

Hydrogen energy four ways

The following formulas for E_n are equivalent.

This is the traditional formula with elementary charge e .

$$E_n = -\frac{\mu}{2n^2} \left(\frac{e^2}{4\pi\epsilon_0\hbar} \right)^2 \quad (1)$$

The following formula eliminates e but still solves the Schrödinger equation directly without any substitutions.

$$E_n = -\frac{\hbar^2}{2n^2\mu a_0^2} \quad (2)$$

This form of E_n is for dividing by \hbar to obtain angular frequency ω .

$$E_n = -\frac{\alpha\hbar c}{2n^2a_0} \quad (3)$$

This form of E_n is irreducible.

$$E_n = -\frac{\alpha^2\mu c^2}{2n^2} \quad (4)$$

Equation (1) reduces to (4) by substituting

$$e^2 = 4\pi\epsilon_0\alpha\hbar c$$

Equations (2) and (3) reduce to (4) by substituting

$$a_0 = \frac{\hbar}{\alpha\mu c}$$