Galilean transformation

Let G be Galilean transformation operators.

$$G_1 = \frac{m}{\hbar}X_1, \quad G_2 = \frac{m}{\hbar}X_2, \quad G_3 = \frac{m}{\hbar}X_3$$

Let U be the unitary transformation

$$U = 1 - i\epsilon G_3 - \tfrac{1}{2}\epsilon^2 G_3{}^2$$

1. Show that to order ϵ^2

$$U^{-1}X_1 U = X_1$$
$$U^{-1}X_2 U = X_2$$
$$U^{-1}X_3 U = X_3$$

2. Show that to order ϵ^2

$$U^{-1}P_1 U = P_1$$
$$U^{-1}P_2 U = P_2$$
$$U^{-1}P_3 U = P_3 - \epsilon m$$

3. Show that to order ϵ^2

$$U^{-1}L_1 U = L_1 - \epsilon m X_1$$
$$U^{-1}L_2 U = L_2 + \epsilon m X_2$$
$$U^{-1}L_3 U = L_3$$

4. Show that to order ϵ^2

$$U^{-1}HU = H - \epsilon P_3 + \tfrac{1}{2}\epsilon^2 m$$

where

$$H = \frac{1}{2m}\left(P_1{}^2 + P_2{}^2 + P_3{}^2\right)$$