

(7.1) For the Lagrangian \mathcal{L} given by

$$\mathcal{L} = \frac{1}{2}\partial^\mu\partial_\mu\phi - \frac{1}{2}m^2\phi^2 - \sum_{n=1}^{\infty}\lambda_n\phi^{2n+2} \quad (7.20)$$

show that the equation of motion is given by

$$(\partial^2 + m^2)\phi + \sum_{n=1}^{\infty}\lambda_n(2n+2)\phi^{2n+1} = 0 \quad (7.21)$$

Recall that

$$\frac{\partial(\partial_\mu\phi)}{\partial\phi} = 0 \quad \text{and} \quad \frac{\partial\phi}{\partial(\partial_\mu\phi)} = 0$$

Hence for \mathcal{L} in (7.20) we have

$$\frac{\partial\mathcal{L}}{\partial\phi} = -m^2\phi - \sum_{n=1}^{\infty}\lambda_n(2n+2)\phi^{2n+1} \quad (1)$$

and

$$\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} = \frac{\partial}{\partial(\partial_\mu\phi)} \left(\frac{1}{2}\partial^\mu\partial_\mu\phi \right) = \partial^\mu\phi \quad (2)$$

This is the Euler-Lagrange equation.

$$\partial_\mu \left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} \right) - \frac{\partial\mathcal{L}}{\partial\phi} = 0 \quad (3)$$

Substitute (1) and (2) into (3) to obtain

$$\partial_\mu\partial^\mu\phi + m^2\phi + \sum_{n=1}^{\infty}\lambda_n(2n+2)\phi^{2n+1} = 0$$

which is equivalent to (7.21).