6-5. The integral over t_c in equation (6.28) can be performed approximately using the method of stationary phase. By studying the application of such a method to this integral, show that most of the contribution to the integral comes from values of t_c near the region $t_c = r_a/u$, the time at which the electron would arrive at the center of the atom if it moved in a classical manner.

$$\begin{aligned} K^{(1)}(b,a) &= -\frac{i}{\hbar}\int K_0(b,c)V(c)K_0(c,a)\,d\tau_c \\ &= -\frac{i}{\hbar}\int\int_0^T \left(\frac{m}{2\pi i\hbar(T-t_c)}\right)^{3/2} \exp\left(\frac{im|\mathbf{x}_b-\mathbf{x}_c|^2}{2\hbar(T-t_c)}\right) \\ &\qquad \times V(\mathbf{x}_c)\left(\frac{m}{2\pi i\hbar t_c}\right)^{3/2} \exp\left(\frac{im|\mathbf{x}_c-\mathbf{x}_a|^2}{2\hbar t_c}\right) dt_c\, d^3\mathbf{x}_c \qquad (6.28) \end{aligned}$$

The phase of the exponential in (6.28) is

$$g(t_c) = \frac{|\mathbf{x}_b-\mathbf{x}_c|^2}{T-t_c} + \frac{|\mathbf{x}_c-\mathbf{x}_a|^2}{t_c}$$

Then for

$$t_c = \frac{T|\mathbf{x}_c-\mathbf{x}_a|}{|\mathbf{x}_b-\mathbf{x}_c|+|\mathbf{x}_c-\mathbf{x}_a|}$$

the phase is stationary, that is,

$$g'(t_c) = \frac{|\mathbf{x}_b-\mathbf{x}_c|^2}{(T-t_c)^2} + \frac{|\mathbf{x}_c-\mathbf{x}_a|^2}{t_c^2} = 0$$

For r_a and $r_b \gg |\mathbf{x}_c|$

$$\begin{aligned} |\mathbf{x}_c-\mathbf{x}_a| &\approx r_a \\ |\mathbf{x}_b-\mathbf{x}_c| &\approx r_b \end{aligned}$$

Hence

$$t_c = \frac{Tr_a}{r_a+r_b}$$