(1.4) Show that

$$\frac{\delta\phi(x)}{\delta\phi(y)} = \delta(x-y) \tag{1.44}$$

and

$$\frac{\delta\dot{\phi}(t)}{\delta\phi(t_0)} = \frac{d}{dt}\delta(t-t_0) \tag{1.45}$$

Let F be the trivial functional $F[\phi] = \phi(x)$.

$$\frac{\delta F}{\delta\phi(y)} = \lim_{\epsilon\to 0}\frac{\phi(x)+\epsilon\delta(x-y)-\phi(x)}{\epsilon} = \delta(x-y)$$

Show (1.45).

$$\frac{\delta\dot{\phi}(t)}{\delta\phi(t_0)} = \lim_{\epsilon\to 0}\frac{1}{\epsilon}\left(\frac{d\big(\phi(t)-\epsilon\delta(t-t_0)\big)}{dt} - \frac{d\phi(t)}{dt}\right) = \frac{d}{dt}\delta(x-y)$$