

Balmer's formula

The following table is from “Note on the spectral lines of hydrogen” by J. J. Balmer dated 1885. Numerical values are wavelengths in units of 10^{-10} meter.

Investigator	H_α	H_β	H_γ	H_δ	H_ϵ	H_ζ	H_η	H_ϑ	H_ι
Van der Willigen	6565.6	4863.94	4342.80	4103.8	—	—	—	—	—
Angstrom	6562.10	4860.74	4340.10	4101.2	—	—	—	—	—
Mendenhall	6561.62	4860.16	—	—	—	—	—	—	—
Mascart	6560.7	4859.8	—	—	—	—	—	—	—
Ditscheiner	6559.5	4859.74	4338.60	4100.0	—	—	—	—	—
Huggins	—	—	—	—	—	3887.5	3834	3795	3767.5
Vogel	—	—	—	—	3969	3887	3834	3795	3769 [†]

([†]The value given in the paper is 6769 which is an obvious typo.)

Balmer discovered the following formula for fitting the data (λ is wavelength).

$$\lambda = \frac{m^2}{m^2 - 2^2} \times 3645.6 \times 10^{-10} \text{ meter}$$

Parameter m is from the following table.

	H_α	H_β	H_γ	H_δ	H_ϵ	H_ζ	H_η	H_ϑ	H_ι
$m =$	3	4	5	6	7	8	9	10	11

Let β be the model coefficient for λ . Using linear regression and the above data we obtain

$$\beta = 3645.3 \times 10^{-10} \text{ meter}$$

The currently accepted value is

$$\beta = \frac{4}{R_H} = 3647.1 \times 10^{-10} \text{ meter}$$

where R_H is the Rydberg constant for hydrogen

$$R_H = 1.09677576 \times 10^7 \text{ meter}^{-1}$$

Balmer's coefficient is within 0.04% of the modern value.

$$100 \times \frac{4/R_H - 3.6456}{4/R_H} = 0.04$$