Exercise 6.6. Assume Charlie has prepared the two spins in the singlet state. This time, Bob measures τ_y and Alice measures σ_x. What is the expectation value of $\sigma_x\tau_y$?

What does this say about the correlation between the two measurements?

Recall that

$$|sing\rangle = \frac{|ud\rangle - |du\rangle}{\sqrt{2}}$$

The expectation value of $\sigma_x\tau_y$ is

$$\langle\sigma_x\tau_y\rangle = \langle sing|\sigma_x\tau_y|sing\rangle$$

Applying the τ_y operator first we have from Table 1 the following result.

$$\langle\sigma_x\tau_y\rangle = \langle sing|\sigma_x\left(\frac{-i|uu\rangle - i|dd\rangle}{\sqrt{2}}\right)$$

Then applying the σ_x operator we have

$$\langle\sigma_x\tau_y\rangle = \langle sing|\left(\frac{-i|du\rangle - i|ud\rangle}{\sqrt{2}}\right)$$

Hence

$$\begin{aligned}\langle\sigma_x\tau_y\rangle &= \frac{1}{2}\left(\langle ud| - \langle du|\right)\left(-i|du\rangle - i|ud\rangle\right)\\ &= \frac{1}{2}\left(-i\langle ud|ud\rangle + i\langle du|du\rangle\right)\\ &= 0 \qquad (1)\end{aligned}$$

From (1) and the result $\langle\sigma_x\rangle = \langle\tau_y\rangle = 0$ given on page 174 we have

$$\langle\sigma_x\tau_y\rangle - \langle\sigma_x\rangle\langle\tau_y\rangle = 0$$

Hence the measurements σ_x and τ_y are uncorrelated.