

(a) Probability of upper state after two measurements:

$$(1 - \alpha t^2)^2 \approx 1 - 2\alpha t^2$$

Probability of upper state after  $2t$ :

$$1 - 4\alpha t^2$$

(b) The probability that the system is still in the upper state is

$$\left(1 - \frac{\alpha T^2}{n^2}\right)^n$$

Let  $y$  be the limit.

$$y = \lim_{n \rightarrow \infty} \left(1 - \frac{\alpha T^2}{n^2}\right)^n = \lim_{n \rightarrow \infty} e^{n \log\left(1 - \frac{\alpha T^2}{n^2}\right)} = \lim_{n \rightarrow \infty} \exp \left[ \frac{\log\left(1 - \frac{\alpha T^2}{n^2}\right)}{\frac{1}{n}} \right]$$

By the composition limit law

$$y = \exp \left[ \lim_{n \rightarrow \infty} \frac{\log\left(1 - \frac{\alpha T^2}{n^2}\right)}{\frac{1}{n}} \right]$$

Apply l'Hôpital's rule.

$$y = \exp \left[ \lim_{n \rightarrow \infty} \frac{\frac{d}{dn} \log\left(1 - \frac{\alpha T^2}{n^2}\right)}{\frac{d}{dn} \frac{1}{n}} \right] = \exp \left( \lim_{n \rightarrow \infty} \frac{\frac{2\alpha T^2}{n^3 - \alpha T^2 n}}{-\frac{1}{n^2}} \right)$$

Hence

$$y = \exp \left( \lim_{n \rightarrow \infty} \frac{2\alpha T^2}{\frac{\alpha T^2}{n} - n} \right) = \exp(0) = 1$$