

2-1. For a free particle $L = (m/2)\dot{x}^2$. Show that the action S_{cl} corresponding to the classical motion of a free particle is

$$S_{cl} = \frac{m}{2} \frac{(x_b - x_a)^2}{t_b - t_a} \quad (2.8)$$

We will need equation (2.7) to determine the classical motion $x(t)$.

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}} = \frac{\partial L}{\partial x} \quad (2.7)$$

For the Lagrangian L given in problem 2-1 we have

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}} = m\ddot{x} \quad \frac{\partial L}{\partial x} = 0$$

By equation (2.7) we have for the classical acceleration $\ddot{x}(t)$

$$\ddot{x}(t) = 0$$

Hence velocity \dot{x} is constant and equals distance divided by time.

$$\dot{x} = \frac{x_b - x_a}{t_b - t_a} \quad (1)$$

Substitute (1) into L .

$$L = \frac{m}{2} \left(\frac{x_b - x_a}{t_b - t_a} \right)^2 \quad (2)$$

Integrate (2) to obtain S_{cl} .

$$\begin{aligned} S_{cl} &= \int_{t_a}^{t_b} L dt \\ &= \frac{m}{2} \left(\frac{x_b - x_a}{t_b - t_a} \right)^2 t \Big|_{t_a}^{t_b} = \frac{m}{2} \frac{(x_b - x_a)^2}{t_b - t_a} \end{aligned}$$