

6-15. Recall that in problem 5-4 we defined a particular integral as the transition amplitude to go from state $\psi(x)$ to state $\chi(x)$. Show that the function λ_{mn} satisfies this definition when the initial state is the eigenfunction $\phi_n(x)$ and the final state is the eigenfunction $\phi_m(x)$.

From problem 5-4 the transition amplitude is

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_m^*(x_b) K_V(x_b, t_b, x_a, t_a) \phi_n(x_a) dx_a dx_b \quad (1)$$

Consider equation (6.68).

$$K_V(b, a) = \sum_m \sum_n \lambda_{mn}(t_b, t_a) \phi_m(x_b) \phi_n^*(x_a) \quad (6.68)$$

Substitute (6.68) into (1).

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_m^*(x_b) \left(\sum_{m'} \sum_{n'} \lambda_{m'n'}(t_b, t_a) \phi_{m'}(x_b) \phi_{n'}^*(x_a) \right) \phi_n(x_a) dx_a dx_b$$

By distributive law

$$= \sum_{m'} \sum_{n'} \lambda_{m'n'}(t_b, t_a) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_m^*(x_b) \phi_{m'}(x_b) \phi_{n'}^*(x_a) \phi_n(x_a) dx_a dx_b$$

By orthogonality of eigenfunctions

$$= \lambda_{mn}(t_b, t_a) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_m^*(x_b) \phi_m(x_b) \phi_n^*(x_a) \phi_n(x_a) dx_a dx_b$$

By normalization of eigenfunctions

$$= \lambda_{mn}(t_b, t_a)$$