Matrix mechanics 4

Let K_1, K_2, and K_3 be the following spatial translation matrices.

$$K_1 = \frac{1}{\hbar} P_1, \quad K_2 = \frac{1}{\hbar} P_2, \quad K_3 = \frac{1}{\hbar} P_3$$

Let U be the unitary transformation

$$U = 1 - i\epsilon K_3 - \tfrac{1}{2}\epsilon^2 {K_3}^2$$

1. Show that to order ϵ^2

$$\begin{aligned} U^{-1} X_1 U &= X_1 \\ U^{-1} X_2 U &= X_2 \\ U^{-1} X_3 U &= X_3 + \epsilon \end{aligned}$$

2. Show that to order ϵ^2

$$\begin{aligned} U^{-1} P_1 U &= P_1 \\ U^{-1} P_2 U &= P_2 \\ U^{-1} P_3 U &= P_3 \end{aligned}$$

3. Show that to order ϵ^2

$$\begin{aligned} U^{-1} L_1 U &= L_1 - \epsilon P_2 \\ U^{-1} L_2 U &= L_2 + \epsilon P_1 \\ U^{-1} L_3 U &= L_3 \end{aligned}$$

4. Show that to order ϵ^2

$$U^{-1} H U = H$$

where

$$H = \frac{1}{2m}\left({P_1}^2 + {P_2}^2 + {P_3}^2\right)$$