

6-3. For a free particle, equation (4.29) reduces to

$$\frac{\partial}{\partial t_b} K_0(b, a) + \frac{i}{\hbar} \left( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_b^2} K_0(b, a) \right) = \delta(t_b - t_a) \delta(x_b - x_a) \quad (6.20)$$

Show, from this result and equation (6.19), that the kernel  $K_V$  satisfies the differential equation

$$\begin{aligned} \frac{\partial}{\partial t_b} K_V(b, a) + \frac{i}{\hbar} \left( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_b^2} K_V(b, a) + V(b) K_V(b, a) \right) \\ = \delta(t_b - t_a) \delta(x_b - x_a) \end{aligned} \quad (6.21)$$

From equation (6.19)

$$K_0(b, a) = K_V(b, a) + \frac{i}{\hbar} \int K_0(b, c) V(c) K_V(c, a) d\tau_c \quad (1)$$

Apply  $\partial/\partial t_b$  and  $\partial^2/\partial x_b^2$  to (1).

$$\frac{\partial}{\partial t_b} K_0(b, a) = \frac{\partial}{\partial t_b} K_V(b, a) + \frac{i}{\hbar} \int \frac{\partial}{\partial t_b} K_0(b, c) V(c) K_V(c, a) d\tau_c \quad (2)$$

$$\frac{\partial^2}{\partial x_b^2} K_0(b, a) = \frac{\partial^2}{\partial x_b^2} K_V(b, a) + \frac{i}{\hbar} \int \frac{\partial^2}{\partial x_b^2} K_0(b, c) V(c) K_V(c, a) d\tau_c \quad (3)$$

Multiply (3) by  $-\hbar^2/2m$ .

$$\begin{aligned} -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_b^2} K_0(b, a) \\ = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_b^2} K_V(b, a) + \frac{i}{\hbar} \int \left( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_b^2} K_0(b, c) \right) V(c) K_V(c, a) d\tau_c \end{aligned} \quad (4)$$

From equation (6.20)

$$\frac{i}{\hbar} \left( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_b^2} K_0(b, a) \right) = -\frac{\partial}{\partial t_b} K_0(b, a) + \delta(t_b - t_a) \delta(x_b - x_a)$$

It follows that

$$\begin{aligned} \frac{i}{\hbar} \int \left( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_b^2} K_0(b, c) \right) V(c) K_V(c, a) d\tau_c \\ = - \int \frac{\partial}{\partial t_b} K_0(b, c) V(c) K_V(c, a) d\tau_c + V(b) K_V(b, a) \end{aligned} \quad (5)$$

where we have used the identity

$$\int \delta(t_b - t_a) \delta(x_b - x_a) V(c) K_V(c, a) d\tau_c = V(b) K_V(b, a)$$

Substitute (5) into (4) to obtain

$$\begin{aligned} -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_b^2} K_0(b, a) &= -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x_b^2} K_V(b, a) \\ &\quad - \int \frac{\partial}{\partial t_b} K_0(b, c) V(c) K_V(c, a) d\tau_c + V(b) K_V(b, a) \quad (6) \end{aligned}$$

Substitute (2) and (6) into (6.20) to obtain (6.21).