

Let  $A$  be the spontaneous emission rate and let  $B\rho(\nu)$  be the stimulated emission rate.

We want to find frequency  $\nu$  such that

$$A = B\rho(\nu)$$

Substitute for  $\rho(\nu)$ .

$$A = B \frac{A/B}{\exp\left(\frac{h\nu}{kT}\right) - 1}$$

Hence

$$\exp\left(\frac{h\nu}{kT}\right) = 2$$

Take the log of both sides.

$$\frac{h\nu}{kT} = \log 2$$

Hence

$$\nu = \frac{kT}{h} \log 2$$

For  $T = 300\text{ K}$  we have

$$\nu = 4.33 \times 10^{12} \text{ Hz}$$

From equation (11.61)

$$\frac{A}{B} \propto \nu^3$$

Hence  $A$  dominates for large  $\nu$ .

Frequency of visible light is around  $10^{15} \text{ Hz}$ . This is well above  $4.33 \times 10^{12} \text{ Hz}$  hence spontaneous emission dominates.