Consider the following eigenstates of a hypothetical quantum system.[1]

$$
\begin{aligned}
|00\rangle &= (1,0,0,0) && \text{no fermions} \\
|10\rangle &= (0,1,0,0) && \text{one fermion in state 1} \\
|01\rangle &= (0,0,1,0) && \text{one fermion in state 2} \\
|11\rangle &= (0,0,0,1) && \text{two fermions, one in state 1, one in state 2}
\end{aligned}
$$

Creation and annihilation operators are formed from outer products of state vectors. Sign changes make the operators antisymmetric.

$$
\begin{aligned}
\hat{b}_1^\dagger &= |10\rangle\langle 00| - |11\rangle\langle 01| && \text{Create one fermion in state 1} \\
\hat{b}_1 &= |00\rangle\langle 10| - |01\rangle\langle 11| && \text{Annihilate one fermion in state 1} \\
\hat{b}_2^\dagger &= |01\rangle\langle 00| + |11\rangle\langle 10| && \text{Create one fermion in state 2} \\
\hat{b}_2 &= |00\rangle\langle 01| + |10\rangle\langle 11| && \text{Annihilate one fermion in state 2}
\end{aligned}
$$

The operators in matrix form.

$$
\hat{b}_1^\dagger = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{pmatrix} \quad
\hat{b}_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad
\hat{b}_2^\dagger = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad
\hat{b}_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
$$

Verify anticommutation relations of the operators.

$$\hat{b}_j\hat{b}_k + \hat{b}_k\hat{b}_j = 0$$

$$\hat{b}_j^\dagger\hat{b}_k^\dagger + \hat{b}_k^\dagger\hat{b}_j^\dagger = 0$$

$$\hat{b}_j\hat{b}_k^\dagger + \hat{b}_k^\dagger\hat{b}_j = \delta_{jk}$$

[1] Adapted from problem 16.1.1 of “Quantum Mechanics for Scientists and Engineers.” `https://ee.stanford.edu/~dabm/QMbook.html`