9-2. Explain why the charge density corresponding to a single charge q located at the point $\mathbf{x}(t) = (x(t), y(t), z(t))$ at time t is

$$\rho(\mathbf{r}, t) = q\delta(r_x - x(t))\delta(r_y - y(t))\delta(r_z - z(t)) = q\delta^3(\mathbf{r} - \mathbf{x}(t))$$

From equation (9.14)

$$\rho(\mathbf{r}, t) = \int \rho_{\mathbf{k}}(t) \exp(i\mathbf{k} \cdot \mathbf{r}) \frac{d^3\mathbf{k}}{(2\pi)^3}$$

For a point charge at $\mathbf{x}(t)$

$$\rho_{\mathbf{k}}(t) = \begin{cases} q & \mathbf{k} \cdot \mathbf{r} = 0 \\ 0 & \mathbf{k} \cdot \mathbf{r} \neq 0 \end{cases}$$

Rewrite as

$$\rho(\mathbf{r}, t) = \\ \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho_{\mathbf{k}}(t) \exp(ik_x r_x) \exp(ik_y r_y) \exp(ik_z r_z)\, dk_x\, dk_y\, dk_z$$