Rutherford scattering data

The following data is from Geiger and Marsden's 1913 paper.[1] Column y is number of scattering events for silver foil.

k	θ	y
1	150	22.2
2	135	27.4
3	120	33.0
4	105	47.3
5	75	136
6	60	320
7	45	989
8	37.5	1760
9	30	5260
10	22.5	20300
11	15	105400

This is the differential cross section for Rutherford scattering.

$$\frac{d\sigma}{d\Omega} \propto \frac{1}{(1-\cos\theta)^2}$$

Let $f(k)$ be the probability of scattering through region θ_k. Then

$$f(k) = \Pr(\theta = \theta_k) \propto \left.\frac{d\sigma}{d\Omega}\right|_{\theta=\theta_k}$$

Hence

$$f(k) = \frac{C}{(1-\cos\theta_k)^2}$$

where C is a normalization constant. To find C let

$$x_k = \frac{1}{(1-\cos\theta_k)^2}$$

By total probability

$$\sum_k f(k) = C\sum_k x_k = 1$$

It follows that

$$C = \frac{1}{\sum_k x_k} = \frac{1}{1132}$$

Hence the scattering probability for angle θ_k is

$$f(k) = \frac{x_k}{1132} = \frac{1}{1132\,(1-\cos\theta_k)^2}$$

[1] www.chemteam.info/Chem-History/GeigerMarsden-1913/GeigerMarsden-1913.html

Let $\hat{y}_k$ be predicted number of scattering events such that

$$\Pr(y = \hat{y}_k) = \Pr(\theta = \theta_k)$$

It follows that

$$\frac{\hat{y}_k}{\sum y} = f(k)$$

Hence

$$\hat{y}_k = f(k) \sum y$$

The following table shows the predicted values.

θ	y	$\hat{y}$
150	22.2	34.1
135	27.4	40.7
120	33.0	52.7
105	47.3	74.9
75	136	216
60	320	474
45	989	1383
37.5	1760	2778
30	5260	6608
22.5	20300	20471
15	105400	102162