

Spin part 1

Let $|s\rangle$ be a spin with polar angle θ and azimuth angle ϕ .

$$|s\rangle = \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2) e^{i\phi} \end{pmatrix}$$

Spin measurement probabilities are the transition probabilities from $|s\rangle$ to an eigenstate.

In the z direction the eigenstates are

$$|z_+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |z_-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Hence in the z direction

$$\begin{aligned} \Pr(+ &)= |\langle z_+ | s \rangle|^2 = \cos^2(\theta/2) = \frac{1}{2} + \frac{1}{2} \cos \theta \\ \Pr(- &)= |\langle z_- | s \rangle|^2 = \sin^2(\theta/2) = \frac{1}{2} - \frac{1}{2} \cos \theta \end{aligned}$$

In the x direction the eigenstates are

$$|x_+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad |x_-\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Hence in the x direction

$$\begin{aligned} \Pr(+ &)= |\langle x_+ | s \rangle|^2 = \frac{1}{2} + \frac{1}{2} \sin \theta \cos \phi \\ \Pr(- &)= |\langle x_- | s \rangle|^2 = \frac{1}{2} - \frac{1}{2} \sin \theta \cos \phi \end{aligned}$$

In the y direction the eigenstates are

$$|y_+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}, \quad |y_-\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

Hence in the y direction

$$\begin{aligned} \Pr(+ &)= |\langle y_+ | s \rangle|^2 = \frac{1}{2} + \frac{1}{2} \sin \theta \sin \phi \\ \Pr(- &)= |\langle y_- | s \rangle|^2 = \frac{1}{2} - \frac{1}{2} \sin \theta \sin \phi \end{aligned}$$

For each direction we have $\Pr(+) + \Pr(-) = 1$ as required by total probability.