

6-17. Interpret equation (6.72) by explaining the meaning of each term. Then explain and verify the equation for the second-order coefficient

$$\begin{aligned} \lambda_{mn}^{(2)} = & \left(-\frac{i}{\hbar}\right)^2 \int_{t_a}^{t_b} \int_{t_a}^{t_c} \sum_j \exp\left(-\frac{i}{\hbar} E_m(t_b - t_c)\right) V_{mj}(t_c) \\ & \times \exp\left(-\frac{i}{\hbar} E_j(t_c - t_d)\right) V_{jn}(t_d) \exp\left(-\frac{i}{\hbar} E_n(t_d - t_a)\right) dt_d dt_c \quad (6.74) \end{aligned}$$

Start with equation (6.13).

$$\begin{aligned} K^{(2)}(b, a) = & \left(-\frac{i}{\hbar}\right)^2 \int_{t_a}^{t_b} \int_{-\infty}^{\infty} \int_{t_a}^{t_c} \int_{-\infty}^{\infty} K_0(b, c) V(c) K_0(c, d) V(d) K_0(d, a) \\ & \times dx_d dt_d dx_c dt_c \end{aligned}$$

Substitute  $K_U$  for  $K_0$ . (See problem 6-1.)

$$\begin{aligned} K^{(2)}(b, a) = & \left(-\frac{i}{\hbar}\right)^2 \int_{t_a}^{t_b} \int_{-\infty}^{\infty} \int_{t_a}^{t_c} \int_{-\infty}^{\infty} K_U(b, c) V(c) K_U(c, d) V(d) K_U(d, a) \\ & \times dx_d dt_d dx_c dt_c \end{aligned}$$

By equation (6.66), substitute sums over eigenstates for  $K_U$ .

$$\begin{aligned} K^{(2)}(b, a) = & \sum_m \sum_j \sum_n \left(-\frac{i}{\hbar}\right)^2 \int_{t_a}^{t_b} \int_{-\infty}^{\infty} \int_{t_a}^{t_c} \int_{-\infty}^{\infty} \\ & \phi_m(x_b) \phi_m^*(x_c) V(c) \phi_j(x_c) \phi_j^*(x_d) V(d) \phi_n(x_d) \phi_n^*(x_a) \\ & \times \exp\left(-\frac{i}{\hbar} E_m(t_b - t_c)\right) \exp\left(-\frac{i}{\hbar} E_j(t_c - t_d)\right) \exp\left(-\frac{i}{\hbar} E_n(t_d - t_a)\right) \\ & \times dx_d dt_d dx_c dt_c \end{aligned}$$

By equation (6.71), substitute  $V_{mn}$  for integrals over  $x$ .

$$\begin{aligned} K^{(2)}(b, a) = & \sum_m \sum_j \sum_n \left(-\frac{i}{\hbar}\right)^2 \int_{t_a}^{t_b} \int_{t_a}^{t_c} \phi_m(x_b) V_{mj}(t_c) V_{jn}(t_d) \phi_n^*(x_a) \\ & \times \exp\left(-\frac{i}{\hbar} E_m(t_b - t_c)\right) \exp\left(-\frac{i}{\hbar} E_j(t_c - t_d)\right) \exp\left(-\frac{i}{\hbar} E_n(t_d - t_a)\right) \\ & \times dt_d dt_c \end{aligned}$$

Finally, by equation (6.68) we obtain  $\lambda_{mn}^{(2)}$ .

$$\begin{aligned} \lambda_{mn}^{(2)}(t_b, t_a) &= \sum_j \left( -\frac{i}{\hbar} \right)^2 \int_{t_a}^{t_b} \int_{t_a}^{t_c} V_{mj}(t_c) V_{jn}(t_d) \\ &\times \exp \left( -\frac{i}{\hbar} E_m(t_b - t_c) \right) \exp \left( -\frac{i}{\hbar} E_j(t_c - t_d) \right) \exp \left( -\frac{i}{\hbar} E_n(t_d - t_a) \right) \\ &\times dt_d dt_c \end{aligned}$$

Here are equations (6.68) and (6.71).

$$K_V(b, a) = \sum_m \sum_n \lambda_{mn}(t_b, t_a) \phi_m(x_b) \phi_n^*(x_a) \quad (6.68)$$

$$V_{mn}(t_c) = \int_{-\infty}^{\infty} \phi_m^*(x_c) V(x_c, t_c) \phi_n(x_c) dx_c \quad (6.71)$$