

Feature index

abs(x)

Returns the absolute value or vector length of x .

```
X = (x,y,z)  
abs(X)
```

$$(x^2 + y^2 + z^2)^{1/2}$$

adj(m)

Returns the adjunct of matrix m . Adjunct is equal to determinant times inverse.

```
A = ((a,b),(c,d))  
adj(A) == det(A) inv(A)
```

1

and(a, b, \dots)

Returns 1 if all arguments are true (nonzero). Returns 0 otherwise.

```
and(1=1,2=2)
```

1

arccos(x)

Returns the arc cosine of x .

```
arccos(1/2)
```

$$\frac{1}{3}\pi$$

arccosh(x)

Returns the arc hyperbolic cosine of x .

arcsin(x)

Returns the arc sine of x .

```
arcsin(1/2)
```

$$\frac{1}{6}\pi$$

arcsinh(x)

Returns the arc hyperbolic sine of x .

arctan(y, x)

Returns the arc tangent of y over x . If x is omitted then $x = 1$ is used.

```
arctan(1,0)
```

$\frac{1}{2}\pi$

arctanh(x)

Returns the arc hyperbolic tangent of x .

arg(z)

Returns the angle of complex z .

```
arg(2 - 3i)
```

```
arctan(-3,2)
```

binding(s)

The result of evaluating a symbol can differ from the symbol's binding. For example, the result may be expanded. The **binding** function returns the actual binding of a symbol.

```
p = quote((x + 1)^2)
p
```

$p = x^2 + 2x + 1$

```
binding(p)
```

$(x + 1)^2$

ceiling(x)

Returns the smallest integer greater than or equal to x .

```
ceiling(1/2)
```

1

check(x)

If x is true (nonzero) then continue, else stop. Expression x can include the relational operators =, ==, <, <=, >, >=. Use the **not** function to test for inequality.

```
A = exp(i pi)
B = -1
check(A == B) -- stop here if A not equal to B
```

choose(n, k)

Returns the binomial coefficient n choose k .

```
choose(52,5) -- number of poker hands
```

2598960

circexp(x)

Returns expression x with circular and hyperbolic functions converted to exponentials.

```
circexp(cos(x) + i sin(x))
```

$\exp(ix)$

clear

Clears all symbol definitions.

clock(z)

Returns complex z in polar form with base of negative 1 instead of e .

```
clock(2 - 3i)
```

$13^{1/2} (-1)^{\arctan(-3,2)/\pi}$

cofactor(m, i, j)

Returns the cofactor of matrix m for row i and column j .

```
A = ((a,b),(c,d))
cofactor(A,1,2) == adj(A)[2,1]
```

1

conj(z)

Returns the complex conjugate of z .

`conj(2 - 3i)`

$2 + 3i$

contract(a, i, j)

Returns tensor a summed over indices i and j . If i and j are omitted then 1 and 2 are used. The expression **contract**(m) computes the trace of matrix m .

`A = ((a,b),(c,d))`
`contract(A)`

$a + d$

cos(x)

Returns the cosine of x .

`cos(pi/4)`

$\frac{1}{2^{1/2}}$

cosh(x)

Returns the hyperbolic cosine of x .

`circexp(cosh(x))`

$\frac{1}{2} \exp(-x) + \frac{1}{2} \exp(x)$

cross(u, v)

Returns the cross product of vectors u and v .

curl(v)

Returns the curl of vector v with respect to symbols \mathbf{x} , \mathbf{y} , and \mathbf{z} .

d(f, x, \dots)

Returns the partial derivative of f with respect to x and any additional arguments.

d(**sin**(**x**),**x**)

$\cos(x)$

Multiderivatives are computed by extending the argument list.

d(**sin**(**x**),**x**,**x**)

$-\sin(x)$

A numeric argument n computes the n th derivative with respect to the previous symbol.

d(**sin**(**x y**),**x**,2,**y**,2)

$x^2y^2 \sin(xy) - 4xy \cos(xy) - 2 \sin(xy)$

Argument f can be a tensor of any rank. Argument x can be a vector. When x is a vector the result is the gradient of f .

