Start with equation (4.14).

$$\frac{\partial\psi}{\partial t} = -\frac{i}{\hbar}H\psi \tag{4.14}$$

Conjugate both sides.

$$\left(\frac{\partial\psi}{\partial t}\right)^* = +\frac{i}{\hbar}(H\psi)^*$$

It is well known that conjugation and differentiation commute, hence

$$\left(\frac{\partial\psi}{\partial t}\right)^* = \frac{\partial\psi^*}{\partial t} = +\frac{i}{\hbar}(H\psi)^*$$

However, just for the fun of it, let us complete the proof without using the commutation rule.

Consider equation (2.22).

$$K(b,a) = \lim_{\epsilon\to 0}\frac{1}{A^N}\int\cdots\int \exp\left(\frac{i}{\hbar}S(b,a)\right)\,dx_1\cdots dx_{N-1} \tag{2.22}$$

Differentiate (2.22) with respect to t_b.

$$\frac{\partial}{\partial t_b}K(b,a) = \lim_{\epsilon\to 0}\frac{1}{A^N}\int\cdots\int \frac{i}{\hbar}\frac{\partial}{\partial t_b}S(b,a)\exp\left(\frac{i}{\hbar}S(b,a)\right)\,dx_1\cdots dx_{N-1}$$

Then conjugate.

$$\left(\frac{\partial}{\partial t_b}K(b,a)\right)^*$$
$$= \lim_{\epsilon\to 0}\frac{1}{A^N}\int\cdots\int -\frac{i}{\hbar}\frac{\partial}{\partial t_b}S(b,a)\exp\left(-\frac{i}{\hbar}S(b,a)\right)\,dx_1\cdots dx_{N-1}$$

Clearly the result is the same for conjugate first then differentiate, hence

$$\left(\frac{\partial}{\partial t_b}K(b,a)\right)^* = \frac{\partial}{\partial t_b}K^*(b,a) \tag{1}$$

Now consider this form of equation (4.2) that has x, t instead of x_b, t_b.

$$\psi(x,t) = \int_\infty^\infty K(x,t,x_a,t_a)\psi(x_a,t_a)\,dx_a \tag{2}$$

Differentiate (2) with respect to t then conjugate. Note that $\psi(x_a,t_a)$ is a constant with respect to t.

$$\left(\frac{\partial}{\partial t}\psi(x,t)\right)^* = \int_\infty^\infty \left(\frac{\partial}{\partial t}K(x,t,x_a,t_a)\right)^*\psi^*(x_a,t_a)\,dx_a$$

Now do the reverse, conjugate (2) then differentiate.

$$\frac{\partial}{\partial t}\psi^*(x,t) = \int_\infty^\infty \frac{\partial}{\partial t}K^*(x,t,x_a,t_a)\psi^*(x_a,t_a)\,dx_a$$

By equation (1) the integrals are equivalent, hence

$$\left(\frac{\partial}{\partial t}\psi(x,t)\right)^* = \frac{\partial}{\partial t}\psi^*(x,t)$$