Consider equation (6.39).

$$v(\breve{\mathbf{p}}) = \int \exp\left(\frac{i\breve{\mathbf{p}}\cdot\mathbf{r}}{\hbar}\right) V(\mathbf{r})\, d^3\mathbf{r} \tag{6.39}$$

Convert (6.39) to polar coordinates.

$$v(\breve{\mathbf{p}}) = \int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} \exp\left(\frac{i\breve{p}r\cos\theta}{\hbar}\right) V(r)\, r^2\sin\theta\, dr\, d\theta\, d\phi$$

Integrate over ϕ.

$$v(\breve{\mathbf{p}}) = 2\pi\int_0^{\pi}\int_0^{\infty} \exp\left(\frac{i\breve{p}r\cos\theta}{\hbar}\right) V(r)\, r^2\sin\theta\, dr\, d\theta$$

Transform the integral over θ to an integral over y where $y = \cos\theta$, $dy = -\sin\theta\, d\theta$.

$$v(\breve{\mathbf{p}}) = -2\pi Ze^2\int_{-1}^{1}\int_0^{\infty} \exp\left(\frac{ipry}{\hbar}\right) r\, dr\, dy$$

Solve the integral over y (note r in the integrand cancels).

$$v(\breve{\mathbf{p}}) = -2\pi Ze^2\int_0^{\infty} \frac{\hbar}{ip}\left[\exp\left(\frac{ipr}{\hbar}\right) - \exp\left(-\frac{ipr}{\hbar}\right)\right] dr$$

Solve the integral over r.

$$v(\breve{\mathbf{p}}) = -2\pi Ze^2\frac{\hbar}{ip}\left[\frac{\hbar}{ip}\exp\left(\frac{ipr}{\hbar}\right) + \frac{\hbar}{ip}\exp\left(-\frac{ipr}{\hbar}\right)\right]_0^{\infty}$$

The first exponential is a problem so go back and multiply the integrand by $\exp(-\epsilon r)$.

$$v(\breve{\mathbf{p}}) = -2\pi Ze^2\int_0^{\infty} \frac{\hbar}{ip}\left[\exp\left(\frac{ipr}{\hbar} - \epsilon r\right) - \exp\left(-\frac{ipr}{\hbar} - \epsilon r\right)\right] dr$$

Solve the integral.

$$v(\breve{\mathbf{p}}) = -2\pi Ze^2\frac{\hbar}{ip}\left[\frac{1}{ip/\hbar - \epsilon}\exp\left(\frac{ipr}{\hbar} - \epsilon r\right) + \frac{1}{ip/\hbar + \epsilon}\exp\left(-\frac{ipr}{\hbar} - \epsilon r\right)\right]_0^{\infty}$$

Evaluate the limits.

$$v(\breve{\mathbf{p}}) = -2\pi Ze^2\frac{\hbar}{ip}\left(-\frac{1}{ip/\hbar - \epsilon} - \frac{1}{ip/\hbar + \epsilon}\right) = -\frac{4\pi Ze^2}{(p/\hbar)^2 + \epsilon^2}$$

Set $\epsilon = 0$ to obtain

$$v(\breve{\mathbf{p}}) = -\frac{4\pi Ze^2\hbar^2}{p^2}$$

By equation (6.44)

$$\frac{d\sigma}{d\Omega} = \left(\frac{m}{2\pi\hbar^2}\right)^2 |v(\breve{\mathbf{p}})|^2 = \frac{4m^2Z^2e^4}{\breve{p}^4}$$