

Feynman and Hibbs problem 4-9

Show from the fact that  $H$  is hermitian that equation (4.46) holds. Hint: Choose  $f = \phi_2$ ,  $g = \phi_1$  in equation (4.30).

From equation (4.42)

$$\begin{aligned}H\phi_1 &= E_1\phi_1 \\H\phi_2 &= E_2\phi_2\end{aligned}$$

Since  $H$  is hermitian we have

$$\int_{-\infty}^{\infty} (Hg)^* f \, dx = \int_{-\infty}^{\infty} g^* (Hf) \, dx \quad (4.30)$$

Substitute  $\phi_1$  into  $g$  and  $\phi_2$  into  $f$ .

$$\int_{-\infty}^{\infty} (H\phi_1)^* \phi_2 \, dx = \int_{-\infty}^{\infty} \phi_1^* (H\phi_2) \, dx$$

Replace  $H$  with the appropriate eigenvalue. (Eigenvalues of  $H$  are real.)

$$E_1 \int_{-\infty}^{\infty} \phi_1^* \phi_2 \, dx = E_2 \int_{-\infty}^{\infty} \phi_1^* \phi_2 \, dx$$

The integrals are identical hence for  $E_1 \neq E_2$  we must have

$$\int_{-\infty}^{\infty} \phi_1^* \phi_2 \, dx = 0$$