5-6. Suppose A, B, and C are the three cartesian coordinates of momentum p_x, p_y, p_z. What is the form of function the $\chi_{a,b,c}(x,y,z)$?

From equation (5.36)

$$F_{a,b,c} = \int_{\mathbb{R}^3} \chi^*_{a,b,c}(\mathbf{x}) f(\mathbf{x})\, dx\, dy\, dz \tag{1}$$

From equation (5.6)

$$\phi(\mathbf{p}) = \int_{\mathbb{R}^3} \exp\left(-\frac{i\mathbf{p}\cdot\mathbf{x}}{\hbar}\right) f(\mathbf{x})\, dx\, dy\, dz \tag{2}$$

where

$$\mathbf{p} = (p_x, p_y, p_z)$$

Equation (1) is the amplitude for the state (a,b,c). Equation (2) is the amplitude for $\mathbf{p}$. Equate (1) with (2) to find $\chi_{a,b,c}$ for $\mathbf{p} = (a,b,c)$.

$$F_{a,b,c} = \int_{\mathbb{R}^3} \chi^*_{a,b,c}(\mathbf{x}) f(\mathbf{x})\, dx\, dy\, dz = \int_{\mathbb{R}^3} \exp\left(-\frac{i\mathbf{p}\cdot\mathbf{x}}{\hbar}\right) f(\mathbf{x})\, dx\, dy\, dz$$

Hence one solution is

$$\chi^*_{a,b,c}(\mathbf{x}) = \exp\left(-\frac{i\mathbf{p}\cdot\mathbf{x}}{\hbar}\right)$$

and

$$\chi_{a,b,c}(\mathbf{x}) = \exp\left(\frac{i\mathbf{p}\cdot\mathbf{x}}{\hbar}\right)$$