Spin state s is a unit vector in $\mathbb{C}^2$.

$$|s\rangle = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}, \quad c_1^* c_1 + c_2^* c_2 = 1$$

Here is $|s\rangle$ as a linear combination of basis states "up" and "down."

$$|s\rangle = c_1|u\rangle + c_2|d\rangle, \quad |u\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |d\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

These are the spin operators.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Expectation of spin operators is a projection of s onto Euclidean space.

$$\langle x\rangle = \langle s|\sigma_x|s\rangle, \quad \langle y\rangle = \langle s|\sigma_y|s\rangle, \quad \langle z\rangle = \langle s|\sigma_z|s\rangle$$

Spin polarization vector $\mathbf{P}$ is

$$\mathbf{P} = \begin{pmatrix} \langle x\rangle \\ \langle y\rangle \\ \langle z\rangle \end{pmatrix} = \langle s|\sigma|s\rangle$$

where

$$\sigma = \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix}$$

Let θ and ϕ be polar and azimuth angles such that

$$\mathbf{P} = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}$$

Then

$$|s\rangle = \begin{pmatrix} \cos(\theta/2) \\ \sin(\theta/2)\exp(i\phi) \end{pmatrix}$$

In component notation $\sigma = \sigma^{\alpha\beta}{}_\gamma$ hence

$$P^\alpha = s^*_\beta \sigma^{\alpha\beta}{}_\gamma s^\gamma$$

A transpose swaps α and β so that summed-over indices are adjacent.

$$P^\alpha = s^*_\beta \sigma^{\beta\alpha}{}_\gamma s^\gamma$$

Hence the Eigenmath code is

```
P = dot(conj(s),transpose(sigma),s)
```