F = (**f**(),**g**(),**h**())

X = (**x**,**y**,**z**)

d(**F**,**X**)

$$\begin{bmatrix} d(f(),x) & d(f(),y) & d(f(),z) \\ d(g(),x) & d(g(),y) & d(g(),z) \\ d(h(),x) & d(h(),y) & d(h(),z) \end{bmatrix}$$

Symbol **d** can be used as a variable name. Doing so does not conflict with function **d**.

Symbol **d** can be redefined as a different function. The function **derivative**, a synonym for **d**, can be used to obtain a partial derivative.

defint(f, x, a, b)

Returns the definite integral of f with respect to x evaluated from a to b . The argument list can be extended for multiple integrals as shown in the following example.

f = (1 + **cos**(**theta**)^2) **sin**(**theta**)

-- integrate over **theta** then over **phi**

defint(**f**, **theta**, 0, **pi**, **phi**, 0, 2 **pi**)

$\frac{16}{3}\pi$

denominator(x)

Returns the denominator of expression x .

```
denominator(a/b)
```

b

det(m)

Returns the determinant of matrix m .

```
A = ((a,b),(c,d))  
det(A)
```

$ad - bc$

dim(a, n)

Returns the dimension of the n th index of tensor a . Index numbering starts with 1.

```
A = ((1,2),(3,4),(5,6))  
dim(A,1)
```

3

div(v)

Returns the divergence of vector v with respect to symbols \mathbf{x} , \mathbf{y} , and \mathbf{z} .

do(a, b, \dots)

Evaluates each argument from left to right. Returns the result of the final argument.

```
do(A=1,B=2,A+B)
```

3

dot(a, b, \dots)

Returns the dot product of vectors, matrices, and tensors. Also known as the matrix product. Arguments are evaluated from right to left. The following example solves for X in $AX = B$.

```
A = ((1,2),(3,4))  
B = (5,6)  
X = dot(inv(A),B)  
X
```

$$\begin{bmatrix} -4 \\ \frac{9}{2} \end{bmatrix}$$

eigenvec(m)

Returns eigenvectors for matrix m . Matrix m is required to be numerical, real, and symmetric. The return value is a matrix with each column an eigenvector. Eigenvalues are obtained as shown.

```
A = ((1,2,3),(2,6,4),(3,4,5))
Q = eigenvec(A)
D = dot(transpose(Q),A,Q) -- eigenvalues on the diagonal of D
dot(Q,D,transpose(Q))
```

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 6 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

eval(f, x, a, y, b, \dots)

Returns f evaluated with x replaced by a , y replaced by b , etc. All arguments can be expressions.

```
f = sqrt(x^2 + y^2)
eval(f,x,3,y,4)
```

5

In the following example, `eval` is used to replace `x` with `cos(theta)`.

```
-- associated legendre of cos theta
P(1,m,x) = test(m < 0, (-1)^m (1 + m)! / (1 - m)! P(1,-m),
               1 / (2^1 1!) sin(theta)^m *
               eval(d((x^2 - 1)^1, x, 1 + m), x, cos(theta)))
```

```
P(2,-1)
```

$$-\frac{1}{2} \cos(\theta) \sin(\theta)$$

exp(x)

Returns the exponential of x .

```
exp(i pi)
```

-1

expcos(*z*)

Returns the cosine of z in exponential form.

expcos(*z*)

$$\frac{1}{2} \exp(iz) + \frac{1}{2} \exp(-iz)$$

expcosh(*z*)

Returns the hyperbolic cosine of z in exponential form.

expcosh(*z*)

$$\frac{1}{2} \exp(-z) + \frac{1}{2} \exp(z)$$

expsin(*z*)

Returns the sine of z in exponential form.

expsin(*z*)

$$-\frac{1}{2}i \exp(iz) + \frac{1}{2}i \exp(-iz)$$

expsinh(*z*)

