For an arbitrary vector $\mathbf{k}'$ choose a rotation matrix $\mathbf{R}$ such that

$$\mathbf{k} = \mathbf{R}\,\mathbf{k}' = \begin{pmatrix} 0 \\ 0 \\ k \end{pmatrix}$$

where $k = |\mathbf{k}'|$.

By the requirement that $\mathbf{a}_\mathbf{k}(t) \cdot \mathbf{k} = 0$, let

$$\mathbf{a}_\mathbf{k}(t) = a(t) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

Then by equation (9.12)

$$\mathbf{A} = \mathbf{a}_\mathbf{k}(t) \exp(i\mathbf{k} \cdot \mathbf{r}) = \begin{pmatrix} a(t) \exp(ikz) \\ a(t) \exp(ikz) \\ 0 \end{pmatrix}$$

By equation (9.9) with $\phi = 0$

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} = -\frac{1}{c}\frac{da(t)}{dt} \begin{pmatrix} \exp(ikz) \\ \exp(ikz) \\ 0 \end{pmatrix}$$

By equation (9.7)

$$\mathbf{B} = \nabla \times \mathbf{A} = \begin{pmatrix} -ia(t) \exp(ikz) \\ ia(t) \exp(ikz) \\ 0 \end{pmatrix}$$

By inspection we have $\mathbf{k} \cdot \mathbf{E} = 0$, $\mathbf{k} \cdot \mathbf{B} = 0$, and $\mathbf{E} \cdot \mathbf{B} = 0$, hence all three vectors are mutually perpendicular.