6-1. Suppose the potential can be written as $U+V$, where V is small but U is large. Suppose further that the kernel for motion in the potential of U alone can be worked out (for example, U might be quadratic in x and independent of time). Show that the motion in the complete potential $U+V$ is described by equations (6.4), (6.11), (6.13), and (6.14) with K_0 replaced by K_U, where K_U is the kernel for motion in the potential U alone.

From equation (6.1) we have

$$K_{U+V}(b,a) = \int_a^b \exp\left(\frac{i}{\hbar}\int_{t_a}^{t_b}\left(\tfrac{1}{2}m\dot{x}^2 - U - V\right)\,dt\right)\mathcal{D}x(t)$$

Consider equation (6.5).

$$K_0(b,a) = \int_a^b \exp\left(\frac{i}{\hbar}\int_{t_a}^{t_b}\tfrac{1}{2}m\dot{x}^2\,dt\right)\mathcal{D}x(t) \tag{6.5}$$

By hypothesis the kernel for U is known hence

$$K_U(b,a) = \int_a^b \exp\left(\frac{i}{\hbar}\int_{t_a}^{t_b}\left(\tfrac{1}{2}m\dot{x}^2 - U\right)\,dt\right)\mathcal{D}x(t)$$

In the expansion of V, replace

$$\exp\left(\frac{i}{\hbar}\int_{t_a}^{t_b}\tfrac{1}{2}m\dot{x}^2\,dt\right)$$

with

$$\exp\left(\frac{i}{\hbar}\int_{t_a}^{t_b}\left(\tfrac{1}{2}m\dot{x}^2 - U\right)\,dt\right)$$

This corresponds to replacing K_0 with K_U in (6.4), etc.