Harmonic oscillator propagator

Show that

$$\left[x^2, p^2\right] = 2i\hbar(xp + px) \tag{1}$$

By the identity

$$[AB, CD] = A[B, C]D + AC[B, D] + [A, C]DB + C[A, D]B$$

we have

$$\left[x^2, p^2\right] = [xx, pp] = x[x, p]p + xp[x, p] + [x, p]px + p[x, p]x$$

Substitute $i\hbar$ for $[x, p]$.

$$\left[x^2, p^2\right] = 2i\hbar(xp + px)$$

Show that

$$\left[x^2, xp + px\right] = 4i\hbar x^2 \tag{2}$$

We have

$$\left[x^2, xp + px\right] = [xx, xp] + [xx, px] = xx[x, p] + x[x, p]x + x[x, p]x + [x, p]xx$$

Substitute $i\hbar$ for $[x, p]$.

$$\left[x^2, xp + px\right] = 4i\hbar x^2$$

Show that

$$\left[p^2, xp + px\right] = -4i\hbar p^2 \tag{3}$$

We have

$$\left[p^2, xp + px\right] = [pp, xp] + [pp, px] = p[p, x]p + [p, x]pp + pp[p, x] + p[p, x]p$$

Substitute $-i\hbar$ for $[p, x]$.

$$\left[p^2, xp + px\right] = -4i\hbar p^2$$