4-9. Show from the fact that H is hermitian that equation (4.46) holds. Hint: Choose $f = \phi_2$, $g = \phi_1$ in equation (4.30).

From equation (4.42)

$$H\phi_1 = E_1\phi_1$$
$$H\phi_2 = E_2\phi_2$$

Since H is hermitian we have from equation (4.30)

$$\int_{-\infty}^{\infty} (Hg)^* f \, dx = \int_{-\infty}^{\infty} g^* (Hf) \, dx \tag{4.30}$$

Substitute ϕ_1 into g and ϕ_2 into f.

$$\int_{-\infty}^{\infty} (H\phi_1)^* \phi_2 \, dx = \int_{-\infty}^{\infty} \phi_1^* (H\phi_2) \, dx$$

Replace H with the corresponding eigenvalue. (Eigenvalues of H are real.)

$$E_1 \int_{-\infty}^{\infty} \phi_1^* \phi_2 \, dx = E_2 \int_{-\infty}^{\infty} \phi_1^* \phi_2 \, dx$$

The integrals are identical hence for $E_1 \neq E_2$ we must have

$$\int_{-\infty}^{\infty} \phi_1^* \phi_2 \, dx = 0$$