Feynman and Hibbs problem 3-5

Use problem 3-2 and equation 3.42 to show that for a free particle wave function ψ

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar}\left(-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}\right)$$

From 3.42

$$\psi(x,t) = \int_{-\infty}^{\infty} K_0(x,t;x_c,t_c)\psi(x_c,t_c)\,dx_c$$

By linearity of differentiation

$$\frac{\partial \psi}{\partial t} = \int_{-\infty}^{\infty} \frac{\partial}{\partial t}\left(K_0(x,t;x_c,t_c)\psi(x_c,t_c)\,dx_c\right)$$
$$\frac{\partial^2 \psi}{\partial x^2} = \int_{-\infty}^{\infty} \frac{\partial^2}{\partial x^2}\left(K_0(x,t;x_c,t_c)\psi(x_c,t_c)\,dx_c\right)$$

By independence of coordinates

$$\frac{\partial \psi}{\partial t} = \int_{-\infty}^{\infty} \frac{\partial K_0}{\partial t}\psi(x_c,t_c)\,dx_c$$
$$\frac{\partial^2 \psi}{\partial x^2} = \int_{-\infty}^{\infty} \frac{\partial^2 K_0}{\partial x^2}\psi(x_c,t_c)\,dx_c$$

$$(1)$$

From problem 3-2

$$\frac{\partial K_0}{\partial t} = -\frac{i}{\hbar}\left(-\frac{\hbar^2}{2m}\frac{\partial^2 K_0}{\partial x^2}\right)$$

Multiply both sides by $\psi(x_c,t_c)$ and integrate over x_c.

$$\int_{-\infty}^{\infty} \frac{\partial K_0}{\partial t}\psi(x_c,t_c)\,dx_c = \int_{-\infty}^{\infty} -\frac{i}{\hbar}\left(-\frac{\hbar^2}{2m}\frac{\partial^2 K_0}{\partial x^2}\right)\psi(x_c,t_c)\,dx_c$$

By the distributive property

$$\int_{-\infty}^{\infty} \frac{\partial K_0}{\partial t}\psi(x_c,t_c)\,dx_c = -\frac{i}{\hbar}\left(-\frac{\hbar^2}{2m}\int_{-\infty}^{\infty} \frac{\partial^2 K_0}{\partial x^2}\psi(x_c,t_c)\,dx_c\right)$$

Then by equation (1)

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar}\left(-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}\right)$$