$$K(b,a) = K_0(b,a) - \frac{i}{\hbar}\int K_0(b,c)V(c)K(c,a)\,d\tau_c$$

Recall

$$\psi(b) = \int K(b,a)\psi(a)\,d\tau_a$$

Hence

$$\psi(b) = \int K_0(b,a)\psi(a)\,d\tau_a + \int \left(-\frac{i}{\hbar}\int K_0(b,c)V(c)K(c,a)\,d\tau_c\right)\psi(a)\,d\tau_a$$

Interchange the order of the integrals.

$$\psi(b) = \int K_0(b,a)\psi(a)\,d\tau_a - \frac{i}{\hbar}\int K_0(b,c)V(c)\left(\int K(c,a)\,d\tau_a\right)\,d\tau_c$$

Substitute $\psi(c)$ for $\int K(c,a)\,d\tau_a$.

$$\psi(b) = \int K_0(b,a)\psi(a)\,d\tau_a - \frac{i}{\hbar}\int K_0(b,c)V(c)\psi(c)\,d\tau_c$$

FIXME