

What is the mean lifetime of  $2p$  hydrogen?

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Let us begin by writing down the wave function  $\psi$  for hydrogen.

$$\psi_{nlm}(r, \theta, \phi) = R_{nl}(r)Y_{lm}(\theta, \phi)$$

where

$$R_{nl}(r) = \frac{2}{n^2} \left( \frac{(n-l-1)!}{(n+l)!} \right)^{1/2} \left( \frac{2r}{na_0} \right)^l L_{n-l-1}^{2l+1} \left( \frac{2r}{na_0} \right) \exp \left( -\frac{r}{na_0} \right) a_0^{-3/2}$$

$$L_n^m(x) = (n+m)! \sum_{k=0}^n \frac{(-x)^k}{(n-k)!(m+k)!k!}$$

$$Y_{lm}(\theta, \phi) = (-1)^m \left( \frac{2l+1}{4\pi} \right)^{1/2} \left( \frac{(l-m)!}{(l+m)!} \right)^{1/2} P_l^m(\cos \theta) \exp(im\phi)$$

$$P_n^m(x) = \frac{1}{2^n n!} (1-x^2)^{m/2} \frac{d^{n+m}}{dx^{n+m}} (x^2-1)^n$$

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{e^2\mu} \approx 0.529 \times 10^{-10} \text{ meter}$$

The state  $2p$  means that  $n = 2$  and  $l = 1$ . That leaves three ways to choose  $m$  hence all of the following processes correspond to the transition  $2p \rightarrow 1s$ .

$$\left. \begin{array}{l} \psi_{2,1,1} \\ \psi_{2,1,0} \\ \psi_{2,1,-1} \end{array} \right\} \rightarrow \psi_{1,0,0}$$