7-11. Show this.

$$\int_{-\infty}^{\infty} \chi^*(x)\frac{\hbar}{i}\frac{\partial\psi(x)}{\partial x}\,dx = \int_{-\infty}^{\infty} \chi^*(p)p\psi(p)\frac{dp}{2\pi\hbar} \tag{7.81}$$

Consider equation (7.80).

$$\chi(p) = \int_{-\infty}^{\infty} \chi(x)\exp\left(-\frac{ipx}{\hbar}\right)\,dx \tag{7.80a}$$

$$\psi(p) = \int_{-\infty}^{\infty} \psi(x)\exp\left(-\frac{ipx}{\hbar}\right)\,dx \tag{7.80b}$$

The associated inverse transforms are

$$\chi(x) = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} \chi(p)\exp\left(\frac{ipx}{\hbar}\right)\,dp$$

$$\psi(x) = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} \psi(p)\exp\left(\frac{ipx}{\hbar}\right)\,dp \tag{1}$$

Complex conjugate (7.80a).

$$\chi^*(p) = \int_{-\infty}^{\infty} \chi^*(x)\exp\left(\frac{ipx}{\hbar}\right)\,dx \tag{2}$$

By equation (1)

$$\frac{\hbar}{i}\frac{\partial\psi(x)}{\partial x} = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} p\psi(p)\exp\left(\frac{ipx}{\hbar}\right)\,dp \tag{3}$$

Substitute (2) and (3) into (7.81).

$$\frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} \chi^*(x)\left(\int_{-\infty}^{\infty} p\psi(p)\exp\left(\frac{ipx}{\hbar}\right)\,dp\right)\,dx$$
$$= \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} \chi^*(x)\exp\left(\frac{ipx}{\hbar}\right)\,dx\right)p\psi(p)\,\frac{dp}{2\pi\hbar}$$

By the distributive law the integrals are equivalent.

$$\frac{1}{2\pi\hbar}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \chi^*(x)p\psi(p)\exp\left(\frac{ipx}{\hbar}\right)\,dp\,dx$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \chi^*(x)\exp\left(\frac{ipx}{\hbar}\right)p\psi(p)\,dx\,\frac{dp}{2\pi\hbar}$$