Exercise 6.1. Prove that if P(a, b) factorizes, then the correlation between a and b is zero.

We have

$$\langle a \rangle \langle b \rangle = \left(\sum_{a} aP(a) \right) \left(\sum_{b} bP(b) \right) = \sum_{a} \sum_{b} abP(a)P(b)$$
 (1)

Let P(a,b) = P(a)P(b). Then

$$\sum_{a} \sum_{b} abP(a)P(b) = \sum_{a} \sum_{b} abP(a,b) = \langle ab \rangle$$
 (2)

By (1) and (2) we have

$$\langle a \rangle \langle b \rangle = \langle ab \rangle$$

Hence a and b are uncorrelated.

Note that P(a, b) = P(a)P(b) is the definition of independence. Hence a and b are uncorrelated by independence.