Electromagnetic tensor

This is the standard model for an EM field.

$$\mathbf{A} = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}, \quad A^{\mu} = \begin{pmatrix} \phi \\ A_x \\ A_y \\ A_z \end{pmatrix}, \quad A_{\mu} = g_{\mu\nu}A^{\nu} = \begin{pmatrix} \phi \\ -A_x \\ -A_y \\ -A_z \end{pmatrix}$$

$$gdd = ((1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1))$$

 $guu = inv(gdd)$

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad \mathbf{E} = -\nabla \phi - \frac{\partial \mathbf{A}}{\partial t}$$

$$B = curl(A)$$

$$E = -d(phi(),(x,y,z)) - d(A,t)$$

Bx = B[1]

By = B[2]

Bz = B[3]

Ex = E[1]

Ey = E[2]

Ez = E[3]

This is the electromagnetic tensor.

$$F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} = A_{\nu,\mu} - A_{\mu,\nu} = \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$X = (t,x,y,z)$$

Add = d(Ad, X)

Fdd = transpose(Add) - Add

check(Fdd == T)

Check the following relations.

$$F_{\mu\nu}F^{\mu\nu} = 2(\mathbf{B}^2 - \mathbf{E}^2), \quad \det(F_{\mu\nu}) = \det(F^{\mu\nu}) = (\mathbf{B} \cdot \mathbf{E})^2$$

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Fuu = dot(guu,Fdd,guu)
T = contract(dot(transpose(Fdd),Fuu))
check(T == 2 dot(B,B) - 2 dot(E,E))

check(det(Fdd) == dot(B,E)^2)
check(det(Fuu) == dot(B,E)^2)
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This is the vector current.

$$J^{\nu} = \partial_{\mu} F^{\mu\nu} = F^{\mu\nu}_{,\mu}$$

Gradient increases rank by one. The new index is the rightmost index, hence the contraction is over the first and third indices.

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Ju = contract(d(Fuu,X),1,3)
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Check the following relations.

$$\partial_{\mu}J^{\mu} = J^{\mu}_{,\mu} = 0, \quad \mathbf{J} = \nabla \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t}$$

check(contract(d(Ju,X)) == 0)

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Jx = Ju[2]
Jy = Ju[3]
Jz = Ju[4]
J = (Jx,Jy,Jz)
check(J == curl(B) - d(E,t))
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