Dirac from boost

This vector represents an electron at rest with spin up along the z axis.

$$u_0 = \sqrt{2m}\begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

This matrix boosts a spinor in the z direction where $E^2 = p^2 + m^2$.

$$\Lambda = \frac{1}{\sqrt{2m(E+m)}}\begin{pmatrix}E+m & 0 & p & 0\\ 0 & E+m & 0 & p\\ p & 0 & E+m & 0\\ 0 & p & 0 & E+m\end{pmatrix}$$

Hence

$$u = \Lambda u_0 = \frac{1}{\sqrt{E+m}}\begin{pmatrix}E+m & 0 & p & 0\\ 0 & E+m & 0 & p\\ p & 0 & E+m & 0\\ 0 & p & 0 & E+m\end{pmatrix}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \frac{1}{\sqrt{E+m}}\begin{pmatrix}E+m\\0\\p\\0\end{pmatrix}$$

This is the Dirac equation in spinor form.

$$\not{p}u = mu$$

Substitute Λu_0 for u.

$$\not{p}\Lambda u_0 = m\Lambda u_0$$

By the identity $\gamma^0 u_0 = u_0$ substitute $\gamma^0 u_0$ for u_0 on the right hand side.

$$\not{p}\Lambda u_0 = m\Lambda\gamma^0 u_0$$

Substitute $\Lambda^{-1}u$ for u_0.

$$\not{p}\Lambda\Lambda^{-1}u = m\Lambda\gamma^0\Lambda^{-1}u$$

Hence

$$\not{p} = m\Lambda\gamma^0\Lambda^{-1}$$

and

$$\Lambda\gamma^0 = m^{-1}\not{p}\Lambda \tag{1}$$

Boost $\gamma^0 u_0 = u_0$ to recover the Dirac equation.

$$\begin{aligned}\Lambda\gamma^0 u_0 &= \Lambda u_0\\ m^{-1}\not{p}u &= u\\ \not{p}u &= mu\end{aligned} \tag{2}$$