(36.1) *An illustration of the reason for anticommutation and spin*

(a) Show that the Dirac equation can be recast in the form

$$i\frac{\partial\psi}{\partial t} = \hat{H}_D\psi \tag{36.33}$$

where $\hat{H}_D = \boldsymbol{\alpha}\cdot\hat{\mathbf{p}} + \beta m$ and find $\boldsymbol{\alpha}$ and β in terms of the γ matrices.

(a) Consider the following form of the Dirac equation.

$$i\left(\gamma^0\frac{\partial}{\partial t} + \gamma^1\frac{\partial}{\partial x} + \gamma^2\frac{\partial}{\partial y} + \gamma^3\frac{\partial}{\partial z}\right)\psi = m\psi$$

Rewrite as

$$i\gamma^0\frac{\partial}{\partial t}\psi = -i\left(\gamma^1\frac{\partial}{\partial x} + \gamma^2\frac{\partial}{\partial y} + \gamma^3\frac{\partial}{\partial z}\right)\psi + m\psi$$

Noting that $(\gamma^0)^2 = I$, multiply both sides by γ^0 to obtain

$$i\frac{\partial}{\partial t}\psi = -i\gamma^0\left(\gamma^1\frac{\partial}{\partial x} + \gamma^2\frac{\partial}{\partial y} + \gamma^3\frac{\partial}{\partial z}\right)\psi + m\gamma^0\psi$$

Hence for $\hat{\mathbf{p}} = -i\nabla$ we have

$$\boldsymbol{\alpha} = \gamma^0\begin{pmatrix}\gamma^1\\ \gamma^2\\ \gamma^3\end{pmatrix}, \quad \beta = \gamma^0$$