Orbital angular momentum 1

These are the orbital angular momentum operators in rectangular coordinates.

$$\begin{aligned} L_1 &= X_2P_3 - X_3P_2 \\ L_2 &= X_3P_1 - X_1P_3 \\ L_3 &= X_1P_2 - X_2P_1 \end{aligned}$$

We also have

$$\mathbf{L} = \mathbf{X} \times \mathbf{P} = \begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix}$$

$$L^2 = |\mathbf{L}|^2 = {L_1}^2 + {L_2}^2 + {L_3}^2$$

$$\mathbf{J} = \frac{1}{\hbar}\mathbf{L}$$

Let

$$X_j = x_j, \quad P_j = -i\hbar\frac{\partial}{\partial x_j}$$

Verify the following equations where $\psi \equiv \psi(x_1, x_2, x_3, t)$.

$$[\mathbf{X}, L_3]\psi = \begin{pmatrix} -i\hbar X_2 \\ i\hbar X_1 \\ 0 \end{pmatrix}\psi \tag{1}$$

$$[\mathbf{P}, L_3]\psi = \begin{pmatrix} -i\hbar P_2 \\ i\hbar P_1 \\ 0 \end{pmatrix}\psi \tag{2}$$

$$[\mathbf{L}, L^2]\psi = 0 \tag{3}$$

$$\mathbf{J} \times \mathbf{J}\psi = i\mathbf{J}\psi \tag{4}$$

$$(\mathbf{P} \times \mathbf{L} + \mathbf{L} \times \mathbf{P})\psi = 2i\hbar\mathbf{P}\psi \tag{5}$$

$$\tfrac{1}{2}(\mathbf{P} \times \mathbf{L} - \mathbf{L} \times \mathbf{P})\psi = (\mathbf{P} \times \mathbf{L} - i\hbar\mathbf{P})\psi \tag{6}$$

$$[L_i, P_j]\psi = i\hbar\sum_k \epsilon_{ijk}P_k\psi \tag{7}$$