The following table of hydrogen transition data is from “Atomic Transition Probabilities,” 1966.

Transition	λ(Å)	E_i(cm^{-1})	E_k(cm^{-1})	g_i	g_k	A_{ki}(sec^{-1})
$1s-2p$	1215.67	0	82259	2	6	6.265×10^8
$1s-3p$	1025.72	0	97492	2	6	1.672×10^8
$1s-4p$	972.537	0	102824	2	6	6.818×10^7
$1s-5p$	949.743	0	105292	2	6	3.437×10^7
$1s-6p$	937.804	0	106632	2	6	1.973×10^7
$2p-3s$	6562.86	82259	97492	6	2	6.313×10^6
$2p-4s$	4861.35	82259	102824	6	2	2.578×10^6
$2p-5s$	4340.48	82259	105292	6	2	1.289×10^6
$2p-6s$	4101.75	82259	106632	6	2	7.350×10^5
$2s-3p$	6562.74	82259	97492	2	6	2.245×10^7
$2s-4p$	4861.29	82259	102824	2	6	9.668×10^6
$2s-5p$	4340.44	82259	105292	2	6	4.948×10^6
$2s-6p$	4101.71	82259	106632	2	6	2.858×10^6
$2p-3d$	6562.81	82259	97492	6	10	6.465×10^7
$2p-4d$	4861.33	82259	102824	6	10	2.062×10^7
$2p-5d$	4340.47	82259	105292	6	10	9.425×10^6
$2p-6d$	4101.74	82259	106632	6	10	5.145×10^6

The $3 \to 2$ transitions emit the bright red H-α line.

Transition	λ (Å)	A_{ki} (second^{-1})
$2p - 3s$	6562.86	6.313×10^6
$2s - 3p$	6562.74	2.245×10^7
$2p - 3d$	6562.81	6.465×10^7

Let us compute the spontaneous emission coefficients A_{ki} for H-α and see if the results match the table.

The orbital names correspond to the following angular momenta.

Letter	Angular momentum ℓ
s	0
p	1
d	2

Because of the magnetic quantum number m_ℓ there are multiple processes for each transition.

There are three processes for the transition $3s \to 2p$.

$$\psi_{3,0,0} \to \psi_{2,1,1}$$
$$\psi_{3,0,0} \to \psi_{2,1,0}$$
$$\psi_{3,0,0} \to \psi_{2,1,-1}$$

There are three processes for the transition $3p \to 2s$.

$$\psi_{3,1,1} \to \psi_{2,0,0}$$
$$\psi_{3,1,0} \to \psi_{2,0,0}$$
$$\psi_{3,1,-1} \to \psi_{2,0,0}$$

Finally, there are fifteen processes for the transition $3d \to 2p$.

$$
\begin{array}{ccc}
\psi_{3,2,2} \to \psi_{2,1,1} & \psi_{3,2,2} \to \psi_{2,1,0} & \psi_{3,2,2} \to \psi_{2,1,-1} \\
\psi_{3,2,1} \to \psi_{2,1,1} & \psi_{3,2,1} \to \psi_{2,1,0} & \psi_{3,2,1} \to \psi_{2,1,-1} \\
\psi_{3,2,0} \to \psi_{2,1,1} & \psi_{3,2,0} \to \psi_{2,1,0} & \psi_{3,2,0} \to \psi_{2,1,-1} \\
\psi_{3,2,-1} \to \psi_{2,1,1} & \psi_{3,2,-1} \to \psi_{2,1,0} & \psi_{3,2,-1} \to \psi_{2,1,-1} \\
\psi_{3,2,-2} \to \psi_{2,1,1} & \psi_{3,2,-2} \to \psi_{2,1,0} & \psi_{3,2,-2} \to \psi_{2,1,-1}
\end{array}
$$

For each process, A_{ki} can be computed using the following Heisenberg formula.

$$A_{ki} = \frac{e^2}{3\pi\varepsilon_0 \hbar c^3}\,\omega_{ki}^3\,|r_{ki}|^2$$

The transition frequency ω_{ki} is given by Bohr's frequency condition.

$$\omega_{ki} = \frac{1}{\hbar}(E_k - E_i)$$

The transition probability (multiplied by a physical constant) is

$$|r_{ki}|^2 = |x_{ki}|^2 + |y_{ki}|^2 + |z_{ki}|^2$$

For wave functions ψ in spherical coordinates we have the following transition amplitudes.

$$
\begin{aligned}
x_{ki} &= \int \psi_k^* \,(r \sin\theta \cos\phi)\, \psi_i \, dV \\
y_{ki} &= \int \psi_k^* \,(r \sin\theta \sin\phi)\, \psi_i \, dV \\
z_{ki} &= \int \psi_k^* \,(r \cos\theta)\, \psi_i \, dV
\end{aligned}
$$

The average A_{ki} is obtained by summing over m_ℓ states and dividing by the number of distinct initial states.

Using Eigenmath we obtain

$$
\begin{aligned}
A_{3s2p} &= 6.31358 \times 10^6 \,\text{second}^{-1} \\
A_{3p2s} &= 2.24483 \times 10^7 \,\text{second}^{-1} \\
A_{3d2p} &= 6.4651 \times 10^7 \,\text{second}^{-1}
\end{aligned}
$$

which is very close to the values shown in the table.