6-21. Consider the special case that the perturbing potential V has no matrix elements except between the two states 1 and 2; and further, suppose these states are degenerate, that is, suppose $E_1 = E_2$. Let $V_{12} = V_{21} = v$ and let V_{11}, V_{22}, and all other V_{mn} be zero. Show that

$$\lambda_{11} = 1 - \frac{v^2T^2}{2\hbar^2} + \frac{v^4T^4}{24\hbar^4} - \cdots \qquad = \cos\frac{vT}{\hbar} \tag{6.81}$$

$$\lambda_{12} = -i\frac{vT}{\hbar} + i\frac{v^3T^3}{6\hbar^3} - \cdots \qquad = -i\sin\frac{vT}{\hbar} \tag{6.82}$$

Consider equation (6.75).

$$\begin{aligned}\lambda_{mn}(t_b,t_a) &= \delta_{mn}\exp\left(-\frac{i}{\hbar}E_m(t_b-t_a)\right)\\ &\quad - \frac{i}{\hbar}\int_{t_a}^{t_b}\exp\left(-\frac{i}{\hbar}E_m(t_b-t_a)\right)\sum_j V_{mj}(t_c)\lambda_{jn}(t_c,t_a)\,dt_c\end{aligned} \tag{6.75}$$

Let $E = E_1 = E_2$ and $T = t_b - t_a$. Then by (6.75) we have

$$\begin{aligned}\lambda_{11}(t_b,t_a) &= \exp\left(-\frac{iET}{\hbar}\right) - \frac{i}{\hbar}\int_{t_a}^{t_b}\exp\left(-\frac{i}{\hbar}E(t_b-t_c)\right)v(t_c)\lambda_{21}(t_c,t_a)\,dt_c\\ \lambda_{12}(t_b,t_a) &= -\frac{i}{\hbar}\int_{t_a}^{t_b}\exp\left(-\frac{i}{\hbar}E(t_b-t_c)\right)v(t_c)\lambda_{21}(t_c,t_a)\,dt_c\\ \lambda_{21}(t_b,t_a) &= -\frac{i}{\hbar}\int_{t_a}^{t_b}\exp\left(-\frac{i}{\hbar}E(t_b-t_c)\right)v(t_c)\lambda_{12}(t_c,t_a)\,dt_c\\ \lambda_{22}(t_b,t_a) &= \exp\left(-\frac{iET}{\hbar}\right) - \frac{i}{\hbar}\int_{t_a}^{t_b}\exp\left(-\frac{i}{\hbar}E(t_b-t_c)\right)v(t_c)\lambda_{12}(t_c,t_a)\,dt_c\end{aligned}$$