Exercise 4.1. Prove that if **U** is unitary, and if $|A\rangle$ and $|B\rangle$ are any two state vectors, then the inner product of $\mathbf{U}|A\rangle$ and $\mathbf{U}|B\rangle$ is the same as the inner product of $|A\rangle$ and $|B\rangle$. One could call this the *conservation of overlaps*. It expresses the fact that the logical relation between states is preserved with time.

If U is unitary then

$$\mathbf{U}^{\dagger}\mathbf{U} = I$$

Hence

$$\langle A|\mathbf{U}^{\dagger}\mathbf{U}|B\rangle = \langle A|I|B\rangle = \langle A|B\rangle$$