Let

$$\bar{a}^c_{1,\mathbf{k}} = \frac{1}{\sqrt{2}}(\bar{a}_{1,\mathbf{k}} + \bar{a}^*_{1,\mathbf{k}})$$
$$\bar{a}^s_{1,\mathbf{k}} = \frac{i}{\sqrt{2}}(\bar{a}_{1,\mathbf{k}} - \bar{a}^*_{1,\mathbf{k}})$$

$$\bar{a}^c_{2,\mathbf{k}} = \frac{1}{\sqrt{2}}(\bar{a}_{2,\mathbf{k}} + \bar{a}^*_{2,\mathbf{k}})$$
$$\bar{a}^s_{2,\mathbf{k}} = \frac{i}{\sqrt{2}}(\bar{a}_{2,\mathbf{k}} - \bar{a}^*_{2,\mathbf{k}})$$

Then

$$\bar{a}_{1,\mathbf{k}} = \frac{1}{\sqrt{2}}(\bar{a}^c_{1,\mathbf{k}} - i\bar{a}^s_{1,\mathbf{k}})$$
$$\bar{a}_{2,\mathbf{k}} = \frac{1}{\sqrt{2}}(\bar{a}^c_{2,\mathbf{k}} - i\bar{a}^s_{2,\mathbf{k}})$$

It follows that

$$\begin{aligned}\bar{a}^*_{1,\mathbf{k}}\bar{a}_{1,\mathbf{k}} &= \frac{1}{2}(\bar{a}^c_{1,\mathbf{k}})^2 + \frac{1}{2}(\bar{a}^s_{1,\mathbf{k}})^2 \\ \bar{a}^*_{2,\mathbf{k}}\bar{a}_{2,\mathbf{k}} &= \frac{1}{2}(\bar{a}^c_{2,\mathbf{k}})^2 + \frac{1}{2}(\bar{a}^s_{2,\mathbf{k}})^2\end{aligned} \tag{1}$$

Substitute (1) into (9.43).

$$\Phi_0 = \exp\left(-\sum_{\mathbf{k}} \frac{kc}{4\hbar}\left((\bar{a}^c_{1,\mathbf{k}})^2 + (\bar{a}^s_{1,\mathbf{k}})^2 + (\bar{a}^c_{2,\mathbf{k}})^2 + (\bar{a}^s_{2,\mathbf{k}})^2\right)\right)$$

FIXME finish showing (9.43) is correct