Exercise 6.9. Prove that the four vectors $|sing\rangle$, $|T_1\rangle$, $|T_2\rangle$, and $|T_3\rangle$ are eigenvectors of $\vec{\sigma} \cdot \vec{\tau}$. What are their eigenvalues?

By Table 1 we obtain the following eigenvalues.

$$|sing\rangle \quad |T_1\rangle \quad |T_2\rangle \quad |T_3\rangle$$
 $\sigma_x \tau_x \quad -1 \quad 1 \quad 1 \quad -1$
 $\sigma_y \tau_y \quad -1 \quad 1 \quad -1 \quad 1$
 $\sigma_z \tau_z \quad -1 \quad -1 \quad 1$
 $\vec{\sigma} \cdot \vec{\tau} \quad -3 \quad 1 \quad 1 \quad 1$

Since $|sing\rangle$, $|T_1\rangle$, $|T_2\rangle$, and $|T_3\rangle$ are eigenvectors of $\sigma_x\tau_x$, $\sigma_y\tau_y$, and $\sigma_z\tau_z$, then they are also eigenvectors of $\vec{\sigma} \cdot \vec{\tau}$ by linearity.