

8-1. The amplitude to go from any state  $\psi(x)$  to another state  $\chi(x)$  is the transition amplitude  $\langle\chi|1|\psi\rangle$  as defined in equation (7.1). Suppose  $\psi(x)$  and  $\chi(x)$  are expanded in terms of the orthogonal functions  $\phi_n(x)$ , the energy solutions to the wave equation associated with the kernel  $K(b, a)$ , as discussed in section 4-2. Thus

$$\psi(x) = \sum_n \psi_n \phi_n(x), \quad \chi(x) = \sum_n \chi_n \phi_n(x) \quad (8.23)$$

Using the coefficients  $\psi_n$  and  $\chi_n$  and equation (4.59), show that the transition amplitude can be written as

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \chi^*(x_b) K(x_b, T; x_a, 0) \psi(x_a) dx_a dx_b = \sum_n \chi_n^* \psi_n \exp\left(-\frac{i}{\hbar} E_n T\right) \quad (8.24)$$

This is equation (4.59).

$$K(x_b, t_b; x_a, t_a) = \begin{cases} \sum_{n=1}^{\infty} \phi_n(x_b) \phi_n^*(x_a) \exp\left(-\frac{i}{\hbar} E_n (t_b - t_a)\right) & t_b > t_a \\ 0 & t_b < t_a \end{cases} \quad (4.59)$$

Let

$$I = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \chi^*(x_b) K(x_b, T; x_a, 0) \psi(x_a) dx_a dx_b$$

By (8.23) and (4.59) we have

$$I = \sum_j \sum_n \sum_k \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \chi_j^* \phi_j^*(x_b) \phi_n(x_b) \phi_n^*(x_a) \exp\left(-\frac{i}{\hbar} E_n T\right) \psi_k \phi_k(x_a) dx_a dx_b$$

The integrals for  $j \neq n$  or  $k \neq n$  vanish by orthogonality of  $\phi_j^* \phi_n$  hence

$$I = \sum_n \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \chi_n^* \phi_n^*(x_b) \phi_n(x_b) \phi_n^*(x_a) \exp\left(-\frac{i}{\hbar} E_n T\right) \psi_n \phi_n(x_a) dx_a dx_b$$

Rewrite as

$$I = \sum_n \chi_n^* \exp\left(-\frac{i}{\hbar} E_n T\right) \psi_n \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi_n^*(x_b) \phi_n(x_b) \phi_n^*(x_a) \phi_n(x_a) dx_a dx_b$$

The remaining integrals are unity by normalization of  $\phi_n^* \phi_n$  hence

$$I = \sum_n \chi_n^* \exp\left(-\frac{i}{\hbar} E_n T\right) \psi_n$$