For simplicity of notation let

$$K(b,a) \equiv K_V(x_b, y_b, t_b; x_a, y_a, t_a)$$

Start with this equation adapted from (3.75).

$$K(b,a) = \int_{y_a}^{y_b} \int_{x_a}^{x_b} \exp\left(\frac{i}{\hbar} \int_{t_a}^{t_b} \frac{m\dot{x}^2}{2}\, dt + \frac{i}{\hbar} \int_{t_a}^{t_b} \frac{M\dot{y}^2}{2}\, dt \right) \\ \times \exp\left(-\frac{i}{\hbar} \int_{t_a}^{t_b} V\big(x(t_c), y(t_c)\big)\, dt_c \right) \mathscr{D}x(t)\, \mathscr{D}y(t)$$

This is the power series expansion of the exponential of V.

$$\exp\left(-\frac{i}{\hbar} \int_{t_a}^{t_b} V\big(x(t_c), y(t_c)\big)\, dt_c \right) = 1 - \frac{i}{\hbar} \int_{t_a}^{t_b} V\big(x(t_c), y(t_c)\big)\, dt_c + \cdots$$

Hence the perturbation expansion of K is

$$K(b,a) = K_0(b,a) + K^{(1)}(b,a) + \cdots$$

where $K_0(b,a)$ is the free particle propagator and

$$K^{(1)}(b,a) = -\frac{i}{\hbar} \int_{y_a}^{y_b} \int_{x_a}^{x_b} \exp\left(\frac{i}{\hbar} \int_{t_a}^{t_b} \frac{m\dot{x}^2}{2}\, dt + \frac{i}{\hbar} \int_{t_a}^{t_b} \frac{M\dot{y}^2}{2}\, dt \right) \\ \times \int_{t_a}^{t_b} V\big(x(t_c), y(t_c)\big)\, dt_c\, \mathscr{D}x(t)\, \mathscr{D}y(t)$$

Interchange the order of the integrals.

$$K^{(1)}(b,a) = -\frac{i}{\hbar} \int_{t_a}^{t_b} \int_{y_a}^{y_b} \int_{x_a}^{x_b} \exp\left(\frac{i}{\hbar} \int_{t_a}^{t_b} \frac{m\dot{x}^2}{2}\, dt + \frac{i}{\hbar} \int_{t_a}^{t_b} \frac{M\dot{y}^2}{2}\, dt \right) \\ \times V\big(x(t_c), y(t_c)\big)\, \mathscr{D}x(t)\, \mathscr{D}y(t)\, dt_c$$

Factor the exponential as follows.

$$K^{(1)}(b,a) = -\frac{i}{\hbar} \int_{t_a}^{t_b} \int_{y_a}^{y_b} \int_{x_a}^{x_b} \exp\left(\frac{i}{\hbar} \int_{t_a}^{t_c} \frac{m\dot{x}^2}{2}\, dt \right) \exp\left(\frac{i}{\hbar} \int_{t_c}^{t_b} \frac{m\dot{x}^2}{2}\, dt \right) \\ \times \exp\left(\frac{i}{\hbar} \int_{t_a}^{t_c} \frac{M\dot{y}^2}{2}\, dt \right) \exp\left(\frac{i}{\hbar} \int_{t_c}^{t_b} \frac{M\dot{y}^2}{2}\, dt \right) V\big(x(t_c), y(t_c)\big)\, \mathscr{D}x(t)\, \mathscr{D}y(t)\, dt_c$$

The exponentials are free particle propagators.

$$\exp\left(\frac{i}{\hbar} \int_{t_a}^{t_c} \frac{m\dot{x}^2}{2}\, dt \right) = K_0\big(x(t_c), t_c, x_a, t_a, m\big) \quad m \text{ propagates from } x_a, t_a \text{ to } x(t_c), t_c$$

$$\exp\left(\frac{i}{\hbar} \int_{t_c}^{t_b} \frac{m\dot{x}^2}{2}\, dt \right) = K_0\big(x_b, t_b, x(t_c), t_c, m\big) \quad m \text{ propagates from } x(t_c), t_c \text{ to } x_b, t_b$$

$$\exp\left(\frac{i}{\hbar} \int_{t_a}^{t_c} \frac{M\dot{y}^2}{2}\, dt \right) = K_0\big(y(t_c), t_c, y_a, t_a, M\big) \quad M \text{ propagates from } y_a, t_a \text{ to } y(t_c), t_c$$

$$\exp\left(\frac{i}{\hbar} \int_{t_c}^{t_b} \frac{M\dot{y}^2}{2}\, dt \right) = K_0\big(y_b, t_b, y(t_c), t_c, M\big) \quad M \text{ propagates from } y(t_c), t_c \text{ to } y_b, t_b$$

Hence

$$K^{(1)}(b,a) = -\frac{i}{\hbar}\int_{t_a}^{t_b}\int_{y_a}^{y_b}\int_{x_a}^{x_b} K_0\big(x(t_c),t_c,x_a,t_a,m\big)K_0\big(x_b,t_b,x(t_c),t_c,m\big)$$
$$\times K_0\big(y(t_c),t_c,y_a,t_a,M\big)K_0\big(y_b,t_b,y(t_c),t_c,M\big)V\big(x(t_c),y(t_c)\big)\,\mathscr{D}x(t)\,\mathscr{D}y(t)\,dt_c$$

