

Atomic transitions 6

From the previous section the transition rate is

$$R_{a \rightarrow b} = \frac{\pi e^2}{3\epsilon_0 \hbar^2} |\langle \psi_b | \mathbf{r} | \psi_a \rangle|^2 \rho(\omega_0)$$

Interchange ψ_a and ψ_b by the identity

$$|\langle \psi_a | \mathbf{r} | \psi_b \rangle|^2 = |\langle \psi_b | \mathbf{r} | \psi_a \rangle|^2$$

to obtain

$$R_{b \rightarrow a} = \frac{\pi e^2}{3\epsilon_0 \hbar^2} |\langle \psi_a | \mathbf{r} | \psi_b \rangle|^2 \rho(\omega_0)$$

The stimulated emission coefficient is

$$B_{b \rightarrow a} = \frac{R_{b \rightarrow a}}{\rho(\omega_0)} = \frac{\pi e^2}{3\epsilon_0 \hbar^2} |\langle \psi_a | \mathbf{r} | \psi_b \rangle|^2$$

The spontaneous emission rate is

$$A_{b \rightarrow a} = \frac{\hbar \omega_0^3}{\pi^2 c^3} B_{b \rightarrow a} = \frac{e^2 \omega_0^3}{3\pi \epsilon_0 \hbar c^3} |\langle \psi_a | \mathbf{r} | \psi_b \rangle|^2 \quad (1)$$

Verify dimensions.

$$A_{b \rightarrow a} \propto \frac{\frac{e^2}{\text{C}^2} \frac{\omega_0^3}{\text{s}^{-3}}}{\frac{\epsilon_0}{\text{C}^2 \text{J}^{-1} \text{m}^{-1}} \frac{\hbar}{\text{J s}} \frac{c^3}{\text{m}^3 \text{s}^{-3}}} \times \frac{|\langle \psi_a | \mathbf{r} | \psi_b \rangle|^2}{\text{m}^2} = \text{s}^{-1}$$