6-15. Recall that in problem 5-4 we defined a particular integral as the transition amplitude to go from state $\psi(x)$ to state $\chi(x)$. Show that the function λ_{mn} satisfies this definition when the initial state is the eigenfunction $\phi_n(x)$ and the final state is the eigenfunction $\phi_m(x)$.

From problem 5-4 the transition amplitude is

$$\langle m|n\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi_m^*(x_b) K_V(b,a) \phi_n(x_a)\, dx_a\, dx_b \tag{1}$$

Consider equation (6.68).

$$K_V(b,a) = \sum_m \sum_n \lambda_{mn}(t_b,t_a)\phi_m(x_b)\phi_n^*(x_a) \tag{6.68}$$

Substitute (6.68) into (1).

$$\begin{aligned}\langle m|n\rangle = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} &\phi_m^*(x_b) \\ &\times \left(\sum_{m'}\sum_{n'} \lambda_{m'n'}(t_b,t_a)\phi_{m'}(x_b)\phi_{n'}^*(x_a)\right)\phi_n(x_a)\, dx_a\, dx_b\end{aligned}$$

By distributive law

$$\langle m|n\rangle = \sum_{m'}\sum_{n'} \lambda_{m'n'}(t_b,t_a)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi_m^*(x_b)\phi_{m'}(x_b)\phi_{n'}^*(x_a)\phi_n(x_a)\, dx_a\, dx_b$$

By orthogonality of eigenfunctions

$$\langle m|n\rangle = \lambda_{mn}(t_b,t_a)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \phi_m^*(x_b)\phi_m(x_b)\phi_n^*(x_a)\phi_n(x_a)\, dx_a\, dx_b$$

By normalization of eigenfunctions

$$\langle m|n\rangle = \lambda_{mn}(t_b,t_a)$$