

Spin measurement

Let $|s\rangle$ be the following spin state given polar angle θ and azimuth ϕ .

$$|s\rangle = \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2) \exp(i\phi) \end{pmatrix}$$

Find spin measurement probabilities for directions x , y , and z .

Eigenstates for the x direction.

$$|x_+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad |x_-\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Eigenstates for the y direction.

$$|y_+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}, \quad |y_-\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

Eigenstates for the z direction.

$$|z_+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |z_-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

For spin measurements in the x direction

$$\begin{aligned} \Pr\left(+\frac{\hbar}{2}\right) &= |\langle x_+ | s \rangle|^2 = \frac{1}{2} + \frac{1}{2} \sin \theta \cos \phi \\ \Pr\left(-\frac{\hbar}{2}\right) &= |\langle x_- | s \rangle|^2 = \frac{1}{2} - \frac{1}{2} \sin \theta \cos \phi \end{aligned}$$

For the y direction

$$\begin{aligned} \Pr\left(+\frac{\hbar}{2}\right) &= |\langle y_+ | s \rangle|^2 = \frac{1}{2} + \frac{1}{2} \sin \theta \sin \phi \\ \Pr\left(-\frac{\hbar}{2}\right) &= |\langle y_- | s \rangle|^2 = \frac{1}{2} - \frac{1}{2} \sin \theta \sin \phi \end{aligned}$$

For the z direction

$$\begin{aligned} \Pr\left(+\frac{\hbar}{2}\right) &= |\langle z_+ | s \rangle|^2 = \frac{1}{2} + \frac{1}{2} \cos \theta \\ \Pr\left(-\frac{\hbar}{2}\right) &= |\langle z_- | s \rangle|^2 = \frac{1}{2} - \frac{1}{2} \cos \theta \end{aligned}$$

Note that for each direction $\Pr\left(+\frac{\hbar}{2}\right) + \Pr\left(-\frac{\hbar}{2}\right) = 1$ as required by total probability.