(a) In equation (5.56) N is the number of atoms and each atom contributes d electrons. In this problem N is the number of nucleons and d is the number of electrons per nucleon.

$$E_{\text{tot}} = \frac{\hbar^2 \left(3\pi^2 N d\right)^{5/3}}{10\pi^2 m} V^{-2/3} \tag{5.56}$$

By $V = 4\pi R^3/3$ we have

$$E_{\text{tot}} = \frac{\hbar^2 \left(3\pi^2 N d\right)^{5/3}}{10\pi^2 m} \left(\frac{4\pi R^3}{3}\right)^{-2/3} = \frac{9}{20}\left(\frac{3\pi^2}{2}\right)^{1/3} \frac{\hbar^2 (Nd)^{5/3}}{mR^2}$$

(b)

$$U = -\frac{3G(NM)^2}{5R}$$

(c) We have

$$\frac{dE_{\text{tot}}}{dR} = -\frac{9}{10}\left(\frac{3\pi^2}{2}\right)^{1/3} \frac{\hbar^2 (Nd)^{5/3}}{mR^3}$$
$$\frac{dU}{dR} = \frac{3G(MN)^2}{5R^2}$$

Find R such that

$$\frac{dE_{\text{tot}}}{dR} + \frac{dU}{dR} = 0$$

Substitute and multiply both sides by R^3.

$$-\frac{9}{10}\left(\frac{3\pi^2}{2}\right)^{1/3} \frac{\hbar^2 (Nd)^{5/3}}{m} + \frac{3G(MN)^2}{5} R = 0$$

Hence

$$R = \frac{9}{10}\left(\frac{3\pi^2}{2}\right)^{1/3} \frac{\hbar^2 (Nd)^{5/3}}{m} \frac{5}{3G(MN)^2} = \left(\frac{9\pi}{4}\right)^{2/3} \frac{\hbar^2 d^{5/3}}{GmM^2 N^{1/3}} \tag{1}$$

(d) For $M_\odot = 1.98892 \times 10^{30}\,\text{kg}$ and $M = m_p = 1.67 \times 10^{-27}\,\text{kg}$ we have

$$N = \frac{M_\odot}{M} = 1.19 \times 10^{57}$$

Hence

$$R = 7160\,\text{km}$$

(e) The fermi energy is

$$E_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N d}{V}\right)^{2/3} = 194\,\text{keV}$$

The rest energy of an electron is

$$m_e c^2 = 511\,\text{keV}$$