The following transitions correspond to the H-α line of the hydrogen spectrum. See "Atomic Transition Probabilities Volume I," issued May 20, 1966, page 2.

Transition	λ (Å)	A_{ki} (second^{-1})
$2p-3s$	6562.86	6.313×10^{6}
$2s-3p$	6562.74	2.245×10^{7}
$2p-3d$	6562.81	6.465×10^{7}

A_{ki} is the spontaneous emission rate where i is the lower state and k is the upper state. Orbital names correspond to the following azimuthal quantum numbers ℓ.

Orbital	ℓ
s	0
p	1
d	2

Each transition in the table has multiple processes due to the magnetic quantum number m_ℓ. (Recall that $m_\ell = 0, \pm1, \ldots, \pm\ell$.)

There are three ways to transition from $3s$ to $2p$.

$$\begin{aligned}\psi_{3,0,0} &\rightarrow \psi_{2,1,1}\\ \psi_{3,0,0} &\rightarrow \psi_{2,1,0}\\ \psi_{3,0,0} &\rightarrow \psi_{2,1,-1}\end{aligned}$$

There are three ways to transition from $3p$ to $2s$.

$$\begin{aligned}\psi_{3,1,1} &\rightarrow \psi_{2,0,0}\\ \psi_{3,1,0} &\rightarrow \psi_{2,0,0}\\ \psi_{3,1,-1} &\rightarrow \psi_{2,0,0}\end{aligned}$$

Finally, there are fifteen ways to transition from $3d$ to $2p$. (Some of these transitions have zero amplitude.)

$$\begin{aligned}
\psi_{3,2,2} &\rightarrow \psi_{2,1,1} & \psi_{3,2,2} &\rightarrow \psi_{2,1,0} & \psi_{3,2,2} &\rightarrow \psi_{2,1,-1}\\
\psi_{3,2,1} &\rightarrow \psi_{2,1,1} & \psi_{3,2,1} &\rightarrow \psi_{2,1,0} & \psi_{3,2,1} &\rightarrow \psi_{2,1,-1}\\
\psi_{3,2,0} &\rightarrow \psi_{2,1,1} & \psi_{3,2,0} &\rightarrow \psi_{2,1,0} & \psi_{3,2,0} &\rightarrow \psi_{2,1,-1}\\
\psi_{3,2,-1} &\rightarrow \psi_{2,1,1} & \psi_{3,2,-1} &\rightarrow \psi_{2,1,0} & \psi_{3,2,-1} &\rightarrow \psi_{2,1,-1}\\
\psi_{3,2,-2} &\rightarrow \psi_{2,1,1} & \psi_{3,2,-2} &\rightarrow \psi_{2,1,0} & \psi_{3,2,-2} &\rightarrow \psi_{2,1,-1}
\end{aligned}$$

For each H-α line, an average A_{ki} is computed by summing over A_{ki} for individual processes and dividing by the number of distinct initial states. For example, $3d \rightarrow 2p$ has five distinct initial states, so the divisor is five.

A_{ki} is computed from the following formula.

$$A_{ki} = \frac{e^2}{3\pi\varepsilon_0\hbar c^3}\,\omega_{ki}^3\,|r_{ki}|^2$$

The transition frequency ω_{ki} is given by Bohr's frequency condition.

$$\omega_{ki} = \frac{1}{\hbar}(E_k - E_i)$$

The transition probability (multiplied by a physical constant) is

$$|r_{ki}|^2 = |x_{ki}|^2 + |y_{ki}|^2 + |z_{ki}|^2$$

These are the transition amplitudes.

$$\begin{aligned}
x_{ki} &= \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} \psi_i^* \, r \sin\theta \cos\phi \, \psi_k \, r^2 \sin\theta \, dr \, d\theta \, d\phi \\
y_{ki} &= \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} \psi_i^* \, r \sin\theta \sin\phi \, \psi_k \, r^2 \sin\theta \, dr \, d\theta \, d\phi \\
z_{ki} &= \int_0^{2\pi} \int_0^{\pi} \int_0^{\infty} \psi_i^* \, r \cos\theta \, \psi_k \, r^2 \sin\theta \, dr \, d\theta \, d\phi
\end{aligned}$$

Using Eigenmath we obtain the following values for average A_{ki}. The results are essentially identical to the values found in "Atomic Transition Probabilities."

$$\begin{aligned}
A_{3s2p} &= 6.31358 \times 10^6 \,\text{second}^{-1} \\
A_{3p2s} &= 2.24483 \times 10^7 \,\text{second}^{-1} \\
A_{3d2p} &= 6.4651 \times 10^7 \,\text{second}^{-1}
\end{aligned}$$

The following table shows $|r_{ki}|^2/a_0^2$ for each transition process where a_0 is Bohr radius and $a_0^2 = 2.8 \times 10^{-21}$ meter^2. Some of the $|r_{ki}|^2$ are zero indicating forbidden transitions.

Initial state	Final state $\psi_{2,0,0}$	Final state $\psi_{2,1,1}$	Final state $\psi_{2,1,0}$	Final state $\psi_{2,1,-1}$
$\psi_{3,0,0}$	–	0.293534	0.293534	0.293534
$\psi_{3,1,1}$	3.13103	–	–	–
$\psi_{3,1,0}$	3.13103	–	–	–
$\psi_{3,1,-1}$	3.13103	–	–	–
$\psi_{3,2,2}$	–	9.01737	0	0
$\psi_{3,2,1}$	–	4.50868	4.50868	0
$\psi_{3,2,0}$	–	1.50289	6.01158	1.50289
$\psi_{3,2,-1}$	–	0	4.50868	4.50868
$\psi_{3,2,-2}$	–	0	0	9.01737