

This is the Schroedinger equation for $2d$ hydrogen.

$$-\frac{\hbar^2}{2\mu} \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \right) \psi_n - \frac{k}{r} \psi_n = E_n \psi_n$$

Symbol μ is reduced electron mass and k/r is potential energy with

$$k = \frac{e^2}{4\pi\epsilon_0}$$

Energy eigenvalues are

$$E_n = -\frac{k^2\mu}{2\hbar^2 \left(n + \frac{1}{2}\right)^2}$$

In matrix form we have

$$H = \frac{P^2}{2\mu} - kV$$

where matrix elements are computed as follows.

$$H_{nm} = E_n \delta_{nm}$$

$$(P^2)_{nm} = -\hbar^2 \int_0^{2\pi} \int_0^\infty \psi_n \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \right) \psi_m r dr d\phi$$

$$V_{nm} = \int_0^{2\pi} \int_0^\infty \psi_n \left(\frac{1}{r} \right) \psi_m r dr d\phi$$

Here are numerical values for 4×4 matrices.

$$H = \begin{pmatrix} -9.68305 & 0 & 0 & 0 \\ 0 & -3.48590 & 0 & 0 \\ 0 & 0 & -1.77852 & 0 \\ 0 & 0 & 0 & -1.07589 \end{pmatrix} \times 10^{-19} \text{ joule}$$

$$P^2 = \begin{pmatrix} 17.6317 & 9.28281 & 5.23021 & 3.49926 \\ 9.28281 & 6.34742 & 3.98524 & 2.46475 \\ 5.23021 & 3.98524 & 3.23848 & 2.21527 \\ 3.49926 & 2.46475 & 2.21527 & 1.95908 \end{pmatrix} \times 10^{-49} \text{ kilogram}^2 \text{ meter}^2 \text{ second}^{-2}$$

$$V = \begin{pmatrix} 83.9421 & 22.0971 & 12.4502 & 8.32975 \\ 22.0971 & 30.2192 & 9.48657 & 5.86716 \\ 12.4502 & 9.48657 & 15.4179 & 5.27330 \\ 8.32975 & 5.86716 & 5.27330 & 9.32690 \end{pmatrix} \times 10^8 \text{ meter}^{-1}$$