Matrix elements for position X and momentum P are the following transition amplitudes.

$$X_{kj} = \int_{-\infty}^{\infty} \psi_k \, x \, \psi_j \, dx$$

$$P_{kj} = \int_{-\infty}^{\infty} \psi_k \left(-i\hbar \frac{d}{dx} \right) \psi_j \, dx$$

For 4×4 matrices we have

$$X = \left(\frac{\hbar}{2m\omega} \right)^{1/2} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & \sqrt{2} & 0 \\ 0 & \sqrt{2} & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}$$

$$P = i \left(\frac{\hbar m\omega}{2} \right)^{1/2} \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & -\sqrt{2} & 0 \\ 0 & \sqrt{2} & 0 & -\sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}$$

$$H = \frac{P^2}{2m} + \frac{1}{2} m\omega^2 X^2 = \begin{pmatrix} \frac{1}{2}\hbar\omega & 0 & 0 & 0 \\ 0 & \frac{3}{2}\hbar\omega & 0 & 0 \\ 0 & 0 & \frac{5}{2}\hbar\omega & 0 \\ 0 & 0 & 0 & \frac{7}{2}\hbar\omega \end{pmatrix}$$

${H^4}_4$ cannot be computed using 4×4 matrices. The value shown is the corrected eigenvalue.