(a)

$$\psi_{100}(r,\theta,\phi) = \frac{1}{\sqrt{\pi a_0^3}} \exp\left(-\frac{r}{a_0}\right)$$

For $\langle r \rangle$ we have

$$\begin{aligned}
\langle r \rangle &= \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} r|\psi_{100}|^2 r^2 \sin\theta \, dr \, d\theta \, d\phi \\
&= \frac{1}{\pi a_0^3} \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} r^3 \exp\left(-\frac{2r}{a_0}\right) \sin\theta \, dr \, d\theta \, d\phi
\end{aligned}$$

Integrate over ϕ (multiply by 2π).

$$\langle r \rangle = \frac{2}{a_0^3} \int_0^{\pi} \int_0^{\infty} r^3 \exp\left(-\frac{2r}{a_0}\right) \sin\theta \, dr \, d\theta$$

Transform the integral over θ to an integral over y where $y = \cos\theta$ and $dy = -\sin\theta \, d\theta$. The minus sign in dy is canceled by interchanging integration limits $\cos 0 = 1$ and $\cos\pi = -1$.

$$\langle r \rangle = \frac{2}{a_0^3} \int_{-1}^{1} \int_0^{\infty} r^3 \exp\left(-\frac{2r}{a_0}\right) \, dr \, dy$$

Integrate over y (multiply by 2).

$$\langle r \rangle = \frac{4}{a_0^3} \int_0^{\infty} r^3 \exp\left(-\frac{2r}{a_0}\right) \, dr$$

Solve the integral over r.

$$\langle r \rangle = \tfrac{3}{2} a_0 \tag{1}$$

For $\langle r^2 \rangle$ we have

$$\langle r^2 \rangle = \frac{4}{a_0^3} \int_0^{\infty} r^4 \exp\left(-\frac{2r}{a_0}\right) \, dr = 3a_0^2 \tag{2}$$

(b) By symmetry we have $\langle x \rangle = 0$ and

$$\langle r^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle + \langle z^2 \rangle = 3\langle x^2 \rangle$$

Hence

$$\langle x^2 \rangle = \tfrac{1}{3} \langle r^2 \rangle = a_0^2$$

(c)

$$\psi_{211} = -\frac{r\sin\theta}{8\sqrt{\pi a_0^5}} \exp\left(-\frac{r}{2a_0} + i\phi\right)$$

For $\langle x^2 \rangle$ we have

$$\begin{aligned}
\langle x^2 \rangle &= \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} (r\sin\theta\cos\phi)^2 |\psi_{211}|^2 r^2 \sin\theta \, dr \, d\theta \, d\phi \\
&= \frac{1}{64\pi a_0^5} \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} r^6 \exp\left(-\frac{r}{a_0}\right) \sin^5\theta \cos^2\phi \, dr \, d\theta \, d\phi
\end{aligned}$$

Integrate over ϕ (multiply by π).

$$\langle x^2 \rangle = \frac{1}{64 a_0^5} \int_0^{\pi} \int_0^{\infty} r^6 \exp\left(-\frac{r}{a_0}\right) \sin^3\theta \, dr \, d\theta$$

Integrate over θ (multiply by $\frac{16}{15}$).

$$\langle x^2 \rangle = \frac{1}{60 a_0^5} \int_0^{\infty} r^6 \exp\left(-\frac{r}{a_0}\right) \, dr$$

Integrate over r.

$$\langle x^2 \rangle = 12 a_0^2$$