$$\dot{c}_a = -\frac{i}{\hbar} H'_{ab} e^{-i\omega_0 t} c_b, \quad \dot{c}_b = -\frac{i}{\hbar} H'_{ba} e^{i\omega_0 t} c_a \tag{11.17}$$

Laplace transform:

$$sC_a(s) - c_a(0) = -\frac{i}{\hbar} H'_{ab} C_b(s + i\omega_0)$$
$$sC_b(s) - c_b(0) = -\frac{i}{\hbar} H'_{ba} C_a(s - i\omega_0)$$

Solve for $C_a(s)$.

$$C_a(s) = \frac{1}{s}\left[-\frac{i}{\hbar} H'_{ab} C_b(s + i\omega_0) + 1\right]$$

Hence

$$C_a(s - i\omega_0) = -\frac{iH'_{ab} C_b(s)}{\hbar(s - i\omega_0)} + \frac{1}{s - i\omega_0}$$

Substitute for $C_a(s - i\omega_0)$ in the equation for $C_b(s)$ to obtain

$$sC_b(s) = -\frac{i}{\hbar} H'_{ba}\left[-\frac{iH'_{ab} C_b(s)}{\hbar(s - i\omega_0)} + \frac{1}{s - i\omega_0}\right]$$

Rewrite as

$$sC_b(s) + \frac{H'_{ab} H'_{ba} C_b(s)}{\hbar^2(s - i\omega_0)} = -\frac{iH'_{ba}}{\hbar(s - i\omega_0)}$$

Hence

$$C_b(s) = -\frac{\dfrac{iH'_{ba}}{\hbar(s - i\omega_0)}}{s + \dfrac{H'_{ab} H'_{ba}}{\hbar^2(s - i\omega_0)}} = -\frac{iH'_{ba}/\hbar}{s^2 - i\omega_0 s + H'_{ab} H'_{ba}/\hbar^2}$$

Inverse Laplace transform:

$$\frac{1}{s^2 + as + b} \quad \Rightarrow \quad \frac{2}{\omega} \sin\left(\frac{\omega t}{2}\right) \exp\left(-\frac{at}{2}\right), \quad \omega = \sqrt{4b - a^2}$$

Hence for $a = -i\omega_0$ and $b = H'_{ab} H'_{ba}/\hbar^2$ we have

$$c_b(t) = -\frac{2iH'_{ba}}{\hbar\omega} \sin\left(\frac{\omega t}{2}\right) \exp\left(\frac{i\omega_0 t}{2}\right), \quad \omega = \sqrt{\frac{4H'_{ab} H'_{ba}}{\hbar^2} + \omega_0^2}$$