

## Dirac from boost

Let  $u_0$  be the spinor for a spin up electron at rest.

$$u_0 = \begin{pmatrix} \sqrt{2m} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Matrix  $A$  boosts  $u_0$  in the  $z$  direction where  $E = \sqrt{p_z^2 + m^2}$ .

$$A = \frac{1}{\sqrt{2m(E+m)}} \begin{pmatrix} E+m & 0 & -p_z & 0 \\ 0 & E+m & 0 & p_z \\ p_z & 0 & E+m & 0 \\ 0 & -p_z & 0 & E+m \end{pmatrix}$$

Hence for boosted spinor  $u$  we have

$$\begin{aligned} u &= Au_0 \\ &= \frac{1}{\sqrt{2m(E+m)}} \begin{pmatrix} E+m & 0 & -p_z & 0 \\ 0 & E+m & 0 & p_z \\ p_z & 0 & E+m & 0 \\ 0 & -p_z & 0 & E+m \end{pmatrix} \begin{pmatrix} \sqrt{2m} \\ 0 \\ 0 \\ 0 \end{pmatrix} \\ &= \frac{1}{\sqrt{E+m}} \begin{pmatrix} E+m \\ 0 \\ p_z \\ 0 \end{pmatrix} \end{aligned}$$

Spinor  $u$  is normalized such that

$$u^\dagger u = 2E$$

We will now derive  $\not{p}$  from  $A$  and the Dirac equation

$$\not{p}u = mu$$

Substitute  $Au_0$  for  $u$  on the right-hand side.

$$\not{p}u = mA u_0$$

By the identity  $\gamma^0 u_0 = u_0$ , substitute  $\gamma^0 u_0$  for  $u_0$ .

$$\not{p}u = mA \gamma^0 u_0$$

Substitute  $A^{-1}u$  for  $u_0$ .

$$\not{p}u = mA \gamma^0 A^{-1}u$$

Cancel  $u$  to obtain

$$\not{p} = mA \gamma^0 A^{-1} \tag{1}$$

To recover the Dirac equation from boost  $A$ , start with the identity

$$\gamma^0 u_0 = u_0$$

Boost both sides of the equation.

$$A\gamma^0 u_0 = Au_0$$

By equation (1) substitute  $(\not{p}/m)A$  for  $A\gamma^0$ .

$$\frac{\not{p}}{m}Au_0 = Au_0$$

Substitute  $u$  for  $Au_0$ .

$$\frac{\not{p}}{m}u = u$$

Multiply both sides by  $m$  to obtain the Dirac equation.

$$\not{p}u = mu \tag{2}$$

Eigenmath script