The first step in the ARAR algorithm requires finding ϕ that minimizes

$$\epsilon = \sum_{t=\tau+1}^{n} [Y_t - \phi Y_{t-\tau}]^2$$

where τ is a lag. Note that ϵ is strictly nonnegative and so is minimized for

$$\frac{d\epsilon}{d\phi} = 0$$

Expand ϵ.

$$\epsilon = \sum_{t=\tau+1}^{n} Y_t^2 - 2\phi \sum_{t=\tau+1}^{n} Y_t Y_{t-\tau} + \phi^2 \sum_{t=\tau+1}^{n} Y_{t-\tau}^2$$

Solve for ϕ in

$$\frac{d\epsilon}{d\phi} = -2 \sum_{t=\tau+1}^{n} Y_t Y_{t-\tau} + 2\phi \sum_{t=\tau+1}^{n} Y_{t-\tau}^2 = 0$$

to obtain

$$\phi = \frac{\sum_{t=\tau+1}^{n} Y_t Y_{t-\tau}}{\sum_{t=\tau+1}^{n} Y_{t-\tau}^2}$$

In R code

$$\sum_{t=\tau+1}^{n} Y_t Y_{t-\tau} = \texttt{sum(y[(tau+1):n]*y[1:(n-tau)])}$$

and

$$\sum_{t=\tau+1}^{n} Y_{t-\tau}^2 = \texttt{sum(y[1:(n-tau)]\^{}2)}$$