

(a) Start with

$$P_{ab}(t) = \int_a^b |\Psi|^2 dx = \int |\Psi|^2 dx \Big|_{x=b} - \int |\Psi|^2 dx \Big|_{x=a}$$

and

$$\frac{\partial}{\partial t} |\Psi|^2 = \frac{\partial}{\partial x} \left[\frac{i\hbar}{2m} \left(\Psi^* \frac{\partial \Psi}{\partial x} - \Psi \frac{\partial \Psi^*}{\partial x} \right) \right] = -\frac{\partial}{\partial x} J(x, t)$$

Hence

$$\frac{d}{dt} P_{ab}(t) = \int -\frac{\partial}{\partial x} J(x, t) dx \Big|_{x=b} - \int -\frac{\partial}{\partial x} J(x, t) dx \Big|_{x=a}$$

Noting that

$$\int -\frac{\partial}{\partial x} J(x, t) dx = -J(x, t)$$

we have

$$\frac{d}{dt} P_{ab}(t) = -J(x, t) \Big|_{x=b} + J(x, t) \Big|_{x=a} = J(a, t) - J(b, t)$$

The units of $J(x, t)$ are hertz (inverse seconds) because $P_{ab}(t)$ is dimensionless. The time derivative of a dimensionless quantity is hertz.

Noting that $\int |\Psi|^2 dx$ is dimensionless, the units of $|\Psi|^2$ must be inverse meters to cancel with dx .

$$\Psi \Psi^* \propto \frac{1}{\text{meter}}$$

Taking the position derivative divides by meter.

$$\Psi \frac{\partial \Psi^*}{\partial x} \propto \frac{1}{\text{meter}^2}$$

Hence

$$\frac{i\hbar}{2m} \left(\Psi \frac{\partial \Psi^*}{\partial x} - \Psi^* \frac{\partial \Psi}{\partial x} \right) \propto \frac{\text{joule second}}{\text{kilogram meter}^2} = \frac{1}{\text{second}} = \text{hertz}$$

(b) FIXME