

Exercise 6.3. Prove that the state  $|sing\rangle$  cannot be written as a product state.

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Consider the formula for the singlet state.

$$|sing\rangle = \frac{1}{\sqrt{2}}(|ud\rangle + |du\rangle)$$

Consider the product state for  $|u\rangle$  and  $|d\rangle$ .

$$\alpha_u\beta_u|uu\rangle + \alpha_u\beta_d|ud\rangle + \alpha_d\beta_u|du\rangle + \alpha_d\beta_d|dd\rangle \quad (6.5)$$

By (6.5) we have for the singlet state

$$\alpha_u\beta_u = 0 \quad (1)$$

$$\alpha_u\beta_d = \frac{1}{\sqrt{2}} \quad (2)$$

$$\alpha_d\beta_u = \frac{1}{\sqrt{2}} \quad (3)$$

$$\alpha_d\beta_d = 0 \quad (4)$$

By (1) we must have  $\alpha_u = 0$  or  $\beta_u = 0$ . If  $\alpha_u = 0$  then (2) fails. If  $\beta_u = 0$  then (3) fails. Hence  $|sing\rangle$  cannot be a product state.