

Feynman and Hibbs problem 2-1

This is the Lagrangian for a free particle.

$$L = \frac{m}{2} \dot{x}^2$$

Show that for a free particle the action S_{cl} is

$$S_{cl} = \frac{m}{2} \frac{(x_b - x_a)^2}{t_b - t_a}$$

For the L given above we have

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) = m \ddot{x}$$

and

$$\frac{\partial L}{\partial x} = 0$$

By equation (2.7) which is

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) = \frac{\partial L}{\partial x}$$

we have

$$\ddot{x} = 0$$

Hence velocity \dot{x} is constant and equals distance divided by time.

$$\dot{x} = \frac{x_b - x_a}{t_b - t_a}$$

By equation (2.1) which is

$$S = \int_{t_a}^{t_b} L dt$$

we have for classical action S_{cl}

$$S_{cl} = \int_{t_a}^{t_b} \frac{m}{2} \dot{x}^2 dt = \int_{t_a}^{t_b} \frac{m}{2} \left(\frac{x_b - x_a}{t_b - t_a} \right)^2 dt$$

Solve the integral.

$$S_{cl} = \frac{m}{2} \left(\frac{x_b - x_a}{t_b - t_a} \right)^2 t \Big|_{t_a}^{t_b} = \frac{m}{2} \frac{(x_b - x_a)^2}{t_b - t_a}$$