{\mathrm{PB}} \\
&\qquad = \frac{\partial}{\partial q_i}\left(\frac{\partial A}{\partial q_i}\frac{\partial B}{\partial p_i} - \frac{\partial A}{\partial p_i}\frac{\partial B}{\partial q_i}\right)\frac{\partial C}{\partial p_i} - \frac{\partial}{\partial p_i}\left(\frac{\partial A}{\partial q_i}\frac{\partial B}{\partial p_i} - \frac{\partial A}{\partial p_i}\frac{\partial B}{\partial q_i}\right)\frac{\partial C}{\partial q_i}
\end{aligned}$$

Eigenmath proof

```
PB(f,g) = d(f,q) d(g,p) - d(f,p) d(g,q)

A = a(p,q)
B = b(p,q)
C = c(p,q)

PB(PB(A,B),C) + PB(PB(C,A),B) + PB(PB(B,C),A) == 0
```