

Exercise 6.4. Use the matrix forms of  $\sigma_z$ ,  $\sigma_x$ , and  $\sigma_y$  and the column vectors for  $|u\rangle$  and  $|d\rangle$  to verify Eqs. 6.6. Then, use Eqs. 6.6 and 6.7 to write the equations that were left out of Eqs. 6.8. Use the appendix to check your answers.

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$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad (3.20)$$

$$|u\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |d\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Hence

$$\begin{aligned} \sigma_z|u\rangle &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |u\rangle \\ \sigma_z|d\rangle &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} = -|d\rangle \\ \sigma_x|u\rangle &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |d\rangle \\ \sigma_x|d\rangle &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |u\rangle \\ \sigma_y|u\rangle &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i \end{pmatrix} = i|d\rangle \\ \sigma_y|d\rangle &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -i \\ 0 \end{pmatrix} = -i|u\rangle \end{aligned}$$

For  $\sigma_z$  of product states we have

$$\begin{aligned} \sigma_z|uu\rangle &= \sigma_z|u\rangle \otimes |u\rangle = |u\rangle \otimes |u\rangle = |uu\rangle \\ \sigma_z|ud\rangle &= \sigma_z|u\rangle \otimes |d\rangle = |u\rangle \otimes |d\rangle = |ud\rangle \\ \sigma_z|du\rangle &= \sigma_z|d\rangle \otimes |u\rangle = -|d\rangle \otimes |u\rangle = -|du\rangle \\ \sigma_z|dd\rangle &= \sigma_z|d\rangle \otimes |d\rangle = -|d\rangle \otimes |d\rangle = -|dd\rangle \end{aligned}$$

For  $\sigma_x$  of product states we have

$$\begin{aligned}\sigma_x|uu\rangle &= \sigma_x|u\rangle \otimes |u\rangle = |d\rangle \otimes |u\rangle = |du\rangle \\ \sigma_x|ud\rangle &= \sigma_x|u\rangle \otimes |d\rangle = |d\rangle \otimes |d\rangle = |dd\rangle \\ \sigma_x|du\rangle &= \sigma_x|d\rangle \otimes |u\rangle = |u\rangle \otimes |u\rangle = |uu\rangle \\ \sigma_x|dd\rangle &= \sigma_x|d\rangle \otimes |d\rangle = |u\rangle \otimes |d\rangle = |ud\rangle\end{aligned}$$

For  $\sigma_y$  of product states we have

$$\begin{aligned}\sigma_y|uu\rangle &= \sigma_y|u\rangle \otimes |u\rangle = i|d\rangle \otimes |u\rangle = i|du\rangle \\ \sigma_y|ud\rangle &= \sigma_y|u\rangle \otimes |d\rangle = i|d\rangle \otimes |d\rangle = i|dd\rangle \\ \sigma_y|du\rangle &= \sigma_y|d\rangle \otimes |u\rangle = -i|u\rangle \otimes |u\rangle = -i|uu\rangle \\ \sigma_y|dd\rangle &= \sigma_y|d\rangle \otimes |d\rangle = -i|u\rangle \otimes |d\rangle = -i|ud\rangle\end{aligned}$$

For  $\tau_z$  of product states we have

$$\begin{aligned}\tau_z|uu\rangle &= |u\rangle \otimes \tau_z|u\rangle = |u\rangle \otimes |u\rangle = |uu\rangle \\ \tau_z|ud\rangle &= |u\rangle \otimes \tau_z|d\rangle = |u\rangle \otimes -|d\rangle = -|ud\rangle \\ \tau_z|du\rangle &= |d\rangle \otimes \tau_z|u\rangle = |d\rangle \otimes |u\rangle = |du\rangle \\ \tau_z|dd\rangle &= |d\rangle \otimes \tau_z|d\rangle = |d\rangle \otimes -|d\rangle = -|dd\rangle\end{aligned}$$

For  $\tau_x$  of product states we have

$$\begin{aligned}\tau_x|uu\rangle &= |u\rangle \otimes \tau_x|u\rangle = |u\rangle \otimes |d\rangle = |ud\rangle \\ \tau_x|ud\rangle &= |u\rangle \otimes \tau_x|d\rangle = |u\rangle \otimes |u\rangle = |uu\rangle \\ \tau_x|du\rangle &= |d\rangle \otimes \tau_x|u\rangle = |d\rangle \otimes |d\rangle = |dd\rangle \\ \tau_x|dd\rangle &= |d\rangle \otimes \tau_x|d\rangle = |d\rangle \otimes |u\rangle = |du\rangle\end{aligned}$$

For  $\tau_y$  of product states we have

$$\begin{aligned}\tau_y|uu\rangle &= |u\rangle \otimes \tau_y|u\rangle = |u\rangle \otimes i|d\rangle = i|ud\rangle \\ \tau_y|ud\rangle &= |u\rangle \otimes \tau_y|d\rangle = |u\rangle \otimes -i|u\rangle = -i|uu\rangle \\ \tau_y|du\rangle &= |d\rangle \otimes \tau_y|u\rangle = |d\rangle \otimes i|d\rangle = i|dd\rangle \\ \tau_y|dd\rangle &= |d\rangle \otimes \tau_y|d\rangle = |d\rangle \otimes -i|u\rangle = -i|du\rangle\end{aligned}$$