2.1. Solve the Klein-Gordon equation.

This is the Klein-Gordon equation.

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{\partial z^2}+\frac{m^2c^2}{\hbar^2}\right)\psi=0$$

A solution is

$$\psi=\exp\left(\frac{i}{\hbar}(Et-p_xx-p_yy-p_zz)\right)$$

where

$$E=\sqrt{m^2c^4+p_x^2c^2+p_y^2c^2+p_z^2c^2}$$

Let us inspect dimensions. We have

$$Et\propto \text{joule}\,\text{second}$$

Hence

$$\frac{Et}{\hbar}\propto\frac{\text{joule}\,\text{second}}{\text{joule}\,\text{second}}=1$$

We also have

$$p_xx\propto\frac{\text{kilogram meter}}{\text{second}}\times\text{meter}=\text{joule}\,\text{second}$$

Hence

$$\frac{p_xx}{\hbar}\propto\frac{\text{joule}\,\text{second}}{\text{joule}\,\text{second}}=1$$

And lastly,

$$\left(\frac{mc}{\hbar}\right)^2\propto\left(\frac{\text{kilogram}\,\text{meter/second}}{\text{joule}\,\text{second}}\right)^2=\left(\frac{\text{joule}}{\text{joule}}\right)^2=1$$