(7.2) Consider a massive scalar field $\phi(x)$ coupled to a source $J(x)$, described by the Lagrangian of eqn 7.10. Show that the equations of motion are those of eqn 7.11.

$$\mathcal{L} = \frac{1}{2}[\partial_u \phi(x)]^2 - \frac{1}{2}m^2[\phi(x)]^2 + J(x)\phi(x) \tag{7.10}$$

$$(\partial_\mu \partial^\mu + m^2)\phi(x) = J(x) \tag{7.11}$$

For the Lagrangian $\mathcal{L}$ given in (7.10) we have

$$\frac{\partial \mathcal{L}}{\partial \phi} = -m^2\phi(x) + J(x)$$

and

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = \partial^\mu \phi(x)$$

Then by the Euler-Lagrange equation we have

$$\partial_\mu \partial^\mu \phi(x) + m^2\phi(x) - J(x) = 0$$

which is equivalent to (7.11).