Equation 7.58.

$$E_r^1 = -\frac{(E_n)^2}{2mc^2}\left[\frac{4n}{\ell+1/2} - 3\right] \tag{7.58}$$

Equation 7.67.

$$E_{\rm so}^1 = \frac{(E_n)^2}{mc^2}\left\{\frac{n[j(j+1)-\ell(\ell+1)-3/4]}{\ell(\ell+1/2)(\ell+1)}\right\} \tag{7.67}$$

The text (p. 303) says add the relativistic correction and the spin-orbit coupling.

$$E_{\rm fs}^1 = -\frac{(E_n)^2}{2mc^2}\left[\frac{4n}{\ell+1/2} - 3\right] + \frac{(E_n)^2}{mc^2}\left\{\frac{n[j(j+1)-\ell(\ell+1)-3/4]}{\ell(\ell+1/2)(\ell+1)}\right\}$$

Rewrite as

$$E_{\rm fs}^1 = \frac{(E_n)^2}{2mc^2}\left(\frac{2n[j(j+1)-\ell(\ell+1)-3/4]}{\ell(\ell+1/2)(\ell+1)} - \frac{4n}{\ell+1/2} + 3\right)$$

To be equivalent to (7.68) requires

$$\frac{2n[j(j+1)-\ell(\ell+1)-3/4]}{\ell(\ell+1/2)(\ell+1)} - \frac{4n}{\ell+1/2} + 3 = 3 - \frac{4n}{j+1/2}$$

Rewrite as

$$\frac{2n[j(j+1)-\ell(\ell+1)-3/4]}{\ell(\ell+1/2)(\ell+1)} - \frac{4n}{\ell+1/2} + \frac{4n}{j+1/2} = 0$$

Divide through by n.

$$\frac{2[j(j+1)-\ell(\ell+1)-3/4]}{\ell(\ell+1/2)(\ell+1)} - \frac{4}{\ell+1/2} + \frac{4}{j+1/2} = 0$$