Angular momentum

Definitions.

$$\mathbf{L} = \mathbf{X} \times \mathbf{P} = \begin{pmatrix} X_2 P_3 - X_3 P_2 \\ X_3 P_1 - X_1 P_3 \\ X_1 P_2 - X_2 P_1 \end{pmatrix}$$

$$L^2 = |\mathbf{L}|^2 = L_1{}^2 + L_2{}^2 + L_3{}^2$$

$$\mathbf{J} = \frac{1}{\hbar} \mathbf{L}$$

Let

$$X_j = x_j, \quad P_j = -i\hbar \frac{\partial}{\partial x_j}$$

Show that

1.

$$[\mathbf{X}, L_3] = \begin{pmatrix} -i\hbar X_2 \\ i\hbar X_1 \\ 0 \end{pmatrix}$$

2.

$$[\mathbf{P}, L_3] = \begin{pmatrix} -i\hbar P_2 \\ i\hbar P_1 \\ 0 \end{pmatrix}$$

3.

$$[\mathbf{L}, L^2] = 0$$

4.

$$\mathbf{J} \times \mathbf{J} = i\mathbf{J}$$

5.

$$\mathbf{P} \times \mathbf{L} + \mathbf{L} \times \mathbf{P} = 2i\hbar\mathbf{P}$$

6.

$$\tfrac{1}{2} \left(\mathbf{P} \times \mathbf{L} - \mathbf{L} \times \mathbf{P} \right) = \mathbf{P} \times \mathbf{L} - i\hbar\mathbf{P}$$

7.

$$[L_i, P_j] = i\hbar \sum_k \epsilon_{ijk} P_k$$