Spherical harmonics

Verify that

$$r^2\nabla^2 Y_{lm}(\theta,\phi) = -l(l+1)Y_{lm}(\theta,\phi)$$

for selected spherical harmonic functions $Y_{lm}(\theta,\phi)$.

Since $Y_{lm}(\theta,\phi)$ is independent of r we have

$$r^2\nabla^2 Y = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial Y}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 Y}{\partial\phi^2}$$