# Stokes's theorem

Stokes's theorem equates a surface integral of the curl of a function with a line integral of the same function. In rectangular coordinates the equivalence is

$$\iint_S (\operatorname{curl}\mathbf{F})\cdot\mathbf{n}\,d\sigma = \oint (P\,dx + Q\,dy + R\,dz)$$

where $\mathbf{F} = (P, Q, R)$. For $S$ parametrized by $x$ and $y$ we have

$$\mathbf{n}\,d\sigma = \left(\frac{\partial S}{\partial x}\times\frac{\partial S}{\partial y}\right)dx\,dy$$

For example, let $\mathbf{F} = (y, z, x)$ and let $S$ be the part of the paraboloid $z = 4 - x^2 - y^2$ that is above the $xy$ plane. The perimeter of the paraboloid is the circle $x^2 + y^2 = 2$. The following script computes both the surface and line integrals. Polar coordinates are used for the line integral so that `defint` can succeed.

```
"Surface integral"
z = 4 - x^2 - y^2
F = (y,z,x)
S = (x,y,z)
z = quote(z) -- clear z for use by curl
f = dot(curl(F),cross(d(S,x),d(S,y)))
x = r cos(theta)
y = r sin(theta)
defint(f r, r, 0, 2, theta, 0, 2 pi)

"Line integral"
x = 2 cos(t)
y = 2 sin(t)
z = 4 - x^2 - y^2
P = y
Q = z
R = x
f = P d(x,t) + Q d(y,t) + R d(z,t)
f = expform(f)
defint(f, t, 0, 2 pi)
```

This is the result when the script runs. Both the surface integral and the line integral yield the same result.

Surface integral
$-4\pi$
Line integral
$-4\pi$