

Spin state

The result of measuring spin is either $+\hbar/2$ or $-\hbar/2$. Let

$$\chi = \begin{pmatrix} \frac{1}{3} - \frac{2}{3}i \\ \frac{2}{3} \end{pmatrix}$$

For spin state χ and all three axes, find the probabilities of measuring $\pm\hbar/2$.

These are the eigenstates for spin.

$$\begin{array}{lll} |x_+\rangle = \frac{1}{\sqrt{2}}(1, 1) & |y_+\rangle = \frac{1}{\sqrt{2}}(1, i) & |z_+\rangle = (1, 0) \\ |x_-\rangle = \frac{1}{\sqrt{2}}(1, -1) & |y_-\rangle = \frac{1}{\sqrt{2}}(1, -i) & |z_-\rangle = (0, 1) \end{array}$$

For the x direction we have

$$\Pr(S_x = +\frac{\hbar}{2}) = |\langle x_+ | \chi \rangle|^2 = \frac{13}{18}, \quad \Pr(S_x = -\frac{\hbar}{2}) = |\langle x_- | \chi \rangle|^2 = \frac{5}{18}$$

For the y direction we have

$$\Pr(S_y = +\frac{\hbar}{2}) = |\langle y_+ | \chi \rangle|^2 = \frac{17}{18}, \quad \Pr(S_y = -\frac{\hbar}{2}) = |\langle y_- | \chi \rangle|^2 = \frac{1}{18}$$

For the z direction we have

$$\Pr(S_z = +\frac{\hbar}{2}) = |\langle z_+ | \chi \rangle|^2 = \frac{5}{9}, \quad \Pr(S_z = -\frac{\hbar}{2}) = |\langle z_- | \chi \rangle|^2 = \frac{4}{9}$$

Find $\langle S_x \rangle$, $\langle S_y \rangle$, and $\langle S_z \rangle$.

$$\begin{array}{ll} \langle x \rangle = \langle \chi | \sigma_x | \chi \rangle = \frac{4}{9}, & \langle S_x \rangle = \frac{\hbar}{2} \langle x \rangle = \frac{2}{9} \hbar \\ \langle y \rangle = \langle \chi | \sigma_y | \chi \rangle = \frac{8}{9}, & \langle S_y \rangle = \frac{\hbar}{2} \langle y \rangle = \frac{4}{9} \hbar \\ \langle z \rangle = \langle \chi | \sigma_z | \chi \rangle = \frac{1}{9}, & \langle S_z \rangle = \frac{\hbar}{2} \langle z \rangle = \frac{1}{18} \hbar \end{array}$$