

Laguerre polynomials

Verify

$$x \frac{d^2}{dx^2} L_{n\alpha}(x) + (\alpha + 1 - x) \frac{d}{dx} L_{n\alpha}(x) + n L_{n\alpha}(x) = 0$$

where $L_{n\alpha}(x)$ is the associated Laguerre polynomial

$$L_{n\alpha}(x) = \frac{e^x}{x^\alpha n!} \frac{d^n}{dx^n} (x^{n+\alpha} e^{-x})$$

For integer α the following formula can be used.

$$L_{n\alpha}(x) = (n + \alpha)! \sum_{k=0}^n \frac{(-x)^k}{(n - k)! (\alpha + k)! k!}$$