Spin operator table

From "Quantum Mechanics" by Susskind and Friedman, p. 350.

Verify the following spin operator table.

	$\vert uu\rangle$	$\vert ud\rangle$	$\vert du\rangle$	$\vert dd\rangle$
σ_z	$\vert uu\rangle$	$\vert ud\rangle$	$-\vert du\rangle$	$-\vert dd\rangle$
σ_x	$\vert du\rangle$	$\vert dd\rangle$	$\vert uu\rangle$	$\vert ud\rangle$
σ_y	$i\vert du\rangle$	$i\vert dd\rangle$	$-i\vert uu\rangle$	$-i\vert ud\rangle$
τ_z	$\vert uu\rangle$	$-\vert ud\rangle$	$\vert du\rangle$	$-\vert dd\rangle$
τ_x	$\vert ud\rangle$	$\vert uu\rangle$	$\vert dd\rangle$	$\vert du\rangle$
τ_y	$i\vert ud\rangle$	$-i\vert uu\rangle$	$i\vert dd\rangle$	$-i\vert du\rangle$

Table elements are final states. For example,

$$\sigma_z|dd\rangle = -|dd\rangle$$

For single spins we have

$$|u\rangle = \begin{pmatrix}1\\0\end{pmatrix}, \quad |d\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

For a system of two spins we use the Kronecker product of single spins.

$$\begin{aligned}|uu\rangle &= |u\rangle \otimes |u\rangle\\ |ud\rangle &= |u\rangle \otimes |d\rangle\\ |du\rangle &= |d\rangle \otimes |u\rangle\\ |dd\rangle &= |d\rangle \otimes |d\rangle\end{aligned}$$

Hence

$$|uu\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}, \quad |ud\rangle = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \quad |du\rangle = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}, \quad |dd\rangle = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$$

The spin operators for single spins are

$$\sigma_z = \tau_z = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}, \quad \sigma_x = \tau_x = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}, \quad \sigma_y = \tau_y = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix}$$

For a system of two spins, the σ operators operate on the first spin and the τ operators operate on the second spin.

$$\sigma_z = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix} \otimes I = \begin{pmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & -1 & 0\\0 & 0 & 0 & -1\end{pmatrix}$$

$$\tau_z = I \otimes \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix} = \begin{pmatrix}1 & 0 & 0 & 0\\0 & -1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & -1\end{pmatrix}$$

Eigenmath script