7-1. If $S\big(x(t)\big) = \int_{t_a}^{t_b} L(\dot{x}, x, t)\,dt$, show that, for any s inside the range t_a to t_b,

$$\frac{\delta S}{\delta x(s)} = -\frac{d}{ds}\left(\frac{\partial L}{\partial \dot{x}}\right) + \frac{\partial L}{\partial x}$$

where the partial derivatives are evaluated at $t = s$.

From equation (2.6)

$$\delta S = \int_{t_a}^{t_b} \left(-\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} + \frac{\partial L}{\partial x}\right)\delta x(t)\,dt$$

Let $t_a = t_b = s$ to obtain

$$\delta S = \left(-\frac{d}{ds}\frac{\partial L}{\partial \dot{x}} + \frac{\partial L}{\partial x}\right)\delta x(s)$$

Divide through by $\delta x(s)$.

$$\frac{\delta S}{\delta x(s)} = -\frac{d}{ds}\frac{\partial L}{\partial \dot{x}} + \frac{\partial L}{\partial x}$$