Example 1. Compute the probability p that 23 people have different birthdays.

$$p = \frac{365}{365} \times \frac{364}{365} \times \frac{363}{365} \times \cdots \times \frac{343}{365} = \frac{365!/(365-23)!}{365^{23}}$$

The probability that at least two people have the same birthday is $1 - p$.

```
"Product method"
p = product(k,1,23,(365-k+1)/365)
float(p)
"Factorial method"
p = 365! / (365 - 23)! / 365^23
float(p)
"Probability of at least one shared birthday"
1.0 - p
```

Example 2. Show that

$$\frac{E^2 + m^2 + p^2 \cos\theta}{8p^4} = \frac{1 - \beta^2 \sin^2\theta/2}{4p^2\beta^2}$$

where $p = \sqrt{E^2 - m^2}$ and $\beta = p/E$.

```
p = sqrt(E^2 - m^2)
beta = p/E
A = (E^2 + m^2 + p^2 cos(theta)) / (8 p^4)
B = (1 - beta^2 sin(theta/2)^2) / (4 p^2 beta^2)
A == B
```

Example 3. Let

$$\mathbf{E} = \begin{pmatrix} A\sin(kz - \omega t + \phi) \\ 0 \\ 0 \end{pmatrix} \qquad \text{and} \qquad \mathbf{B} = \begin{pmatrix} 0 \\ A\sin(kz - \omega t + \phi) \\ 0 \end{pmatrix}$$

where $k = \omega/c$. Verify that $\mathbf{E}$ and $\mathbf{B}$ are solutions to the free-field Maxwell equations

$$\nabla \cdot \mathbf{E} = 0$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial}{\partial t}\mathbf{B} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$
$$\nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial}{\partial t}\mathbf{E} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

```
k = omega/c
E = (A sin(k z - omega t + phi), 0, 0)
B = (0, A sin(k z - omega t + phi), 0)
div(E) == 0
div(B) == 0
curl(E) + d(B,t)/c == 0
curl(B) - d(E,t)/c == 0
```

Example 4. Let

$$\psi = \exp(ik_x x + ik_y y + ik_z z - i\omega t)$$

where

$$\omega = \sqrt{k_x^2 + k_y^2 + k_z^2 + m^2}$$

Verify that ψ is a solution to the Klein-Gordon equation

$$\frac{\partial^2}{\partial t^2}\psi - \frac{\partial^2}{\partial x^2}\psi - \frac{\partial^2}{\partial y^2}\psi - \frac{\partial^2}{\partial z^2}\psi + m^2\psi = 0$$

```
omega = sqrt(kx^2 + ky^2 + kz^2 + m^2)
psi = exp(i kx x + i ky y + i kz z - i omega t)
d(psi,t,t) - d(psi,x,x) - d(psi,y,y) - d(psi,z,z) + m^2 psi == 0
```

Example 5. Show that

$$u_1\bar{u}_1 + u_2\bar{u}_2 = (E + m)(\not{p} + m)$$

where

$$E = \sqrt{p_x^2 + p_y^2 + p_z^2 + m^2}$$

The spinors are

$$u_1 = \begin{pmatrix} E + m \\ 0 \\ p_z \\ p_x + ip_y \end{pmatrix} \qquad u_2 = \begin{pmatrix} 0 \\ E + m \\ p_x - ip_y \\ -p_z \end{pmatrix}$$

The adjoint of u is $\bar{u} = u^\dagger\gamma^0$. Note that $\bar{u}$ is a row vector hence $u\bar{u}$ is an outer product.

```
E = sqrt(px^2 + py^2 + pz^2 + m^2)
u1 = (E + m, 0, pz, px + i py)
u2 = (0, E + m, px - i py, -pz)
p = (E, px, py, pz)
I = ((1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1))
gmunu = ((1,0,0,0),(0,-1,0,0),(0,0,-1,0),(0,0,0,-1))
gamma0 = ((1,0,0,0),(0,1,0,0),(0,0,-1,0),(0,0,0,-1))
gamma1 = ((0,0,0,1),(0,0,1,0),(0,-1,0,0),(-1,0,0,0))
gamma2 = ((0,0,0,-i),(0,0,i,0),(0,i,0,0),(-i,0,0,0))
```

```
gamma3 = ((0,0,1,0),(0,0,0,-1),(-1,0,0,0),(0,1,0,0))
gamma = (gamma0,gamma1,gamma2,gamma3)
pslash = dot(gmunu,p,gamma)
bar(u) = dot(conj(u),gamma0)
A = outer(u1,bar(u1)) + outer(u2,bar(u2))
B = (E + m) (pslash + m I)
A == B
```