Harmonic oscillator propagator

For a quantum harmonic oscillator, this is the propagator (kernel) to go from x_a at time t_a to x_b at time t_b where $T = t_b - t_a$.

$$K = \left(\frac{m\omega}{2\pi i\hbar \sin(\omega T)}\right)^{\frac{1}{2}} \exp\left[\frac{im\omega}{2\hbar \sin(\omega T)}\left(x_a^2 \cos(\omega T) - 2x_a x_b + x_b^2 \cos(\omega T)\right)\right]$$

Exercises

1. Verify that $\phi^* \phi = \psi_1^*(x_b)\psi_1(x_b)$ where

$$\phi = \int_{-\infty}^{\infty} K\psi_1(x_a)\, dx_a$$