5-3. Assume $f(x)$ is normalized, that is,

$$\int_{-\infty}^{\infty} f^*(x) f(x)\, dx = 1$$

Under this constraint, show that the state $f(x)$ which has the highest probability of having property G is $f(x) = g(x)$.

Consider equation (5.32).

$$P(G) = \left| \int_{-\infty}^{\infty} g^*(x) f(x)\, dx \right|^2 \tag{5.32}$$

If $f(x) = g(x)$ then

$$P(G) = \left| \int_{-\infty}^{\infty} f^*(x) f(x)\, dx \right|^2 = 1$$

which is the highest probability.