

6-7. Suppose the potential energy $V(\mathbf{r}) = -e\phi(\mathbf{r})$ is the result of a charge distribution $\rho(\mathbf{r})$ so that

$$\nabla^2\phi(\mathbf{r}) = -4\pi\rho(\mathbf{r}) \tag{6.48}$$

In rectangular coordinates

$$\nabla^2\phi(\mathbf{r}) = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}$$

Hence

$$-4\pi \int_{\mathbb{R}^3} \rho(\mathbf{r}) \exp\left(\frac{ip_x x}{\hbar}\right) \exp\left(\frac{ip_y y}{\hbar}\right) \exp\left(\frac{ip_z z}{\hbar}\right) dx dy dz$$