Balmer's formula

The following table is from "Note on the spectral lines of hydrogen" by J. J. Balmer dated 1885. Numerical values are wavelengths in units of 10^{-10} meter.

Investigator	H_α	H_β	H_γ	H_δ	H_ϵ	H_ζ	H_η	H_ϑ	H_ι
Van der Willigen	6565.6	4863.94	4342.80	4103.8	–	–	–	–	–
Angstrom	6562.10	4860.74	4340.10	4101.2	–	–	–	–	–
Mendenhall	6561.62	4860.16	–	–	–	–	–	–	–
Mascart	6560.7	4859.8	–	–	–	–	–	–	–
Ditscheiner	6559.5	4859.74	4338.60	4100.0	–	–	–	–	–
Huggins	–	–	–	–	–	3887.5	3834	3795	3767.5
Vogel	–	–	–	–	3969	3887	3834	3795	3769†

(†The value given in the paper is 6769 which is an obvious typo.)

Balmer discovered the following formula for fitting the data (λ is wavelength).

$$\lambda = \frac{m^2}{m^2 - 2^2} \times 3645.6 \times 10^{-10} \, \text{meter}$$

Parameter m is from the following table.

$$
\begin{array}{cccccccccc}
 & H_\alpha & H_\beta & H_\gamma & H_\delta & H_\epsilon & H_\zeta & H_\eta & H_\vartheta & H_\iota \\
m = & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11
\end{array}
$$

Let β be the model coefficient for λ. Using linear regression and the above data we obtain

$$\beta = 3645.3 \times 10^{-10} \, \text{meter}$$

The currently accepted value is

$$\beta = \frac{4}{R_H} = 3647.1 \times 10^{-10} \, \text{meter}$$

where R_H is the Rydberg constant for hydrogen

$$R_H = 1.09677576 \times 10^7 \, \text{meter}^{-1}$$

Balmer's coefficient is within 0.04% of the modern value.

$$100 \times \frac{4/R_H - 3.6456}{4/R_H} = 0.04$$

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