3-1. The probability that a particle arrives at the point b is by definition proportional to the absolute square of the kernel $K(b, a)$. For the free-particle kernel of equation (3.3) this is

$$P(b)\,dx = \frac{m}{2\pi\hbar(t_b - t_1)}\,dx \tag{3.6}$$

Clearly this is a relative probability, since the integral over the complete range of x diverges. What does the particular normalization mean? Show that this corresponds to a classical picture in which a particle starts from the point a with all momenta equally likely. Show that the corresponding relative probability that the momentum of the particle lies in the range dp is $dp/2\pi\hbar$.

Let the classical momentum at $x = a$ be somewhere between zero and p. Then from $p = mv$ we have the following maximum distance d.

$$d = \frac{p}{m}(t_b - t_a)$$

Hence the normalization constant C is

$$\begin{aligned}
C &= \int_a^{a+d} \frac{m}{2\pi\hbar(t_b - t_a)}\,dx \\
&= \frac{mx}{2\pi\hbar(t_b - t_a)}\bigg|_a^{a+d} \\
&= \frac{m(a+d)}{2\pi\hbar(t_b - t_a)} - \frac{ma}{2\pi\hbar(t_b - t_a)} \\
&= \frac{md}{2\pi\hbar(t_b - t_a)} \\
&= \frac{p}{2\pi\hbar}
\end{aligned}$$

Hence diverging normalization corresponds to unrestricted momentum p.

Given

$$x + dx = \frac{p + dp}{m}(t_b - t_a)$$

we have

$$dx = \frac{dp}{m}(t_b - t_a)$$

It follows that

$$\frac{m}{2\pi\hbar(t_b - t_a)}\,dx = \frac{dp}{2\pi\hbar}$$