The Maxwell-Boltzmann distribution is a normal (Gaussian) distribution with zero mean. The following joint probability density function is the Maxwell-Boltzmann distribution for velocity components v_x, v_y, and v_z.

$$f(v_x, v_y, v_z) = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{m(v_x^2 + v_y^2 + v_z^2)}{2kT}\right)$$

In spherical coordinates with $v^2 = v_x^2 + v_y^2 + v_z^2$ we have

$$f(v, \theta, \phi) = \left(\frac{m}{2\pi kT}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT}\right)$$

Integrate over θ and ϕ to obtain the following probability density function which is Maxwell's χ^2 speed distribution.

$$f(v) = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2kT}\right)$$

Noting that

$$dv_x\, dv_y\, dv_z = v^2 \sin\theta\, dv\, d\theta\, d\phi$$

and

$$\int_0^\pi \sin\theta\, d\theta = \cos(0) - \cos(\pi) = 2$$

the integral can be done by inspection.

$$f(v) = \int_0^{2\pi} \int_0^\pi f(v, \theta, \phi)\, v^2 \sin\theta\, d\theta\, d\phi = 4\pi v^2 f(v, \theta, \phi)$$

Historically, the speed distribution came first.[1] Maxwell derived it in 1867. Boltzmann extended Maxwell's work a year later in 1868.

[1] https://mathshistory.st-andrews.ac.uk/Projects/Johnson/chapter-6/