

Let  $|\Psi\rangle$  be a coherent state where  $\bar{n}$  is the expected number of photons.

$$|\Psi\rangle = \sum_{n=0}^{\infty} \sqrt{\frac{\bar{n}^n \exp(-\bar{n})}{n!}} \exp\left(-i\left(n + \frac{1}{2}\right)\omega t\right) |n\rangle$$

It can be shown that<sup>1</sup>

$$\hat{a}|\Psi\rangle = \sqrt{\bar{n}} \exp(-i\omega t)|\Psi\rangle$$

It follows that

$$\langle\Psi|\hat{a}^\dagger = (\hat{a}|\Psi\rangle)^\dagger = \sqrt{\bar{n}} \exp(i\omega t)\langle\Psi|$$

Let  $\hat{E}$  be the electric field operator

$$\hat{E} = i\sqrt{\frac{\hbar\omega}{2\epsilon_0}}(\hat{a} - \hat{a}^\dagger)$$

The expected electric field for the coherent state is

$$\langle\hat{E}\rangle = \langle\Psi|\hat{E}|\Psi\rangle = i\sqrt{\frac{\hbar\omega}{2\epsilon_0}}\langle\Psi|(\hat{a} - \hat{a}^\dagger)|\Psi\rangle$$

Hence

$$\langle\hat{E}\rangle = i\sqrt{\frac{\hbar\omega}{2\epsilon_0}} (\sqrt{\bar{n}} \exp(-i\omega t)\langle\Psi|\Psi\rangle - \sqrt{\bar{n}} \exp(i\omega t)\langle\Psi|\Psi\rangle)$$

By  $\langle\Psi|\Psi\rangle = 1$  we have

$$\langle\hat{E}\rangle = i\sqrt{\frac{\hbar\omega}{2\epsilon_0}} (\sqrt{\bar{n}} \exp(-i\omega t) - \sqrt{\bar{n}} \exp(i\omega t))$$

Recalling that

$$2\sin(\omega t) = i\exp(-i\omega t) - i\exp(i\omega t)$$

we have

$$\langle\hat{E}\rangle = \sqrt{\frac{2\hbar\omega\bar{n}}{\epsilon_0}} \sin(\omega t)$$

Hence the peak amplitude is proportional to  $\sqrt{\bar{n}}$ .

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<sup>1</sup>See *Quantum Mechanics for Scientists and Engineers* problem 15.6.1.