2-4. Classically, the momentum is defined as

$$p = \frac{\partial L}{\partial \dot{x}} \tag{2.10}$$

Show that the momentum at a final point is

$$\left(\frac{\partial L}{\partial \dot{x}}\right)_{x=x_b} = +\frac{\partial S_{cl}}{\partial x_b} \tag{2.11}$$

while the momentum at an initial point is

$$\left(\frac{\partial L}{\partial \dot{x}}\right)_{x=x_a} = -\frac{\partial S_{cl}}{\partial x_a} \tag{2.11}$$

Hint: Consider the effect on equation (2.6) of a change in the end points.

$$\delta S = \left(\delta x(t)\frac{\partial L}{\partial \dot{x}}\right)_{t_a}^{t_b} - \int_{t_a}^{t_b} \delta x(t)\left(\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x}\right) dt \tag{2.6}$$

Consider equation (2.7) for the classical path.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0 \tag{2.7}$$

Substitute (2.7) into (2.6) to obtain

$$\delta S_{cl} = \left(\delta x(t)\frac{\partial L}{\partial \dot{x}}\right)_{t_a}^{t_b} = \delta x_b\left(\frac{\partial L}{\partial \dot{x}}\right)_{x=x_b} - \delta x_a\left(\frac{\partial L}{\partial \dot{x}}\right)_{x=x_a} \tag{1}$$

For the endpoints x_a and x_b we have $\delta S_{cl} = 0$ by equation (2.3).

$$\delta x(t_a) = \delta x(t_b) = 0 \tag{2.3}$$

However, consider a variation of the endpoint x_b. Then by equation (1)

$$\delta S_{cl} = \delta x_b\left(\frac{\partial L}{\partial \dot{x}}\right)_{x=x_b}$$

Hence

$$\lim \frac{\delta S_{cl}}{\delta x_b} = \frac{\partial S_{cl}}{\partial x_b} = \left(\frac{\partial L}{\partial \dot{x}}\right)_{x=x_b}$$

Likewise, a variation of x_a yields

$$\lim \frac{\delta S_{cl}}{\delta x_a} = \frac{\partial S_{cl}}{\partial x_a} = -\left(\frac{\partial L}{\partial \dot{x}}\right)_{x=x_a}$$