

# Hydrogen energy four ways

The following formulas for  $E_n$  are equivalent.

This is the traditional formula with elementary charge  $e$ .

$$E_n = -\frac{\mu}{2n^2} \left( \frac{e^2}{4\pi\epsilon_0\hbar} \right)^2 \quad (1)$$

The following formula eliminates  $e$  but still solves the Schrödinger equation directly without any substitutions.

$$E_n = -\frac{\hbar^2}{2n^2\mu a_0^2} \quad (2)$$

This form of  $E_n$  is for dividing by  $\hbar$  to obtain angular frequency  $\omega$ .

$$E_n = -\frac{\alpha\hbar c}{2n^2 a_0} \quad (3)$$

This form of  $E_n$  is irreducible.

$$E_n = -\frac{\alpha^2\mu c^2}{2n^2} \quad (4)$$

Equation (1) reduces to (4) by substituting

$$e^2 = 4\pi\epsilon_0\alpha\hbar c$$

Equations (2) and (3) reduce to (4) by substituting

$$a_0 = \frac{\hbar}{\alpha\mu c}$$