

## Laplacian

The Laplacian  $\nabla^2$  is the divergence of the gradient of scalar function  $f$ .

$$\nabla^2 f = \nabla \cdot \nabla f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

`div(grad(f()))`

$$d(d(f(), x), x) + d(d(f(), y), y) + d(d(f(), z), z)$$

This is the vector Laplacian.

$$\nabla^2 \mathbf{A} = \nabla \cdot \nabla \mathbf{A} - \nabla \times (\nabla \times \mathbf{A}) = \begin{pmatrix} \nabla^2 A_x \\ \nabla^2 A_y \\ \nabla^2 A_z \end{pmatrix} = \frac{\partial^2 \mathbf{A}}{\partial x^2} + \frac{\partial^2 \mathbf{A}}{\partial y^2} + \frac{\partial^2 \mathbf{A}}{\partial z^2}$$

`A = (Ax(), Ay(), Az())`  
`div(grad(A)) - curl(curl(A)) == d(A,x,x) + d(A,y,y) + d(A,z,z)`

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Show that

$$\nabla \cdot \nabla \mathbf{A} = \nabla(\nabla \cdot \mathbf{A})$$

`div(grad(A)) == grad(div(A))`

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