Take (5.38) and prime the indices.

$$f(x,y,z,\ldots) = \sum_{a'}\sum_{b'}\sum_{c'}\cdots F'_{a',b',c',\ldots}\chi_{a',b',c',\ldots}(x,y,z,\ldots) \tag{1}$$

This is equation (5.36).

$$F_{a,b,c,\ldots} = \int_{-\infty}^{\infty} \chi^*_{a,b,c,\ldots}(x)f(x)\,dx \tag{5.36}$$

Substitute the right-hand side of (1) for $f(x)$ in (5.36).

$$\begin{aligned} F_{a,b,c,\ldots} = \int_{\mathbb{R}^n} &\chi^*_{a,b,c,\ldots}(x,y,z,\ldots) \\ &\times \left(\sum_{a'}\sum_{b'}\sum_{c'}\cdots F'_{a',b',c',\ldots}\chi_{a',b',c',\ldots}(x,y,z,\ldots)\right) dx\,dy\,dz\,\cdots \end{aligned}$$

Combine factors.

$$\begin{aligned} F_{a,b,c,\ldots} = \int_{\mathbb{R}^n} \sum_{a'}\sum_{b'}\sum_{c'}\cdots &\chi^*_{a,b,c,\ldots}(x,y,z,\ldots) \\ &\times F'_{a',b',c',\ldots}\chi_{a',b',c',\ldots}(x,y,z,\ldots)\,dx\,dy\,dz\,\cdots \end{aligned}$$

Interchange the order of integration and summation.

$$\begin{aligned} F_{a,b,c,\ldots} = \sum_{a'}\sum_{b'}\sum_{c'}\cdots \int_{\mathbb{R}^n} &\chi^*_{a,b,c,\ldots}(x,y,z,\ldots) \\ &\times F'_{a',b',c',\ldots}\chi_{a',b',c',\ldots}(x,y,z,\ldots)\,dx\,dy\,dz\,\cdots \end{aligned}$$

Factor out F'.

$$\begin{aligned} F_{a,b,c,\ldots} = \sum_{a'}\sum_{b'}\sum_{c'}\cdots &F'_{a',b',c',\ldots} \\ &\times \left(\int_{\mathbb{R}^n} \chi^*_{a,b,c,\ldots}(x,y,z,\ldots)\chi_{a',b',c',\ldots}(x,y,z,\ldots)\,dx\,dy\,dz\,\cdots\right) \end{aligned}$$

By equation (5.35) the integral becomes a product of delta functions.

$$F_{a,b,c,\ldots} = \sum_{a'}\sum_{b'}\sum_{c'}\cdots F'_{a',b',c',\ldots}\delta(a-a')\delta(b-b')\delta(c-c')\cdots$$

Hence for $F_{a,b,c,\ldots} \neq 0$ we must have $a = a'$, $b = b'$, etc. Therefore

$$F_{a,b,c,\ldots} = F'_{a',b',c',\ldots} = F'_{a,b,c,\ldots}$$