

This is the Schrodinger equation for $2d$ hydrogen.

$$-\frac{\hbar^2}{2\mu} \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \right) \psi_n - \frac{k}{r} \psi_n = E_n \psi_n$$

Symbol μ is reduced electron mass and k/r is potential energy with

$$k = \frac{e^2}{4\pi\epsilon_0}$$

Energy eigenvalues are

$$E_n = -\frac{k^2\mu}{2\hbar^2 \left(n + \frac{1}{2}\right)^2}$$

In matrix form we have

$$H = \frac{P^2}{2\mu} - kV$$

where matrix elements are computed as follows.

$$H_{nm} = E_n \delta_{nm}$$

$$(P^2)_{nm} = -\hbar^2 \int_0^{2\pi} \int_0^\infty \psi_n \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} \right) \psi_m r dr d\phi$$

$$V_{nm} = \int_0^{2\pi} \int_0^\infty \psi_n \left(\frac{1}{r} \right) \psi_m r dr d\phi$$

Here are numerical values for 4×4 matrices.

$$H = \begin{pmatrix} -87.1474 & 0 & 0 & 0 \\ 0 & -9.68305 & 0 & 0 \\ 0 & 0 & -3.48590 & 0 \\ 0 & 0 & 0 & -1.77852 \end{pmatrix} \times 10^{-19} \text{ joule}$$

$$P^2 = \begin{pmatrix} 158.685 & 45.8085 & 21.0271 & 12.6515 \\ 45.8085 & 17.6317 & 9.28281 & 5.23021 \\ 21.0271 & 9.28281 & 6.34742 & 3.98524 \\ 12.6515 & 5.23021 & 3.98524 & 3.23848 \end{pmatrix} \times 10^{-49} \text{ kilogram}^2 \text{ meter}^2 \text{ second}^{-2}$$

$$V = \begin{pmatrix} 755.479 & 109.044 & 50.0534 & 30.1160 \\ 109.044 & 83.9421 & 22.0971 & 12.4502 \\ 50.0534 & 22.0971 & 30.2192 & 9.48657 \\ 30.1160 & 12.4502 & 9.48657 & 15.4179 \end{pmatrix} \times 10^8 \text{ meter}^{-1}$$