7-2. If $F\big(x(t)\big) = x(t)$, show that

$$\frac{\delta F}{\delta x(s)} = \delta(t - s)$$

Consider equation (7.20).

$$F\big(x(t) + \eta(t)\big) = F\big(x(t)\big) + \int \frac{\delta F}{\delta x(s)} \eta(s)\, ds + \cdots \qquad (7.20)$$

For $F\big(x(t)\big) = x(t)$ we have

$$F\big(x(t) + \eta(t)\big) = x(t) + \eta(t)$$

Then by equation (7.20) to first order

$$\eta(t) = \int \frac{\delta F}{\delta x(s)} \eta(s)\, ds$$

Hence

$$\frac{\delta F}{\delta x(s)} = \begin{cases} 1 & t = s \\ 0 & t \neq s \end{cases}$$