Let ϕ be the field

$$\phi(x,y,z,t) = p_x x + p_y y + p_z z - Et$$

where

$$E = \sqrt{p_x^2 c^2 + p_y^2 c^2 + p_z^2 c^2 + m^2 c^4}$$

The following solutions to the Dirac equation are used to model fermions.

$$\psi_1 = \underbrace{\begin{pmatrix} E/c + mc \\ 0 \\ p_z \\ p_x + ip_y \end{pmatrix}}_{\text{fermion spin up}} \exp\left(\frac{i\phi}{\hbar}\right) \qquad \psi_2 = \underbrace{\begin{pmatrix} 0 \\ E/c + mc \\ p_x - ip_y \\ -p_z \end{pmatrix}}_{\text{fermion spin down}} \exp\left(\frac{i\phi}{\hbar}\right)$$

$$\psi_7 = \underbrace{\begin{pmatrix} p_z \\ p_x + ip_y \\ E/c + mc \\ 0 \end{pmatrix}}_{\text{anti-fermion spin up}} \exp\left(-\frac{i\phi}{\hbar}\right) \qquad \psi_8 = \underbrace{\begin{pmatrix} p_x - ip_y \\ -p_z \\ 0 \\ E/c + mc \end{pmatrix}}_{\text{anti-fermion spin down}} \exp\left(-\frac{i\phi}{\hbar}\right)$$

A spinor is the vector part of each solution. The following eight spinors are used for scattering calculations. The u spinors are fermions from ψ_1 and ψ_2. The v spinors are anti-fermions from ψ_7 and ψ_8. The last digit of the u or v subscript is 1 for spin up and 2 for spin down.

$$u_{11} = \begin{pmatrix} E_1/c + m_1 c \\ 0 \\ p_{1z} \\ p_{1x} + ip_{1y} \end{pmatrix} \quad v_{21} = \begin{pmatrix} p_{2z} \\ p_{2x} + ip_{2y} \\ E_2/c + m_2 c \\ 0 \end{pmatrix} \quad u_{31} = \begin{pmatrix} E_3/c + m_3 c \\ 0 \\ p_{3z} \\ p_{3x} + ip_{3y} \end{pmatrix} \quad v_{41} = \begin{pmatrix} p_{4z} \\ p_{4x} + ip_{4y} \\ E_4/c + m_4 c \\ 0 \end{pmatrix}$$

$$u_{12} = \begin{pmatrix} 0 \\ E_1/c + m_1 c \\ p_{1x} - ip_{1y} \\ -p_{1z} \end{pmatrix} \quad v_{22} = \begin{pmatrix} p_{2x} - ip_{2y} \\ -p_{2z} \\ 0 \\ E_2/c + m_2 c \end{pmatrix} \quad u_{32} = \begin{pmatrix} 0 \\ E_3/c + m_3 c \\ p_{3x} - ip_{3y} \\ -p_{3z} \end{pmatrix} \quad v_{42} = \begin{pmatrix} p_{4x} - ip_{4y} \\ -p_{4z} \\ 0 \\ E_4/c + m_4 c \end{pmatrix}$$

These are the associated momentum vectors.

$$p_1 = \begin{pmatrix} E_1/c \\ p_{1x} \\ p_{1y} \\ p_{1z} \end{pmatrix} \quad p_2 = \begin{pmatrix} E_2/c \\ p_{2x} \\ p_{2y} \\ p_{2z} \end{pmatrix} \quad p_3 = \begin{pmatrix} E_3/c \\ p_{3x} \\ p_{3y} \\ p_{3z} \end{pmatrix} \quad p_4 = \begin{pmatrix} E_4/c \\ p_{4x} \\ p_{4y} \\ p_{4z} \end{pmatrix}$$

Spinors are solutions to the following momentum-space Dirac equation with $\not{p} = p \cdot (\gamma^0, \gamma^1, \gamma^2, \gamma^3)$.

$$(\not{p} - mc)u = 0 \qquad (\not{p} + mc)v = 0$$

Up and down spinors have the following "completeness property."

$$u_{11}\bar{u}_{11} + u_{12}\bar{u}_{12} = (E_1/c + m_1 c)(\not{p}_1 + m_1 c) \qquad v_{21}\bar{v}_{21} + v_{22}\bar{v}_{22} = (E_2/c + m_2 c)(\not{p}_2 - m_2 c)$$

The adjoint of a spinor is $\bar{u} = u^\dagger \gamma^0$. The adjoint is a row vector hence $u\bar{u}$ is an outer product.