

Exercise 9.2. Prove Eq. 9.10 by expanding each side and comparing the results.

We are given

$$\mathbf{X} = x, \quad \mathbf{P} = -i\hbar \frac{\partial}{\partial x}, \quad \mathbf{P}^2 = -\hbar^2 \frac{\partial^2}{\partial x^2}$$

and

$$[\mathbf{P}^2, \mathbf{X}] = \mathbf{P}[\mathbf{P}, \mathbf{X}] + [\mathbf{P}, \mathbf{X}]\mathbf{P} \quad (9.10)$$

Note that

$$\mathbf{P}\mathbf{X} = -i\hbar \frac{\partial}{\partial x} x = -i\hbar, \quad \mathbf{P}^2\mathbf{X} = -\hbar^2 \frac{\partial^2}{\partial x^2} x = 0$$

We have

$$[\mathbf{P}^2, \mathbf{X}] = \mathbf{P}^2\mathbf{X} - \mathbf{X}\mathbf{P}^2 = -\mathbf{X}\mathbf{P}^2 \quad (1)$$

$$\mathbf{P}[\mathbf{P}, \mathbf{X}] = \mathbf{P}\mathbf{P}\mathbf{X} - \mathbf{P}\mathbf{X}\mathbf{P} = i\hbar\mathbf{P} \quad (2)$$

$$[\mathbf{P}, \mathbf{X}]\mathbf{P} = \mathbf{P}\mathbf{X}\mathbf{P} - \mathbf{X}\mathbf{P}\mathbf{P} = -i\hbar\mathbf{P} - \mathbf{X}\mathbf{P}^2 \quad (3)$$

Substitute (1), (2), and (3) into (9.10) to obtain

$$-\mathbf{X}\mathbf{P}^2 = -\mathbf{X}\mathbf{P}^2$$

Hence (9.10) is proved.