Consider the harmonic oscillator

$$\psi_n(x) = \frac{1}{\sqrt{2^n n!}} \left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{4}} H_n\left(\sqrt{\frac{m\omega}{\hbar}}\, x\right) \exp\left(-\frac{m\omega x^2}{2\hbar}\right)$$

and its propagator

$$K(x_b, t_b, x_a, t_a) = \left(\frac{m\omega}{2\pi i\hbar \sin(\omega T)}\right)^{\frac{1}{2}} \exp\left[\frac{im\omega}{2\hbar\sin(\omega T)}\left(x_a^2\cos(\omega T) - 2x_a x_b + x_b^2\cos(\omega T)\right)\right]$$

where $T = t_b - t_a$.

We should have

$$\psi_n(x_b) = \int_{-\infty}^{\infty} K(x_b, t_b, x_a, t_a)\psi_n(x_a)\, dx_a$$

Try for $n = 1$.

$$\psi_1(x_a) = \sqrt{2}\left(\frac{m^2\omega^3}{\pi\hbar^3}\right)^{\frac{1}{4}} x_a \exp\left(-\frac{m\omega x_a^2}{2\hbar}\right)$$

The path integral is

$$I = \int_{-\infty}^{\infty} K(x_b, t_b, x_a, t_a)\psi_1(x_a)\, dx_a = \frac{1}{2}\left(-\frac{m^9\omega^9}{\pi\hbar^9}\right)^{\frac{1}{4}} \left(\frac{m\omega}{2\hbar}\sin(\omega T) - \frac{im\omega}{2\hbar}\cos(\omega T)\right)^{-\frac{3}{2}}$$
$$\times\, x_b \exp\left(-\frac{m^2\omega^2 x_b^2}{4\hbar^2\left(\frac{m\omega}{2\hbar}\sin^2(\omega T) - \frac{im\omega}{2\hbar}\sin(\omega T)\cos(\omega T)\right)} + \frac{im\omega x_b^2\cos(\omega T)}{2\hbar\sin(\omega T)}\right)$$

Let $T = \pi/(2\omega)$ to cancel time dependency.

$$I = \psi_1(x_b)\exp\left(\frac{i\pi}{4}\right)$$

The phase shift is inconsequential and can be discarded.

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