

Feynman and Hibbs problem 4-5

Using the relation

$$K(b, a) = \int_{-\infty}^{\infty} K(b, c)K(c, a) dx_c \quad (4.26)$$

with $t_c - t_a = \epsilon$, an infinitesimal, show that if t_b is greater than t_a , the kernel K satisfies

$$\frac{\partial}{\partial t_a} K(b, a) = +\frac{i}{\hbar} H_a^* K(b, a)$$

where H_a now operates on the a variables only.

Expand the arguments of K in (4.26).

$$K(x_b, t_b, x_a, t_a) = \int_{-\infty}^{\infty} K(x_b, t_b, x_c, t_c)K(x_c, t_c, x_a, t_a) dx_c$$

Substitute $t_a = t_c - \epsilon$.

$$K(x_b, t_b, x_a, t_a) = \int_{-\infty}^{\infty} K(x_b, t_b, x_c, t_c)K(x_c, t_c, x_a, t_c - \epsilon) dx_c$$

Consider the following Taylor series expansion of $K(x_c, t_a + \epsilon, x_a, t_a)$.

$$K(x_c, t_a + \epsilon, x_a, t_a) \approx K(x_c, t_a, x_a, t_a) + \epsilon \frac{\partial}{\partial t_a} K(x_c, t_a, x_a, t_a)$$

$$\lim_{\epsilon \rightarrow 0} \int_{-\infty}^{\infty} K(x_b, t_b, x_c, t_c)K(x_c, t_c, x_a, t_c - \epsilon) dx_c = \int_{-\infty}^{\infty} K(x_b, t_b, x_c, t_c) dx_c$$