Spin flip

An electron is at rest in the following magnetic field.

$$\mathbf{B} = B_0 \cos(\omega t) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

What is the minimum B_0 that flips electron spin in the x direction?

These are the spin operators.

$$S_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad S_y = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad S_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Let $\mathbf{S}$ be the spin angular momentum operator

$$\mathbf{S} = \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}$$

Let H be the Hamiltonian

$$H = \frac{ge}{2m} \mathbf{S} \cdot \mathbf{B} = \frac{ge}{2m} S_z B_0 \cos(\omega t)$$

Let $s(t)$ be the spin state

$$s(t) = \begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix}$$

By the Schrodinger equation

$$i\hbar \frac{\partial}{\partial t} s(t) = H s(t)$$

we have

$$\begin{aligned} i\hbar \frac{\partial}{\partial t} c_1(t) &= \frac{ge\hbar}{4m} B_0 \cos(\omega t) c_1(t) \\ i\hbar \frac{\partial}{\partial t} c_2(t) &= -\frac{ge\hbar}{4m} B_0 \cos(\omega t) c_2(t) \end{aligned}$$

Hence

$$\begin{aligned} c_1(t) &= C \exp\left(-\frac{ige}{4m\omega} B_0 \sin(\omega t)\right) \\ c_2(t) &= -C \exp\left(-\frac{ige}{4m\omega} B_0 \sin(\omega t)\right) \end{aligned} \tag{1}$$

By the normalization requirement $|s(t)| = 1$ we have

$$C = \frac{1}{\sqrt{2}}$$

This is the expectation value for S_x.

$$\langle S_x \rangle = \langle s|S_x|s \rangle = \frac{\hbar}{2} \cos\left(\frac{ge}{2m\omega} B_0 \sin(\omega t)\right) \tag{2}$$

Note that the cosine is positive for all time t when

$$\frac{ge}{2m\omega} B_0 \leq \frac{\pi}{2}$$

Hence the minimum B_0 to make $\langle S_x \rangle$ negative for some time t is

$$B_0 > \frac{\pi m\omega}{ge}$$