Since the free-particle lagrangian is quadratic, show that

$$K(b,a) = F(t_b, t_a) \exp\left(\frac{im(x_b - x_a)^2}{2\hbar(t_b - t_a)}\right)$$

and give an argument to show that F can depend only on the difference $F(t_b - t_a)$.

From equation 3.51

$$K(b,a) = \exp\left(\frac{i}{\hbar} S_{cl}(b,a)\right) F(t_b, t_a)$$

From problem 2.1

$$S_{cl} = \frac{m}{2} \frac{(x_b - x_a)^2}{t_b - t_a}$$

Hence by substitution of S_{cl}

$$K(b,a) = F(t_b, t_a) \exp\left(\frac{im(x_b - x_a)^2}{2\hbar(t_b - t_a)}\right)$$

The argument for why F can only depend on $t_b - t_a$ is invariance under time translation. For any constant s we should have

$$F(t_b, t_a) = F(t_b + s, t_a + s)$$

For example, we should have

$$F(1, 0) = F(3, 2)$$

Hence F can only depend on the difference $t_b - t_a$ and not the specific values of t_a and t_b.