3-5. Let ψ be a free-particle wave function. Use problem 3-2 and equation (3.42) to show that

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar}\left(-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}\right)$$

From equation (3.42)

$$\psi(x,t) = \int_{-\infty}^{\infty} K_0(x,t;x_c,t_c)\psi(x_c,t_c)\,dx_c$$

By linearity of differentiation

$$\begin{aligned}
\frac{\partial \psi}{\partial t} &= \int_{-\infty}^{\infty} \frac{\partial}{\partial t}\Big(K_0(x,t;x_c,t_c)\psi(x_c,t_c)\,dx_c\Big) \\
\frac{\partial^2 \psi}{\partial x^2} &= \int_{-\infty}^{\infty} \frac{\partial^2}{\partial x^2}\Big(K_0(x,t;x_c,t_c)\psi(x_c,t_c)\,dx_c\Big)
\end{aligned}$$

By independence of coordinates

$$\begin{aligned}
\frac{\partial \psi}{\partial t} &= \int_{-\infty}^{\infty} \frac{\partial K_0}{\partial t}\psi(x_c,t_c)\,dx_c \\
\frac{\partial^2 \psi}{\partial x^2} &= \int_{-\infty}^{\infty} \frac{\partial^2 K_0}{\partial x^2}\psi(x_c,t_c)\,dx_c
\end{aligned} \tag{1}$$

From problem 3-2

$$\frac{\partial K_0}{\partial t} = -\frac{i}{\hbar}\left(-\frac{\hbar^2}{2m}\frac{\partial^2 K_0}{\partial x^2}\right)$$

Multiply both sides by $\psi(x_c,t_c)$ and integrate over x_c.

$$\int_{-\infty}^{\infty} \frac{\partial K_0}{\partial t}\psi(x_c,t_c)\,dx_c = \int_{-\infty}^{\infty} -\frac{i}{\hbar}\left(-\frac{\hbar^2}{2m}\frac{\partial^2 K_0}{\partial x^2}\right)\psi(x_c,t_c)\,dx_c$$

By the distributive property

$$\int_{-\infty}^{\infty} \frac{\partial K_0}{\partial t}\psi(x_c,t_c)\,dx_c = -\frac{i}{\hbar}\left(-\frac{\hbar^2}{2m}\int_{-\infty}^{\infty} \frac{\partial^2 K_0}{\partial x^2}\psi(x_c,t_c)\,dx_c\right)$$

Then by equation (1)

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar}\left(-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}\right)$$