

The file q4.txt defines kets, operators, and a measurement function for simulating a four qbit quantum computer. See [eigenmath.org/q.c](http://eigenmath.org/q.c) for the program that generates q4.txt.

Kets are unit vectors in  $\mathbb{C}^{16}$ . The dimension is 16 because four qbits have  $2^4 = 16$  basis states. Qbit numbering is  $|q_3q_2q_1q_0\rangle$ . The following basis kets are defined in q4.txt.

$$\begin{aligned} |0\rangle &= |0000_2\rangle = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \\ |1\rangle &= |0001_2\rangle = (0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \\ |2\rangle &= |0010_2\rangle = (0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \\ |3\rangle &= |0011_2\rangle = (0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \\ &\vdots \\ |15\rangle &= |1111_2\rangle = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1) \end{aligned}$$

Operators are  $16 \times 16$  matrices that rotate ket vectors. (A ket always has unit length.) The following operators are defined in q4.txt.

|                |   |
|----------------|---|
| $C_{mn}$       | Controlled not (CNOT) operator, $m$ is the control bit, $n$ is the target bit.              |
| $H_n$          | Hadamard operator on bit $n$ .  |
| $I$            | Identity matrix.  |
| $P_{mn}(\phi)$ | Controlled phase shift, $m$ is the control bit, $n$ is the target bit, $\phi$ is the phase. |
| $Q$            | Quantum Fourier transform.  |
| $R$            | Inverse quantum Fourier transform.  |
| $S_{mn}$       | Swap bits $m$ and $n$ .   |
| $X_n$          | Pauli X (NOT) operator on bit $n$ .   |
| $Y_n$          | Pauli Y operator on bit $n$ .   |
| $Z_n$          | Pauli Z operator on bit $n$ .   |

Function  $M$  measures the final state by drawing a graph of the probability for each of 16 states.

$$M(\psi)$$

Quantum algorithms are expressed as sequences of operators applied to the initial state  $|0\rangle$ . The operator sequence should be read backwards, from right to left, although the direction makes no difference mathematically.

## Deutsch-Jozsa algorithm

Let  $f(q_0, q_1, q_2)$  be an operator ( $16 \times 16$  matrix) that operates on  $q_3$  in a manner consistent with a constant or balanced oracle. Then the Deutsch-Jozsa algorithm for identifying  $f$  is

$$\psi = H_2 H_1 H_0 f(q_0, q_1, q_2) H_3 X_3 H_2 H_1 H_0 |0\rangle$$

## Bernstein-Vazirani algorithm

Let  $f(q_0, q_1, q_2)$  be an operator ( $16 \times 16$  matrix) that operates on  $q_3$ . Then the Bernstein-Vazirani algorithm for identifying  $f$  is

$$\psi = H_2 H_1 H_0 f(q_0, q_1, q_2) Z_3 H_3 H_2 H_1 H_0 |0\rangle$$