

9-1. Show that  $\mathbf{E}$ ,  $\mathbf{B}$ , and  $\mathbf{k}$  are mutually perpendicular for this plane-wave solution.

$$\mathbf{A}(\mathbf{r}, t) = \mathbf{a}_{\mathbf{k}}(t) \exp(i\mathbf{k} \cdot \mathbf{r}) \quad (9.12)$$

For an arbitrary direction vector  $\mathbf{k}'$ , choose a rotation matrix  $\mathbf{R}$  such that

$$\mathbf{k} = \mathbf{R} \mathbf{k}' = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

By the requirement that  $\mathbf{a}_{\mathbf{k}} \cdot \mathbf{k} = 0$ , let

$$\mathbf{a}_{\mathbf{k}} = a(t) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

Then by equation (9.12)

$$\mathbf{A} = \mathbf{a}_{\mathbf{k}} \exp(i\mathbf{k} \cdot \mathbf{r}) = \begin{pmatrix} a(t) \exp(iz) \\ a(t) \exp(iz) \\ 0 \end{pmatrix}$$

By equation (9.9) with  $\phi = 0$

$$\mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} = -\frac{1}{c} \frac{da(t)}{dt} \begin{pmatrix} \exp(iz) \\ \exp(iz) \\ 0 \end{pmatrix}$$

By equation (9.7)

$$\mathbf{B} = \nabla \times \mathbf{A} = \begin{pmatrix} -ia(t) \exp(iz) \\ ia(t) \exp(iz) \\ 0 \end{pmatrix}$$

By inspection we have  $\mathbf{k} \cdot \mathbf{E} = 0$ ,  $\mathbf{k} \cdot \mathbf{B} = 0$ , and  $\mathbf{E} \cdot \mathbf{B} = 0$ , hence all three vectors are mutually perpendicular.