

Canonical commutation relation in one dimension:

$$XP - PX = i\hbar$$

Let

$$X = x, \quad P = -i\hbar \frac{\partial}{\partial x}$$

Then

$$\begin{aligned} (XP - PX)\psi(x, t) &= XP\psi(x, t) - PX\psi(x, t) \\ &= x \left(-i\hbar \frac{\partial}{\partial x} \psi(x, t) \right) + i\hbar \frac{\partial}{\partial x} (x\psi(x, t)) \\ &= -i\hbar x \frac{\partial}{\partial x} \psi(x, t) + i\hbar \left(\frac{\partial}{\partial x} x \right) \psi(x, t) + i\hbar x \frac{\partial}{\partial x} \psi(x, t) \\ &= i\hbar \psi(x, t) \end{aligned}$$

Eigenmath code:

```
X(f) = x f
P(f) = -i hbar d(f,x)
X(P(psi(x,t))) - P(X(psi(x,t)))
```

Result:

$$i\hbar\psi(x, t)$$

Another example: Show that

$$[X^2, P^2] = 2i\hbar(XP + PX)$$

Eigenmath code:

```
A = X(X(P(P(psi(x,t)))) - P(P(X(X(psi(x,t)))))
B = 2 i hbar (X(P(psi(x,t))) + P(X(psi(x,t))))
check(A == B)
"ok"
```

Result:

ok

(The statement `check(A == B)` halts if A not equal B .)