Noting that
$$[z, [H, z]]\psi = z([H, z]\psi) - ([H, z]\psi)z = 0$$
we have
$$zHz = \frac{1}{2}(Hz^2 + z^2 H)$$
Hence
$$\begin{aligned}
\langle z\hat{H}z \rangle &= \frac{1}{2}\langle \psi_{100}|(\hat{H}z^2 + z^2\hat{H})|\psi_{100}\rangle \\
&= \frac{1}{2}\langle \psi_{100}|(E_1 z^2 + z^2 E_1)|\psi_{100}\rangle \\
&= E_1 \langle z^2 \rangle
\end{aligned}$$

From problem 4.15
$$E_1 \langle z^2 \rangle = E_1 a_0^2 \tag{1}$$