(a)

$$\psi(x) = A\sin(kx) + B\cos(kx) \tag{5.66}$$

$$A\sin(ka) = \left[e^{iqa} - \cos(ka)\right] B \tag{5.70}$$

By equation (5.70)

$$B = \frac{e^{-iqa} A\sin(ka)}{1 - e^{-iqa}\cos(ka)}$$

Substitute B into (5.66) to obtain

$$\psi(x) = A\sin(kx) + \frac{e^{-iqa} A\sin(ka)}{1 - e^{-iqa}\cos(ka)}\cos(kx)$$

Rewrite as

$$\psi(x) = \frac{A\sin(kx)[1 - e^{-iqa}\cos(ka)]}{1 - e^{-iqa}\cos(ka)} + \frac{e^{-iqa} A\sin(ka)}{1 - e^{-iqa}\cos(ka)}\cos(kx)$$

Since A and $1 - e^{-iqa}\cos(ka)$ are constant they can be factored into C, hence

$$\psi(x) \propto \sin(kx)[1 - e^{-iqa}\cos(ka)] + e^{-iqa}\sin(ka)\cos(kx)$$

Rewrite as

$$\psi(x) \propto \sin(kx) + e^{-iqa}[-\sin(kx)\cos(ka) + \sin(ka)\cos(kx)]$$

By the trigonometric identity

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$$

we have for $\alpha = ka$ and $\beta = kx$

$$-\sin(kx)\cos(ka) + \sin(ka)\cos(kx) = \sin(ka - kx)$$

Hence

$$\psi(x) \propto \sin(kx) + e^{-iqa}\sin[k(a - x)]$$

or equivalently

$$\psi(x) = C\left\{\sin(kx) + e^{-iqa}\sin[k(a - x)]\right\}$$

(b) By equation (5.70)

$$B = \frac{A\sin(ka)}{e^{iqa} - \cos(ka)}$$

For $z = j\pi = ka$ we have

$$B = \frac{A\sin(j\pi)}{e^{iqa} - \cos(j\pi)} = 0$$

By equation (5.66) with $B = 0$ and $k = j\pi/a$

$$\psi(x) = A\sin(kx) + B\cos(kx) = A\sin\left(\frac{j\pi x}{a}\right)$$

At each delta function $\sin(j\pi x/a) = 0$ hence $\psi(x) = 0$.