

Bohr model

By an argument that is no longer relevant the Bohr model has for hydrogen energy levels

$$E_n = -\frac{\alpha^2 mc^2}{2n^2}$$

By the kinetic energy relation

$$v^2 = -\frac{2E_n}{m}$$

we have for velocity v

$$v = \frac{\alpha c}{n}$$

The Bohr model quantizes orbital angular momentum as

$$mvr_n = n\hbar$$

Hence

$$r_n = \frac{n\hbar}{mv} = \frac{n^2\hbar}{\alpha mc}$$

For $n = 1$ and $m = m_e$ we have

$$\begin{aligned} E_1 &= -13.6057 \text{ eV} \\ r_1 &= 5.29177 \times 10^{-11} \text{ meter} \end{aligned}$$

For reduced electron mass $m = \mu$

$$\begin{aligned} E_1 &= -13.5983 \text{ eV} \\ r_1 &= 5.29465 \times 10^{-11} \text{ meter} \end{aligned}$$