In polar form

$$\hat{H}' = eE(t)\, r \cos\theta$$

The matrix elements are

$$\begin{aligned}
\langle \psi_{2,0,0} | \hat{H}' | \psi_{1,0,0} \rangle &= 0 \\
\langle \psi_{2,1,0} | \hat{H}' | \psi_{1,0,0} \rangle &= \tfrac{128}{243}\sqrt{2} a_0 e E(t) \\
\langle \psi_{2,1,1} | \hat{H}' | \psi_{1,0,0} \rangle &= 0 \\
\langle \psi_{2,1,-1} | \hat{H}' | \psi_{1,0,0} \rangle &= 0
\end{aligned}$$

The unit of electric field is volts per meter.

The dimension of $a_0 e E(t)$ is joules. So somehow this has to get cancelled to get a dimensionless probability.

Answer: Divide by $\hbar$ to get hertz, then integrate over time.

$$\Pr(t) = \left| \int_0^t e^{i\Delta\omega t'} \frac{\tfrac{128}{243}\sqrt{2} a_0 e E(t')}{i\hbar}\, dt' \right|^2$$