This is how Einstein tensor $G_{\mu\nu}$ is calculated from metric tensor $g_{\mu\nu}$.

1. Calculate inverse of $g_{\mu\nu}$.

$$g^{\mu\nu} = (g_{\mu\nu})^{-1}$$

2. Calculate connection coefficients.

$$\Gamma^{\alpha}{}_{\beta\gamma} = \frac{1}{2} g^{\alpha\mu} (g_{\mu\beta,\gamma} + g_{\mu\gamma,\beta} - g_{\beta\gamma,\mu})$$

3. Calculate Riemann tensor.

$$R^{\alpha}{}_{\beta\gamma\delta} = \Gamma^{\alpha}{}_{\beta\delta,\gamma} - \Gamma^{\alpha}{}_{\beta\gamma,\delta} + \Gamma^{\alpha}{}_{\mu\gamma}\Gamma^{\mu}{}_{\beta\delta} - \Gamma^{\alpha}{}_{\mu\delta}\Gamma^{\mu}{}_{\beta\gamma}$$

4. Calculate Ricci tensor.

$$R_{\mu\nu} = R^{\alpha}{}_{\mu\alpha\nu}$$

5. Calculate Ricci scalar.

$$R = R^{\mu}{}_{\mu}$$

6. Calculate Einstein tensor.

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

This is the Schwarzschild metric.

$$g_{\mu\nu} = \begin{pmatrix} 2M/r - 1 & 0 & 0 & 0 \\ 0 & (1 - 2M/r)^{-1} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix}$$

The Einstein tensor vanishes for the Schwarzschild metric.

$$G_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$