Returns the hyperbolic sine of z in exponential form.

expsinh(*z*)

$$-\frac{1}{2} \exp(-z) + \frac{1}{2} \exp(z)$$

exptan(*z*)

Returns the tangent of z in exponential form.

exptan(*z*)

$$\frac{i}{\exp(2iz) + 1} - \frac{i \exp(2iz)}{\exp(2iz) + 1}$$

exptanh(*z*)

Returns the hyperbolic tangent of z in exponential form.

exptanh(*z*)

$$-\frac{1}{\exp(2z) + 1} + \frac{\exp(2z)}{\exp(2z) + 1}$$

factorial(n)

Returns the factorial of n . The expression $\mathbf{n!}$ can also be used.

`20!`

2432902008176640000

float(x)

Returns expression x with rational numbers and integers converted to floating point values. The symbol `pi` and the natural number are also converted.

`float(212^17)`

3.52947×10^{39}

floor(x)

Returns the largest integer less than or equal to x .

`floor(1/2)`

0

for(i, j, k, a, b, \dots)

For i equals j through k evaluate a, b , etc.

`for(k,1,3,A=k,print(A))`

$A = 1$

$A = 2$

$A = 3$

Note: The original value of i is restored after `for` completes. If symbol `i` is used for index variable i then the imaginary unit is overridden in the scope of `for`.

grad(f)

Returns the gradient $\mathbf{d(f, (x,y,z))}$.

`grad(f())`

$$\begin{bmatrix} d(f(), x) \\ d(f(), y) \\ d(f(), z) \end{bmatrix}$$

hadamard(a, b, \dots)

Returns the Hadamard (element-wise) product.

```
X = (a,b,c)
hadamard(X,X)
```

$$\begin{bmatrix} a^2 \\ b^2 \\ c^2 \end{bmatrix}$$

i

Symbol **i** is initialized to the imaginary unit $\sqrt{-1}$.

```
exp(i pi)
-1
```

Note: It is ok to clear or redefine **i** and use the symbol for something else.

imag(z)

Returns the imaginary part of complex z .

```
imag(2 - 3i)
-3
```

infixform(x)

Converts expression x to a string and returns the result.

```
p = (x + 1)^2
infixform(p)
x^2 + 2 x + 1
```

inner(a, b, \dots)

Returns the inner product of vectors, matrices, and tensors. Also known as the matrix product.

```
A = ((a,b),(c,d))
B = (x,y)
inner(A,B)
```

$$\begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

Note: **inner** and **dot** are the same function.

integral(f, x)

Returns the integral of f with respect to x .

`integral(x^2,x)`

$$\frac{1}{3}x^3$$

inv(m)

Returns the inverse of matrix m .

`A = ((1,2),(3,4))`

`inv(A)`

$$\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$$

j

Set `j=sqrt(-1)` to use `j` for the imaginary unit instead of `i`.

`j = sqrt(-1)`

`1/sqrt(-1)`

$$-j$$

kronecker(a, b, \dots)

Returns the Kronecker product of vectors and matrices.

`A = ((1,2),(3,4))`

`B = ((a,b),(c,d))`

`kronecker(A,B)`

$$\begin{bmatrix} a & b & 2a & 2b \\ c & d & 2c & 2d \\ 3a & 3b & 4a & 4b \\ 3c & 3d & 3c & 4d \end{bmatrix}$$

last

The result of the previous calculation is stored in **last**.

```
212^17
```

```
3529471145760275132301897342055866171392
```

```
last^(1/17)
```

```
212
```

Symbol **last** is an implied argument when a function has no argument list.

```
212^17
```

```
3529471145760275132301897342055866171392
```

```
float
```

```
3.52947 × 1039
```

log(*x*)

Returns the natural logarithm of *x*.

```
log(x^y)
```

```
y log(x)
```

mag(*z*)

