

7-1. If  $S(x(t)) = \int_{t_a}^{t_b} L(\dot{x}, x, t) dt$ , show that, for any  $s$  inside the range  $t_a$  to  $t_b$ ,

$$\frac{\delta S}{\delta x(s)} = -\frac{d}{ds} \left( \frac{\partial L}{\partial \dot{x}} \right) + \frac{\partial L}{\partial x}$$

where the partial derivatives are evaluated at  $t = s$ .

From equation (2.6)

$$\delta S = \int_{t_a}^{t_b} \left( -\frac{d}{dt} \frac{\partial L}{\partial \dot{x}} + \frac{\partial L}{\partial x} \right) \delta x(t) dt$$

Let  $t_a = t_b = s$  to obtain

$$\delta S = \left( -\frac{d}{ds} \frac{\partial L}{\partial \dot{x}} + \frac{\partial L}{\partial x} \right) \delta x(s)$$

Divide through by  $\delta x(s)$ .

$$\frac{\delta S}{\delta x(s)} = -\frac{d}{ds} \frac{\partial L}{\partial \dot{x}} + \frac{\partial L}{\partial x}$$