Exercise 6.1. Prove that if $P(a, b)$ factorizes, then the correlation between a and b is zero.

We have

$$\langle a\rangle\langle b\rangle = \left(\sum_a aP(a)\right)\left(\sum_b bP(b)\right) = \sum_a\sum_b abP(a)P(b) \tag{1}$$

Let $P(a, b) = P(a)P(b)$. Then

$$\sum_a\sum_b abP(a)P(b) = \sum_a\sum_b abP(a, b) = \langle ab\rangle \tag{2}$$

By (1) and (2) we have

$$\langle a\rangle\langle b\rangle = \langle ab\rangle$$

Hence a and b are uncorrelated.

Note that $P(a, b) = P(a)P(b)$ is the definition of independence. Hence a and b are uncorrelated by independence.