

Start with equation (6.1).

$$K_V(b, a) = \int_a^b \exp \left(\frac{i}{\hbar} \int_{t_a}^{t_b} \left(\frac{1}{2} m \dot{x}^2 - V(x, t) \right) dt \right) \mathcal{D}x(t) \quad (6.1)$$

Partition the integral.

$$K_V(b, a) = \int_a^b \exp \left(\frac{i}{\hbar} \int_{t_a}^{t_b} \frac{1}{2} m \dot{x}^2 dt - \frac{i}{\hbar} \int_{t_a}^{t_b} V(x, t) dt \right) \mathcal{D}x(t)$$

Factor the exponential.

$$K_V(b, a) = \int_a^b \exp \left(\frac{i}{\hbar} \int_{t_a}^{t_b} \frac{1}{2} m \dot{x}^2 dt \right) \exp \left(-\frac{i}{\hbar} \int_{t_a}^{t_b} V(x, t) dt \right) \mathcal{D}x(t)$$

Make the second exponential a power series.

$$\begin{aligned} K_V(b, a) &= \int_a^b \exp \left(\frac{i}{\hbar} \int_{t_a}^{t_b} \frac{1}{2} m \dot{x}^2 dt \right) \\ &\quad \times \left(1 - \frac{i}{\hbar} \int_{t_a}^{t_b} V(x, t) dt + \frac{1}{2} \left(-\frac{i}{\hbar} \int_{t_a}^{t_b} V(x, t) dt \right)^2 + \dots \right) \mathcal{D}x(t) \end{aligned}$$

Hence

$$\begin{aligned} K_V(b, a) &= \int_a^b \exp \left(\frac{i}{\hbar} \int_{t_a}^{t_b} \frac{1}{2} m \dot{x}^2 dt \right) \mathcal{D}x(t) \\ &\quad - \frac{i}{\hbar} \int_a^b \exp \left(\frac{i}{\hbar} \int_{t_a}^{t_b} \frac{1}{2} m \dot{x}^2 dt \right) \left(\int_{t_a}^{t_b} V(x, t) dt \right) \mathcal{D}x(t) \\ &\quad - \frac{1}{2\hbar^2} \int_a^b \exp \left(\frac{i}{\hbar} \int_{t_a}^{t_b} \frac{1}{2} m \dot{x}^2 dt \right) \left(\int_{t_a}^{t_b} V(x, t) dt \right)^2 \mathcal{D}x(t) + \dots \end{aligned}$$

Let

$$\begin{aligned} K_0(b, a) &= \int_a^b \exp \left(\frac{i}{\hbar} \int_{t_a}^{t_b} \frac{1}{2} m \dot{x}^2 dt \right) \mathcal{D}x(t) \\ K^{(1)}(b, a) &= -\frac{i}{\hbar} \int_a^b \exp \left(\frac{i}{\hbar} \int_{t_a}^{t_b} \frac{1}{2} m \dot{x}^2 dt \right) \left(\int_{t_a}^{t_b} V(x, t) dt \right) \mathcal{D}x(t) \\ K^{(2)}(b, a) &= -\frac{1}{2\hbar^2} \int_a^b \exp \left(\frac{i}{\hbar} \int_{t_a}^{t_b} \frac{1}{2} m \dot{x}^2 dt \right) \left(\int_{t_a}^{t_b} V(x, t) dt \right)^2 \mathcal{D}x(t) \end{aligned}$$

Then equation (6.4) follows.

$$K_V(b, a) = K_0(b, a) + K^{(1)}(b, a) + K^{(2)}(b, a) + \dots \quad (6.4)$$