The file q4.txt defines kets, operators, and a measurement function for simulating a four bit quantum computer. See eigenmath.org/q.c for the program that generates q4.txt.

Kets are unit vectors in $\mathbb{C}^{16}$. The dimension is 16 because a four bit quantum computer has $2^4 = 16$ eigenstates. The following basis kets that represent eigenstates are defined in q4.txt.

$$
\begin{aligned}
&|0\rangle = |0000_2\rangle = (1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)\\
&|1\rangle = |0001_2\rangle = (0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0)\\
&|2\rangle = |0010_2\rangle = (0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0)\\
&|3\rangle = |0011_2\rangle = (0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0)\\
&\vdots\\
&|15\rangle = |1111_2\rangle = (0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1)
\end{aligned}
$$

Operators are 16×16 matrices that rotate ket vectors. The following operators and the measurement function $M(\psi)$ are defined in q4.txt.

H_n	Hadamard operator on bit n.
I	Identity matrix.
$M(\psi)$	Measurement function (not an operator).
$P_n(\phi)$	Phase shift ϕ on bit n.
$P_{mn}(\phi)$	Controlled phase shift, m is the control bit, n is the target bit, ϕ is the phase.
Q	Quantum Fourier transform.
R	Inverse quantum Fourier transform.
S_{mn}	Swap bits m and n.
X_n	Pauli X (NOT) operator on bit n.
X_{mn}	Controlled X (CNOT) operator, m is the control bit, n is the target bit.
Y_n	Pauli Y operator on bit n.
Z_n	Pauli Z operator on bit n.

Measurement function $M(\psi)$ shows, for all $k = 0 \dots 15$, the probability P_k of observing eigenstate k given that the quantum computer is in state ψ.

$$\psi = \sum_{k=0}^{15} c_k |k\rangle, \quad |\psi|^2 = 1, \quad P_k = c_k c_k^*$$

Quantum algorithms are expressed as sequences of operators applied to the initial state $|0\rangle$. The operator sequence should be read backwards, from right to left, although the direction makes no difference mathematically.

Deutsch-Jozsa algorithm

Let $f(q_0, q_1, q_2)$ be an operator (16×16 matrix) that operates on q_3 in a manner consistent with a constant or balanced oracle. Then the Deutsch-Jozsa algorithm for identifying f is

$$\psi = H_2 \; H_1 \; H_0 \; f(q_0, q_1, q_2) \; H_3 \; X_3 \; H_2 \; H_1 \; H_0 \; |0\rangle$$

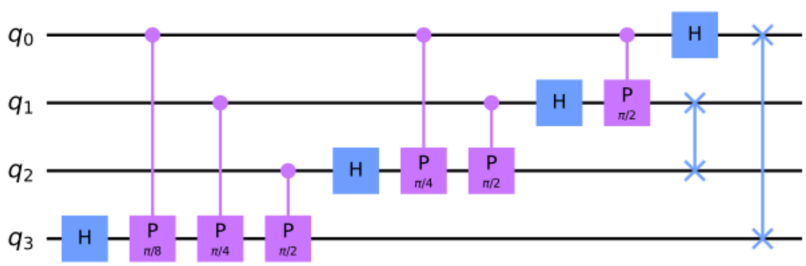
Bernstein-Vazirani algorithm

Let $f(q_0, q_1, q_2)$ be an operator (16×16 matrix) that operates on q_3. Then the Bernstein-Vazirani algorithm for identifying f is

$$\psi = H_2 \; H_1 \; H_0 \; f(q_0, q_1, q_2) \; Z_3 \; H_3 \; H_2 \; H_1 \; H_0 \; |0\rangle$$

Quantum Fourier transform

The following circuit diagram[1] shows how to implement the QFT.



This is how the QFT operator Q is defined in q4.txt.

```
Q = dot(
S03,            -- Swap qubits 0 and 3
S12,            -- Swap qubits 1 and 2
H0,             -- Hadamard qubit 0
P01(pi/2),      -- Controlled phase shift (0 control, 1 target)
H1,             -- Hadamard on qubit 1
P12(pi/2),      -- Controlled phase shift (1 control, 2 target)
P02(pi/4),      -- Controlled phase shift (0 control, 2 target)
H2,             -- Hadamard qubit 2
P23(pi/2),      -- Controlled phase shift (2 control, 3 target)
P13(pi/4),      -- Controlled phase shift (1 control, 3 target)
P03(pi/8),      -- Controlled phase shift (0 control, 3 target)
H3)             -- Hadamard qubit 3
```

The inverse QFT operator R is defined similarly except the operators appear in reverse order and the phase shifts are negated.

[1] qiskit.org/textbook/ch-algorithms/quantum-fourier-transform.html