2-1. For a free particle $L = (m/2)\dot{x}^2$. Show that the action S_{cl} corresponding to the classical motion of a free particle is

$$S_{cl} = \frac{m}{2}\frac{(x_b - x_a)^2}{t_b - t_a} \tag{2.8}$$

We will need the following equations.

$$S = \int_{t_a}^{t_b} L\,dt \tag{2.1}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = \frac{\partial L}{\partial x} \tag{2.7}$$

For $L = (m/2)\dot{x}^2$ we have

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = m\ddot{x} \qquad \frac{\partial L}{\partial x} = 0$$

By equation (2.7)

$$\ddot{x} = 0$$

Hence velocity $\dot{x}$ is constant and equals distance divided by time.

$$\dot{x} = \frac{x_b - x_a}{t_b - t_a} \tag{1}$$

Substitute (1) into the Lagrangian.

$$L = \frac{m}{2}\left(\frac{x_b - x_a}{t_b - t_a}\right)^2 \tag{2}$$

Substitute (2) into (2.1) to obtain

$$S_{cl} = \int_{t_a}^{t_b} \frac{m}{2}\left(\frac{x_b - x_a}{t_b - t_a}\right)^2 dt \tag{3}$$

Solve the integral in (3).

$$S_{cl} = \frac{m}{2}\left(\frac{x_b - x_a}{t_b - t_a}\right)^2 t\,\Bigg|_{t_a}^{t_b} = \frac{m}{2}\frac{(x_b - x_a)^2}{t_b - t_a}$$