Feynman and Hibbs problem 5-5

Assume that the function $f(x, y, z, \ldots)$ can be represented by

$$f(x, y, z, \ldots) = \sum_a \sum_b \sum_c \cdots F'_{a,b,c,\ldots} \chi_{a,b,c,\ldots}(x, y, z, \ldots) \tag{5.38}$$

By substituting this relation into equation (5.36), and using the orthogonal properties of $\chi$ as defined by equation (5.35), show that $F'_{a,b,c,\ldots} = F_{a,b,c,\ldots}$.

Take (5.38) and prime the indices.

$$f(x, y, z, \ldots) = \sum_{a'} \sum_{b'} \sum_{c'} \cdots F'_{a',b',c',\ldots} \chi_{a',b',c',\ldots}(x, y, z, \ldots) \tag{1}$$

This is equation (5.36).

$$F_{a,b,c,\ldots} = \int_{-\infty}^{\infty} \chi^*_{a,b,c,\ldots}(x) f(x)\, dx \tag{5.36}$$

Substitute the right-hand side of (1) for $f(x)$ in (5.36).

$$\begin{aligned} F_{a,b,c,\ldots} = \int_{\mathbb{R}^n} & \chi^*_{a,b,c,\ldots}(x, y, z, \ldots) \\ & \times \left( \sum_{a'} \sum_{b'} \sum_{c'} \cdots F'_{a',b',c',\ldots} \chi_{a',b',c',\ldots}(x, y, z, \ldots) \right) dx\, dy\, dz\, \cdots \end{aligned}$$

Combine factors.

$$\begin{aligned} F_{a,b,c,\ldots} = \int_{\mathbb{R}^n} & \sum_{a'} \sum_{b'} \sum_{c'} \cdots \\ & \chi^*_{a,b,c,\ldots}(x, y, z, \ldots) F'_{a',b',c',\ldots} \chi_{a',b',c',\ldots}(x, y, z, \ldots)\, dx\, dy\, dz\, \cdots \end{aligned}$$

Interchange the order of integration and summation.

$$\begin{aligned} F_{a,b,c,\ldots} = \sum_{a'} \sum_{b'} \sum_{c'} \cdots & \int_{\mathbb{R}^n} \\ & \chi^*_{a,b,c,\ldots}(x, y, z, \ldots) F'_{a',b',c',\ldots} \chi_{a',b',c',\ldots}(x, y, z, \ldots)\, dx\, dy\, dz\, \cdots \end{aligned}$$

Factor out $F'$.

$$F_{a,b,c,\ldots} = \sum_{a'} \sum_{b'} \sum_{c'} \cdots F'_{a',b',c',\ldots}$$
$$\times \left( \int_{\mathbb{R}^n} \chi^*_{a,b,c,\ldots}(x,y,z,\ldots) \chi_{a',b',c',\ldots}(x,y,z,\ldots)\, dx\, dy\, dz\, \cdots \right)$$

By equation (5.35) the integral becomes a product of delta functions.

$$F_{a,b,c,\ldots} = \sum_{a'} \sum_{b'} \sum_{c'} \cdots F'_{a',b',c',\ldots} \delta(a-a')\delta(b-b')\delta(c-c')\cdots$$

Hence for $F_{a,b,c,\ldots} \neq 0$ we must have $a = a'$, $b = b'$, etc. Therefore

$$F_{a,b,c,\ldots} = F'_{a',b',c',\ldots} = F'_{a,b,c,\ldots}$$