

Feynman and Hibbs problem 3-5

Let ψ be a free-particle wave function. Use problem 3-2 and equation (3.42) to show that

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar} \left(-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} \right)$$

From equation (3.42)

$$\psi(x, t) = \int_{-\infty}^{\infty} K_0(x, t; x_c, t_c) \psi(x_c, t_c) dx_c$$

By linearity of differentiation

$$\begin{aligned} \frac{\partial \psi}{\partial t} &= \int_{-\infty}^{\infty} \frac{\partial}{\partial t} \left(K_0(x, t; x_c, t_c) \psi(x_c, t_c) dx_c \right) \\ \frac{\partial^2 \psi}{\partial x^2} &= \int_{-\infty}^{\infty} \frac{\partial^2}{\partial x^2} \left(K_0(x, t; x_c, t_c) \psi(x_c, t_c) dx_c \right) \end{aligned}$$

By independence of coordinates

$$\begin{aligned} \frac{\partial \psi}{\partial t} &= \int_{-\infty}^{\infty} \frac{\partial K_0}{\partial t} \psi(x_c, t_c) dx_c \\ \frac{\partial^2 \psi}{\partial x^2} &= \int_{-\infty}^{\infty} \frac{\partial^2 K_0}{\partial x^2} \psi(x_c, t_c) dx_c \end{aligned} \tag{1}$$

From problem 3-2

$$\frac{\partial K_0}{\partial t} = -\frac{i}{\hbar} \left(-\frac{\hbar^2}{2m} \frac{\partial^2 K_0}{\partial x^2} \right)$$

Multiply both sides by $\psi(x_c, t_c)$ and integrate over x_c .

$$\int_{-\infty}^{\infty} \frac{\partial K_0}{\partial t} \psi(x_c, t_c) dx_c = \int_{-\infty}^{\infty} -\frac{i}{\hbar} \left(-\frac{\hbar^2}{2m} \frac{\partial^2 K_0}{\partial x^2} \right) \psi(x_c, t_c) dx_c$$

By the distributive property

$$\int_{-\infty}^{\infty} \frac{\partial K_0}{\partial t} \psi(x_c, t_c) dx_c = -\frac{i}{\hbar} \left(-\frac{\hbar^2}{2m} \int_{-\infty}^{\infty} \frac{\partial^2 K_0}{\partial x^2} \psi(x_c, t_c) dx_c \right)$$

Then by equation (1)

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar} \left(-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} \right)$$