7-3. If

$$F(j(\mathbf{r},t)) = \exp\left(\frac{1}{2} \int \int \int \int j(\mathbf{r}_1,t_1) j(\mathbf{r}_2,t_2) R(\mathbf{r}_2 - \mathbf{r}_1,t_2 - t_1) d^3 \mathbf{r}_2 dt_2 d^3 \mathbf{r}_1 dt_1\right)$$

where the integrals extend over all space and time, show that

$$\frac{\delta F}{\delta j(\mathbf{x},s)} = F \int \int j(\mathbf{r},t) \frac{1}{2} \left( R(\mathbf{r} - \mathbf{x},t-s) + R(\mathbf{x} - \mathbf{r},s-t) \right) d^3 \mathbf{r} dt \quad (7.27)$$

By equation (7.20) to first order we have

$$F(j(\mathbf{r},t) + \eta(\mathbf{r},t)) - F(j(\mathbf{r},t)) = \int \int \frac{\delta F}{\delta j(\mathbf{x},s)} \eta(\mathbf{x},s) d^3 \mathbf{x} ds$$

Let

$$I(f_1, f_2) = \frac{1}{2} \int \int \int \int f_1(\mathbf{r}_1, t_1) f_2(\mathbf{r}_2, t_2) R(\mathbf{r}_2 - \mathbf{r}_1, t_2 - t_1) d^3 \mathbf{r}_2 dt_2 d^3 \mathbf{r}_1 dt_1$$

Then

$$F(j(\mathbf{r},t) + \eta(\mathbf{r},t)) = F(j(\mathbf{r},t)) \exp(I(j,\eta)) \exp(I(\eta,j)) \exp(I(\eta,\eta))$$

Use the approximation  $\exp(x) = 1 + x$  to obtain to first order in  $\eta$ 

$$F(j(\mathbf{r},t) + \eta(\mathbf{r},t)) = F(j(\mathbf{r},t))(1 + I(j,\eta) + I(\eta,j))$$

Then by equation (7.20)

$$F(j(\mathbf{r},t))I(j,\eta) + F(j(\mathbf{r},t))I(\eta,j) = \int \int \frac{\delta F}{\delta j(\mathbf{x},s)} \eta(\mathbf{x},s) d^3 \mathbf{x} ds \quad (1)$$

Substitute (7.27) into (1). The  $F(j(\mathbf{r},t))$  cancel leaving

$$\begin{split} I(j,\eta) + I(\eta,j) &= \\ \frac{1}{2} \int \int \int \int j(\mathbf{r},t) R(\mathbf{r} - \mathbf{x}, t - s) \eta(\mathbf{x}, s) \, d^3 \mathbf{r} \, dt \, d^3 \mathbf{x} \, ds \\ &+ \frac{1}{2} \int \int \int \int \int j(\mathbf{r}, t) R(\mathbf{x} - \mathbf{r}, s - t) \eta(\mathbf{x}, s) \, d^3 \mathbf{r} \, dt \, d^3 \mathbf{x} \, ds \end{split}$$

which is easily verified.