

3-1. The probability that a particle arrives at the point  $b$  is by definition proportional to the absolute square of the kernel  $K(b, a)$ . For the free-particle kernel of equation (3.3) this is

$$P(b) dx = \frac{m}{2\pi\hbar(t_b - t_a)} dx \quad (3.6)$$

Clearly this is a relative probability, since the integral over the complete range of  $x$  diverges. What does the particular normalization mean? Show that this corresponds to a classical picture in which a particle starts from the point  $a$  with all momenta equally likely. Show that the corresponding relative probability that the momentum of the particle lies in the range  $dp$  is  $dp/2\pi\hbar$ .

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Compute the normalization constant  $C$ .

$$\begin{aligned} C &= \int_{x_a}^{x_b} P(b) dx \\ &= \frac{m}{2\pi\hbar(t_b - t_a)} x \Big|_{x_a}^{x_b} \\ &= \frac{m}{2\pi\hbar} \left( \frac{x_b - x_a}{t_b - t_a} \right) \end{aligned}$$

From  $v = (x_b - x_a)/(t_b - t_a)$  and  $p = mv$  we have

$$C = \frac{p}{2\pi\hbar}$$

Hence diverging normalization  $C$  corresponds to unrestricted momentum  $p$ .

From  $v = p/m$  we have

$$\frac{x_b - x_a}{t_b - t_a} + \frac{dx}{t_b - t_a} = \frac{p}{m} + \frac{dp}{m}$$

It follows that

$$dx = \frac{dp}{m}(t_b - t_a)$$

Multiply both sides by  $P(b)$ .

$$P(b) dx = \frac{dp}{2\pi\hbar}$$