8-1. The amplitude to go from any state $\psi(x)$ to another state $\chi(x)$ is the transition amplitude $\langle\chi|1|\psi\rangle$ as defined in equation (7.1). Suppose $\psi(x)$ and $\chi(x)$ are expanded in terms of the orthogonal functions $\phi_n(x)$, the energy solutions to the wave equation associated with the kernel $K(b,a)$, as discussed in section 4-2. Thus

$$\psi(x) = \sum_n \psi_n \phi_n(x), \qquad \chi(x) = \sum_n \chi_n \phi_n(x) \tag{8.23}$$

Using the coefficients ψ_n and χ_n and equation (4.59), show that the transition amplitude can be written as

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \chi^*(x_b) K(x_b,T;x_a,0)\psi(x_a)\,dx_a\,dx_b = \sum_n \chi_n^*\psi_n \exp\left(-\frac{i}{\hbar}E_n T\right) \tag{8.24}$$

This is equation (4.59).

$$K(x_b,t_b;x_a,t_a) = \begin{cases} \sum_{n=1}^{\infty} \phi_n(x_b)\phi_n^*(x_a)\exp\left(-\frac{i}{\hbar}E_n(t_b-t_a)\right) & t_b > t_a \\ 0 & t_b < t_a \end{cases} \tag{4.59}$$

Let

$$I = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \chi^*(x_b) K(x_b,T;x_a,0)\psi(x_a)\,dx_a\,dx_b$$

By (8.23) and (4.59) we have

$$I = \sum_j \sum_n \sum_k \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}$$
$$\chi_j^*\phi_j^*(x_b) \times \phi_n(x_b)\phi_n^*(x_a)\exp\left(-\frac{i}{\hbar}E_n T\right) \times \psi_k\phi_k(x_a)\,dx_a\,dx_b$$

The integrals involving $\phi_j^*\phi_n$ and $\phi_n^*\phi_k$ vanish by orthogonality for $j \neq n$ and $k \neq n$, hence

$$I = \sum_n \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}$$
$$\chi_n^*\phi_n^*(x_b) \times \phi_n(x_b)\phi_n^*(x_a)\exp\left(-\frac{i}{\hbar}E_n T\right) \times \psi_n\phi_n(x_a)\,dx_a\,dx_b$$

The integrals over $\phi_n^*\phi_n$ are unity by normalization hence

$$I = \sum_n \chi_n^* \exp\left(-\frac{i}{\hbar}E_n T\right)\psi_n$$