Exercise 6.3. Prove that the state $|sing\rangle$ cannot be written as a product state.

Consider the formula for the singlet state.

$$|sing\rangle = \frac{1}{\sqrt{2}}\left(|ud\rangle + |du\rangle\right)$$

Consider the product state for $|u\rangle$ and $|d\rangle$.

$$\alpha_u\beta_u|uu\rangle + \alpha_u\beta_d|ud\rangle + \alpha_d\beta_u|du\rangle + \alpha_d\beta_d|dd\rangle \tag{6.5}$$

By (6.5) we have for the singlet state

$$\alpha_u\beta_u = 0 \tag{1}$$

$$\alpha_u\beta_d = \frac{1}{\sqrt{2}} \tag{2}$$

$$\alpha_d\beta_u = \frac{1}{\sqrt{2}} \tag{3}$$

$$\alpha_d\beta_d = 0 \tag{4}$$

By (1) we must have $\alpha_u = 0$ or $\beta_u = 0$. If $\alpha_u = 0$ then (2) fails. If $\beta_u = 0$ then (3) fails. Hence $|sing\rangle$ cannot be a product state.