

Feynman and Hibbs problem 4-3

Show that the complex conjugate function ψ^* , defined as the function ψ with every i changed to $-i$, satisfies

$$\frac{\partial \psi^*}{\partial t} = +\frac{i}{\hbar}(H\psi)^*$$

Start with equation (4.14)

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar}H\psi$$

Conjugate both sides.

$$\left(\frac{\partial \psi}{\partial t}\right)^* = +\frac{i}{\hbar}(H\psi)^*$$

It is well known that conjugation and differentiation commute, hence

$$\left(\frac{\partial \psi}{\partial t}\right)^* = \frac{\partial \psi^*}{\partial t} = +\frac{i}{\hbar}(H\psi)^*$$

However, just for the fun of it, let us complete the proof without using the commutation rule.

From equation (2.22) we have

$$K(b, a) = \lim_{\epsilon \rightarrow 0} \frac{1}{A^N} \int \cdots \int \exp\left(\frac{i}{\hbar}S(b, a)\right) dx_1 \cdots dx_{N-1} \quad (1)$$

Differentiate (1).

$$\frac{\partial}{\partial t}K(b, a) = \lim_{\epsilon \rightarrow 0} \frac{1}{A^N} \int \cdots \int \frac{i}{\hbar} \frac{\partial}{\partial t}S(b, a) \exp\left(\frac{i}{\hbar}S(b, a)\right) dx_1 \cdots dx_{N-1} \quad (2)$$

Conjugate (2).

$$\begin{aligned} & \left(\frac{\partial}{\partial t}K(b, a)\right)^* \\ &= \lim_{\epsilon \rightarrow 0} \frac{1}{A^N} \int \cdots \int -\frac{i}{\hbar} \frac{\partial}{\partial t}S(b, a) \exp\left(-\frac{i}{\hbar}S(b, a)\right) dx_1 \cdots dx_{N-1} \end{aligned}$$

Clearly the result is the same for conjugate first then differentiate, hence

$$\left(\frac{\partial}{\partial t}K(b, a)\right)^* = \frac{\partial}{\partial t}K^*(b, a) \quad (3)$$

Now consider this form of equation (4.2)

$$\psi(x, t) = \int_{-\infty}^{\infty} K(x, t, x_a, t_a) \psi(x_a, t_a) dx_a \quad (4)$$

Differentiate (4) then conjugate.

$$\left(\frac{\partial}{\partial t}\psi(x, t)\right)^* = \int_{-\infty}^{\infty} \left(\frac{\partial}{\partial t}K(x, t, x_a, t_a)\right)^* \psi^*(x_a, t_a) dx_a$$

Conjugate (4) then differentiate.

$$\frac{\partial}{\partial t}\psi^*(x, t) = \int_{-\infty}^{\infty} \frac{\partial}{\partial t}K^*(x, t, x_a, t_a) \psi^*(x_a, t_a) dx_a$$

Then by (3)

$$\left(\frac{\partial}{\partial t}\psi(x, t)\right)^* = \frac{\partial}{\partial t}\psi^*(x, t)$$