(a)

$$\psi_{100}(r,\theta,\phi) = \frac{1}{\sqrt{\pi a^3}} \exp\left(-\frac{r}{a}\right) \tag{4.80}$$

$$|\psi_{100}|^2 = \frac{1}{\pi a^3} \exp\left(-\frac{2r}{a}\right)$$

$$\Pr(0<r<b) = \frac{1}{\pi a^3} \int_0^b \int_0^\pi \int_0^{2\pi} \exp\left(-\frac{2r}{a}\right) r^2 \sin\theta \, dr \, d\theta \, d\phi$$

Integrate over ϕ (multiply by 2π).

$$\Pr(0<r<b) = \frac{2}{a^3} \int_0^b \int_0^\pi \exp\left(-\frac{2r}{a}\right) r^2 \sin\theta \, dr \, d\theta$$

Transform the integral over θ to an integral over y where $y = \cos\theta$ and $dy = -\sin\theta \, d\theta$. The minus sign in dy is canceled by interchanging integration limits $\cos 0 = 1$ and $\cos\pi = -1$.

$$\Pr(0<r<b) = \frac{2}{a^3} \int_0^b \int_{-1}^1 \exp\left(-\frac{2r}{a}\right) r^2 \, dr \, dy$$

Integrate over y (multiply by 2).

$$\Pr(0<r<b) = \frac{4}{a^3} \int_0^b \exp\left(-\frac{2r}{a}\right) r^2 \, dr$$

Solve the integral over r.

$$\Pr(0<r<b) = 1 - \left(\frac{2b^2}{a^2} + \frac{2b}{a} + 1\right) \exp\left(-\frac{2b}{a}\right) \tag{1}$$

(b) Using the approximation

$$\exp(-\epsilon) = 1 - \epsilon + \tfrac{1}{2}\epsilon^2 - \tfrac{1}{6}\epsilon^3$$

we have

$$\Pr(0<r<b) = 1 - \left(\tfrac{1}{2}\epsilon^2 + \epsilon + 1\right)\left(1 - \epsilon + \tfrac{1}{2}\epsilon^2 - \tfrac{1}{6}\epsilon^3\right)$$

Hence

$$\Pr(0<r<b) = \tfrac{1}{6}\epsilon^3 - \tfrac{1}{12}\epsilon^4 + \tfrac{1}{12}\epsilon^5 \tag{2}$$

To lowest order

$$\Pr(0<r<b) = \tfrac{1}{6}\epsilon^3 = \frac{4b^3}{3a^3}$$

(c) We have

$$|\psi(0)|^2 = \frac{1}{\pi a^3}$$

Hence

$$\Pr(0<r<b) = \frac{4}{3}\pi b^3 |\psi(0)|^2 = \frac{4b^3}{3a^3}$$

(d)

$$\frac{4b^3}{3a^3} = 1.07 \times 10^{-14}$$