7-11. Show this.

$$\int_{-\infty}^{\infty} \chi^*(x) \frac{\hbar}{i} \frac{\partial \psi(x)}{\partial x} \, dx = \int_{-\infty}^{\infty} \chi^*(p) p \psi(p) \frac{dp}{2\pi\hbar} \qquad (7.81)$$

Consider equation (7.80).

$$\chi(p) = \int_{-\infty}^{\infty} \chi(x) \exp\left(-\frac{ipx}{\hbar}\right) dx \qquad (7.80a)$$

$$\psi(p) = \int_{-\infty}^{\infty} \psi(x) \exp\left(-\frac{ipx}{\hbar}\right) dx \qquad (7.80b)$$

The associated inverse transforms are

$$\chi(x) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \chi(p) \exp\left(\frac{ipx}{\hbar}\right) dp$$

$$\psi(x) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \psi(p) \exp\left(\frac{ipx}{\hbar}\right) dp \qquad (1)$$

Complex conjugate (7.80a).

$$\chi^*(p) = \int_{-\infty}^{\infty} \chi^*(x) \exp\left(\frac{ipx}{\hbar}\right) dx \qquad (2)$$

By equation (1)

$$\frac{\hbar}{i} \frac{\partial \psi(x)}{\partial x} = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} p\psi(p) \exp\left(\frac{ipx}{\hbar}\right) dp \qquad (3)$$

Substitute (2) and (3) into (7.81).

$$\frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \chi^*(x) \left(\int_{-\infty}^{\infty} p\psi(p) \exp\left(\frac{ipx}{\hbar}\right) dp\right) dx$$

$$= \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} \chi^*(x) \exp\left(\frac{ipx}{\hbar}\right) dx\right) p\psi(p) \frac{dp}{2\pi\hbar}$$

By the distributive law the integrals are equivalent.

$$\frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \chi^*(x) p\psi(p) \exp\left(\frac{ipx}{\hbar}\right) dp \, dx$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \chi^*(x) \exp\left(\frac{ipx}{\hbar}\right) p\psi(p) \, dx \frac{dp}{2\pi\hbar}$$

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