

Galilean transformation

Let G be the Galilean transformation operators

$$G_1 = \frac{m}{\hbar}X_1, \quad G_2 = \frac{m}{\hbar}X_2, \quad G_3 = \frac{m}{\hbar}X_3$$

Let U be the unitary transformation

$$U = 1 - i\epsilon G_1 - \frac{1}{2}\epsilon^2 G_1^2$$

Show that to order ϵ^2

$$U^{-1}P_1U = P_1 - \epsilon m, \quad U^{-1}P_2U = P_2, \quad U^{-1}P_3U = P_3 \quad (1)$$

and

$$U^{-1}HU = H - \epsilon P_1 + \frac{1}{2}\epsilon^2 m \quad (2)$$

where

$$H = \frac{1}{2m} (P_1^2 + P_2^2 + P_3^2)$$