Let

$$f = \frac{1}{\sqrt{1 + \left(\frac{\alpha}{A + \sqrt{B^2 - \alpha^2}}\right)^2}}$$

Expand f as a Taylor series.

$$f = 1 - \frac{\alpha^2}{2(A+B)^2} - \frac{\alpha^4}{2B(A+B)^3} + \frac{3\alpha^4}{8(A+B)^4} + \mathcal{O}(\alpha^6)$$

Substitute $A = n - (j + 1/2)$ and $B = j + 1/2$ to obtain $A + B = n$ and

$$f = 1 - \frac{\alpha^2}{2n^2} - \frac{\alpha^4}{2n^3(j+1/2)} + \frac{3\alpha^4}{8n^4} \tag{1}$$

Hence

$$E_{nj} = mc^2(f-1) = mc^2\left(-\frac{\alpha^2}{2n^2} - \frac{\alpha^4}{2n^3(j+1/2)} + \frac{3\alpha^4}{8n^4}\right)$$

Factor out $-\alpha^2/(2n^2)$.

$$E_{nj} = -\frac{mc^2\alpha^2}{2n^2}\left(1 + \frac{\alpha^2}{n(j+1/2)} - \frac{3\alpha^2}{4n^2}\right) \tag{2}$$

Rewrite as

$$E_{nj} = -\frac{mc^2\alpha^2}{2n^2}\left[1 + \frac{\alpha^2}{n^2}\left(\frac{n}{j+1/2} - \frac{3}{4}\right)\right]$$