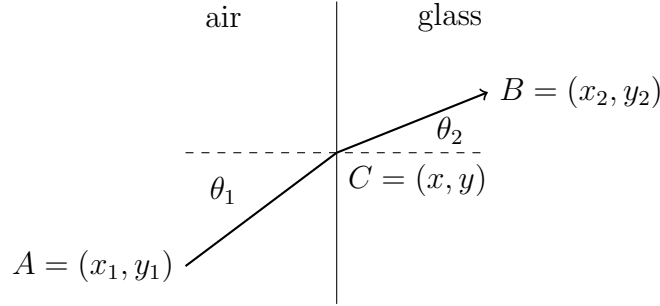


(1.1) Use Fermat's principle of least time to derive Snell's law.

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A light ray travels from  $A$  to  $B$  by going through  $C$ . Let  $d_1$  be the distance from  $A$  to  $C$  and let  $d_2$  be the distance from  $C$  to  $B$ .

$$d_1 = \sqrt{(x - x_1)^2 + (y - y_1)^2} \quad d_2 = \sqrt{(x - x_2)^2 + (y - y_2)^2}$$

Let  $v_1$  be the velocity of light through air and let  $v_2$  be the velocity of light through glass. Then the time  $t$  to go from  $A$  to  $B$  is

$$t = \frac{d_1}{v_1} + \frac{d_2}{v_2}$$

Differentiate  $t$  with respect to  $y$  and set the result to zero to obtain an equation that minimizes  $t$ . (The  $x$  coordinate of  $C$  is fixed by the boundary between air and glass.)

$$\frac{dt}{dy} = \frac{y - y_1}{v_1 d_1} + \frac{y - y_2}{v_2 d_2} = 0$$

Rewrite as

$$\frac{y - y_1}{v_1 d_1} = \frac{y_2 - y}{v_2 d_2}$$

Hence

$$\frac{\sin \theta_1}{v_1} = \frac{\sin \theta_2}{v_2}$$

Convert velocity to refractive indices.

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

where

$$n_1 = \frac{c}{v_1} \quad n_2 = \frac{c}{v_2}$$