Spin part 1

Let $|s\rangle$ be a spin with polar angle θ and azimuth angle ϕ.

$$|s\rangle = \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2)\, e^{i\phi} \end{pmatrix}$$

Spin measurement probabilities are the transition probabilities from $|s\rangle$ to an eigenstate.

In the z direction the eigenstates are

$$|z_+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |z_-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Hence in the z direction

$$\begin{aligned} \Pr(+) &= |\langle z_+|s\rangle|^2 = \cos^2(\theta/2) = \tfrac{1}{2} + \tfrac{1}{2}\cos\theta \\ \Pr(-) &= |\langle z_-|s\rangle|^2 = \sin^2(\theta/2) = \tfrac{1}{2} - \tfrac{1}{2}\cos\theta \end{aligned}$$

In the x direction the eigenstates are

$$|x_+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad |x_-\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Hence in the x direction

$$\begin{aligned} \Pr(+) &= |\langle x_+|s\rangle|^2 = \tfrac{1}{2} + \tfrac{1}{2}\sin\theta\cos\phi \\ \Pr(-) &= |\langle x_-|s\rangle|^2 = \tfrac{1}{2} - \tfrac{1}{2}\sin\theta\cos\phi \end{aligned}$$

In the y direction the eigenstates are

$$|y_+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \qquad |y_-\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

Hence in the y direction

$$\begin{aligned} \Pr(+) &= |\langle y_+|s\rangle|^2 = \tfrac{1}{2} + \tfrac{1}{2}\sin\theta\sin\phi \\ \Pr(-) &= |\langle y_-|s\rangle|^2 = \tfrac{1}{2} - \tfrac{1}{2}\sin\theta\sin\phi \end{aligned}$$

For each direction we have $\Pr(+) + \Pr(-) = 1$ as required by total probability.