This is equation (4.59).

$$K(x_b, t_b; x_a, t_a) = \begin{cases} \sum_{n=1}^{\infty} \phi_n(x_b)\phi_n^*(x_a) \exp\left(-\frac{i}{\hbar}E_n(t_b - t_a)\right) & t_b > t_a \\ 0 & t_b < t_a \end{cases} \tag{4.59}$$

Let

$$I = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \chi^*(x_b) K(x_b, T; x_a, 0) \psi(x_a)\, dx_a\, dx_b$$

By (8.23) and (4.59) we have

$$I = \sum_j \sum_n \sum_k \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \chi_j^* \phi_j^*(x_b) \\ \times \phi_n(x_b)\phi_n^*(x_a) \exp\left(-\frac{i}{\hbar}E_n T\right) \psi_k \phi_k(x_a)\, dx_a\, dx_b$$

The integrals for $j, k \neq n$ vanish by orthogonality hence

$$I = \sum_n \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \chi_n^* \phi_n^*(x_b)\; \phi_n(x_b)\phi_n^*(x_a) \exp\left(-\frac{i}{\hbar}E_n T\right)\; \psi_n \phi_n(x_a)\, dx_a\, dx_b$$

Rewrite as

$$I = \sum_n \chi_n^* \exp\left(-\frac{i}{\hbar}E_n T\right) \psi_n \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_n^*(x_b)\phi_n(x_b)\phi_n^*(x_a)\phi_n(x_a)\, dx_a\, dx_b$$

The remaining integrals are unity by normalization hence

$$I = \sum_n \chi_n^* \exp\left(-\frac{i}{\hbar}E_n T\right) \psi_n$$

FIXME problem cont'd