

$$f_A(\check{\mathbf{p}}) = C_A \int \exp\left(\frac{i\check{\mathbf{p}} \cdot \mathbf{r}}{\hbar}\right) V_A(\mathbf{r}) d^3\mathbf{r}$$

$$f_B(\check{\mathbf{p}}) = C_B \int \exp\left(\frac{i\check{\mathbf{p}} \cdot \mathbf{r}}{\hbar}\right) V_B(\mathbf{r}) d^3\mathbf{r}$$

This is the combined potential.

$$V(\mathbf{r}) = V_A(\mathbf{r} - \mathbf{a}) + V_B(\mathbf{r} - \mathbf{b})$$

Hence

$$K^{(1)} = C_A \int \exp\left(\frac{i\check{\mathbf{p}} \cdot \mathbf{r}}{\hbar}\right) V_A(\mathbf{r} - \mathbf{a}) d^3\mathbf{r} + C_B \int \exp\left(\frac{i\check{\mathbf{p}} \cdot \mathbf{r}}{\hbar}\right) V_B(\mathbf{r} - \mathbf{b}) d^3\mathbf{r}$$

Substitute $\mathbf{r} + \mathbf{a}$ for \mathbf{r} in the first integral and $\mathbf{r} + \mathbf{b}$ for \mathbf{r} in the second integral.

$$K^{(1)} = C_A \int \exp\left(\frac{i\check{\mathbf{p}} \cdot (\mathbf{r} + \mathbf{a})}{\hbar}\right) V_A(\mathbf{r}) d^3\mathbf{r} + C_B \int \exp\left(\frac{i\check{\mathbf{p}} \cdot (\mathbf{r} + \mathbf{b})}{\hbar}\right) V_B(\mathbf{r}) d^3\mathbf{r}$$

Factor the exponentials.

$$K^{(1)} = C_A \exp\left(\frac{i\check{\mathbf{p}} \cdot \mathbf{a}}{\hbar}\right) \int \exp\left(\frac{i\check{\mathbf{p}} \cdot \mathbf{r}}{\hbar}\right) V_A(\mathbf{r}) d^3\mathbf{r}$$

$$+ C_B \exp\left(\frac{i\check{\mathbf{p}} \cdot \mathbf{b}}{\hbar}\right) \int \exp\left(\frac{i\check{\mathbf{p}} \cdot \mathbf{r}}{\hbar}\right) V_B(\mathbf{r}) d^3\mathbf{r}$$

Hence

$$K^{(1)} = \exp\left(\frac{i\check{\mathbf{p}} \cdot \mathbf{a}}{\hbar}\right) f_A(\check{\mathbf{p}}) + \exp\left(\frac{i\check{\mathbf{p}} \cdot \mathbf{b}}{\hbar}\right) f_B(\check{\mathbf{p}})$$

The probability of scattering is

$$|K^{(1)}|^2 = f_A^2 + f_B^2 + f_A f_B \exp\left(\frac{i\check{\mathbf{p}} \cdot (\mathbf{a} - \mathbf{b})}{\hbar}\right) + f_A f_B \exp\left(\frac{i\check{\mathbf{p}} \cdot (\mathbf{b} - \mathbf{a})}{\hbar}\right)$$

Substitute \mathbf{d} for $\mathbf{a} - \mathbf{b}$.

$$|K^{(1)}|^2 = f_A^2 + f_B^2 + f_A f_B \exp\left(\frac{i\check{\mathbf{p}} \cdot \mathbf{d}}{\hbar}\right) + f_A f_B \exp\left(-\frac{i\check{\mathbf{p}} \cdot \mathbf{d}}{\hbar}\right)$$

Change complex exponentials to rectangular form.

$$|K^{(1)}|^2 = f_A^2 + f_B^2 + f_A f_B \left[\cos\left(\frac{\check{\mathbf{p}} \cdot \mathbf{d}}{\hbar}\right) + i \sin\left(\frac{\check{\mathbf{p}} \cdot \mathbf{d}}{\hbar}\right) \right]$$

$$+ f_A f_B \left[\cos\left(\frac{\check{\mathbf{p}} \cdot \mathbf{d}}{\hbar}\right) - i \sin\left(\frac{\check{\mathbf{p}} \cdot \mathbf{d}}{\hbar}\right) \right]$$

The sine functions cancel.

$$|K^{(1)}|^2 = f_A^2 + f_B^2 + 2f_A f_B \cos\left(\frac{\check{\mathbf{p}} \cdot \mathbf{d}}{\hbar}\right)$$