By equation (7.20) to first order we have

$$F\big(j(\mathbf{r},t) + \eta(\mathbf{r},t)\big) - F\big(j(\mathbf{r},t)\big) = \iint \frac{\delta F}{\delta j(\mathbf{x},s)} \eta(\mathbf{x},s)\, d^3\mathbf{x}\, ds$$

Let

$$I(f_1, f_2) = \frac{1}{2} \iiiint f_1(\mathbf{r}_1, t_1) f_2(\mathbf{r}_2, t_2) R(\mathbf{r}_2 - \mathbf{r}_1, t_2 - t_1)\, d^3\mathbf{r}_2\, dt_2\, d^3\mathbf{r}_1\, dt_1$$

Then

$$F\big(j(\mathbf{r},t) + \eta(\mathbf{r},t)\big) = F\big(j(\mathbf{r},t)\big) \exp\big(I(j,\eta)\big) \exp\big(I(\eta,j)\big) \exp\big(I(\eta,\eta)\big)$$

Use the approximation $\exp(x) = 1 + x$ to obtain to first order in η

$$F\big(j(\mathbf{r},t) + \eta(\mathbf{r},t)\big) = F\big(j(\mathbf{r},t)\big)\big(1 + I(j,\eta) + I(\eta,j)\big)$$

Then by equation (7.20)

$$F\big(j(\mathbf{r},t)\big)I(j,\eta) + F\big(j(\mathbf{r},t)\big)I(\eta,j) = \iint \frac{\delta F}{\delta j(\mathbf{x},s)} \eta(\mathbf{x},s)\, d^3\mathbf{x}\, ds \tag{1}$$

Substitute (7.27) into (1). The $F\big(j(\mathbf{r},t)\big)$ cancel leaving

$$I(j,\eta) + I(\eta,j) = \frac{1}{2} \iiiint j(\mathbf{r},t) R(\mathbf{r} - \mathbf{x}, t - s) \eta(\mathbf{x},s)\, d^3\mathbf{r}\, dt\, d^3\mathbf{x}\, ds$$

$$+ \frac{1}{2} \iiiint j(\mathbf{r},t) R(\mathbf{x} - \mathbf{r}, s - t) \eta(\mathbf{x},s)\, d^3\mathbf{r}\, dt\, d^3\mathbf{x}\, ds$$

which is easily verified.