

Canonical commutation relation

Consider the canonical commutation relation in one dimension.

$$XP - PX = i\hbar$$

Let

$$X = x, \quad P = -i\hbar \frac{\partial}{\partial x}$$

Show that

$$(XP - PX)\psi(x, t) = i\hbar\psi(x, t)$$

Eigenmath code:

```
X(f) = x f
P(f) = -i hbar d(f,x)
X(P(psi(x,t))) - P(X(psi(x,t)))
```

Result:

$$i\hbar\psi(x, t)$$

In three dimensions:

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \otimes, \quad P = -i\hbar \nabla$$

Eigenmath code:

```
X(f) = outer((x1,x2,x3),f)
P(f) = -i hbar d(f,(x1,x2,x3))
X(P(psi(x1,x2,x3,t))) - P(X(psi(x1,x2,x3,t)))
```

Result:

$$\begin{bmatrix} i\hbar\psi(x_1, x_2, x_3, t) & 0 & 0 \\ 0 & i\hbar\psi(x_1, x_2, x_3, t) & 0 \\ 0 & 0 & i\hbar\psi(x_1, x_2, x_3, t) \end{bmatrix}$$