

# Canonical commutation relation

Consider the canonical commutation relation in one dimension.

$$XP - PX = i\hbar$$

Let

$$X = x, \quad P = -i\hbar \frac{\partial}{\partial x}$$

Show that

$$(XP - PX)\psi(x, t) = i\hbar\psi(x, t)$$

Eigenmath code:

```
X(f) = x f
P(f) = -i hbar d(f,x)
X(P(psi(x,t))) - P(X(psi(x,t)))
```

Result:

$$i\hbar\psi(x, t)$$

In three dimensions (symbol  $\otimes$  is outer product,  $\nabla$  is gradient)

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \otimes, \quad P = -i\hbar \nabla$$

Eigenmath code:

```
X(f) = outer((x,y,z),f)
P(f) = -i hbar d(f,(x,y,z))
X(P(psi(x,y,z,t))) - P(X(psi(x,y,z,t)))
```

Result:

$$\begin{bmatrix} i\hbar\psi(x, y, z, t) & 0 & 0 \\ 0 & i\hbar\psi(x, y, z, t) & 0 \\ 0 & 0 & i\hbar\psi(x, y, z, t) \end{bmatrix}$$