# Elements of Deductive Logic

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## 1 Mathematical Induction

Proof by mathematical induction is a powerful method of reasoning. As we will see, there are numerous formulations of induction.

**Weak Principle of Induction (WPI)** If (i) some $P$ is true of the first member of a sequence $S$ (ordered by the natural numbers), and (ii) if $P$ is true of the $n^{\text{th}}$ member of $S$, then $P$ is true of the $(n+1)^{\text{th}}$ member of $S$; then for every $x \in S$, $P$ is true of $x$.

We call condition (i) the *base case* and condition (ii) the *induction case*.

Notation-wise, we can write $P(x)$ for the statement '$P$ is true for $x$'. This way, we can rewrite WPI symbolically as:

$$P(s_0) \wedge (P(s_n) \rightarrow P(s_{n+1})) \rightarrow (\forall x \in S)P(x)$$

**The Strong Principle of Induction (SPI)** If ; then for every $x \in S$

**The Least Number Principle (LNP)** For a non-empty subset $M$ of $\mathbb{N}$, $M$ has a least member.