Integrate using polar coordinates.

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{\infty}\frac{4\pi\rho_{\mathbf{k}}(t)}{k^2}\exp(ikR)\,k^2\sin\theta\,dk\,d\theta\,d\phi=\frac{16\pi^2 i\rho_{\mathbf{k}}(t)}{R} \tag{1}$$

The result is a Coulomb potential for $\rho_{\mathbf{k}}(t)\propto -iq$.

The following integrals show how (1) is obtained.

$$\int_0^{\infty}\exp(-ax)\,dx=\frac{1}{a}\qquad\int_0^{\pi}\sin\theta\,d\theta=2\qquad\int_0^{2\pi}d\phi=2\pi$$

Note: For multiple charges q_i we have

$$\begin{aligned}\rho(\mathbf{r},t)&=\sum_i q_i\delta(R_i)\\ \phi(\mathbf{r},t)&=\sum_i\frac{q_i}{R_i}\end{aligned}$$