Gordon decomposition

Show that

$$\bar{u}(p_2, s_2)\gamma^\mu u(p_1, s_1) = \bar{u}(p_2, s_2)G^\mu u(p_1, s_1)$$

where

$$G^\mu = \frac{(p_1 + p_2)^\mu + i\sigma^{\mu\nu}(p_2 - p_1)_\nu}{m_1 + m_2}$$

Start by introducing the cast of characters. First, the momentum vectors.

$$p_1 = \begin{pmatrix} E_1 \\ p_{1x} \\ p_{1y} \\ p_{1z} \end{pmatrix}, \quad p_2 = \begin{pmatrix} E_2 \\ p_{2x} \\ p_{2y} \\ p_{2z} \end{pmatrix}$$

Spinors for particle one (normalized as $|u|^2 = 2E$).

$$u(p_1, 1) = \frac{1}{\sqrt{E_1 + m_1}} \underbrace{\begin{pmatrix} E_1 + m_1 \\ 0 \\ p_{1z} \\ p_{1x} + ip_{1y} \end{pmatrix}}_{\text{spin up}}, \quad u(p_1, 2) = \frac{1}{\sqrt{E_1 + m_1}} \underbrace{\begin{pmatrix} 0 \\ E_1 + m_1 \\ p_{1x} - ip_{1y} \\ -p_{1z} \end{pmatrix}}_{\text{spin down}}$$

Spinors for particle two.

$$u(p_2, 1) = \frac{1}{\sqrt{E_2 + m_2}} \underbrace{\begin{pmatrix} E_2 + m_2 \\ 0 \\ p_{2z} \\ p_{2x} + ip_{2y} \end{pmatrix}}_{\text{spin up}}, \quad u(p_2, 2) = \frac{1}{\sqrt{E_2 + m_2}} \underbrace{\begin{pmatrix} 0 \\ E_2 + m_2 \\ p_{2x} - ip_{2y} \\ -p_{2z} \end{pmatrix}}_{\text{spin down}}$$

Relativistic energy.

$$E_1 = \sqrt{p_{1x}^2 + p_{1y}^2 + p_{1z}^2 + m_1^2}, \quad E_2 = \sqrt{p_{2x}^2 + p_{2y}^2 + p_{2z}^2 + m_2^2}$$

This is the definition for tensor $\sigma^{\mu\nu}$.

$$\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right] = \frac{i}{2}\left(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\right)$$

In component notation

$$\sigma^{\mu\alpha\nu}{}_\beta = \frac{i}{2}\left(\gamma^{\mu\alpha}{}_\rho\gamma^{\nu\rho}{}_\beta - \gamma^{\nu\alpha}{}_\rho\gamma^{\mu\rho}{}_\beta\right)$$

Let $T^{\mu\nu} = \gamma^\mu\gamma^\nu$. In component notation

$$T^{\mu\alpha\nu}{}_\beta = \gamma^{\mu\alpha}{}_\rho\gamma^{\nu\rho}{}_\beta$$

In Eigenmath code (transpose to interchange ν and ρ of $\gamma^{\nu\rho}{}_\beta$)

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T = dot(gamma,transpose(gamma))
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Calculate $\sigma^{\mu\nu}$ (transpose to interchange μ and ν of $T^{\mu\alpha\nu}{}_\beta$).

$$\texttt{sigmamunu} = \texttt{i/2 (T - transpose(T,1,3))}$$

Transpose $\sigma^{\mu\alpha\nu}{}_\beta$ to $\sigma^{\mu\alpha}{}_\beta{}^\nu$.

$$\texttt{sigmamunu} = \texttt{transpose(sigmamunu,3,4)}$$

In component notation

$$\sigma^{\mu\nu}(p_2 - p_1)_\nu = \sigma^{\mu\alpha}{}_\beta{}^\nu g_{\nu\rho}(p_2 - p_1)^\rho$$

In Eigenmath code

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dot(sigmamunu, gmunu, p2 - p1)
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