2.1. Solve the Klein-Gordon equation.

This is the Klein-Gordon equation.

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} + \frac{m^2c^2}{\hbar^2}\right)\psi = 0$$

One solution is

$$\psi = \exp\left(-\frac{i}{\hbar}(Et - p_x x - p_y y - p_z z)\right) \tag{1}$$

where

$$E = \sqrt{p_x^2c^2 + p_y^2c^2 + p_z^2c^2 + m^2c^4}$$

Let us inspect dimensions. The dimensions of Et are joule seconds.

$$Et \propto \text{joule second}$$

Hence $Et/\hbar$ is dimensionless.

$$\frac{Et}{\hbar} \propto \frac{\text{joule second}}{\text{joule second}} = 1$$

The dimensions of $p_x x$ are joule seconds.

$$p_x x \propto \frac{\text{kilogram meter}}{\text{second}} \times \text{meter} = \text{joule second}$$

Hence $p_x x/\hbar$ is dimensionless.

$$\frac{p_x x}{\hbar} \propto \frac{\text{joule second}}{\text{joule second}} = 1$$

Solution (1) can be written in a more compact form as

$$\psi = \exp\left(-\frac{i}{\hbar}P \cdot X\right)$$

where

$$P = \begin{pmatrix} E/c \\ p_x \\ p_y \\ p_z \end{pmatrix}, \quad X = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

and

$$P \cdot X = P^\mu g_{\mu\nu} X^\nu = Et - p_x x - p_y y - p_z z$$