Let θ_{12} be the angular distance between $P_1 = (r_1, \theta_1, \phi_1)$ and $P_2 = (r_2, \theta_2, \phi_2)$. Then

$$\cos\theta_{12} = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \cos(\phi_1 - \phi_2)$$

$$\cos\theta_{12} = \frac{r_1^2 + r_2^2 - r_{12}^2}{2 r_1 r_2}$$

$$\cos\theta_{12} = \frac{x_1 x_2 + y_1 y_2 + z_1 z_2}{r_1 r_2}$$

where

$$r_{12}^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$$

and

$$\begin{aligned} x_1 &= r_1 \sin\theta_1 \cos\phi_1 \\ y_1 &= r_1 \sin\theta_1 \sin\phi_1 \\ z_1 &= r_1 \cos\theta_1 \end{aligned}$$

$$\begin{aligned} x_2 &= r_2 \sin\theta_2 \cos\phi_2 \\ y_2 &= r_2 \sin\theta_2 \sin\phi_2 \\ z_2 &= r_2 \cos\theta_2 \end{aligned}$$