# Spin sign change

An electron is at rest in the following magnetic field.

$$\mathbf{B} = B_0 \cos(\omega t) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

What is the minimum $B_0$ that changes the sign of spin in the $x$ direction?

These are the spin operators.

$$S_x = \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad S_y = \frac{\hbar}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad S_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

This is the spin angular momentum operator.

$$\mathbf{S} = \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}$$

This is the Hamiltonian.

$$H = \frac{ge}{2m} \mathbf{S} \cdot \mathbf{B} = \frac{ge}{2m} S_z B_0 \cos(\omega t)$$

Let $s(t)$ be the spin state

$$s(t) = \begin{pmatrix} c_1(t) \\ c_2(t) \end{pmatrix}$$

By the Schrodinger equation

$$i\hbar \frac{\partial}{\partial t} s(t) = H s(t)$$

we have

$$\begin{aligned} i\hbar \frac{\partial}{\partial t} c_1(t) &= \frac{ge\hbar}{4m} B_0 \cos(\omega t) c_1(t) \\ i\hbar \frac{\partial}{\partial t} c_2(t) &= -\frac{ge\hbar}{4m} B_0 \cos(\omega t) c_2(t) \end{aligned}$$

Hence

$$\begin{aligned} c_1(t) &= C \exp\left( -\frac{ige}{4m\omega} B_0 \sin(\omega t) \right) \\ c_2(t) &= C \exp\left( \frac{ige}{4m\omega} B_0 \sin(\omega t) \right) \end{aligned} \tag{1}$$

By the normalization requirement $|s(t)| = 1$ we have

$$C = \frac{1}{\sqrt{2}}$$

In rectangular form

$$c_1(t) = \frac{1}{\sqrt{2}}\cos\left(\frac{ge}{4m\omega}B_0\sin(\omega t)\right) - \frac{i}{\sqrt{2}}\sin\left(\frac{ge}{4m\omega}B_0\sin(\omega t)\right)$$
$$c_2(t) = \frac{1}{\sqrt{2}}\cos\left(\frac{ge}{4m\omega}B_0\sin(\omega t)\right) + \frac{i}{\sqrt{2}}\sin\left(\frac{ge}{4m\omega}B_0\sin(\omega t)\right)$$

This is the expectation value for $S_x$.

$$\langle S_x\rangle = \langle s(t)|S_x|s(t)\rangle = \frac{\hbar}{2}\cos\left(\frac{ge}{2m\omega}B_0\sin(\omega t)\right) \tag{2}$$

As a result of $-1 \le \sin(\omega t) \le 1$ the cosine in (2) is always positive for

$$\frac{ge}{2m\omega}B_0 \le \frac{\pi}{2}$$

Hence to change the sign of $\langle S_x\rangle$ requires

$$B_0 > \frac{\pi m\omega}{ge}$$