

Feynman and Hibbs problem 5-6

Suppose A , B , and C are the three cartesian coordinates of momentum p_x , p_y , p_z . What is the form of function the $\chi_{a,b,c}(x, y, z)$?

From equation (5.36)

$$F_{a,b,c} = \int_{\mathbb{R}^3} \chi_{a,b,c}^*(\mathbf{x}) f(\mathbf{x}) dx dy dz \quad (1)$$

From equation (5.6)

$$\phi(\mathbf{p}) = \int_{\mathbb{R}^3} \exp\left(-\frac{i\mathbf{p} \cdot \mathbf{x}}{\hbar}\right) f(\mathbf{x}) dx dy dz \quad (2)$$

where

$$\mathbf{p} = (p_x, p_y, p_z)$$

Equation (1) is the amplitude for the state (a, b, c) . Equation (2) is the amplitude for \mathbf{p} . Equate (1) with (2) to find $\chi_{a,b,c}$ for $\mathbf{p} = (a, b, c)$.

$$F_{a,b,c} = \int_{\mathbb{R}^3} \chi_{a,b,c}^*(\mathbf{x}) f(\mathbf{x}) dx dy dz = \int_{\mathbb{R}^3} \exp\left(-\frac{i\mathbf{p} \cdot \mathbf{x}}{\hbar}\right) f(\mathbf{x}) dx dy dz$$

Hence one solution is

$$\chi_{a,b,c}^*(\mathbf{x}) = \exp\left(-\frac{i\mathbf{p} \cdot \mathbf{x}}{\hbar}\right)$$

and

$$\chi_{a,b,c}(\mathbf{x}) = \exp\left(\frac{i\mathbf{p} \cdot \mathbf{x}}{\hbar}\right)$$