Method of stationary phase

The method of stationary phase can solve integrals of the form

$$I = \int_a^b f(x) e^{i\lambda g(x)}\, dx$$

where $\lambda \gg 1$. The solution is

$$I \approx e^{i\pi\,\mathrm{sgn}(g''(c))/4} \left(\frac{2\pi}{\lambda |g''(c)|}\right)^{1/2} f(c) e^{i\lambda g(c)}$$

where c is the critical point such that

$$g'(c) = 0$$

For $\mathrm{sgn}(g''(c)) = 1$ the solution can be written more simply as

$$I \approx \left(\frac{2\pi i}{\lambda |g''(c)|}\right)^{1/2} f(c) e^{i\lambda g(c)}$$

For example, let I be the integral

$$I = \int_0^{t_b} \left(\frac{m}{2\pi i\hbar(t_b - t_c)}\right)^{3/2} \exp\left(\frac{imR_{bc}^2}{2\hbar(t_b - t_c)}\right) \exp\left(-\frac{ip^2 t_c}{2m\hbar}\right) dt_c$$

Let

$$\begin{aligned} f(t_c) &= \left(\frac{m}{2\pi i\hbar(t_b - t_c)}\right)^{3/2} \\ g(t_c) &= \frac{mR_{bc}^2}{2(t_b - t_c)} - \frac{p^2 t_c}{2m} \\ \lambda &= \frac{1}{\hbar} \end{aligned}$$

The phase of the exponential is stationary ($g'(c) = 0$) for

$$c = t_b - \frac{mR_{bc}}{p}$$

Hence

$$I \approx \left(\frac{2\pi i}{\lambda |g''(c)|}\right)^{1/2} f(c) \exp(i\lambda g(c))$$