

## Spin measurement

Let  $|s\rangle$  be the following spin state where  $\theta$  and  $\phi$  are Bloch sphere angles.

$$|s\rangle = \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2) \exp(i\phi) \end{pmatrix}$$

Find spin measurement probabilities for directions  $x$ ,  $y$ , and  $z$ .

Eigenstates for the  $x$  direction.

$$|x_+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad |x_-\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Eigenstates for the  $y$  direction.

$$|y_+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix}, \quad |y_-\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

Eigenstates for the  $z$  direction.

$$|z_+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |z_-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

For spin measurements in the  $x$  direction

$$\begin{aligned} \Pr\left(+\frac{\hbar}{2}\right) &= |\langle x_+ | s \rangle|^2 = \frac{1}{2} + \frac{1}{2} \sin \theta \cos \phi \\ \Pr\left(-\frac{\hbar}{2}\right) &= |\langle x_- | s \rangle|^2 = \frac{1}{2} - \frac{1}{2} \sin \theta \cos \phi \end{aligned}$$

For the  $y$  direction

$$\begin{aligned} \Pr\left(+\frac{\hbar}{2}\right) &= |\langle y_+ | s \rangle|^2 = \frac{1}{2} + \frac{1}{2} \sin \theta \sin \phi \\ \Pr\left(-\frac{\hbar}{2}\right) &= |\langle y_- | s \rangle|^2 = \frac{1}{2} - \frac{1}{2} \sin \theta \sin \phi \end{aligned}$$

For the  $z$  direction

$$\begin{aligned} \Pr\left(+\frac{\hbar}{2}\right) &= |\langle z_+ | s \rangle|^2 = \frac{1}{2} + \frac{1}{2} \cos \theta \\ \Pr\left(-\frac{\hbar}{2}\right) &= |\langle z_- | s \rangle|^2 = \frac{1}{2} - \frac{1}{2} \cos \theta \end{aligned}$$

Note that for each direction  $\Pr\left(+\frac{\hbar}{2}\right) + \Pr\left(-\frac{\hbar}{2}\right) = 1$  as required by total probability.