

(a) For the stationary states we have

$$\begin{aligned}\psi_{211} &= -\frac{r \sin \theta}{8\sqrt{\pi a_0^5}} \exp\left(-\frac{r}{2a_0} + i\phi\right) \\ \psi_{21-1} &= \frac{r \sin \theta}{8\sqrt{\pi a_0^5}} \exp\left(-\frac{r}{2a_0} - i\phi\right) = -\psi_{211}^*\end{aligned}$$

The general solution to the Schrödinger equation is

$$\sum c_n \psi_n(\mathbf{r}) \exp\left(-\frac{iE_n t}{\hbar}\right) \quad (4.9)$$

Hence

$$\Psi(\mathbf{r}, t) = \frac{r \sin \theta}{8\sqrt{2\pi a_0^5}} \exp\left(-\frac{r}{2a_0} - \frac{iE_2 t}{\hbar}\right) (\exp(-i\phi) - \exp(i\phi))$$

(b) For the potential energy we have

$$V(r) = -\frac{e^2}{4\pi\epsilon_0} \frac{1}{r} \quad (4.52)$$

The expectation value is

$$\begin{aligned}\langle V \rangle &= \langle \Psi | V | \Psi \rangle \\ &= \int_0^\infty \int_0^\pi \int_0^{2\pi} \Psi^* V \Psi r^2 \sin \theta \, dr \, d\theta \, d\phi \\ &= -\frac{e^2}{16\pi\epsilon_0 a_0}\end{aligned} \quad (1)$$

Convert to electron volts.

$$\langle V \rangle = -6.8 \text{ eV}$$

The expectation value $\langle V \rangle$ does not depend on t .