

The file q4.txt defines kets, operators, and a measurement function for simulating a four bit quantum computer. See eigenmath.org/q.c for the program that generates q4.txt.

Kets are unit vectors in \mathbb{C}^{16} . The dimension is 16 because a four bit quantum computer has $2^4 = 16$ eigenstates. The following kets that represent eigenstates are defined in q4.txt.

$$\begin{aligned} |0\rangle &= |0000_2\rangle = (1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \\ |1\rangle &= |0001_2\rangle = (0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \\ |2\rangle &= |0010_2\rangle = (0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \\ |3\rangle &= |0011_2\rangle = (0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) \\ &\vdots \\ |15\rangle &= |1111_2\rangle = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1) \end{aligned}$$

Operators are 16×16 matrices that rotate ket vectors. The following operators and the measurement function $M(\psi)$ are defined in q4.txt.

H_n	Hadamard operator on bit n .
I	Identity matrix.
$M(\psi)$	Measurement function (not an operator).
$P_n(\phi)$	Phase shift ϕ on bit n .
$P_{mn}(\phi)$	Controlled phase shift, m is the control bit, n is the target bit, ϕ is the phase.
Q	Quantum Fourier transform.
V	Inverse quantum Fourier transform.
W_{mn}	Swap bits m and n .
X_n	Pauli X (NOT) operator on bit n .
X_{mn}	Controlled X (CNOT) operator, m is the control bit, n is the target bit.
Y_n	Pauli Y operator on bit n .
Z_n	Pauli Z operator on bit n .

Measurement function $M(\psi)$ shows, for all $k = 0 \dots 15$, the probability P_k of observing eigenstate k given that the quantum computer is in state ψ .

$$\psi = \sum_{k=0}^{15} c_k |k\rangle, \quad |\psi| = 1, \quad P_k = c_k c_k^*$$

Quantum algorithms are expressed as sequences of operators applied to the initial state $|0\rangle$. The operator sequence should be read backwards, from right to left, although the direction makes no difference mathematically.

Deutsch-Jozsa algorithm

Let $f(q_0, q_1, q_2)$ be an operator (16×16 matrix) that operates on q_3 in a manner consistent with a constant or balanced oracle. Then the Deutsch-Jozsa algorithm for identifying f is

$$\psi = H_2 H_1 H_0 f(q_0, q_1, q_2) H_3 X_3 H_2 H_1 H_0 |0\rangle$$

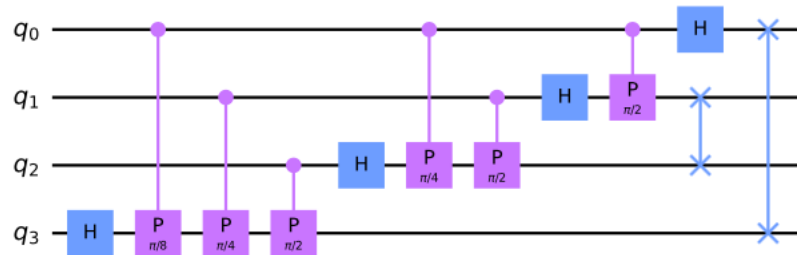
Bernstein-Vazirani algorithm

Let $f(q_0, q_1, q_2)$ be an operator (16×16 matrix) that operates on q_3 . Then the Bernstein-Vazirani algorithm for identifying f is

$$\psi = H_2 H_1 H_0 f(q_0, q_1, q_2) Z_3 H_3 H_2 H_1 H_0 |0\rangle$$

Quantum Fourier transform

The following circuit diagram shows how the QFT is implemented.¹



This is how the QFT operator Q is defined in q4.txt.

```
Q = dot(
W03,          -- Swap qubits 0 and 3
W12,          -- Swap qubits 1 and 2
H0,           -- Hadamard qubit 0
P01(pi/2),    -- Controlled phase shift (0 control, 1 target)
H1,           -- Hadamard qubit 1
P12(pi/2),    -- Controlled phase shift (1 control, 2 target)
P02(pi/4),    -- Controlled phase shift (0 control, 2 target)
H2,           -- Hadamard qubit 2
P23(pi/2),    -- Controlled phase shift (2 control, 3 target)
P13(pi/4),    -- Controlled phase shift (1 control, 3 target)
P03(pi/8),    -- Controlled phase shift (0 control, 3 target)
H3)
```

The inverse QFT operator V is defined similarly except the operators appear in reverse order and the phase shifts are negated.

¹qiskit.org/textbook/ch-algorithms/quantum-fourier-transform.html