(37.2) Show that both choices of sign $\mathbf{K} = \pm\frac{i\boldsymbol{\sigma}}{2}$ obey the necessary commutation relations.

These are the commutation relations from page 336.

$$[J^i, K^j] = i\varepsilon^{ijk}K^k$$
$$[K^i, K^j] = -i\varepsilon^{ijk}J^k$$

On page 337 we are given

$$\mathbf{J} = \tfrac{1}{2}\boldsymbol{\sigma}$$

and

$$[\sigma^i, \sigma^j] = 2i\varepsilon^{ijk}\sigma^k \tag{1}$$

For the first commutation relation we have

$$[J^i, \pm K^j] = \left[\tfrac{1}{2}\sigma^i, \pm\tfrac{i}{2}\sigma^j\right] = \pm\tfrac{i}{4}[\sigma^i, \sigma^j]$$

By equation (1)

$$\pm\tfrac{i}{4}[\sigma^i, \sigma^j] = \pm\tfrac{i}{4}\big(2i\varepsilon^{ijk}\sigma^k\big) = \mp\tfrac{1}{2}\varepsilon^{ijk}\sigma^k = \pm i\varepsilon^{ijk}K^k$$

For the second commutation relation we have

$$[K^i, K^j] = \left[\pm\tfrac{i}{2}\sigma^i, \pm\tfrac{i}{2}\sigma^j\right] = -\tfrac{1}{4}[\sigma^i, \sigma^j]$$

By equation (1)

$$-\tfrac{1}{4}[\sigma^i, \sigma^j] = -\tfrac{1}{4}\big(2i\varepsilon^{ijk}\sigma^k\big) = -i\varepsilon^{ijk}J^k$$