8-4. Show that the ground-state wave function for the Lagrangian of equation (8.78) can be written

$$\Phi_0 = A \exp\left(-\frac{1}{2\hbar} \sum_{\alpha=1}^{N-1} \omega_\alpha Q_\alpha^* Q_\alpha \right) \tag{8.83}$$

(where A is a constant) by starting with the wave function in terms of the real variables Q_α^c and Q_α^s.

Consider equations (8.67) and (8.82).

$$L = \frac{1}{2} \sum_{j=1}^{N} \dot{q}_j^2 - \frac{\nu^2}{2} \sum_{j=1}^{N-1} (q_{j+1} - q_j)^2 \tag{8.67}$$

$$q_j(t) = \sqrt{\frac{2}{N}} \left(\frac{1}{2} Q_0^c(t) + \sum_{\alpha=1}^{(N-1)/2} \left(Q_\alpha^c(t) \cos\frac{2\pi\alpha j}{N} + Q_\alpha^s(t) \sin\frac{2\pi\alpha j}{N} \right) \right) \tag{8.82}$$