

Exercise 4.1. Prove that if \mathbf{U} is unitary, and if $|A\rangle$ and $|B\rangle$ are any two state vectors, then the inner product of $\mathbf{U}|A\rangle$ and $\mathbf{U}|B\rangle$ is the same as the inner product of $|A\rangle$ and $|B\rangle$. One could call this the *conservation of overlaps*. It expresses the fact that the logical relation between states is preserved with time.

If \mathbf{U} is unitary then

$$\mathbf{U}^\dagger \mathbf{U} = I$$

Hence

$$\langle A | \mathbf{U}^\dagger \mathbf{U} | B \rangle = \langle A | I | B \rangle = \langle A | B \rangle$$