

Consider the following anova program and its output.

```
data ;
input trt $ y @@ ;
datalines ;
A 6    A 0    A 2    A 8    A 11
A 4    A 13   A 1    A 8    A 0
B 0    B 2    B 3    B 1    B 18
B 4    B 14   B 9    B 1    B 9
C 13   C 10   C 18   C 5    C 23
C 12   C 5    C 16   C 1    C 20
;

proc anova ;
model y = trt ;
means trt / lsd ttest ;
```

Analysis of Variance						
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F	
Model	2	293.60000000	146.80000000	3.98	0.0305	
Error	27	995.10000000	36.85555556			
Total	29	1288.70000000				
		R-Square	Coeff Var	Root MSE	Y Mean	
		0.227826	76.846553	6.070878	7.900000	
Source	DF	Anova SS	Mean Square	F Value	Pr > F	
TRT	2	293.60000000	146.80000000	3.98	0.0305	
Mean Response						
	TRT	N	Mean Y	95% CI MIN	95% CI MAX	
	A	10	5.300000	1.360937	9.239063	
	B	10	6.100000	2.160937	10.039063	
	C	10	12.300000	8.360937	16.239063	
Least Significant Difference Test						
TRT	TRT	Delta Y	95% CI MIN	95% CI MAX	t Value	Pr > t
A	B	-0.800000	-6.370677	4.770677	-0.29	0.7705
A	C	-7.000000	-12.570677	-1.429323	-2.58	0.0157 *
B	A	0.800000	-4.770677	6.370677	0.29	0.7705
B	C	-6.200000	-11.770677	-0.629323	-2.28	0.0305 *
C	A	7.000000	1.429323	12.570677	2.58	0.0157 *
C	B	6.200000	0.629323	11.770677	2.28	0.0305 *
Two Sample t-Test						
TRT	TRT	Delta Y	95% CI MIN	95% CI MAX	t Value	Pr > t
A	B	-0.800000	-5.922307	4.322307	-0.33	0.7466
A	C	-7.000000	-12.664270	-1.335730	-2.60	0.0182 *
B	A	0.800000	-4.322307	5.922307	0.33	0.7466
B	C	-6.200000	-12.467653	0.067653	-2.08	0.0523
C	A	7.000000	1.335730	12.664270	2.60	0.0182 *
C	B	6.200000	-0.067653	12.467653	2.08	0.0523

Let us take a closer look at the mean response table.

Mean Response				
TRT	N	Mean Y	95% CI MIN	95% CI MAX
A	10	5.300000	1.360937	9.239063
B	10	6.100000	2.160937	10.039063
C	10	12.300000	8.360937	16.239063

Recall that the confidence interval for a treatment mean is

$$\bar{y} \pm t(1 - \alpha/2, \text{dfe}) \cdot \sqrt{\frac{\text{MSE}}{n}}$$

Recall that MSE is an estimate of model variance. From the analysis of variance table at the top of the output we have

Source	DF	Sum of Squares	Mean Square
Error	27	995.10000000	36.85555556

Hence

$$\text{dfe} = 27, \quad \text{MSE} = 36.8556$$

The confidence interval for the mean of treatment A can be checked by typing the following into R.

```
ybar = 5.3
n = 10
MSE = 36.8556
dfe = 27
alpha = 0.05
t = qt(1 - alpha/2, dfe)
ybar - t * sqrt(MSE/n)
ybar + t * sqrt(MSE/n)
```

R prints the following results.

```
[1] 1.360934
[1] 9.239066
```

The R results match the mean response table for treatment A.

TRT	N	Mean Y	95% CI MIN	95% CI MAX
A	10	5.300000	1.360937	9.239063

Let us now consider the first result in the least significant difference table.

Least Significant Difference Test						
TRT	TRT	Delta Y	95% CI MIN	95% CI MAX	t Value	Pr > t
A	B	-0.800000	-6.370677	4.770677	-0.29	0.7705

The least significant difference of two treatment means \bar{y}_A and \bar{y}_B is

$$\text{LSD} = t(1 - \alpha/2, \text{dfe}) \times \sqrt{\text{MSE} \times \left(\frac{1}{n_A} + \frac{1}{n_B} \right)}$$

The corresponding confidence interval is

$$\bar{y}_A - \bar{y}_B \pm \text{LSD}$$

The confidence interval in the LSD table can be checked by typing the following into R.

```
ybarA = 5.3
ybarB = 6.1
nA = 10
nB = 10
MSE = 36.8556
dfe = 27
alpha = 0.05
LSD = qt(1 - alpha/2, dfe) * sqrt(MSE * (1/nA + 1/nB))
ybarA - ybarB - LSD
ybarA - ybarB + LSD
```

R prints the following results.

```
[1] -6.37068
[1] 4.77068
```

The R results match the confidence interval in the LSD table.

TRT	TRT	Delta Y	95% CI MIN	95% CI MAX	t Value	Pr > t
A	B	-0.800000	-6.370677	4.770677	-0.29	0.7705