

6-5. The integral over t_c in equation (6.28) can be performed approximately using the method of stationary phase. By studying the application of such a method to this integral, show that most of the contribution to the integral comes from values of t_c near the region $t_c = r_a/u$, the time at which the electron would arrive at the center of the atom if it moved in a classical manner.

$$\begin{aligned}
K^{(1)}(b, a) &= -\frac{i}{\hbar} \int K_0(b, c) V(c) K_0(c, a) d\tau_c \\
&= -\frac{i}{\hbar} \int \int_0^T \left(\frac{m}{2\pi i \hbar (T - t_c)} \right)^{3/2} \exp \left(\frac{im |\mathbf{x}_b - \mathbf{x}_c|^2}{2\hbar (T - t_c)} \right) \\
&\quad \times V(\mathbf{x}_c) \left(\frac{m}{2\pi i \hbar t_c} \right)^{3/2} \exp \left(\frac{im |\mathbf{x}_c - \mathbf{x}_a|^2}{2\hbar t_c} \right) dt_c d^3 \mathbf{x}_c \quad (6.28)
\end{aligned}$$

The phase of the exponential in (6.28) is

$$g(t_c) = \frac{|\mathbf{x}_b - \mathbf{x}_c|^2}{T - t_c} + \frac{|\mathbf{x}_c - \mathbf{x}_a|^2}{t_c}$$

Then for

$$t_c = \frac{T |\mathbf{x}_c - \mathbf{x}_a|}{|\mathbf{x}_b - \mathbf{x}_c| + |\mathbf{x}_c - \mathbf{x}_a|}$$

the phase is stationary, that is,

$$g'(t_c) = \frac{|\mathbf{x}_b - \mathbf{x}_c|^2}{(T - t_c)^2} + \frac{|\mathbf{x}_c - \mathbf{x}_a|^2}{t_c^2} = 0$$

For r_a and $r_b \gg |\mathbf{x}_c|$

$$\begin{aligned}
|\mathbf{x}_c - \mathbf{x}_a| &\approx r_a \\
|\mathbf{x}_b - \mathbf{x}_c| &\approx r_b
\end{aligned}$$

Hence

$$t_c = \frac{T r_a}{r_a + r_b}$$