Prove the following Gordon decomposition by direct calculation. Momentum vectors p_1 and p_2 have the same rest mass m. Each of the spins s_1 and s_2 can be either up or down.

$$\bar{u}(p_2, s_2)\gamma^\mu u(p_1, s_1) = \bar{u}(p_2, s_2)\left[\frac{(p_2 + p_1)^\mu}{2m} + i\sigma^{\mu\nu}\frac{(p_2 - p_1)_\nu}{2m}\right]u(p_1, s_1)$$

The following vectors and spinors are used. Spinors u_{11} and u_{21} are spin up, u_{12} and u_{22} are spin down.

$$p_1 = \begin{pmatrix} E_1 \\ p_{1x} \\ p_{1y} \\ p_{1z} \end{pmatrix} \quad u_{11} = \begin{pmatrix} E_1 + m \\ 0 \\ p_{1z} \\ p_{1x} + ip_{1y} \end{pmatrix} \quad u_{12} = \begin{pmatrix} 0 \\ E_1 + m \\ p_{1x} - ip_{1y} \\ -p_{1z} \end{pmatrix} \quad E_1 = \sqrt{p_{1x}^2 + p_{1y}^2 + p_{1z}^2 + m^2}$$

$$p_2 = \begin{pmatrix} E_2 \\ p_{2x} \\ p_{2y} \\ p_{2z} \end{pmatrix} \quad u_{21} = \begin{pmatrix} E_2 + m \\ 0 \\ p_{2z} \\ p_{2x} + ip_{2y} \end{pmatrix} \quad u_{22} = \begin{pmatrix} 0 \\ E_2 + m \\ p_{2x} - ip_{2y} \\ -p_{2z} \end{pmatrix} \quad E_2 = \sqrt{p_{2x}^2 + p_{2y}^2 + p_{2z}^2 + m^2}$$

Tensor $\sigma^{\mu\nu}$ is defined as

$$\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right] = \frac{i}{2}\left(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\right)$$

In component notation we have

$$\sigma^{\mu\nu\alpha}{}_\beta = \frac{i}{2}\left(\gamma^{\mu\alpha}{}_\rho\gamma^{\nu\rho}{}_\beta - \gamma^{\nu\alpha}{}_\rho\gamma^{\mu\rho}{}_\beta\right)$$

Let $T^{\mu\nu} = \gamma^\mu\gamma^\nu$. Transpose the first two indices of $\gamma^{\nu\rho}{}_\beta$ to form a dot product.

$$T^{\mu\nu\alpha}{}_\beta = \gamma^{\mu\alpha}{}_\rho\gamma^{\rho\nu}{}_\beta$$

Convert to code. The transpose on the second and third indices interchanges α and ν.

$$T^{\mu\nu\alpha}{}_\beta = \texttt{transpose(dot(gamma,transpose(gamma)),2,3)}$$

Hence

$$\sigma^{\mu\nu} = \texttt{i/2 (T - transpose(T))}$$

where $\texttt{T} = T^{\mu\nu\alpha}{}_\beta$. Now convert $\sigma^{\mu\nu}(p_2 - p_1)_\nu$ to code.

$$\sigma^{\mu\nu}(p_2 - p_1)_\nu = \sigma^{\mu\alpha}{}_\beta{}^\nu g_{\nu\rho}(p_2 - p_1)^\rho = \texttt{dot(S,gmunu,p2 - p1)}$$

where $\texttt{S} = \sigma^{\mu\alpha}{}_\beta{}^\nu = \texttt{transpose(transpose(sigmamunu,2,3),3,4)}$.