Returns the magnitude of complex *z*. Function **mag** treats undefined symbols as real while **abs** does not.

```
mag(x + i y)
```

```
(x2 + y2)1/2
```

minor(*m*, *i*, *j*)

Returns the minor of matrix *m* for row *i* and column *j*.

```
A = ((1,2,3),(4,5,6),(7,8,9))
```

```
minor(A,1,1) == det(minormatrix(A,1,1))
```

1

minormatrix(m, i, j)

Returns a copy of matrix m with row i and column j removed.

```
A = ((1,2,3),(4,5,6),(7,8,9))  
minormatrix(A,1,1)
```

$$\begin{bmatrix} 5 & 6 \\ 8 & 9 \end{bmatrix}$$

noexpand(x)

Evaluates expression x without expanding products of sums.

```
noexpand((x + 1)^2 / (x + 1))
```

$$x + 1$$

not(x)

Returns 0 if x is true (nonzero). Returns 1 otherwise.

```
not(1=1)
```

$$0$$

nroots(p, x)

Returns the approximate roots of polynomials with real or complex coefficients. Multiple roots are returned as a vector.

```
p = x^5 - 1  
nroots(p,x)
```

$$\begin{bmatrix} 1 \\ -0.809017 + 0.587785 i \\ -0.809017 - 0.587785 i \\ 0.309017 + 0.951057 i \\ 0.309017 - 0.951057 i \end{bmatrix}$$

numerator(x)

Returns the numerator of expression x .

```
numerator(a/b)
```

$$a$$

or(a, b, \dots)

Returns 1 if at least one argument is true (nonzero). Returns 0 otherwise.

`or(1=1,2=2)`

1

outer(a, b, \dots)

Returns the outer product of vectors, matrices, and tensors.

`A = (a,b,c)`

`B = (x,y,z)`

`outer(A,B)`

$$\begin{bmatrix} ax & ay & az \\ bx & by & bz \\ cx & cy & cz \end{bmatrix}$$

pi

Symbol for π .

`exp(i pi)`

-1

polar(z)

Returns complex z in polar form.

`polar(x - i y)`

$$(x^2 + y^2)^{1/2} \exp(i \arctan(-y, x))$$

power

Use `^` to raise something to a power. Use parentheses for negative powers.

`x^(-2)`

$$\frac{1}{x^2}$$

print(a, b, \dots)

Evaluate expressions and print the results. Useful for printing from inside a **for** loop.

```
for(j,1,3,print(j))
```

$j = 1$

$j = 2$

$j = 3$

product(i, j, k, f)

For i equals j through k evaluate f . Returns the product of all f .

```
product(j,1,3,x + j)
```

$x^3 + 6x^2 + 11x + 6$

The original value of i is restored after **product** completes. If symbol **i** is used for index variable i then the imaginary unit is overridden in the scope of **product**.

product(y)

Returns the product of components of y .

```
y = (1,2,3,4)
```

```
product(y)
```

24

quote(x)

Returns expression x without evaluating it first.

```
quote((x + 1)^2)
```

$(x + 1)^2$

rank(a)

Returns the number of indices that tensor a has.

```
A = ((a,b),(c,d))
```

```
rank(A)
```

2

rationalize(x)

Returns expression x with everything over a common denominator.

```
rationalize(1/a + 1/b + 1/2)
```

$$\frac{2a + ab + 2b}{2ab}$$

Note: **rationalize** returns an unexpanded expression. If the result is assigned to a symbol, evaluating the symbol will expand the result. Use **binding** to retrieve the unexpanded expression.

```
f = rationalize(1/a + 1/b + 1/2)
binding(f)
```

$$\frac{2a + ab + 2b}{2ab}$$

real(z)

Returns the real part of complex z .

```
real(2 - 3i)
```

2

rect(z)

Returns complex z in rectangular form.

```
rect(exp(i x))
```

$$\cos(x) + i \sin(x)$$

roots(p, x)

