Problem 15.6.2. Using the results of Prob. 15.6.1, find an expression for the expectation value of the "position" ξ_λ for the coherent state in Eq. (15.94) in terms of $\bar{n}$, ω_λ and time t.

The expectation value $\bar{\xi}_\lambda$ for the coherent state $|\Psi_{\lambda\bar{n}}\rangle$ is

$$\bar{\xi}_\lambda = \langle\Psi_{\lambda\bar{n}}|\hat{\xi}_\lambda|\Psi_{\lambda\bar{n}}\rangle$$

The operator $\hat{\xi}_\lambda$ is given by equation (15.78).

$$\hat{\xi}_\lambda \equiv \frac{1}{\sqrt{2}}\left(\hat{a}_\lambda + \hat{a}_\lambda^\dagger\right) \tag{15.78}$$

Hence

$$\bar{\xi}_\lambda = \frac{1}{\sqrt{2}}\langle\Psi_{\lambda\bar{n}}|\left(\hat{a}_\lambda + \hat{a}_\lambda^\dagger\right)|\Psi_{\lambda\bar{n}}\rangle$$

Expand the right-hand side.

$$\bar{\xi}_\lambda = \frac{1}{\sqrt{2}}\langle\Psi_{\lambda\bar{n}}|\hat{a}_\lambda|\Psi_{\lambda\bar{n}}\rangle + \frac{1}{\sqrt{2}}\langle\Psi_{\lambda\bar{n}}|\hat{a}_\lambda^\dagger|\Psi_{\lambda\bar{n}}\rangle \tag{1}$$

From problem 15.6.1 we have

$$\hat{a}_\lambda|\Psi_{\lambda\bar{n}}\rangle = \sqrt{\bar{n}}\exp(-i\omega_\lambda t)|\Psi_{\lambda\bar{n}}\rangle \tag{2}$$

It follows that

$$\langle\Psi_{\lambda\bar{n}}|\hat{a}_\lambda^\dagger = (\hat{a}_\lambda|\Psi_{\lambda\bar{n}}\rangle)^\dagger = \sqrt{\bar{n}}\exp(i\omega_\lambda t)\langle\Psi_{\lambda\bar{n}}| \tag{3}$$

Substitute (2) and (3) into (1) to obtain

$$\bar{\xi}_\lambda = \sqrt{\frac{\bar{n}}{2}}\exp(-i\omega_\lambda t)\langle\Psi_{\lambda\bar{n}}|\Psi_{\lambda\bar{n}}\rangle + \sqrt{\frac{\bar{n}}{2}}\exp(i\omega_\lambda t)\langle\Psi_{\lambda\bar{n}}|\Psi_{\lambda\bar{n}}\rangle$$

Noting that $\langle\Psi_{\lambda\bar{n}}|\Psi_{\lambda\bar{n}}\rangle = 1$ we have

$$\bar{\xi}_\lambda = \sqrt{\frac{\bar{n}}{2}}\exp(-i\omega_\lambda t) + \sqrt{\frac{\bar{n}}{2}}\exp(i\omega_\lambda t)$$

Finally, by the identity

$$2\cos(\omega_\lambda t) = \exp(-i\omega_\lambda t) + \exp(i\omega_\lambda t)$$

we have

$$\bar{\xi}_\lambda = \sqrt{2\bar{n}}\cos(\omega_\lambda t)$$