

Feynman and Hibbs problem 3-8

This is the Lagrangian for a harmonic oscillator.

$$L = \frac{m}{2} \dot{x}^2 - \frac{m\omega^2}{2} x^2$$

Show that the resulting kernel is

$$K = F(T) \exp \left( \frac{im\omega}{2\hbar \sin(\omega T)} ((x_b^2 + x_a^2) \cos(\omega T) - 2x_b x_a) \right)$$

where  $T = t_b - t_a$ .

From problem 2-2 we have

$$S_{cl} = \frac{m\omega}{2\hbar \sin(\omega T)} ((x_b^2 + x_a^2) \cos(\omega T) - 2x_b x_a)$$

From equation (3.51)

$$K = F(T) \exp \left( \frac{iS_{cl}}{\hbar} \right)$$

By substitution of  $S_{cl}$

$$K = F(T) \exp \left( \frac{im\omega}{2\hbar \sin(\omega T)} ((x_b^2 + x_a^2) \cos(\omega T) - 2x_b x_a) \right)$$