

Canonical commutation relation

This is the canonical commutation relation in one dimension.

$$XP - PX = i\hbar$$

Let

$$X = x, \quad P = -i\hbar \frac{\partial}{\partial x}$$

Then

$$\begin{aligned}(XP - PX)\psi(x, t) &= XP\psi(x, t) - PX\psi(x, t) \\&= x \left(-i\hbar \frac{\partial}{\partial x} \psi(x, t) \right) + i\hbar \frac{\partial}{\partial x} (x\psi(x, t)) \\&= -i\hbar x \frac{\partial}{\partial x} \psi(x, t) + i\hbar \left(\frac{\partial}{\partial x} x \right) \psi(x, t) + i\hbar x \frac{\partial}{\partial x} \psi(x, t) \\&= i\hbar \psi(x, t)\end{aligned}$$

Eigenmath code:

```
X(f) = x f
P(f) = -i hbar d(f,x)
X(P(psi(x,t))) - P(X(psi(x,t)))
```

Result:

$$i\hbar\psi(x, t)$$

Another example: Show that

$$[X^2, P^2] = 2i\hbar(XP + PX)$$

Eigenmath code:

```
X2(f) = X(X(f))
P2(f) = P(P(f))
A = X2(P2(psi(x,t))) - P2(X2(psi(x,t)))
B = 2 i hbar (X(P(psi(x,t))) + P(X(psi(x,t))))
check(A == B)
"ok"
```

Result:

ok

(The statement `check(A == B)` halts if A not equal B .)