From equation (6.19)

$$K_0(b,a) = K_V(b,a) + \frac{i}{\hbar}\int K_0(b,c)V(c)K_V(c,a)\,d\tau_c \tag{1}$$

Apply $\partial/\partial t_b$ and $\partial^2/\partial x_b^2$ to (1).

$$\frac{\partial}{\partial t_b}K_0(b,a) = \frac{\partial}{\partial t_b}K_V(b,a) + \frac{i}{\hbar}\int \frac{\partial}{\partial t_b}K_0(b,c)V(c)K_V(c,a)\,d\tau_c \tag{2}$$

$$\frac{\partial^2}{\partial x_b^2}K_0(b,a) = \frac{\partial^2}{\partial x_b^2}K_V(b,a) + \frac{i}{\hbar}\int \frac{\partial^2}{\partial x_b^2}K_0(b,c)V(c)K_V(c,a)\,d\tau_c \tag{3}$$

Multiply (3) by $-\hbar^2/2m$.

$$\begin{aligned}-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_b^2}K_0(b,a)&\\ &= -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_b^2}K_V(b,a) + \frac{i}{\hbar}\int\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_b^2}K_0(b,c)\right)V(c)K_V(c,a)\,d\tau_c\end{aligned} \tag{4}$$

From equation (6.20)

$$\frac{i}{\hbar}\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_b^2}K_0(b,a)\right) = -\frac{\partial}{\partial t_b}K_0(b,a) + \delta(t_b-t_a)\delta(x_b-x_a)$$

It follows that

$$\begin{aligned}\frac{i}{\hbar}\int\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_b^2}K_0(b,c)\right)V(c)K_V(c,a)\,d\tau_c&\\ &= -\int\frac{\partial}{\partial t_b}K_0(b,c)V(c)K_V(c,a)\,d\tau_c + V(b)K_V(b,a)\end{aligned} \tag{5}$$

where we have used the identity

$$\int \delta(t_b-t_a)\delta(x_b-x_a)V(c)K_V(c,a)\,d\tau_c = V(b)K_V(b,a)$$

Substitute (5) into (4) to obtain

$$\begin{aligned}-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_b^2}K_0(b,a) &= -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_b^2}K_V(b,a)\\ &\quad -\int\frac{\partial}{\partial t_b}K_0(b,c)V(c)K_V(c,a)\,d\tau_c + V(b)K_V(b,a)\end{aligned} \tag{6}$$

Substitute (2) and (6) into (6.20) to obtain (6.21).