# Spin flip

Consider an electron at rest in the following magnetic field.

$$\mathbf{B} = B_0 \cos(\omega t) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Find the minimum $B_0$ such that the sign of $\langle S_x \rangle$ flips periodically.

Start with the Hamiltonian for the electron.

$$H = \frac{ge}{2m}\mathbf{B} \cdot \mathbf{S} = \frac{ge}{2m} B_0 \cos(\omega t) S_z$$

Let $|s\rangle$ be the following spin state.

$$|s\rangle = \begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix}$$

Consider the Schrodinger equation for spin state $|s\rangle$.

$$i\hbar \frac{\partial}{\partial t}|s\rangle = H|s\rangle$$

By the Schrodinger equation we have for $c_1(t)$ and $c_2(t)$

$$\begin{aligned} i\hbar \frac{\partial}{\partial t} c_1(t) &= \frac{ge\hbar}{4m} B_0 \cos(\omega t) c_1(t) \\ i\hbar \frac{\partial}{\partial t} c_2(t) &= -\frac{ge\hbar}{4m} B_0 \cos(\omega t) c_2(t) \end{aligned}$$

The solutions for $c_1(t)$ and $c_2(t)$ are

$$\begin{aligned} c_1(t) &= C_1 \exp\left(-\frac{ige}{4m\omega} B_0 \sin(\omega t)\right) \\ c_2(t) &= C_2 \exp\left(\frac{ige}{4m\omega} B_0 \sin(\omega t)\right) \end{aligned} \tag{1}$$

where $C_1$ and $C_2$ are complex coefficients

$$C_1 = a_1 \exp(i\theta_1), \quad C_2 = a_2 \exp(i\theta_2), \quad |c_1(t)|^2 + |c_2(t)|^2 = a_1^2 + a_2^2 = 1$$

From spin operator

$$S_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

and (1) we have

$$\langle S_x \rangle = \langle s|S_x|s\rangle = a_1 a_2 \hbar \cos\left(\frac{ge}{2m\omega} B_0 \sin(\omega t) - \theta_1 + \theta_2\right) \tag{2}$$

Assume $\theta_1 = \theta_2$ for this problem so that

$$\langle S_x \rangle = a_1 a_2 \hbar \cos\left(\frac{ge}{2m\omega} B_0 \sin(\omega t)\right)$$

Note that if the sign of $\langle S_x \rangle$ is constant in time then

$$-\frac{\pi}{2} \leq \frac{ge}{2m\omega} B_0 \sin(\omega t) \leq \frac{\pi}{2}$$

For $t$ such that $\sin(\omega t) = 1$ we have

$$\frac{ge}{2m\omega} B_0 \leq \frac{\pi}{2}$$

Hence the sign of $\langle S_x \rangle$ changes periodically for

$$B_0 > \frac{\pi m\omega}{ge}$$

See exercise 10.6 of *Quantum Mechanics* (Lulu edition) by Richard Fitzpatrick.