Let $\psi(x, y)$ be the wave function of two electrons in a one dimensional box of length L.

$$\psi(x, y) = \frac{1}{\sqrt{2}}\big(\phi_1(x)\phi_2(y) - \phi_1(y)\phi_2(x)\big)$$

$$\phi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$$

Wave function $\psi(x, y)$ is antisymmetric with respect to interchange of electron coordinates.

$$\psi(x, y) = -\psi(y, x)$$

For $L = 10^{-9}$ meter the expected potential energy $\langle V \rangle$ is

$$\langle V \rangle = \frac{e^2}{4\pi\epsilon_0}\int_0^L\int_0^L \frac{\psi^*(x, y)\psi(x, y)}{|x - y|}\,dx\,dy = 4.67\,\text{eV}$$

Next calculate the expected potential energy $\langle V_s \rangle$ for a symmetric wave function.

$$\langle V_s \rangle = \frac{e^2}{4\pi\epsilon_0}\int_0^L\int_0^L \frac{\phi_1^*(x)\phi_2^*(y)\phi_1(x)\phi_2(y)}{|x - y|}\,dx\,dy = 12.80\,\text{eV}$$

The difference is the exchange energy.

$$\langle V_{ex} \rangle = \langle V \rangle - \langle V_s \rangle = -8.13\,\text{eV}$$

Note that the formula for $\langle V_s \rangle$ has a singularity at $x = y$. The computed value shown above is the result of an arbitrary cutoff in numerical integration. The actual value of $\langle V_s \rangle$ goes to infinity.

Note also that there is a singularity at $x = y$ in the formula for $\langle V \rangle$. However, due to antisymmetry we have $\psi(x, x) = 0$ and hence the integral converges.