Let

$$\delta(t) = \begin{cases} 1/\epsilon, & 0 < t \le \epsilon \\ 0, & \text{otherwise} \end{cases}$$

The height of the rectangle is $1/\epsilon$ for unit area. Then for $0 < t \le \epsilon$ we have

$$H'_{ab} = U_{ab}\delta(t) = \frac{\alpha}{\epsilon}, \quad H'_{ba} = U_{ba}\delta(t) = \frac{\alpha^*}{\epsilon}$$

Use these matrix elements and the result from problem 11.3 to obtain for $t \ge \epsilon$

$$c_a(t) = \left[\cos\left(\frac{1}{2}\sqrt{\frac{4|\alpha|^2}{\hbar^2} + \epsilon^2\omega_0^2}\right) + \frac{i\omega_0}{\sqrt{\frac{4|\alpha|^2}{\epsilon^2\hbar^2} + \omega_0^2}}\sin\left(\frac{1}{2}\sqrt{\frac{4|\alpha|^2}{\hbar^2} + \epsilon^2\omega_0^2}\right)\right]\exp\left(-\frac{i\epsilon\omega_0}{2}\right)$$

$$c_b(t) = -\frac{2i\alpha^*}{\sqrt{4|\alpha|^2 + \epsilon^2\hbar^2\omega_0^2}}\sin\left(\frac{1}{2}\sqrt{\frac{4|\alpha|^2}{\hbar^2} + \epsilon^2\omega_0^2}\right)\exp\left(\frac{i\epsilon\omega_0}{2}\right)$$

Let $\epsilon \to 0$ to obtain for $t > 0$

$$c_a(t) = \cos\frac{|\alpha|}{\hbar}$$
$$c_b(t) = -\frac{i\alpha^*}{|\alpha|}\sin\frac{|\alpha|}{\hbar}$$

Hence

$$P_{a\to b} = |c_b(t)|^2 = \sin^2\frac{|\alpha|}{\hbar}$$