4-5. Using the relation

$$K(b,a) = \int_{-\infty}^{\infty} K(b,c)K(c,a)\,dx_c \tag{4.26}$$

with $t_c - t_a = \epsilon$, an infinitesimal, show that if t_b is greater than t_a, the kernel K satisfies

$$\frac{\partial}{\partial t_a}K(b,a) = +\frac{i}{\hbar}H_a^* K(b,a)$$

where H_a now operates on the a variables only.

Expand the arguments of K in (4.26).

$$K(x_b,t_b,x_a,t_a) = \int_{-\infty}^{\infty} K(x_b,t_b,x_c,t_c)K(x_c,t_c,x_a,t_a)\,dx_c$$

Substitute $t_a = t_c - \epsilon$.

$$K(x_b,t_b,x_a,t_a) = \int_{-\infty}^{\infty} K(x_b,t_b,x_c,t_c)K(x_c,t_c,x_a,t_c-\epsilon)\,dx_c$$

Consider the following Taylor series expansion of $K(x_c, t_a+\epsilon, x_a, t_a)$.

$$K(x_c,t_a+\epsilon,x_a,t_a) \approx K(x_c,t_a,x_a,t_a) + \epsilon\frac{\partial}{\partial t_a}K(x_c,t_a,x_a,t_a)$$

$$\lim_{\epsilon\to 0}\int_{-\infty}^{\infty} K(x_b,t_b,x_c,t_c)K(x_c,t_c,x_a,t_c-\epsilon)\,dx_c = \int_{-\infty}^{\infty} K(x_b,t_b,x_c,t_c)\,dx_c$$