

# Translation

Let  $K_1$ ,  $K_2$ , and  $K_3$  be translation operators.

$$K_1 = \frac{1}{\hbar}P_1, \quad K_2 = \frac{1}{\hbar}P_2, \quad K_3 = \frac{1}{\hbar}P_3$$

Let  $U$  be the unitary transformation

$$U = 1 - i\epsilon K_3 - \frac{1}{2}\epsilon^2 K_3^2$$

1. Show that to order  $\epsilon^2$

$$U^{-1}X_1U = X_1$$

$$U^{-1}X_2U = X_2$$

$$U^{-1}X_3U = X_3 + \epsilon$$

2. Show that to order  $\epsilon^2$

$$U^{-1}P_1U = P_1$$

$$U^{-1}P_2U = P_2$$

$$U^{-1}P_3U = P_3$$

3. Show that to order  $\epsilon^2$

$$U^{-1}L_1U = L_1 - \epsilon P_2$$

$$U^{-1}L_2U = L_2 + \epsilon P_1$$

$$U^{-1}L_3U = L_3$$