Fine structure 2

Let E_n be the Bohr energy for hydrogen.

$$E_n = -\frac{\mu c^2 \alpha^2}{2n^2}$$

Let E_r be the first order relativistic correction.

$$E_r = -\frac{E_n^2}{2\mu c^2}\left(\frac{4n}{l+\frac{1}{2}} - 3\right)$$

Let E_{so} be the first order spin-orbit correction.

$$E_{so} = \frac{nE_n^2}{\mu c^2}\left(\frac{j(j+1) - l(l+1) - \frac{3}{4}}{l\left(l+\frac{1}{2}\right)(l+1)}\right)$$

Show that

$$E_n + E_r + E_{so} = E_n\left(1 + \frac{\alpha^2}{n\left(j+\frac{1}{2}\right)} - \frac{3\alpha^2}{4n^2}\right) \tag{1}$$

for $j = l \pm 1/2$.