

Exercise 8.1. Prove that \mathbf{X} and \mathbf{D} are linear operators.

We are given

$$\mathbf{X}\psi(x) = x\psi(x) \tag{8.5}$$

$$\mathbf{D}\psi(x) = \frac{d\psi(x)}{\psi(x)} \tag{8.6}$$

For \mathbf{X} we have

$$\mathbf{X}(f + g) = x(f + g) = xf + xg = \mathbf{X}f + \mathbf{X}g$$

and

$$\mathbf{X}(af) = x(af) = axf = a\mathbf{X}f$$

For \mathbf{D} we have

$$\mathbf{D}(f + g) = \frac{d(f + g)}{dx} = \frac{df}{dx} + \frac{dg}{dx} = \mathbf{D}f + \mathbf{D}g$$

and

$$\mathbf{D}(af) = \frac{d(af)}{dx} = a \frac{df}{dx} = a\mathbf{D}f$$

Therefore \mathbf{X} and \mathbf{D} are linear operators.