

7-7. Show that for any quadratic action

$$\langle x(t) \rangle = \bar{x}(t) \langle 1 \rangle \quad (7.57)$$

From p. 59, $\bar{x}(t)$ is the classical path.

The following solution is similar to problem 2-1.

The general quadratic Lagrangian is given by equation (3.44).

$$L = a(t)\dot{x}^2 + b(t)\dot{x}x + c(t)x^2 + d(t)\dot{x} + e(t)x + f(t) \quad (3.44)$$

It follows that

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}} = 2 \frac{d}{dt} a(t) \dot{x} + 2a(t) \ddot{x} + \frac{d}{dt} b(t) x + b(t) \dot{x} + \frac{d}{dt} d(t) \quad (1)$$

and

$$\frac{\partial L}{\partial x} = b(t) \dot{x} + 2c(t)x + e(t) \quad (2)$$

Consider equation (2.7) which determines the classical path $\bar{x}(t)$.

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}} = \frac{\partial L}{\partial x} \quad (2.7)$$

By equation (2.7) and (1) and (2) we have

$$\ddot{x} = 0$$

Hence velocity \dot{x} is constant and equals distance divided by time.

$$\dot{x} = \frac{x_b - x_a}{t_b - t_a}$$

It follows that

$$\bar{x}(t) = x_a + \frac{t}{t_b - t_a} (x_b - x_a) \quad (3)$$

Consider equation (7.56).

$$\langle x(t) \rangle = \left(x_a + \frac{t}{T} (x_b - x_a) \right) \langle 1 \rangle \quad (7.56)$$

Substitute (3) into (7.56) to obtain (7.57).

$$\langle x(t) \rangle = \bar{x}(t) \langle 1 \rangle \quad (7.57)$$