

The following data is from “Note on the spectral lines of hydrogen” by J. J. Balmer dated 1885. The numerical values are wavelengths in units of  $10^{-10}$  meter.

	$H_\alpha$	$H_\beta$	$H_\gamma$	$H_\delta$	$H_\epsilon$	$H_\zeta$	$H_\eta$	$H_\theta$	$H_\iota$
Van der Willigen	6565.6	4863.94	4342.80	4103.8	—	—	—	—	—
Angstrom	6562.10	4860.74	4340.10	4101.2	—	—	—	—	—
Mendenhall	6561.2	4860.16	—	—	—	—	—	—	—
Mascart	6560.7	4859.8	—	—	—	—	—	—	—
Ditscheiner	6559.5	4859.74	4338.60	4100.0	—	—	—	—	—
Huggins	—	—	—	—	—	3887.5	3834	3795	3767.5
Vogel	—	—	—	—	3969	3887	3834	3795	3769 <sup>†</sup>

(<sup>†</sup>The value given in the paper is 6769 which is an obvious typo.)

From this data, Balmer determined that

$$\hat{y} = \frac{m^2}{m^2 - 2^2} \times 3645.6 \times 10^{-10} \text{ meter}$$

where  $\hat{y}$  is the predicted wavelength and  $m$  is determined by the hydrogen line according to the following table.

$$m = \begin{array}{ccccccccc} H_\alpha & H_\beta & H_\gamma & H_\delta & H_\epsilon & H_\zeta & H_\eta & H_\theta & H_\iota \\ 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \end{array}$$

Just for the fun of it, use linear regression in R to compute the model coefficient.

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m = c(3,3,3,3,3,4,4,4,4,4,5,5,5,6,6,6,7,8,8,9,9,10,10,11,11)
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x = m^2 / (m^2 - 4)
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y = c(
6565.60,6562.10,6561.62,6560.70,6559.50,
4863.94,4860.74,4860.16,4859.80,4859.74,
4342.80,4340.10,4338.60,4103.80,4101.20,
4100.00,3969.00,3887.50,3887.00,3834.00,
3834.00,3795.00,3795.00,3767.50,3769.00)
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coef(lm(y ~ 0 + x))
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The result is

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3645.296
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which is a little bit smaller than Balmer’s value.

The actual value is now known to be

$$\frac{4}{R_H} = 3647.05 \times 10^{-10} \text{ meter}$$

where  $R_H$  is the Rydberg constant for hydrogen

$$R_H = 1.096776 \times 10^7 \text{ meter}^{-1}$$