

Harmonic oscillator kernel

For a quantum harmonic oscillator, this is the kernel (propagator) to go from x_a at time t_a to x_b at time t_b where $T = t_b - t_a$.

$$K = \left(\frac{m\omega}{2\pi i \hbar \sin(\omega T)} \right)^{\frac{1}{2}} \exp \left(\frac{im\omega}{2\hbar \sin(\omega T)} (x_a^2 \cos(\omega T) - 2x_a x_b + x_b^2 \cos(\omega T)) \right)$$

Exercises

1. Verify that

$$\left| \int_{-\infty}^{\infty} K \psi_1(x_a) dx_a \right|^2 = |\psi_1(x_b)|^2$$