(10.1) Show that  $[\phi(x), P^{\alpha}] = i\partial^{\alpha}\phi(x)$ , where  $P^{\alpha}$  is the conserved charges from spacetime translation (eqn 10.28).

We have

$$[\phi(x), P] = iD\phi(x)$$

where  $D\phi$  is a directional derivative.

In component notation

$$[\phi(x), a_{\mu}P^{\mu}] = ia_{\mu}\partial^{\mu}\phi(x)$$

Hence

$$[\phi(x), P^{\mu}] = i\partial^{\mu}\phi(x)$$