Feynman and Hibbs problem 2-2

This is the Lagrangian for a harmonic oscillator.

$$L = \frac{m}{2}(\dot{x}^2 - \omega^2 x^2)$$

Let $T = t_b - t_a$. Show that the classical action is

$$S_{cl} = \frac{m\omega}{2\sin\omega T}\left((x_b^2 + x_a^2)\cos(\omega T) - 2x_b x_a\right)$$

This is the equation of motion.

$$x(t) = x_a + \frac{x_b - x_a}{2}\sin(\omega t)$$

From equation 2.1

$$S_{cl} = \frac{m}{2}\int_{t_a}^{t_b}(\dot{x}^2 - \omega^2 x^2)\,dt$$

Rewrite as two integrals.

$$S_{cl} = \frac{m}{2}\int_{t_a}^{t_b}\dot{x}^2\,dt - \frac{m}{2}\int_{t_a}^{t_b}\omega^2 x^2\,dt$$