

Exercise 9.2. Prove Eq. 9.10 by expanding each side and comparing the results.

We are given

$$[\mathbf{P}^2, \mathbf{X}] = \mathbf{P}[\mathbf{P}, \mathbf{X}] + [\mathbf{P}, \mathbf{X}]\mathbf{P} \quad (9.10)$$

Expanding the commutators in (9.10) we have

$$[\mathbf{P}^2, \mathbf{X}] = \mathbf{P}\mathbf{P}\mathbf{X} - \mathbf{X}\mathbf{P}\mathbf{P}$$

$$\mathbf{P}[\mathbf{P}, \mathbf{X}] = \mathbf{P}\mathbf{P}\mathbf{X} - \mathbf{P}\mathbf{X}\mathbf{P}$$

$$[\mathbf{P}, \mathbf{X}]\mathbf{P} = \mathbf{P}\mathbf{X}\mathbf{P} - \mathbf{X}\mathbf{P}\mathbf{P}$$

Substitute the expansions back into (9.10) to obtain

$$\mathbf{P}\mathbf{P}\mathbf{X} - \mathbf{X}\mathbf{P}\mathbf{P} = \mathbf{P}\mathbf{P}\mathbf{X} - \mathbf{P}\mathbf{X}\mathbf{P} + \mathbf{P}\mathbf{X}\mathbf{P} - \mathbf{X}\mathbf{P}\mathbf{P}$$

The $\mathbf{P}\mathbf{X}\mathbf{P}$ cancel and (9.10) is proved.