Feynman and Hibbs problem 3-8

This is the Lagrangian for a harmonic oscillator.

$$L = \frac{m}{2}\dot{x}^2 - \frac{m\omega^2}{2}x^2$$

Show that the resulting kernel is

$$K = F(T)\exp\left(\frac{im\omega}{2\hbar\sin(\omega T)}\left((x_b^2 + x_a^2)\cos(\omega T) - 2x_b x_a\right)\right)$$

where $T = t_b - t_a$.

From problem 2-2 we have

$$S_{cl} = \frac{m\omega}{2\hbar\sin(\omega T)}\left((x_b^2 + x_a^2)\cos(\omega T) - 2x_b x_a\right)$$

From equation (3.51)

$$K = F(T)\exp\left(\frac{iS_{cl}}{\hbar}\right)$$

By substitution of S_{cl}

$$K = F(T)\exp\left(\frac{im\omega}{2\hbar\sin(\omega T)}\left((x_b^2 + x_a^2)\cos(\omega T) - 2x_b x_a\right)\right)$$