The file q4.txt defines kets, operators, and a measurement function for simulating a four bit quantum computer. See eigenmath.org/q.c for the program that generates q4.txt.

Kets are unit vectors in $\mathbb{C}^{16}$. The dimension is 16 because a four bit quantum computer has $2^4 = 16$ eigenstates. The following basis kets that represent eigenstates are defined in q4.txt.

$$|0\rangle = |0000_2\rangle = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$$
$$|1\rangle = |0001_2\rangle = (0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$$
$$|2\rangle = |0010_2\rangle = (0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$$
$$|3\rangle = |0011_2\rangle = (0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$$
$$\vdots$$
$$|15\rangle = |1111_2\rangle = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1)$$

Operators are 16×16 matrices that rotate ket vectors. The following operators and the measurement function $M(\psi)$ are defined in q4.txt.

H_n Hadamard operator on bit n.

I Identity matrix.

$M(\psi)$ Measurement function (not an operator).

$P_{mn}(\phi)$ Controlled phase shift, m is the control bit, n is the target bit, ϕ is the phase.

Q Quantum Fourier transform.

R Inverse quantum Fourier transform.

S_{mn} Swap bits m and n.

X_n Pauli X (NOT) operator on bit n.

X_{mn} Controlled X (CNOT) operator, m is the control bit, n is the target bit.

Y_n Pauli Y operator on bit n.

Z_n Pauli Z operator on bit n.

Measurement function $M(\psi)$ shows, for all $k = 0 \ldots 15$, the probability P_k of observing eigenstate k given that the quantum computer is in state ψ.

$$\psi = \sum_{k=0}^{15} c_k |k\rangle, \quad |\psi|^2 = 1, \quad P_k = c_k c_k^*$$

Quantum algorithms are expressed as sequences of operators applied to the initial state $|0\rangle$. The operator sequence should be read backwards, from right to left, although the direction makes no difference mathematically.

Deutsch-Jozsa algorithm

Let $f(q_0, q_1, q_2)$ be an operator (16×16 matrix) that operates on q_3 in a manner consistent with a constant or balanced oracle. Then the Deutsch-Jozsa algorithm for identifying f is

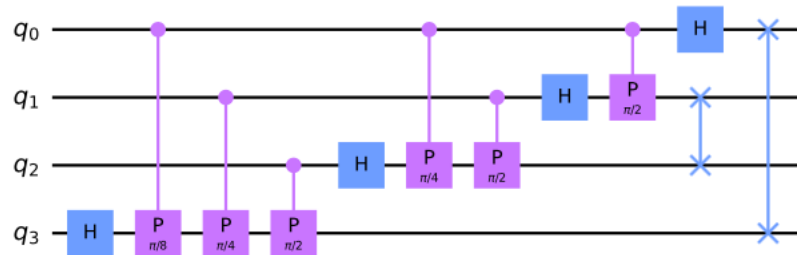$$\psi = H_2 \, H_1 \, H_0 \, f(q_0, q_1, q_2) \, H_3 \, X_3 \, H_2 \, H_1 \, H_0 \, |0\rangle$$

Bernstein-Vazirani algorithm

Let $f(q_0, q_1, q_2)$ be an operator (16×16 matrix) that operates on q_3. Then the Bernstein-Vazirani algorithm for identifying f is

$$\psi = H_2 \, H_1 \, H_0 \, f(q_0, q_1, q_2) \, Z_3 \, H_3 \, H_2 \, H_1 \, H_0 \, |0\rangle$$

Quantum Fourier transform

The following circuit diagram[1] shows how to implement the QFT.

This is how the QFT operator Q is defined in q4.txt.

```
Q = dot(
S03,
S12,
H0,
P01(pi/2),
H1,
P12(pi/2),
P02(pi/4),
H2,
P23(pi/2),
P13(pi/4),
P03(pi/8),
H3)
```

The inverse QFT operator R is defined similarly except the operators appear in reverse order and the phase shifts are negated.

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[1] qiskit.org/textbook/ch-algorithms/quantum-fourier-transform.html