

7-4. Discuss why the integrated part vanishes.

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The authors are referring to integration by parts in equation (7.32).

$$\int \frac{\partial F}{\partial x_k} \exp\left(\frac{i}{\hbar} S(x(t))\right) \mathcal{D}x(t) = -\frac{i}{\hbar} \int F \frac{\partial S}{\partial x_k} \exp\left(\frac{i}{\hbar} S(x(t))\right) \mathcal{D}x(t) \quad (7.32)$$

where

$$x_k = x(t_k)$$

Note that integration by parts in (7.32) uses  $x_k$ , not  $x(t)$ .

Let

$$u = \exp\left(\frac{i}{\hbar} S(x(t))\right) \mathcal{D}x(t)$$

$$dv = \frac{\partial F}{\partial x_k}$$

Then

$$du = \frac{i}{\hbar} \frac{\partial S}{\partial x_k} \exp\left(\frac{i}{\hbar} S(x(t))\right) \mathcal{D}x(t)$$

$$v = F$$

Integrate by parts.

$$\int u dv = uv - \int v du$$

$$= F \exp\left(\frac{i}{\hbar} S(x(t))\right) \mathcal{D}x(t) - \frac{i}{\hbar} \int F \frac{\partial S}{\partial x_k} \exp\left(\frac{i}{\hbar} S(x(t))\right) \mathcal{D}x(t)$$

The term  $uv$  vanishes because there is no integral over  $\mathcal{D}x(t)$ . There has to be an integration interval to obtain a nonzero transition amplitude. The concept is similar to a probability density function. A probability density function must be integrated over an interval to obtain a nonzero probability.