6-13. Assume that $V(\mathbf{r},t)$ is independent of time. Substitute the free particle kernel K_0 into equation (6.61) and integrate over t_c to show that

$$\psi(b) = \exp\left(-\frac{iE_a t_b}{\hbar}\right)\exp\left(\frac{i\mathbf{p}_a\cdot\mathbf{x}_b}{\hbar}\right) - \exp\left(-\frac{iE_a t_b}{\hbar}\right)\frac{m}{2\pi\hbar^2}\int\frac{1}{R_{bc}}\exp\left(\frac{ipR_{bc}}{\hbar}\right)V(\mathbf{x}_c)\exp\left(\frac{i\mathbf{p}_a\cdot\mathbf{x}_c}{\hbar}\right)d^3\mathbf{x}_c \tag{6.62}$$

where R_{bc} is the distance from the variable point of integration $\mathbf{x}_c$ to the final point $\mathbf{x}_b$ and p is the magnitude of the momentum of the electron.

This is equation (6.61).

$$\psi(b) = \exp\left(-\frac{iE_a t_b}{\hbar}\right)\exp\left(\frac{i\mathbf{p}_a\cdot\mathbf{x}_b}{\hbar}\right) - \frac{i}{\hbar}\int_0^{t_b}\int K_0(b,c)V(c)\exp\left(-\frac{iE_a t_c}{\hbar}\right)\exp\left(\frac{i\mathbf{p}_a\cdot\mathbf{x}_c}{\hbar}\right)d^3\mathbf{x}_c\,dt_c \tag{6.61}$$

This is the integral over t_c from (6.61) with $V(c)$ independent of time.

$$I = \int_0^{t_b} K_0(b,c)\exp\left(-\frac{iE_a t_c}{\hbar}\right)dt_c$$

Substitute $E_a = p^2/2m$.

$$I = \int_0^{t_b} K_0(b,c)\exp\left(-\frac{ip^2 t_c}{2m\hbar}\right)dt_c$$

Substitute K_0 from problem 4-12.

$$I = \int_0^{t_b}\left(\frac{m}{2\pi i\hbar(t_b-t_c)}\right)^{3/2}\exp\left(\frac{imR_{bc}^2}{2\hbar(t_b-t_c)}\right)\exp\left(-\frac{ip^2 t_c}{2m\hbar}\right)dt_c$$

Let

$$f = \left(\frac{m}{2\pi i\hbar(t_b-t_c)}\right)^{3/2}$$
$$g = \frac{mR_{bc}^2}{2(t_b-t_c)} - \frac{p^2 t_c}{2m}$$
$$\lambda = \frac{1}{\hbar}$$

Then

$$I = \int_0^{t_b} f \exp(i\lambda g)\, dt_c$$

The phase of the exponential is stationary (i.e., $g' = 0$) for

$$t_c = t_b - \frac{mR_{bc}}{p}$$

By the method of stationary phase

$$I \approx \pm \left(\frac{2\pi i}{\lambda g''} \right)^{1/2} f \exp(i\lambda g) \bigg|_{t_c}$$

Hence

$$I \approx -\frac{im}{2\pi\hbar R_{bc}} \exp\left(\frac{ipR_{bc}}{\hbar} - \frac{ip^2 t_b}{2m\hbar} \right)$$

The integral can also be written as

$$-\frac{im}{2\pi\hbar} \frac{1}{R_{bc}} \exp\left(\frac{ipR_{bc}}{\hbar} \right) \exp\left(-\frac{iE_a t_b}{\hbar} \right) \tag{1}$$

Substitute (1) into (6.61) to obtain (6.62).

Note that the method of stationary phase requires $0 < t_c < t_b$ so the above solution is valid for physical values that satisfy

$$0 < \frac{mR_{bc}}{p} < t_b$$

Hence (6.62) is not true in general but is *probably* true if t_b is large.

Just for the fun of it, check physical dimensions.

$$\frac{mR_{bc}}{p} \propto \frac{\text{mass} \times \text{length}}{\text{mass} \times \text{length/time}} = \text{time}$$