8-4. Show that the ground-state wave function for the Lagrangian of equation (8.78) can be written

$$\Phi_0 = A \exp\left(-\frac{1}{2\hbar}\sum_{\alpha=1}^{N-1}\omega_\alpha Q_\alpha^* Q_\alpha\right) \tag{8.83}$$

(where A is a constant) by starting with the wave function in terms of the real variables Q_α^c and Q_α^s.

Consider equation (8.82).

$$q_j(t) = \sqrt{\frac{2}{N}}\left(\frac{1}{2}Q_0^c(t) + \sum_{\alpha=1}^{(N-1)/2}\left(Q_\alpha^c(t)\cos\frac{2\pi\alpha j}{N} - Q_\alpha^s(t)\sin\frac{2\pi\alpha j}{N}\right)\right) \tag{8.82}$$

$$L = \frac{1}{2}\sum_{\alpha=0}^{N-1}\left(\dot{Q}_\alpha^*\dot{Q}_\alpha - \omega_\alpha^2 Q_\alpha^* Q_\alpha\right) \tag{8.78}$$

$$Q_\alpha^c = \frac{1}{\sqrt{2}}(Q_\alpha + Q_{-\alpha}) \tag{8.79}$$

$$Q_\alpha^s = \frac{i}{\sqrt{2}}(Q_\alpha - Q_{-\alpha}) \tag{8.80}$$

Consider this part of equation (8.63).

$$\Phi_0 = \exp\left(-\frac{1}{2\hbar}\sum_{\alpha=1}^{n}\omega_\alpha Q_\alpha^2\right) \tag{8.63}$$

How to go from Lagrangian to wave function?