

# Hydrogen alpha line

The following transitions correspond to the H- $\alpha$  line of the hydrogen spectrum. See “Atomic Transition Probabilities Volume I,” issued May 20, 1966, page 2.  $A_{ki}$  is the spontaneous emission rate where  $k$  is the upper state and  $i$  is the lower state.

Transition	$\lambda$ (Å)	$A_{ki}$ (second <sup>-1</sup> )
$2p - 3s$	6562.86	$6.313 \times 10^6$
$2s - 3p$	6562.74	$2.245 \times 10^7$
$2p - 3d$	6562.81	$6.465 \times 10^7$

Orbital names correspond to azimuthal quantum numbers  $\ell$ .

Orbital	$\ell$
$s$	0
$p$	1
$d$	2

Each transition in the table has multiple processes due to the magnetic quantum number  $m_\ell$ . (Recall that  $m_\ell = 0, \pm 1, \dots, \pm \ell$ .)

There are three ways to transition from  $3s$  to  $2p$ .

$$\begin{aligned}\psi_{3,0,0} &\rightarrow \psi_{2,1,1} \\ \psi_{3,0,0} &\rightarrow \psi_{2,1,0} \\ \psi_{3,0,0} &\rightarrow \psi_{2,1,-1}\end{aligned}$$

There are three ways to transition from  $3p$  to  $2s$ .

$$\begin{aligned}\psi_{3,1,1} &\rightarrow \psi_{2,0,0} \\ \psi_{3,1,0} &\rightarrow \psi_{2,0,0} \\ \psi_{3,1,-1} &\rightarrow \psi_{2,0,0}\end{aligned}$$

Finally, there are fifteen ways to transition from  $3d$  to  $2p$ . (Some of these transitions have zero amplitude.)

$$\begin{array}{lll}\psi_{3,2,2} \rightarrow \psi_{2,1,1} & \psi_{3,2,2} \rightarrow \psi_{2,1,0} & \psi_{3,2,2} \rightarrow \psi_{2,1,-1} \\ \psi_{3,2,1} \rightarrow \psi_{2,1,1} & \psi_{3,2,1} \rightarrow \psi_{2,1,0} & \psi_{3,2,1} \rightarrow \psi_{2,1,-1} \\ \psi_{3,2,0} \rightarrow \psi_{2,1,1} & \psi_{3,2,0} \rightarrow \psi_{2,1,0} & \psi_{3,2,0} \rightarrow \psi_{2,1,-1} \\ \psi_{3,2,-1} \rightarrow \psi_{2,1,1} & \psi_{3,2,-1} \rightarrow \psi_{2,1,0} & \psi_{3,2,-1} \rightarrow \psi_{2,1,-1} \\ \psi_{3,2,-2} \rightarrow \psi_{2,1,1} & \psi_{3,2,-2} \rightarrow \psi_{2,1,0} & \psi_{3,2,-2} \rightarrow \psi_{2,1,-1}\end{array}$$

For each H- $\alpha$  line, an average  $A_{ki}$  is computed by summing over  $A_{ki}$  for individual processes and dividing by the number of distinct initial states. For example,  $3d \rightarrow 2p$  has five distinct initial states, so the divisor is five.

$A_{ki}$  is computed from the following formula.

$$A_{ki} = \frac{e^2}{3\pi\epsilon_0\hbar c^3} \omega_{ki}^3 |r_{ki}|^2$$

The transition frequency  $\omega_{ki}$  is given by Bohr's frequency condition

$$\omega_{ki} = \frac{E_k - E_i}{\hbar}$$

The transition probability is

$$|r_{ki}|^2 = |x_{ki}|^2 + |y_{ki}|^2 + |z_{ki}|^2$$

These are the transition amplitudes.

$$\begin{aligned} x_{ki} &= \int_0^\infty \int_0^\pi \int_0^{2\pi} r \sin \theta \cos \phi \psi_i^* \psi_k r^2 \sin \theta dr d\theta d\phi \\ y_{ki} &= \int_0^\infty \int_0^\pi \int_0^{2\pi} r \sin \theta \sin \phi \psi_i^* \psi_k r^2 \sin \theta dr d\theta d\phi \\ z_{ki} &= \int_0^\infty \int_0^\pi \int_0^{2\pi} r \cos \theta \psi_i^* \psi_k r^2 \sin \theta dr d\theta d\phi \end{aligned}$$

Using Eigenmath, the results are essentially identical to the book values.

$$\begin{aligned} A_{3s2p} &= 6.31358 \times 10^6 \text{ second}^{-1} \\ A_{3p2s} &= 2.24483 \times 10^7 \text{ second}^{-1} \\ A_{3d2p} &= 6.4651 \times 10^7 \text{ second}^{-1} \end{aligned}$$