4-8. Show from the fact that H is hermitian that E is real. Hint: Choose $f = g = \phi$ in equation (4.30).

The Hamiltonian H is an eigenfunction with corresponding eigenvalue E.

$$H\phi(x) = E\phi(x) \tag{4.42}$$

Since H is hermitian we have from equation (4.30)

$$\int_{-\infty}^{\infty} (Hg)^* f \, dx = \int_{-\infty}^{\infty} g^* (Hf) \, dx \tag{4.30}$$

Substitute ϕ into f and g.

$$\int_{-\infty}^{\infty} (H\phi)^* \phi \, dx = \int_{-\infty}^{\infty} \phi^* (H\phi) \, dx$$

Replace H with eigenvalue E.

$$\int_{-\infty}^{\infty} (E\phi)^* \phi \, dx = \int_{-\infty}^{\infty} \phi^* E\phi \, dx$$

Since E is a constant it can be factored out of the integrands.

$$E^* \int_{-\infty}^{\infty} \phi^* \phi \, dx = E \int_{-\infty}^{\infty} \phi^* \phi \, dx$$

The integrals are identical hence

$$E^* = E$$