Addition of angular momentum

Let $\mathbf{J}$ be the sum of orbital angular momentum $\mathbf{L}$ and spin angular momentum $\mathbf{S}$.

$$\mathbf{J} = \mathbf{L} + \mathbf{S}$$

Recall that

$$L_x \psi = -i\hbar \left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y} \right) \psi \qquad\qquad S_x \chi = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \chi$$

$$L_y \psi = -i\hbar \left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z} \right) \psi \qquad\qquad S_y \chi = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \chi$$

$$L_z \psi = -i\hbar \left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x} \right) \psi \qquad\qquad S_z \chi = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \chi$$

Let ϕ be the product of wave function ψ and electron spinor χ.

$$\phi = \psi\chi$$

Then

$$\mathbf{J}\phi = \mathbf{L}\phi + \mathbf{S}\phi$$

Let J^2 be the magnitude-squared of total angular momentum.

$$J^2 = \mathbf{J} \cdot \mathbf{J} = J_x^2 + J_y^2 + J_z^2$$

Operator J^2 can be decomposed as

$$J^2 = (\mathbf{L} + \mathbf{S}) \cdot (\mathbf{L} + \mathbf{S}) = L^2 + S^2 + 2\mathbf{L} \cdot \mathbf{S}$$

In Eigenmath code

$$\mathbf{L} \cdot \mathbf{S}\phi = \texttt{contract(L(S(phi)))} \tag{1}$$

The commutation relations for J^2 are

$$[J^2, L^2] = 0$$
$$[J^2, S^2] = 0$$

$$[J^2, J_x] = 0$$
$$[J^2, J_y] = 0$$
$$[J^2, J_z] = 0$$

$$[J^2, L_x] \neq 0$$
$$[J^2, L_y] \neq 0$$
$$[J^2, L_z] \neq 0$$

$$[J^2, S_x] \neq 0$$
$$[J^2, S_y] \neq 0$$
$$[J^2, S_z] \neq 0$$