2-1. For a free particle $L = (m/2)\dot{x}^2$. Show that the action S_{cl} corresponding to the classical motion of a free particle is

$$S_{cl} = \frac{m}{2}\frac{(x_b - x_a)^2}{t_b - t_a} \tag{2.8}$$

We will need (2.7) to determine the classical equation of motion $x(t)$.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = \frac{\partial L}{\partial x} \tag{2.7}$$

For the Lagrangian L given in problem 2-1 we have

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} = m\ddot{x} \qquad \frac{\partial L}{\partial x} = 0$$

By equation (2.7) we have for the classical acceleration $\ddot{x}(t)$

$$\ddot{x}(t) = 0$$

Hence velocity $\dot{x}$ is constant and equals distance divided by time.

$$\dot{x} = \frac{x_b - x_a}{t_b - t_a} \tag{1}$$

Substitute (1) into L.

$$L = \frac{m}{2}\left(\frac{x_b - x_a}{t_b - t_a}\right)^2 \tag{2}$$

Integrate (2) to obtain S_{cl}.

$$\begin{aligned} S_{cl} &= \int_{t_a}^{t_b} L\,dt \\ &= \frac{m}{2}\left(\frac{x_b - x_a}{t_b - t_a}\right)^2 t\,\bigg|_{t_a}^{t_b} = \frac{m}{2}\frac{(x_b - x_a)^2}{t_b - t_a} \end{aligned}$$