The authors are referring to integration by parts in equation (7.32).

$$\int \frac{\partial F}{\partial x_k} \exp\left(\frac{i}{\hbar} S\big(x(t)\big)\right) \mathcal{D}x(t) = -\frac{i}{\hbar} \int F \frac{\partial S}{\partial x_k} \exp\left(\frac{i}{\hbar} S\big(x(t)\big)\right) \mathcal{D}x(t) \tag{7.32}$$

where

$$x_k = x(t_k)$$

Note that integration by parts in (7.32) uses x_k, not $x(t)$.

Let

$$\begin{aligned} u &= \exp\left(\frac{i}{\hbar} S\big(x(t)\big)\right) \mathcal{D}x(t) \\ dv &= \frac{\partial F}{\partial x_k} \end{aligned}$$

Then

$$\begin{aligned} du &= \frac{i}{\hbar} \frac{\partial S}{\partial x_k} \exp\left(\frac{i}{\hbar} S\big(x(t)\big)\right) \mathcal{D}x(t) \\ v &= F \end{aligned}$$

Integrate by parts.

$$\begin{aligned} \int u\, dv &= uv - \int v\, du \\ &= F \exp\left(\frac{i}{\hbar} S\big(x(t)\big)\right) \mathcal{D}x(t) - \frac{i}{\hbar} \int F \frac{\partial S}{\partial x_k} \exp\left(\frac{i}{\hbar} S\big(x(t)\big)\right) \mathcal{D}x(t) \end{aligned}$$

The term uv vanishes because there is no integral over $\mathcal{D}x(t)$. There has to be an integration interval to obtain a nonzero transition amplitude. The concept is similar to a probability density function. A probability density function must be integrated over an interval to obtain a nonzero probability.