

Consider a light wave propagating in the z direction. For simplicity let the light be linearly polarized with electric field vector \vec{E} pointing in the x direction.

$$\vec{E}(t, x, y, z) = \begin{pmatrix} E_x \cos(kz - \omega t) \\ 0 \\ 0 \end{pmatrix}$$

Symbol ω is angular frequency and k is the wave number $k = \omega/c$.

The corresponding wave function is

$$\psi = A \left| n - \frac{1}{2} \right\rangle + B \left| n + \frac{1}{2} \right\rangle$$

where n is the number of photons per unit volume and

$$A = \exp \left(-i \left(n - \frac{1}{2} \right) \omega t \right)$$

$$B = \exp \left(-i \left(n + \frac{1}{2} \right) \omega t \right)$$

The electric field operator is

$$\hat{\mathcal{E}} = C\hat{a} + C^*\hat{a}^\dagger$$

where \hat{a} and \hat{a}^\dagger are the lowering and raising operators such that

$$a \left| n + \frac{1}{2} \right\rangle = \sqrt{n} \left| n - \frac{1}{2} \right\rangle$$

$$a^\dagger \left| n - \frac{1}{2} \right\rangle = \sqrt{n} \left| n + \frac{1}{2} \right\rangle$$

The quantity C is

$$C = \sqrt{\frac{\hbar\omega}{2V\varepsilon_0}} \exp(ikz)$$

where V is a unit volume.

Apply electric field operator $\hat{\mathcal{E}}$ to wave function ψ .

$$\begin{aligned} \hat{\mathcal{E}}\psi &= C\hat{a}\psi + C^*\hat{a}^\dagger\psi \\ &= CA\sqrt{n-1} \left| n - \frac{3}{2} \right\rangle + CB\sqrt{n} \left| n - \frac{1}{2} \right\rangle + C^*A\sqrt{n} \left| n + \frac{1}{2} \right\rangle + C^*B\sqrt{n+1} \left| n + \frac{3}{2} \right\rangle \end{aligned}$$

The observed electric field is the eigenvalue \mathcal{E} such that $\hat{\mathcal{E}}\psi = \mathcal{E}\psi$.

$$\begin{aligned} \mathcal{E} &= \psi^\dagger \hat{\mathcal{E}} \psi \\ &= \left\langle n - \frac{1}{2} \right| A^* C B \sqrt{n} \left| n - \frac{1}{2} \right\rangle + \left\langle n + \frac{1}{2} \right| B^* C^* A \sqrt{n} \left| n + \frac{1}{2} \right\rangle \\ &= \sqrt{n} C \exp(-i\omega t) + \sqrt{n} C^* \exp(i\omega t) \\ &= \sqrt{\frac{2n\hbar\omega}{V\varepsilon_0}} \cos(kz - \omega t) \end{aligned}$$

Identifying \mathcal{E} as the first component of \vec{E} we have

$$E_x \cos(kz - \omega t) = \sqrt{\frac{2n\hbar\omega}{V\varepsilon_0}} \cos(kz - \omega t)$$

Hence the electric field amplitude E_x is proportional to the square root of photon density.

$$E_x = \sqrt{\frac{2n\hbar\omega}{V\varepsilon_0}}$$