Surface area

Let S be a surface parameterized by x and y. That is, let $S = (x, y, z)$ where $z = f(x, y)$. The tangent lines at a point on S form a tiny parallelogram. The area a of the parallelogram is given by the magnitude of the cross product.

$$a = \left| \frac{\partial S}{\partial x} \times \frac{\partial S}{\partial y} \right|$$

By summing over all the parallelograms we obtain the total surface area A. Hence

$$A = \int\!\!\int dA = \int\!\!\int a \, dx \, dy$$

The following example computes the surface area of a unit disk parallel to the xy plane.

```
z = 2
S = (x,y,z)
a = abs(cross(d(S,x),d(S,y)))
defint(a,y,-sqrt(1 - x^2),sqrt(1 - x^2),x,-1,1)
```

π

The result is π, the area of a unit circle, which is what we expect. The following example computes the surface area of $z = x^2 + 2y$ over a unit square.

```
z = x^2 + 2y
S = (x,y,z)
a = abs(cross(d(S,x),d(S,y)))
defint(a,x,0,1,y,0,1)
```

$\frac{5}{8} \log(5) + \frac{3}{2}$

The following exercise is from *Multivariable Mathematics* by Williamson and Trotter, p. 598. Find the area of the spiral ramp defined by

$$S = \begin{pmatrix} u \cos v \\ u \sin v \\ v \end{pmatrix}, \quad 0 \le u \le 1, \quad 0 \le v \le 3\pi$$

```
x = u cos(v)
y = u sin(v)
z = v
S = (x,y,z)
a = circexp(abs(cross(d(S,u),d(S,v))))
defint(a,u,0,1,v,0,3pi)
```

$$\frac{3\pi}{2^{1/2}} + \frac{3}{2}\pi \log\left(2^{1/2} + 1\right)$$

```
float
```

10.8177