9-3. Prove that the relation $\phi_{\mathbf{k}} = 4\pi\rho_{\mathbf{k}}/k^2$ simply means that $\phi_{\mathbf{k}}$ at any instant is the Coulomb potential from the charges at that instant, so that, for example, if ρ comes from a number of charges q_i at distances R_i from a point, the potential at the point is $\phi = \sum_i q_i/R_i$.

Integrate using polar coordinates.

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{\infty} \frac{4\pi\rho_{\mathbf{k}}(t)}{k^2}\exp(ikR)\,k^2\sin\theta\,dk\,d\theta\,d\phi = \frac{16\pi^2 i\rho_{\mathbf{k}}(t)}{R} \tag{1}$$

The result is Coulomb's law for

$$\rho_{\mathbf{k}}(t) = -\frac{iq}{64\pi^3\varepsilon_0}$$

The following integrals show how (1) is obtained.

$$\int_0^{\infty}\exp(-ax)\,dx = \frac{1}{a} \qquad \int_0^{\pi}\sin\theta\,d\theta = 2 \qquad \int_0^{2\pi} d\phi = 2\pi$$

Note that for multiple charges q_i we have

$$\rho(\mathbf{r},t) = \sum_i q_i\delta(R_i)$$
$$\phi(\mathbf{r},t) = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{R_i}$$