

Use problem 3-2 and equation 3.42 to show that for a free particle wave function ψ we have

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar} \left(-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} \right)$$

From 3.42

$$\psi(x, t) = \int_{-\infty}^{\infty} K_0(x, t; x_c, t_c) \psi(x_c, t_c) dx_c$$

By linearity of differentiation

$$\begin{aligned} \frac{\partial \psi}{\partial t} &= \int_{-\infty}^{\infty} \frac{\partial}{\partial t} \left(K_0(x, t; x_c, t_c) \psi(x_c, t_c) dx_c \right) \\ \frac{\partial^2 \psi}{\partial x^2} &= \int_{-\infty}^{\infty} \frac{\partial^2}{\partial x^2} \left(K_0(x, t; x_c, t_c) \psi(x_c, t_c) dx_c \right) \end{aligned}$$

By independence of $\psi(x_c, t_c)$ and dx_c

$$\begin{aligned} \frac{\partial \psi}{\partial t} &= \int_{-\infty}^{\infty} \frac{\partial K_0}{\partial t} \psi(x_c, t_c) dx_c \\ \frac{\partial^2 \psi}{\partial x^2} &= \int_{-\infty}^{\infty} \frac{\partial^2 K_0}{\partial x^2} \psi(x_c, t_c) dx_c \end{aligned}$$

From problem 3-2

$$\frac{\partial K_0}{\partial t} = -\frac{i}{\hbar} \left(-\frac{\hbar^2}{2m} \frac{\partial^2 K_0}{\partial x^2} \right)$$

Hence by linearity

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar} \left(-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} \right)$$