

The transition rate from state n to m is

$$A_{nm} = \frac{e^2}{3\pi\epsilon_0\hbar c^3} \omega_{nm}^3 |\langle r_{nm} \rangle|^2$$

where

$$\omega_{nm} = \frac{1}{\hbar}(E_n - E_m), \quad E_n = -\frac{\mu}{2n^2} \left(\frac{e^2}{4\pi\epsilon_0\hbar} \right)^2$$

Symbol μ is the reduced electron mass.

The radial density is

$$|\langle r_{nm} \rangle|^2 = |\langle x_{nm} \rangle|^2 + |\langle y_{nm} \rangle|^2 + |\langle z_{nm} \rangle|^2$$

where

$$\begin{aligned} \langle x_{nm} \rangle &= \int \psi_m^* (r \sin \theta \cos \phi) \psi_n dV \\ \langle y_{nm} \rangle &= \int \psi_m^* (r \sin \theta \sin \phi) \psi_n dV \\ \langle z_{nm} \rangle &= \int \psi_m^* (r \cos \theta) \psi_n dV \end{aligned}$$

Let us compute A_{21} for the hydrogen atom. For $n = 2$ there are four possible states.

n	ℓ	m
2	1	1
2	1	-1
2	1	0
2	0	0

The following table shows calculations for every possible transition.

	$(2, 1, 1) \rightarrow (1, 0, 0)$	$(2, 1, -1) \rightarrow (1, 0, 0)$	$(2, 1, 0) \rightarrow (1, 0, 0)$	$(2, 0, 0) \rightarrow (1, 0, 0)$
$\langle x_{nm} \rangle =$	$-\frac{128}{243} a_0$	$\frac{128}{243} a_0$	0	0
$\langle y_{nm} \rangle =$	$-\frac{128}{243} i a_0$	$-\frac{128}{243} i a_0$	0	0
$\langle z_{nm} \rangle =$	0	0	$\frac{128}{243} \sqrt{2} a_0$	0
$ \langle r_{nm} \rangle ^2 =$	$\frac{32768}{59049} a_0^2$	$\frac{32768}{59049} a_0^2$	$\frac{32768}{59049} a_0^2$	0

Note that the transition $(2, 0, 0) \rightarrow (1, 0, 0)$ is not allowed. For the allowed transitions, the result is independent of ℓ and m .

Symbol a_0 is the Bohr radius

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{e^2\mu}$$

We have

$$\omega_{12} = \frac{1}{\hbar}(E_2 - E_1) = \frac{3e^4\mu}{128\pi^2\epsilon_0^2\hbar^3}$$

Hence

$$A_{12} = \frac{e^2}{3\pi\varepsilon_0\hbar c^3} \left(\frac{3e^4\mu}{128\pi^2\varepsilon_0^2\hbar^3} \right)^3 \frac{32768}{59049} \left(\frac{4\pi\varepsilon_0\hbar^2}{e^2\mu} \right)^2 = \frac{e^{10}\mu}{26244\pi^5\varepsilon_0^5\hbar^6c^3} = 6.27 \times 10^{-8} \text{ second}^{-1}$$