$$r_1 = \sqrt{x_1^2 + y_1^2 + z_1^2}$$
$$r_2 = \sqrt{x_2^2 + y_2^2 + z_2^2}$$
$$r_{12} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

The Halleraas coordinates are

$$s = r_1 + r_2$$
$$t = r_2 - r_1$$
$$u = r_{12}$$

Hence

$$\frac{\partial s}{\partial x_1} = \frac{x_1}{r_1} \qquad \frac{\partial t}{\partial x_1} = -\frac{x_1}{r_1} \qquad \frac{\partial u}{\partial x_1} = \frac{x_1 - x_2}{r_{12}}$$

$$\frac{\partial s}{\partial y_1} = \frac{y_1}{r_1} \qquad \frac{\partial t}{\partial y_1} = -\frac{y_1}{r_1} \qquad \frac{\partial u}{\partial y_1} = \frac{y_1 - y_2}{r_{12}}$$

$$\frac{\partial s}{\partial z_1} = \frac{z_1}{r_1} \qquad \frac{\partial t}{\partial z_1} = -\frac{z_1}{r_1} \qquad \frac{\partial u}{\partial z_1} = \frac{z_1 - z_2}{r_{12}}$$

Start by converting the Laplacian terms to Halleraas coordinates. The first Laplacian is

$$\nabla_1^2 \psi = \left(\frac{\partial \psi}{\partial x_1}\right)^2 + \left(\frac{\partial \psi}{\partial y_1}\right)^2 + \left(\frac{\partial \psi}{\partial z_1}\right)^2$$

By the chain rule

$$\frac{\partial \psi}{\partial x_1} = \frac{\partial \psi}{\partial s}\frac{\partial s}{\partial x_1} + \frac{\partial \psi}{\partial t}\frac{\partial t}{\partial x_1} + \frac{\partial \psi}{\partial t}\frac{\partial t}{\partial x_1}$$

Hence by (?)

$$\frac{\partial \psi}{\partial x_1} = \frac{x_1}{r_1}\frac{\partial \psi}{\partial s} - \frac{x_1}{r_1}\frac{\partial \psi}{\partial t} - \frac{x_2 - x_1}{u}\frac{\partial \psi}{\partial u}$$

And by symmetry for y_1 and z_1.

$$\frac{\partial \psi}{\partial y_1} = \frac{y_1}{r_1}\frac{\partial \psi}{\partial s} - \frac{y_1}{r_1}\frac{\partial \psi}{\partial t} - \frac{y_2 - y_1}{u}\frac{\partial \psi}{\partial u}$$
$$\frac{\partial \psi}{\partial z_1} = \frac{z_1}{r_1}\frac{\partial \psi}{\partial s} - \frac{z_1}{r_1}\frac{\partial \psi}{\partial t} - \frac{z_2 - z_1}{u}\frac{\partial \psi}{\partial u}$$

Hence

$$\nabla_1^2 \psi = \left(\frac{\partial \psi}{\partial s}\right)^2 + \left(\frac{\partial \psi}{\partial t}\right)^2 + \left(\frac{\partial \psi}{\partial u}\right)^2 - 2\frac{\partial \psi}{\partial s}\frac{\partial \psi}{\partial t} + 2\left(\frac{\partial \psi}{\partial s}\frac{\partial \psi}{\partial u} + \frac{\partial \psi}{\partial t}\frac{\partial \psi}{\partial u}\right) \text{(more stuff here)}$$