

9-8. For the state for which there is just one photon present in level 1,  $\mathbf{k}$ , all of the factors in the wave function are  $\phi_0$  except one, which is  $\phi_1$ . But for an oscillator  $\phi_1(x) = \sqrt{2}x\phi_0(x)$ . The wave function representing an excited running wave is a linear superposition of the state with the cosine mode excited and  $i$  times the state with the sine wave excited, so show that the unnormalized wave function for just one photon present in 1,  $\mathbf{k}$  is  $\bar{a}_{1,\mathbf{k}}^* \Phi_0$ . The normalization is  $\int \Phi_0^* \bar{a}_{1,\mathbf{k}} \bar{a}_{1,\mathbf{k}}^* \Phi_0 d\bar{a}$ , or the expectation of  $\bar{a}_{1,\mathbf{k}} \bar{a}_{1,\mathbf{k}}^*$  for the vacuum, which we have seen in the preceding problem is  $\hbar/2kc$ . Hence the normalized one-photon state is  $\sqrt{2kc/\hbar} \bar{a}_{1,\mathbf{k}}^* \Phi_0$ .

From problem 9-6, let

$$\Phi_0 = \exp \left( -\frac{kc}{4\hbar} (\bar{a}_{1,\mathbf{k}}^c)^2 - \frac{kc}{4\hbar} (\bar{a}_{1,\mathbf{k}}^s)^2 \right)$$

It follows that

$$\Phi_0^* \Phi_0 = \exp \left( -\frac{kc}{2\hbar} (\bar{a}_{1,\mathbf{k}}^c)^2 - \frac{kc}{2\hbar} (\bar{a}_{1,\mathbf{k}}^s)^2 \right)$$

The expectation of  $\Phi_0$  is

$$\langle \Phi_0 \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi_0^* \Phi_0 d\bar{a}_{1,\mathbf{k}}^c d\bar{a}_{1,\mathbf{k}}^s = \frac{2\pi\hbar}{kc} \quad (1)$$

Let

$$\Phi_1 = \bar{a}_{1,\mathbf{k}}^* \Phi_0$$

Then

$$\Phi_1^* \Phi_1 = \Phi_0^* \frac{(\bar{a}_{1,\mathbf{k}}^c)^2 + (\bar{a}_{1,\mathbf{k}}^s)^2}{2} \Phi_0$$

The expectation of  $\Phi_1$  is

$$\langle \Phi_1 \rangle = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi_1^* \Phi_1 d\bar{a}_{1,\mathbf{k}}^c d\bar{a}_{1,\mathbf{k}}^s = \frac{\hbar}{kc} \frac{2\pi\hbar}{kc} \quad (2)$$

Hence the expectation of  $\Phi_1$  is  $\hbar/kc$  times  $\Phi_0$ .

FIXME: Should be  $\hbar/2kc$ .