Let $x_c = x(t_c)$ and $y_c = y(t_c)$. Since we are integrating over all possible paths $x(t)$ and $y(t)$, we have $-\infty < x_c < \infty$ and $-\infty < y_c < \infty$. Hence the integral can be transformed into an integral over x_c and y_c.

$$K^{(1)}(b,a) = -\frac{i}{\hbar}\int_{t_a}^{t_b}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K_0(x_c,t_c,x_a,t_a,m)K_0(x_b,t_b,x_c,t_c,m)$$
$$\times K_0(y_c,t_c,y_a,t_a,M)K_0(y_b,t_b,y_c,t_c,M)V(x_c,y_c)\,dx_c\,dy_c\,dt_c$$

These are the free particle propagators in three dimensions.

$$K_0(x_c,t_c,x_a,t_a,m) = \left(\frac{m}{2\pi i\hbar(t_c-t_a)}\right)^{\frac{3}{2}}\exp\left(\frac{im|\mathbf{x}_c-\mathbf{x}_a|^2}{2\hbar(t_c-t_a)}\right)\qquad m\text{ from }x_a,t_a\text{ to }x_c,t_c$$
$$K_0(x_b,t_b,x_c,t_c,m) = \left(\frac{m}{2\pi i\hbar(t_b-t_c)}\right)^{\frac{3}{2}}\exp\left(\frac{im|\mathbf{x}_b-\mathbf{x}_c|^2}{2\hbar(t_b-t_c)}\right)\qquad m\text{ from }x_c,t_c\text{ to }x_b,t_b$$
$$K_0(y_c,t_c,y_a,t_a,M) = \left(\frac{M}{2\pi i\hbar(t_c-t_a)}\right)^{\frac{3}{2}}\exp\left(\frac{iM|\mathbf{y}_c-\mathbf{y}_a|^2}{2\hbar(t_c-t_a)}\right)\qquad M\text{ from }y_a,t_a\text{ to }y_c,t_c$$
$$K_0(y_b,t_b,y_c,t_c,M) = \left(\frac{M}{2\pi i\hbar(t_b-t_c)}\right)^{\frac{3}{2}}\exp\left(\frac{iM|\mathbf{y}_b-\mathbf{y}_c|^2}{2\hbar(t_b-t_c)}\right)\qquad M\text{ from }y_c,t_c\text{ to }y_b,t_b$$

Hence

$$K^{(1)}(b,a) = -\frac{i}{\hbar}\int_{t_a}^{t_b}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{m}{2\pi i\hbar(t_c-t_a)}\right)^{\frac{3}{2}}\exp\left(\frac{im|\mathbf{x}_c-\mathbf{x}_a|^2}{2\hbar(t_c-t_a)}\right)$$
$$\times\left(\frac{m}{2\pi i\hbar(t_b-t_c)}\right)^{\frac{3}{2}}\exp\left(\frac{im|\mathbf{x}_b-\mathbf{x}_c|^2}{2\hbar(t_b-t_c)}\right)$$
$$\times\left(\frac{M}{2\pi i\hbar(t_c-t_a)}\right)^{\frac{3}{2}}\exp\left(\frac{iM|\mathbf{y}_c-\mathbf{y}_a|^2}{2\hbar(t_c-t_a)}\right)$$
$$\times\left(\frac{M}{2\pi i\hbar(t_b-t_c)}\right)^{\frac{3}{2}}\exp\left(\frac{iM|\mathbf{y}_b-\mathbf{y}_c|^2}{2\hbar(t_b-t_c)}\right)V(\mathbf{x}_c,\mathbf{y}_c)\,d\mathbf{x}_c\,d\mathbf{y}_c\,dt_c$$

Let $t_a = 0$ and $t_b = T$.

$$
\begin{aligned}
K^{(1)}(b,a) = -\frac{i}{\hbar}\int_0^T \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} &\left(\frac{m}{2\pi i\hbar t_c}\right)^{\frac{3}{2}} \exp\left(\frac{im|\mathbf{x}_c - \mathbf{x}_a|^2}{2\hbar t_c}\right) \\
&\times \left(\frac{m}{2\pi i\hbar (T - t_c)}\right)^{\frac{3}{2}} \exp\left(\frac{im|\mathbf{x}_b - \mathbf{x}_c|^2}{2\hbar (T - t_c)}\right) \\
&\times \left(\frac{M}{2\pi i\hbar t_c}\right)^{\frac{3}{2}} \exp\left(\frac{iM|\mathbf{y}_c - \mathbf{y}_a|^2}{2\hbar t_c}\right) \\
&\times \left(\frac{M}{2\pi i\hbar (T - t_c)}\right)^{\frac{3}{2}} \exp\left(\frac{iM|\mathbf{y}_b - \mathbf{y}_c|^2}{2\hbar (T - t_c)}\right) V(\mathbf{x}_c, \mathbf{y}_c)\, d\mathbf{x}_c\, d\mathbf{y}_c\, dt_c
\end{aligned}
$$