

Exercise 8.1. Prove that  $\mathbf{X}$  and  $\mathbf{D}$  are linear operators.

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We are given

$$\mathbf{X}\psi(x) = x\psi(x) \tag{8.5}$$

$$\mathbf{D}\psi(x) = \frac{d}{dx}\psi(x) \tag{8.6}$$

For operator  $\mathbf{X}$  we have

$$\mathbf{X}(f + g) = x(f + g) = xf + xg = \mathbf{X}f + \mathbf{X}g$$

and

$$\mathbf{X}(af) = x(af) = axf = a\mathbf{X}f$$

For operator  $\mathbf{D}$  we have

$$\mathbf{D}(f + g) = \frac{d(f + g)}{dx} = \frac{df}{dx} + \frac{dg}{dx} = \mathbf{D}f + \mathbf{D}g$$

and

$$\mathbf{D}(af) = \frac{d(af)}{dx} = a\frac{df}{dx} = a\mathbf{D}f$$

Therefore  $\mathbf{X}$  and  $\mathbf{D}$  are linear operators.