Matrix mechanics 2

From "Quantum Mechanics in Matrix Form" by Günter Ludyk, p. 73.

Verify that for hydrogen the components of angular momentum matrices are

$$
L_1 = \begin{array}{c} \\ \psi_{1,0,0} \\ \psi_{2,1,-1} \\ \psi_{2,1,0} \\ \psi_{2,1,1} \\ \\ \end{array}
\begin{array}{cccc} \psi_{1,0,0} & \psi_{2,1,-1} & \psi_{2,1,0} & \psi_{2,1,1} \\ \end{array}
\left(\begin{array}{ccccc}
0 & 0 & 0 & 0 & \cdots \\
0 & 0 & \frac{\hbar}{\sqrt{2}} & 0 & \cdots \\
0 & \frac{\hbar}{\sqrt{2}} & 0 & \frac{\hbar}{\sqrt{2}} & \cdots \\
0 & 0 & \frac{\hbar}{\sqrt{2}} & 0 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{array}\right)
$$

$$
L_2 = \begin{array}{c} \\ \psi_{1,0,0} \\ \psi_{2,1,-1} \\ \psi_{2,1,0} \\ \psi_{2,1,1} \\ \\ \end{array}
\begin{array}{cccc} \psi_{1,0,0} & \psi_{2,1,-1} & \psi_{2,1,0} & \psi_{2,1,1} \\ \end{array}
\left(\begin{array}{ccccc}
0 & 0 & 0 & 0 & \cdots \\
0 & 0 & \frac{i\hbar}{\sqrt{2}} & 0 & \cdots \\
0 & -\frac{i\hbar}{\sqrt{2}} & 0 & \frac{i\hbar}{\sqrt{2}} & \cdots \\
0 & 0 & -\frac{i\hbar}{\sqrt{2}} & 0 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{array}\right)
$$

$$
L_3 = \begin{array}{c} \\ \psi_{1,0,0} \\ \psi_{2,1,-1} \\ \psi_{2,1,0} \\ \psi_{2,1,1} \\ \\ \end{array}
\begin{array}{cccc} \psi_{1,0,0} & \psi_{2,1,-1} & \psi_{2,1,0} & \psi_{2,1,1} \\ \end{array}
\left(\begin{array}{ccccc}
0 & 0 & 0 & 0 & \cdots \\
0 & -\hbar & 0 & 0 & \cdots \\
0 & 0 & 0 & 0 & \cdots \\
0 & 0 & 0 & \hbar & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{array}\right)
$$

and

$$
\mathbf{L}^2 = \begin{array}{c} \\ \psi_{1,0,0} \\ \psi_{2,1,-1} \\ \psi_{2,1,0} \\ \psi_{2,1,1} \\ \\ \end{array}
\begin{array}{cccc} \psi_{1,0,0} & \psi_{2,1,-1} & \psi_{2,1,0} & \psi_{2,1,1} \\ \end{array}
\left(\begin{array}{ccccc}
0 & 0 & 0 & 0 & \cdots \\
0 & 2\hbar^2 & 0 & 0 & \cdots \\
0 & 0 & 2\hbar^2 & 0 & \cdots \\
0 & 0 & 0 & 2\hbar^2 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{array}\right)
$$

Verify

$$\mathbf{L}^2 = L_1{}^2 + L_2{}^2 + L_3{}^2$$

and

$$L_2 L_3 - L_3 L_2 = i\hbar L_1$$
$$L_3 L_1 - L_1 L_3 = i\hbar L_2$$
$$L_1 L_2 - L_2 L_1 = i\hbar L_3$$

In spherical coordinates the angular momentum operators are

$$\hat{L}_1 = i\hbar \left(\sin\phi \frac{\partial}{\partial\theta} + \cot\theta \cos\phi \frac{\partial}{\partial\phi} \right)$$

$$\hat{L}_2 = i\hbar \left(-\cos\phi \frac{\partial}{\partial\theta} + \cot\theta \sin\phi \frac{\partial}{\partial\phi} \right)$$

$$\hat{L}_3 = -i\hbar \frac{\partial}{\partial\phi}$$

and

$$\hat{\mathbf{L}}^2 = -\hbar^2 \left(\frac{1}{\sin\theta} \frac{\partial}{\partial\theta} \left(\sin\theta \frac{\partial}{\partial\theta} \right) + \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial\phi^2} \right)$$