6-17. Interpret equation (6.72) by explaining the meaning of each term. Then explain and verify the equation for the second-order coefficient

$$\lambda_{mn}^{(2)} = \left(-\frac{i}{\hbar}\right)^2 \int_{t_a}^{t_b} \int_{t_a}^{t_c} \sum_j \exp\left(-\frac{i}{\hbar}E_m(t_b - t_c)\right) V_{mj}(t_c)$$
$$\times \exp\left(-\frac{i}{\hbar}E_j(t_c - t_d)\right) V_{jn}(t_d) \exp\left(-\frac{i}{\hbar}E_n(t_d - t_a)\right) dt_d\, dt_c \qquad (6.74)$$

Start with equation (6.13).

$$K^{(2)}(b,a) = \left(-\frac{i}{\hbar}\right)^2 \int_{t_a}^{t_b} \int_{-\infty}^{\infty} \int_{t_a}^{t_c} \int_{-\infty}^{\infty} K_0(b,c)V(c)K_0(c,d)V(d)K_0(d,a)$$
$$\times\, dx_d\, dt_d\, dx_c\, dt_c$$

Substitute K_U for K_0. (See problem 6-1 for justification.)

$$K^{(2)}(b,a) = \left(-\frac{i}{\hbar}\right)^2 \int_{t_a}^{t_b} \int_{-\infty}^{\infty} \int_{t_a}^{t_c} \int_{-\infty}^{\infty} K_U(b,c)V(c)K_U(c,d)V(d)K_U(d,a)$$
$$\times\, dx_d\, dt_d\, dx_c\, dt_c$$

By equation (6.66) substitute sums over eigenstates for K_U.

$$K^{(2)}(b,a) = \sum_m \sum_j \sum_n \left(-\frac{i}{\hbar}\right)^2 \int_{t_a}^{t_b} \int_{-\infty}^{\infty} \int_{t_a}^{t_c} \int_{-\infty}^{\infty}$$
$$\phi_m(x_b)\phi_m^*(x_c) \times V(c) \times \phi_j(x_c)\phi_j^*(x_d) \times V(d) \times \phi_n(x_d)\phi_n^*(x_a)$$
$$\times \exp\left(-\frac{i}{\hbar}E_m(t_b - t_c)\right) \exp\left(-\frac{i}{\hbar}E_j(t_c - t_d)\right) \exp\left(-\frac{i}{\hbar}E_n(t_d - t_a)\right)$$
$$\times\, dx_d\, dt_d\, dx_c\, dt_c$$

By equation (6.71) substitute V_{mn} for integrals over x.

$$K^{(2)}(b,a) = \sum_m \sum_j \sum_n \left(-\frac{i}{\hbar}\right)^2 \int_{t_a}^{t_b} \int_{t_a}^{t_c} \phi_m(x_b)V_{mj}(t_c)V_{jn}(t_d)\phi_n^*(x_a)$$
$$\times \exp\left(-\frac{i}{\hbar}E_m(t_b - t_c)\right) \exp\left(-\frac{i}{\hbar}E_j(t_c - t_d)\right) \exp\left(-\frac{i}{\hbar}E_n(t_d - t_a)\right)$$
$$\times\, dt_d\, dt_c$$

Finally, by equation (6.68) we obtain $\lambda_{mn}^{(2)}$.

$$\lambda_{mn}^{(2)}(t_b, t_a) = \sum_j \left(-\frac{i}{\hbar}\right)^2 \int_{t_a}^{t_b} \int_{t_a}^{t_c} V_{mj}(t_c) V_{jn}(t_d)$$
$$\times \exp\left(-\frac{i}{\hbar} E_m (t_b - t_c)\right) \exp\left(-\frac{i}{\hbar} E_j (t_c - t_d)\right) \exp\left(-\frac{i}{\hbar} E_n (t_d - t_a)\right)$$
$$\times \, dt_d \, dt_c$$

Here are equations (6.68) and (6.71).

$$K_V(b, a) = \sum_m \sum_n \lambda_{mn}(t_b, t_a) \phi_m(x_b) \phi_n^*(x_a) \tag{6.68}$$

$$V_{mn}(t_c) = \int_{-\infty}^{\infty} \phi_m^*(x_c) V(x_c, t_c) \phi_n(x_c) \, dx_c \tag{6.71}$$