(a) From equations (11.14) and (11.15)

$$\begin{aligned}\dot{c}_a &= -\frac{i}{\hbar}\left(c_a H'_{aa} + c_b H'_{ab} e^{-i\omega_0 t}\right)\\ \dot{c}_b &= -\frac{i}{\hbar}\left(c_b H'_{bb} + c_a H'_{ba} e^{i\omega_0 t}\right)\end{aligned} \tag{1}$$

Zeroth order:

$$c_a(t) = 1, \quad c_b(t) = 0$$

First order:

$$\begin{aligned}\dot{c}_a^{(1)} &= -\frac{i}{\hbar}\left[c_a H'_{aa} + c_b H'_{ab} e^{-i\omega_0 t}\right]_{c_a=1, c_b=0} &&= -\frac{i}{\hbar} H'_{aa}\\ \dot{c}_b^{(1)} &= -\frac{i}{\hbar}\left[c_b H'_{bb} + c_a H'_{ba} e^{i\omega_0 t}\right]_{c_a=1, c_b=0} &&= -\frac{i}{\hbar} H'_{ba} e^{i\omega_0 t}\end{aligned}$$

Hence

$$\begin{aligned}c_a^{(1)} &= 1 - \frac{i}{\hbar}\int_0^t H'_{aa}(t')\,dt'\\ c_b^{(1)} &= 0 - \frac{i}{\hbar}\int_0^t H'_{ba}(t') e^{i\omega_0 t'}\,dt'\end{aligned}$$

Hence to first order (discard $|H'_{aa}|^2$ and $|H'_{ba}|^2$)

$$\begin{aligned}\left|c_a^{(1)}\right|^2 &= 1 + \left[-\frac{i}{\hbar}\int_0^t H'_{aa}(t')\,dt'\right]\left[-\frac{i}{\hbar}\int_0^t H'_{aa}(t')\,dt'\right]^* &&= 1\\ \left|c_b^{(1)}\right|^2 &= \left[-\frac{i}{\hbar}\int_0^t H'_{ba}(t') e^{i\omega_0 t'}\,dt'\right]\left[-\frac{i}{\hbar}\int_0^t H'_{ba}(t') e^{i\omega_0 t'}\,dt'\right]^* &&= 0\end{aligned}$$

(b) Let

$$A = \frac{i}{\hbar}\int_0^t H'_{aa}(t')\,dt', \quad B = \frac{i}{\hbar}\int_0^t H'_{bb}(t')\,dt'$$

so that

$$d_a = e^A c_a, \quad d_b = e^B c_b \tag{2}$$

For time derivatives we have

$$\begin{aligned}\dot{d}_a &= \frac{de^A}{dt} c_a + e^A \dot{c}_a &&= \frac{i}{\hbar} H'_{aa} e^A c_a + e^A \dot{c}_a\\ \dot{d}_b &= \frac{de^B}{dt} c_b + e^B \dot{c}_b &&= \frac{i}{\hbar} H'_{bb} e^B c_b + e^B \dot{c}_b\end{aligned}$$

By equation (1) substitute for $\dot{c}_a$ and $\dot{c}_b$.

$$\begin{aligned}\dot{d}_a &= \frac{i}{\hbar} H'_{aa} e^A c_a - \frac{i}{\hbar} e^A \left(c_a H'_{aa} + c_b H'_{ab} e^{-i\omega_0 t}\right)\\ \dot{d}_b &= \frac{i}{\hbar} H'_{bb} e^B c_b - \frac{i}{\hbar} e^B \left(c_b H'_{bb} + c_a H'_{ba} e^{i\omega_0 t}\right)\end{aligned}$$

Expand right-hand sides.

$$\dot{d}_a = \frac{i}{\hbar} H'_{aa} e^{A} c_a - \frac{i}{\hbar} H'_{aa} e^{A} c_a - \frac{i}{\hbar} e^{A} c_b H'_{ab} e^{-i\omega_0 t}$$
$$\dot{d}_b = \frac{i}{\hbar} H'_{bb} e^{B} c_b - \frac{i}{\hbar} H'_{bb} e^{B} c_b - \frac{i}{\hbar} e^{B} c_a H'_{ba} e^{i\omega_0 t}$$

Cancel terms.

$$\dot{d}_a = -\frac{i}{\hbar} e^{A} c_b H'_{ab} e^{-i\omega_0 t}$$
$$\dot{d}_b = -\frac{i}{\hbar} e^{B} c_a H'_{ba} e^{i\omega_0 t}$$

By equation (2) we have for c_a and c_b

$$c_a = e^{-A} d_a, \quad c_b = e^{-B} d_b$$

Hence by substitution for c_a and c_b

$$\dot{d}_a = -\frac{i}{\hbar} e^{A} e^{-B} H'_{ab} e^{-i\omega_0 t} d_b$$
$$\dot{d}_b = -\frac{i}{\hbar} e^{B} e^{-A} H'_{ba} e^{i\omega_0 t} d_a$$

Let

$$e^{i\phi} = e^{A} e^{-B} = e^{A-B} = \exp\left(\frac{i}{\hbar} \int_0^t \left[H'_{aa}(t') - H'_{bb}(t')\right] dt'\right)$$

Then

$$\dot{d}_a = -\frac{i}{\hbar} e^{i\phi} H'_{ab} e^{-i\omega_0 t} d_b$$
$$\dot{d}_b = -\frac{i}{\hbar} e^{-i\phi} H'_{ba} e^{i\omega_0 t} d_a$$