Returns the rational roots of a polynomial. Multiple roots are returned as a vector.

```
p = (x + 1) (x - 2)
roots(p,x)
```

$$\begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

If no roots are found then **nil** is returned. A **nil** result is not printed so the following example uses **infixform** to print **nil** as a string.

```
p = x^2 + 1
infixform(roots(p,x))
```

nil

rotate(u, s, k, \dots)

Rotates vector u and returns the result. Vector u is required to have 2^n elements where n is an integer from 1 to 15. Arguments s, k, \dots are a sequence of rotation codes where s is an upper case letter and k is a qubit number from 0 to $n - 1$. Rotations are evaluated from left to right. See the section on quantum computing for a list of rotation codes.

```
psi = (1,0,0,0)
rotate(psi,H,0)
```

$$\begin{bmatrix} \frac{1}{2^{1/2}} \\ \frac{1}{2^{1/2}} \\ 0 \\ 0 \end{bmatrix}$$

run(x)

Run script x where x evaluates to a filename string. Useful for importing function libraries.

```
run("/Users/heisenberg/EVA2.txt")
```

simplify(x)

Returns expression x in a simpler form.

```
simplify(sin(x)^2 + cos(x)^2)
```

1

sin(x)

Returns the sine of x .

```
sin(pi/4)
```

$$\frac{1}{2^{1/2}}$$

sinh(x)

Returns the hyperbolic sine of x .

```
circexp(sinh(x))
```

$$-\frac{1}{2} \exp(-x) + \frac{1}{2} \exp(x)$$

sqrt(x)

Returns the square root of x .

`sqrt(10!)`

720 $7^{1/2}$

stop

In a script, it does what it says.

sum(i, j, k, f)

For i equals j through k evaluate f . Returns the sum of all f .

`sum(j,1,5,x^j)`

$x^5 + x^4 + x^3 + x^2 + x$

The original value of i is restored after **sum** completes. If symbol **i** is used for index variable i then the imaginary unit is overridden in the scope of **sum**.

sum(y)

Returns the sum of components of y .

`y = (1,2,3,4)`
`sum(y)`

10

tan(x)

Returns the tangent of x .

`simplify(tan(x) - sin(x)/cos(x))`

0

tanh(x)

Returns the hyperbolic tangent of x .

`circexp(tanh(x))`

$$-\frac{1}{\exp(2x) + 1} + \frac{\exp(2x)}{\exp(2x) + 1}$$

test(a, b, c, d, \dots)

If argument a is true (nonzero) then b is returned, else if c is true then d is returned, etc. If the number of arguments is odd then the final argument is returned if all else fails. Expressions can include the relational operators $=$, $==$, $<$, $<=$, $>$, $>=$. Use the **not** function to test for inequality. (The equality operator $==$ is available for contexts in which $=$ is the assignment operator.)

```
A = 1
B = 1
test(A=B, "yes", "no")
```

yes

trace

Set **trace**=1 in a script to print the script as it is evaluated. Useful for debugging.

```
trace = 1
```

Note: The **contract** function is used to obtain the trace of a matrix.

transpose(a, i, j)

Returns the transpose of tensor a with respect to indices i and j . If i and j are omitted then 1 and 2 are used. Hence a matrix can be transposed with a single argument.

```
A = ((a,b),(c,d))
transpose(A)
```

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

Note: The argument list can be extended for multiple transpose operations. Arguments are evaluated from left to right. For example, **transpose**(A,1,2,2,3) is equivalent to **transpose**(**transpose**(A,1,2),2,3)

tty

Set **tty**=1 to show results in string format. Set **tty**=0 to turn off. Can be useful when displayed results exceed window size.

```
tty = 1
(x + 1)^2
```

$x^2 + 2x + 1$

unit(n)

Returns an n by n identity matrix.

`unit(3)`

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

zero(i, j, \dots)

Returns a null tensor with dimensions i, j , etc. Useful for creating a tensor and then setting component values.

```
A = zero(3,3)
for(k,1,3,A[k,k]=k)
A
```

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$