This is a state with the cosine mode excited.

$$\bar{a}^c_{1,\mathbf{k}}\Phi_0$$

This is a state with the sine mode excited.

$$\bar{a}^s_{1,\mathbf{k}}\Phi_0$$

This is a superposition of the cosine state and i times the sine state. The factor $1/\sqrt{2}$ is for normalization, i.e., $|1+i| = \sqrt{2}$.

$$\frac{1}{\sqrt{2}}\left(\bar{a}^c_{1,\mathbf{k}}\Phi_0 + i\bar{a}^s_{1,\mathbf{k}}\Phi_0\right) = \bar{a}^*_{1,\mathbf{k}}\Phi_0$$

Here are some additional results.

From equation (9.43)

$$|\Phi_0|^2 = \Phi_0^*\Phi_0 = \exp\left(-\frac{kc}{\hbar}(\bar{a}^c_{1,\mathbf{k}})^2 - \frac{kc}{\hbar}(\bar{a}^s_{1,\mathbf{k}})^2 - \frac{kc}{\hbar}(\bar{a}^c_{2,\mathbf{k}})^2 - \frac{kc}{\hbar}(\bar{a}^s_{2,\mathbf{k}})^2\right)$$

For simplicity of notation, let

$$d\bar{a} = d\bar{a}^c_{1,\mathbf{k}}\,d\bar{a}^s_{1,\mathbf{k}}\,d\bar{a}^c_{2,\mathbf{k}}\,d\bar{a}^s_{2,\mathbf{k}}$$

The expectation of Φ_0 is

$$\langle\Phi_0\rangle = \int\limits_{-\infty}^{\infty}\cdots\int\limits_{-\infty}^{\infty}|\Phi_0|^2\,d\bar{a} = \left(\frac{\pi\hbar}{kc}\right)^2 \tag{1}$$

Let

$$\Phi_1 = \bar{a}^*_{1,\mathbf{k}}\Phi_0$$

Then

$$|\Phi_1|^2 = \Phi_1^*\Phi_1 = \left(\frac{(\bar{a}^c_{1,\mathbf{k}})^2 + (\bar{a}^s_{1,\mathbf{k}})^2}{2}\right)|\Phi_0|^2$$

The expectation of Φ_1 is

$$\langle\Phi_1\rangle = \int\limits_{-\infty}^{\infty}\cdots\int\limits_{-\infty}^{\infty}\left(\frac{(\bar{a}^c_{1,\mathbf{k}})^2 + (\bar{a}^s_{1,\mathbf{k}})^2}{2}\right)|\Phi_0|^2\,d\bar{a} = \frac{\hbar}{2kc}\left(\frac{\pi\hbar}{kc}\right)^2 \tag{2}$$

The expectation for n photons is

$$\langle\Phi_n\rangle = \int\limits_{-\infty}^{\infty}\cdots\int\limits_{-\infty}^{\infty}\left(\frac{(\bar{a}^c_{1,\mathbf{k}})^2 + (\bar{a}^s_{1,\mathbf{k}})^2}{2}\right)^n|\Phi_0|^2\,d\bar{a}$$

By a result from problem 9-7

$$\langle\Phi_n\rangle = n!\left(\frac{\hbar}{2kc}\right)^n\left(\frac{\pi\hbar}{kc}\right)^2 = n!\left(\frac{\hbar}{2kc}\right)^n\langle\Phi_0\rangle \tag{3}$$