

6-1. Suppose the potential can be written as  $U+V$ , where  $V$  is small but  $U$  is large. Suppose further that the kernel for motion in the potential of  $U$  alone can be worked out (for example,  $U$  might be quadratic in  $x$  and independent of time). Show that the motion in the complete potential  $U+V$  is described by equations (6.4), (6.11), (6.13), and (6.14) with  $K_0$  replaced by  $K_U$ , where  $K_U$  is the kernel for motion in the potential  $U$  alone.

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From equation (6.1) we have

$$K_{U+V}(b, a) = \int_a^b \exp \left( \frac{i}{\hbar} \int_{t_a}^{t_b} \left( \frac{1}{2} m \dot{x}^2 - U - V \right) dt \right) \mathcal{D}x(t)$$

Consider equation (6.5).

$$K_0(b, a) = \int_a^b \exp \left( \frac{i}{\hbar} \int_{t_a}^{t_b} \frac{1}{2} m \dot{x}^2 dt \right) \mathcal{D}x(t) \quad (6.5)$$

By hypothesis the kernel for  $U$  is known hence

$$K_U(b, a) = \int_a^b \exp \left( \frac{i}{\hbar} \int_{t_a}^{t_b} \left( \frac{1}{2} m \dot{x}^2 - U \right) dt \right) \mathcal{D}x(t)$$

In the expansion of  $V$ , replace

$$\exp \left( \frac{i}{\hbar} \int_{t_a}^{t_b} \frac{1}{2} m \dot{x}^2 dt \right)$$

with

$$\exp \left( \frac{i}{\hbar} \int_{t_a}^{t_b} \left( \frac{1}{2} m \dot{x}^2 - U \right) dt \right)$$

This corresponds to replacing  $K_0$  with  $K_U$  in (6.4), etc.