

Electron spin state s is a unit vector in \mathbb{C}^2 .

$$s = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}, \quad |s| = s^\dagger s = c_1^* c_1 + c_2^* c_2 = 1$$

These are the spin operators.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Expectation of spin operators is a projection of s onto Euclidean space.

$$\langle x \rangle = \langle s | \sigma_x | s \rangle = s^\dagger \sigma_x s, \quad \langle y \rangle = \langle s | \sigma_y | s \rangle = s^\dagger \sigma_y s, \quad \langle z \rangle = \langle s | \sigma_z | s \rangle = s^\dagger \sigma_z s$$

Let

$$\sigma = \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix}$$

Then the spin polarization vector P is

$$P = \begin{pmatrix} \langle x \rangle \\ \langle y \rangle \\ \langle z \rangle \end{pmatrix} = s^\dagger \sigma s$$

In component notation $\sigma = \sigma^{\alpha\beta}{}_\gamma$ and

$$P^\alpha = s_\beta^* \sigma^{\alpha\beta}{}_\gamma s^\gamma$$

In Eigenmath a transpose is needed to swap α and β so that summed-over indices are adjacent.

$$P^\alpha = s_\beta^* \sigma^{\beta\alpha}{}_\gamma s^\gamma$$

Hence the code is

$$P = \text{dot}(\text{conj}(s), \text{transpose}(\text{sigma}), s)$$

Let θ and ϕ be polar and azimuth angles such that

$$P = \begin{pmatrix} \sin \theta \cos \phi \\ \sin \theta \sin \phi \\ \cos \theta \end{pmatrix}$$

The corresponding spin state is

$$s = \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2) \exp(i\phi) \end{pmatrix}$$