

We are given

$$\psi(x, 0) = C \exp\left(\frac{ipx}{\hbar}\right)$$

For a free particle we have

$$K = \left(\frac{m}{2\pi i\hbar(t_b - t_a)}\right)^{1/2} \exp\left(\frac{im(x_b - x_a)^2}{2\hbar(t_b - t_a)}\right) \quad (3.3)$$

From

$$p = \frac{m(x_b - x_a)}{t_b - t_a}$$

we have

$$\frac{m(x_b - x_a)^2}{t_b - t_a} = \frac{p^2(t_b - t_a)}{m}$$

Hence

$$K = \left(\frac{m}{2\pi i\hbar(t_b - t_a)}\right)^{1/2} \exp\left(\frac{ip^2(t_b - t_a)}{2m\hbar} + \frac{ipx}{\hbar}\right)$$

By equation (3.42) we have

$$\begin{aligned} \psi(x, t_b) &= K\psi(x, 0) \\ &= C \left(\frac{m}{2\pi i\hbar(t_b - t_a)}\right)^{1/2} \exp\left(\frac{ip^2(t_b - t_a)}{2m\hbar} + \frac{ipx}{\hbar}\right) \end{aligned}$$

Finally, set  $t_a = 0$  and change  $t_b$  to  $t$  to obtain

$$\psi(x, t) = C \left(\frac{m}{2\pi i\hbar t}\right)^{1/2} \exp\left(\frac{ip^2 t}{2m\hbar} + \frac{ipx}{\hbar}\right)$$