What is $d\sigma$?

$d\sigma$ is an *unnormalized* probability density function. Integrate $d\sigma$ to obtain a cumulative distribution function $F(\theta)$. Differentiate $F(\theta)$ to obtain a *normalized* probability density function $f(\theta)$.

$$f(\theta) = \frac{dF(\theta)}{d\theta} \propto d\sigma$$

For example, the well known cross section for Bhabha scattering is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2(\hbar c)^2}{4s}\left(\frac{\cos^2\theta + 3}{\cos\theta - 1}\right)^2$$

Recalling that $d\Omega = \sin\theta\, d\theta\, d\phi$ we can write

$$d\sigma = \frac{\alpha^2(\hbar c)^2}{4s}\left(\frac{\cos^2\theta + 3}{\cos\theta - 1}\right)^2 \sin\theta\, d\theta\, d\phi$$

Let $I(\theta)$ be the following integral of $d\sigma$.

$$I(\theta) = \int \left(\frac{\cos^2\theta + 3}{\cos\theta - 1}\right)^2 \sin\theta\, d\theta$$

The integral is

$$I(\theta) = \frac{16}{\cos\theta - 1} - \frac{\cos^3\theta}{3} - \cos^2\theta - 9\cos\theta - 16\log(1 - \cos\theta)$$

The cumulative distribution function is

$$F(\theta) = \frac{I(\theta) - I(a)}{I(\pi) - I(a)}, \quad a \le \theta \le \pi$$

Angular support is reduced by an arbitrary angle $a > 0$ because $I(0)$ is undefined. (For an actual detector used in an experiment, angle a would be the minimum angle and π would be replaced by the maximum angle.)

The probability of observing scattering events in the interval θ_1 to θ_2 is

$$P(\theta_1 \le \theta \le \theta_2) = F(\theta_2) - F(\theta_1)$$

Let N be the total number of scattering events from an experiment. Then the number of scattering events in the interval θ_1 to θ_2 is predicted to be

$$NP(\theta_1 \le \theta \le \theta_2)$$

The probability density function is

$$f(\theta) = \frac{dF(\theta)}{d\theta} = \frac{1}{I(\pi) - I(a)}\left(\frac{\cos^2\theta + 3}{\cos\theta - 1}\right)^2 \sin\theta$$

Note that if we had carried through the $\alpha^2(\hbar c)^2/4s\,d\phi$ in $I(\theta)$, it would have canceled out in $F(\theta)$.

The raw data from scattering experiments are counts per angular bin. The raw data are processed to produce numbers that can be compared directly with $d\sigma$. For example, here is Bhabha scattering data from DESY.

x	y
−0.7300	0.10115
−0.6495	0.12235
−0.5495	0.11258
−0.4494	0.09968
−0.3493	0.14749
−0.2491	0.14017
−0.1490	0.18190
−0.0488	0.22964
0.0514	0.25312
0.1516	0.30998
0.2520	0.40898
0.3524	0.62695
0.4529	0.91803
0.5537	1.51743
0.6548	2.56714
0.7323	4.30279

Data x and y have the following relationship with the cross section formula.

$$x = \cos\theta, \quad y = \frac{d\sigma}{d\Omega} \text{ in nanobarns}$$

The Bhabha scattering cross section formula is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\left(\frac{\cos^2\theta + 3}{\cos\theta - 1}\right)^2 \times (\hbar c)^2$$

To compute predicted values $\hat{y}$, multiply by 10^{37} to convert square meters to nanobarns.

$$\hat{y} = \frac{\alpha^2}{4s}\left(\frac{x^2 + 3}{x - 1}\right)^2 \times (\hbar c)^2 \times 10^{37}$$

The following table shows predicted values $\hat{y}$ for $s = (14.0\,\text{GeV})^2$.

x	y	$\hat{y}$
-0.7300	0.10115	0.110296
-0.6495	0.12235	0.113816
-0.5495	0.11258	0.120101
-0.4494	0.09968	0.129075
-0.3493	0.14749	0.141592
-0.2491	0.14017	0.158934
-0.1490	0.18190	0.182976
-0.0488	0.22964	0.216737
0.0514	0.25312	0.264989
0.1516	0.30998	0.335782
0.2520	0.40898	0.443630
0.3524	0.62695	0.615528
0.4529	0.91803	0.907700
0.5537	1.51743	1.451750
0.6548	2.56714	2.609280
0.7323	4.30279	4.615090

The coefficient of determination R^2 measures how well predicted values fit the data.

$$R^2 = 1 - \frac{\sum(y - \hat{y})^2}{\sum(y - \bar{y})^2} = 0.995$$

The result indicates that 99.5% of the variance in the data is explained by $d\sigma$.