

### Feynman and Hibbs problem 3-6

Since the free-particle lagrangian is quadratic, show that

$$K(b, a) = F(t_b, t_a) \exp \left( \frac{im(x_b - x_a)^2}{2\hbar(t_b - t_a)} \right)$$

and give an argument to show that  $F$  can depend only on the difference  $F(t_b - t_a)$ .

From equation 3.51

$$K(b, a) = \exp \left( \frac{i}{\hbar} S_{cl}(b, a) \right) F(t_b, t_a)$$

From problem 2.1

$$S_{cl} = \frac{m}{2} \frac{(x_b - x_a)^2}{t_b - t_a}$$

Hence by substitution of  $S_{cl}$

$$K(b, a) = F(t_b, t_a) \exp \left( \frac{im(x_b - x_a)^2}{2\hbar(t_b - t_a)} \right)$$

The argument for why  $F$  can only depend on  $t_b - t_a$  is invariance under time translation. For any constant  $s$  we should have

$$F(t_b, t_a) = F(t_b + s, t_a + s)$$

For example, we should have

$$F(1, 0) = F(3, 2)$$

Hence  $F$  can only depend on the difference  $t_b - t_a$  and not the specific values of  $t_a$  and  $t_b$ .