2B Concept formation according to APOS theory

Try to identify the individual A-P-O-S phases in the following sequence of steps to *form the concept of the inverse matrix* to an invertible matrix A:

$\begin{bmatrix} 1x+2y=3 \\ 4x+5y=6 \end{bmatrix}$	Row image
$\downarrow$	1. compression via concept of vector (addition)
$\begin{bmatrix} 1 \\ 4 \end{bmatrix} x + \begin{bmatrix} 2 \\ 5 \end{bmatrix} y = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$	Vector image
$\downarrow$	2. compression via concept of matrix (multiplication)
$\begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix} . \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$	Matrix image
$\downarrow$	3. compression via symbolisation
A.X=B	
$\downarrow$	…problemsolving via concept of analogy [1, p.10] *using CAS*
$X = A^{-1}.B = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$	
$\downarrow$	…and abstraction gives
•	Concept image
$A^{-1} = \frac{1}{3}\begin{bmatrix} -5 & 2 \\ 4 & -1 \end{bmatrix}$	**of inverse** A^{-1}

Exercise:

a) First test and explain the following interactive EIGENMATH mini learning environment (LE) to support *the formation of the concept of the inverse matrix* as shown above:

```
# EIGENMATH
A = ((1,2),(4,5))
X = (x,y)
B = (3,6)               -- dot(.)=..*..
dot(A,X)                -- LHS of linear system A*X=B
X = dot(inv(A),B)       -- X = 1/A*B = A^(-1)*B
dot(A,X) == B           -- verify solution X = A^-1*B
```

Note: EIGENMATH use *dot* for the scalar/matrix product and *inv*(A) for the inverse matrix A^{-1}.

b) In your opinion, what exploration opportunities are offered to the student in the above CAS-LE? Which *phenomena* could be specifically observed? In what respect is this LE *interactive*? What additional didactical and methodological options are there compared to a CAS-free access to the formation of the concept ‚inverse matrix'?

c) Design a small A.C.E cycle based on the above CAS-LE to form the concept of the inverse matrix.

d) Construct an A.C.E cycle within a suitable CAS-LE to create a concept for a self-chosen concept from linear algebra/analytic geometry or analysis.

Invoke Eigenmath here: https://georgeweigt.github.io/eigenmath-demo.html

References:

[1] DUBINSKY, E., and MCDONALD, M.: APOS: A Constructivist Theory of Learning in UME Research. o. O., 2000

[2] DUBINSKY, E., and TALL, D.: Advanced Mathematical Thinking and the Computer.
In ***Advanced Mathematical Thinking***, D. Tall, Ed., vol. 5. Kluwer, Dordrecht, 1991, pp. 231–250.

[3] TALL, D., and VINNER, S.: Concept Images and Concept Definition in Mathematics with Particular reference to Limits and Continuity. ***Educational Studies in Mathematics,*** 12 (1991), 49–63

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