

Spin operator table

From “Quantum Mechanics” by Susskind and Friedman, p. 350.

Verify the following spin operator table.

	$ uu\rangle$	$ ud\rangle$	$ du\rangle$	$ dd\rangle$
σ_z	$ uu\rangle$	$ ud\rangle$	$- du\rangle$	$- dd\rangle$
σ_x	$ du\rangle$	$ dd\rangle$	$ uu\rangle$	$ ud\rangle$
σ_y	$i du\rangle$	$i dd\rangle$	$-i uu\rangle$	$-i ud\rangle$
τ_z	$ uu\rangle$	$- ud\rangle$	$ du\rangle$	$- dd\rangle$
τ_x	$ ud\rangle$	$ uu\rangle$	$ dd\rangle$	$ du\rangle$
τ_y	$i ud\rangle$	$-i uu\rangle$	$i dd\rangle$	$-i du\rangle$

Table elements are final states. For example,

$$\sigma_z|dd\rangle = -|dd\rangle$$

For single spins we have

$$|u\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |d\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

For a system of two spins we use the Kronecker product of single spins.

$$\begin{aligned} |uu\rangle &= |u\rangle \otimes |u\rangle \\ |ud\rangle &= |u\rangle \otimes |d\rangle \\ |du\rangle &= |d\rangle \otimes |u\rangle \\ |dd\rangle &= |d\rangle \otimes |d\rangle \end{aligned}$$

Hence

$$|uu\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad |ud\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad |du\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad |dd\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

The spin operators for single spins are

$$\sigma_z = \tau_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_x = \tau_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \tau_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

For a system of two spins, the σ operators operate on the first spin and the τ operators operate on the second spin.

$$\begin{aligned} \sigma_z &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \\ \tau_z &= I \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \end{aligned}$$