

7-1. If $S(x(t)) = \int_{t_a}^{t_b} L(\dot{x}, x, t) dt$, show that, for any s inside the range t_a to t_b ,

$$\frac{\delta S}{\delta x(s)} = -\frac{d}{ds} \left(\frac{\partial L}{\partial \dot{x}} \right) + \frac{\partial L}{\partial x}$$

where the partial derivatives are evaluated at $t = s$.

From equation (2.6)

$$\delta S = \int_{t_a}^{t_b} \left(-\frac{d}{dt} \frac{\partial L}{\partial \dot{x}} + \frac{\partial L}{\partial x} \right) \delta x(t) dt$$

Let $t_a = t_b = s$.

$$\delta S = \left(-\frac{d}{ds} \frac{\partial L}{\partial \dot{x}} + \frac{\partial L}{\partial x} \right) \delta x(s)$$

Hence

$$\frac{\delta S}{\delta x(s)} = -\frac{d}{ds} \frac{\partial L}{\partial \dot{x}} + \frac{\partial L}{\partial x}$$