2-1. This is the Lagrangian for a free particle.

$$L = \frac{m}{2}\dot{x}^2$$

Show that for a free particle the action S_{cl} is

$$S_{cl} = \frac{m}{2}\frac{(x_b - x_a)^2}{t_b - t_a}$$

For the L given above we have

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = m\ddot{x}$$

and

$$\frac{\partial L}{\partial x} = 0$$

By equation (2.7) which is

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x}$$

we have

$$\ddot{x} = 0$$

Hence velocity $\dot{x}$ is constant and equals distance divided by time.

$$\dot{x} = \frac{x_b - x_a}{t_b - t_a}$$

By equation (2.1) which is

$$S = \int_{t_a}^{t_b} L\,dt$$

we have for classical action S_{cl}

$$S_{cl} = \int_{t_a}^{t_b} \frac{m}{2}\dot{x}^2\,dt = \int_{t_a}^{t_b} \frac{m}{2}\left(\frac{x_b - x_a}{t_b - t_a}\right)^2 dt$$

Solve the integral.

$$S_{cl} = \frac{m}{2}\left(\frac{x_b - x_a}{t_b - t_a}\right)^2 t\,\Bigg|_{t_a}^{t_b} = \frac{m}{2}\frac{(x_b - x_a)^2}{t_b - t_a}$$