

Spin flip

Consider an electron at rest in the following magnetic field.

$$\mathbf{B} = B_0 \cos(\omega t) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Find the minimum B_0 such that the sign of $\langle S_x \rangle$ flips periodically.

Start with the Hamiltonian for an electron in a magnetic field.

$$H = \frac{ge}{2m} \mathbf{B} \cdot \mathbf{S} = \frac{ge}{2m} B_0 \cos(\omega t) S_z$$

Let $|s\rangle$ be the spin state

$$|s\rangle = \begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix}$$

The Schrodinger equation

$$i\hbar \frac{\partial}{\partial t} |s\rangle = H |s\rangle$$

is equivalent to the differential equations

$$\begin{aligned} i\hbar \frac{\partial}{\partial t} c_1(t) &= \frac{ge\hbar}{4m} B_0 \cos(\omega t) c_1(t) \\ i\hbar \frac{\partial}{\partial t} c_2(t) &= -\frac{ge\hbar}{4m} B_0 \cos(\omega t) c_2(t) \end{aligned}$$

The solutions are

$$\begin{aligned} c_1(t) &= C_1 \exp\left(-\frac{ige}{4m\omega} B_0 \sin(\omega t)\right) \\ c_2(t) &= C_2 \exp\left(\frac{ige}{4m\omega} B_0 \sin(\omega t)\right) \end{aligned} \tag{1}$$

where complex coefficients C_1 and C_2 have the general polar forms

$$C_1 = a_1 \exp(i\theta_1), \quad C_2 = a_2 \exp(i\theta_2), \quad |C_1|^2 + |C_2|^2 = a_1^2 + a_2^2 = 1$$

For $\langle S_x \rangle$ we have

$$\langle S_x \rangle = \langle s | S_x | s \rangle = a_1 a_2 \hbar \cos\left(\frac{ge}{2m\omega} B_0 \sin(\omega t) - \theta_1 + \theta_2\right) \tag{2}$$

Assume $\theta_1 = \theta_2$ for this problem so that

$$\langle S_x \rangle = a_1 a_2 \hbar \cos\left(\frac{ge}{2m\omega} B_0 \sin(\omega t)\right)$$

Note that if the sign of $\langle S_x \rangle$ is constant in time then

$$-\frac{\pi}{2} \leq \frac{ge}{2m\omega} B_0 \sin(\omega t) \leq \frac{\pi}{2}$$

For t such that $\sin(\omega t) = 1$ we have

$$\frac{ge}{2m\omega} B_0 \leq \frac{\pi}{2}$$

Hence the sign of $\langle S_x \rangle$ changes periodically for

$$B_0 > \frac{\pi m \omega}{ge}$$

See exercise 10.6 of *Quantum Mechanics* (Lulu edition) by Richard Fitzpatrick.