This is the Schroedinger equation for $2d$ hydrogen.

$$-\frac{\hbar^2}{2\mu}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}\right)\psi_n-\frac{k}{r}\psi_n=E_n\psi_n$$

Symbol μ is reduced electron mass and k/r is potential energy with

$$k=\frac{e^2}{4\pi\varepsilon_0}$$

Energy eigenvalues are

$$E_n=-\frac{k^2\mu}{2\hbar^2\left(n+\frac{1}{2}\right)^2}$$

In matrix form we have

$$H=\frac{P^2}{2\mu}-kV$$

where matrix elements are computed as follows.

$$H_{nm}=E_n\,\delta_{nm}$$

$$\left(P^2\right)_{nm}=-\hbar^2\int_0^{2\pi}\int_0^{\infty}\psi_n\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}\right)\psi_m\,r\,dr\,d\phi$$

$$V_{nm}=\int_0^{2\pi}\int_0^{\infty}\psi_n\left(\frac{1}{r}\right)\psi_m\,r\,dr\,d\phi$$

Here are numerical values for 4×4 matrices.

$$H=\begin{pmatrix}-9.68305 & 0 & 0 & 0\\ 0 & -3.48590 & 0 & 0\\ 0 & 0 & -1.77852 & 0\\ 0 & 0 & 0 & -1.07589\end{pmatrix}\times10^{-19}\,\text{joule}$$

$$P^2=\begin{pmatrix}17.6317 & 9.28281 & 5.23021 & 3.49926\\ 9.28281 & 6.34742 & 3.98524 & 2.46475\\ 5.23021 & 3.98524 & 3.23848 & 2.21527\\ 3.49926 & 2.46475 & 2.21527 & 1.95908\end{pmatrix}\times10^{-49}\,\text{kilogram}^2\,\text{meter}^2\,\text{second}^{-2}$$

$$V=\begin{pmatrix}83.9421 & 22.0971 & 12.4502 & 8.32975\\ 22.0971 & 30.2192 & 9.48657 & 5.86716\\ 12.4502 & 9.48657 & 15.4179 & 5.27330\\ 8.32975 & 5.86716 & 5.27330 & 9.32690\end{pmatrix}\times10^{8}\,\text{meter}^{-1}$$