(7.1) For the Lagrangian $\mathcal{L}$ given by

$$\mathcal{L} = \frac{1}{2}\partial^\mu \partial_\mu \phi - \frac{1}{2}m^2\phi^2 - \sum_{n=1}^{\infty} \lambda_n \phi^{2n+2} \tag{7.20}$$

show that the equation of motion is given by

$$(\partial^2 + m^2)\phi + \sum_{n=1}^{\infty} \lambda_n (2n+2)\phi^{2n+1} = 0 \tag{7.21}$$

For the Lagrangian $\mathcal{L}$ given in (7.20) we have

$$\frac{\partial \mathcal{L}}{\partial \phi} = -m^2\phi - \sum_{n=1}^{\infty} \lambda_n (2n+2)\phi^{2n+1} \tag{1}$$

and

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = \frac{\partial}{\partial(\partial_\mu \phi)} \left(\frac{1}{2}\partial^2 \phi \right) = \partial^\mu \phi \tag{2}$$

Consider the following Euler-Lagrange equation.

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \tag{3}$$

Substitute (1) and (2) into (3) to obtain

$$\partial_\mu \partial^\mu \phi + m^2\phi + \sum_{n=1}^{\infty} \lambda_n (2n+2)\phi^{2n+1} = 0$$

which is equivalent to (7.21).

Note: Recall that

$$\partial^\mu \partial_\mu \phi = \partial_\mu \partial^\mu \phi = (\partial_0^2 + \partial_1^2 + \partial_2^2 + \partial_3^2)\phi$$