

3-8. For a harmonic oscillator the Lagrangian is

$$L = \frac{m}{2}\dot{x}^2 - \frac{m\omega^2}{2}x^2 \quad (3.58)$$

Show that the resulting kernel is (see problem 2-2)

$$K = F(T) \exp \left(\frac{im\omega}{2\hbar \sin(\omega T)} ((x_b^2 + x_a^2) \cos(\omega T) - 2x_b x_a) \right) \quad (3.59)$$

where $T = t_b - t_a$. Note that the multiplicative function $F(T)$ has not been explicitly worked out. It can be obtained by other means, and for the harmonic oscillator it is (see section 3-11)

$$F(T) = \left(\frac{m\omega}{2\pi i \hbar \sin \omega T} \right)^{1/2} \quad (3.60)$$

From problem 2-2 we have

$$S_d = \frac{m\omega}{2\hbar \sin(\omega T)} ((x_b^2 + x_a^2) \cos(\omega T) - 2x_b x_a) \quad (1)$$

From equation (3.51)

$$K = F(T) \exp \left(\frac{iS_d}{\hbar} \right) \quad (2)$$

Substitute (1) into (2).

$$K = F(T) \exp \left(\frac{im\omega}{2\hbar \sin(\omega T)} ((x_b^2 + x_a^2) \cos(\omega T) - 2x_b x_a) \right)$$