3-8. For a harmonic oscillator the Lagrangian is

$$L = \frac{m}{2}\dot{x}^2 - \frac{m\omega^2}{2}x^2 \tag{3.58}$$

Show that the resulting kernel is (see problem 2-2)

$$K = F(T)\exp\left(\frac{im\omega}{2\hbar\sin(\omega T)}\left((x_b^2 + x_a^2)\cos(\omega T) - 2x_b x_a\right)\right) \tag{3.59}$$

where $T = t_b - t_a$. Note that the multiplicative function $F(T)$ has not been explicitly worked out. It can be obtained by other means, and for the harmonic oscillator it is (see section 3-11)

$$F(T) = \left(\frac{m\omega}{2\pi i\hbar\sin\omega T}\right)^{1/2} \tag{3.60}$$

From problem 2-2 we have

$$S_{cl} = \frac{m\omega}{2\hbar\sin(\omega T)}\left((x_b^2 + x_a^2)\cos(\omega T) - 2x_b x_a\right) \tag{1}$$

From equation (3.51)

$$K = F(T)\exp\left(\frac{iS_{cl}}{\hbar}\right) \tag{2}$$

Substitute (1) into (2).

$$K = F(T)\exp\left(\frac{im\omega}{2\hbar\sin(\omega T)}\left((x_b^2 + x_a^2)\cos(\omega T) - 2x_b x_a\right)\right)$$