Feynman and Hibbs problem 5-4

Suppose the wave function for a system is $\psi(x)$ at time $t_a$. Suppose further that the behavior of the system described by the kernel $K(x_b, t_b, x_a, t_a)$ for motions in the interval $t_b \geq t \geq t_a$. Show that the probability that the system is found to be in state $\chi(x)$ at time $t_b$ is given by the square of the integral

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \chi^*(x_b) K(x_b, t_b, x_a, t_a) \psi(x_a) \, dx_a \, dx_b \tag{1}$$

We call this integral the "transition amplitude" to go from state $\psi(x)$ to state $\chi(x)$.

From equation (3.42)

$$\psi(x_b, t_b) = \int_{-\infty}^{\infty} K(x_b, t_b, x_a, t_a) \psi(x_a, t_a) \, dx_a$$

Hence the integral over $x_a$ in (1) is equivalent to $\psi(x_b)$.

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \chi^*(x_b) K(x_b, t_b, x_a, t_a) \psi(x_a) \, dx_a \, dx_b = \int_{-\infty}^{\infty} \chi^*(x_b) \psi(x_b) \, dx_b$$

Then by equation (5.32)

$$P(\mathrm{X}) = \left| \int_{-\infty}^{\infty} \chi^*(x_b) \psi(x_b) \, dx_b \right|^2$$

where X is upper-case $\chi$.