Use problem 3-2 and equation 3.42 to show that for a free particle wave function ψ

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar}\left(-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}\right)$$

From 3.42

$$\psi(x,t) = \int_{-\infty}^{\infty} K_0(x,t;x_c,t_c)\psi(x_c,t_c)\,dx_c$$

By linearity of differentiation

$$\frac{\partial \psi}{\partial t} = \int_{-\infty}^{\infty} \frac{\partial}{\partial t}\Big(K_0(x,t;x_c,t_c)\psi(x_c,t_c)\,dx_c\Big)$$
$$\frac{\partial^2 \psi}{\partial x^2} = \int_{-\infty}^{\infty} \frac{\partial^2}{\partial x^2}\Big(K_0(x,t;x_c,t_c)\psi(x_c,t_c)\,dx_c\Big)$$

By indepedence of $\psi(x_c,t_c)$ and dx_c

$$\frac{\partial \psi}{\partial t} = \int_{-\infty}^{\infty} \frac{\partial K_0}{\partial t}\psi(x_c,t_c)\,dx_c$$
$$\frac{\partial^2 \psi}{\partial x^2} = \int_{-\infty}^{\infty} \frac{\partial^2 K_0}{\partial x^2}\psi(x_c,t_c)\,dx_c$$

From problem 3-2

$$\frac{\partial K_0}{\partial t} = -\frac{i}{\hbar}\left(-\frac{\hbar^2}{2m}\frac{\partial^2 K_0}{\partial x^2}\right)$$

Hence by linearity

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar}\left(-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}\right)$$