Exercise 6.9. Prove that the four vectors  $|sing\rangle$ ,  $|T_1\rangle$ ,  $|T_2\rangle$ , and  $|T_3\rangle$  are eigenvectors of  $\vec{\sigma} \cdot \vec{\tau}$ . What are their eigenvalues?

Since  $|sing\rangle$ ,  $|T_1\rangle$ ,  $|T_2\rangle$ , and  $|T_3\rangle$  are eigenvectors of  $\sigma_x\tau_x$ ,  $\sigma_y\tau_y$ , and  $\sigma_z\tau_z$ , then they are also eigenvectors of  $\vec{\sigma} \cdot \vec{\tau}$  by linearity.

We have the following eigenvalues.

$$|sing\rangle \quad |T_1\rangle \quad |T_2\rangle \quad |T_3\rangle$$

$$\sigma_x \tau_x \quad -1 \quad 1 \quad 1 \quad -1$$

$$\sigma_y \tau_y \quad -1 \quad 1 \quad -1 \quad 1$$

$$\sigma_z \tau_z \quad -1 \quad -1 \quad 1 \quad 1$$

$$\vec{\sigma} \cdot \vec{\tau} \quad -3 \quad 1 \quad 1 \quad 1$$