Schrodinger from free-particle propagator

Consider the free-particle propagator

$$K_0(x,t,x_a,t_a) = \left(\frac{m}{2\pi i\hbar(t-t_a)}\right)^{\frac{1}{2}} \exp\left(\frac{im(x-x_a)^2}{2\hbar(t-t_a)}\right)$$

Let $\psi(x,t)$ be a free-particle wave function. Then

$$\psi(x,t) = \int_{-\infty}^{\infty} K_0(x,t,x_a,t_a)\psi(x_a,t_a)\,dx_a \tag{1}$$

By (1) and noting that $\psi(x_a,t_a)$ does not depend on t we have

$$\frac{\partial}{\partial t}\psi(x,t) = \int_{-\infty}^{\infty} \frac{\partial K_0}{\partial t}\psi(x_a,t_a)\,dx_a \tag{2}$$

By (1) and noting that $\psi(x_a,t_a)$ does not depend on x we have

$$\frac{\partial^2}{\partial x^2}\psi(x,t) = \int_{-\infty}^{\infty} \frac{\partial^2 K_0}{\partial x^2}\psi(x_a,t_a)\,dx_a \tag{3}$$

By computer algebra (see demo)

$$\frac{\partial K_0}{\partial t} = \frac{i\hbar}{2m}\frac{\partial^2 K_0}{\partial x^2} \tag{4}$$

Multiply both sides of (4) by $\psi(x_a,t_a)$ and integrate over x_a.

$$\underbrace{\int_{-\infty}^{\infty} \frac{\partial K_0}{\partial t}\psi(x_a,t_a)\,dx_a}_{(2)} = \frac{i\hbar}{2m}\underbrace{\int_{-\infty}^{\infty} \frac{\partial^2 K_0}{\partial x^2}\psi(x_a,t_a)\,dx_a}_{(3)} \tag{5}$$

Substitute (2) and (3) into (5) to obtain

$$\frac{\partial}{\partial t}\psi(x,t) = \frac{i\hbar}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t)$$

Multiply both sides by $i\hbar$ to obtain the free-particle Schrodinger equation

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t)$$

Ref. Feynman and Hibbs problem 3-5.