Math189R SU17
Homework 6
Wednesday, June 7, 2017

Feel free to work with other students, but make sure you write up the homework and code on your own (no copying homework *or* code; no pair programming). Feel free to ask students or instructors for help debugging code or whatever else, though.

The starter files for problem 2 can be found under the Resource tab on course website. Please print out all the graphs generated by your own code and submit them together with the written part, and make sure you upload the code to your Github repository.

1 (Murphy 11.2 - EM for Mixtures of Gaussians) Show that the M step for ML estimation of a mixture of Gaussians is given by

$$\mu_k = \frac{\sum_i r_{ik}\mathbf{x}_i}{r_k}$$

$$\Sigma_k = \frac{1}{r_k}\sum_i r_{ik}(\mathbf{x}_i - \mu_k)(\mathbf{x}_i - \mu_k)^\top = \frac{1}{r_k}\sum_i r_{ik}\mathbf{x}_i\mathbf{x}_i^\top - r_k\mu_k\mu_k^\top.$$

Looked at solution to know where to start.
From the textbook, we know the expected log likelihood is given by:

$$\ell(\vec{\mu}_k, \Sigma_k) = -\frac{1}{2}\sum_i r_{ik}\left[\log|\Sigma_k| + (x-\mu_k)^T \Sigma_k^{-1}(x_i-\mu_k)\right]$$

Because we want to maximize ℓ wrt μ_k & Σ_k, we will take the partials wrt μ_k & Σ_k.

$$\frac{\partial \ell}{\partial \mu_k} = -\frac{1}{2}\sum_i r_{ik}\left[2(x-\mu_k)\Sigma_k^{-1}\right]$$

$$0 = \cancel{\Sigma_k^{-1}} \sum_i r_{ik}(x-\mu_k)$$

$$\sum_i r_{ik} x = \sum_i r_{ik}\mu_k \quad \text{Because} \quad \sum_i r_{ik} = r_k$$

$$\boxed{\mu_k = \frac{\sum_i r_{ik} x}{r_k} \quad \text{as desired}}$$

Now we will take the partial w.r.t. Σ_k

Needed solution for derivative.

$$\frac{\partial l}{\partial \Sigma_k} = -\frac{1}{2}\sum_i r_{ik}\left(\Sigma_k^{-1} - \Sigma_k^{-1}(x_i-\mu_k)(x_i-\mu_k)^T\Sigma_k^{-1}\right) = 0$$

$$\sum_i r_{ik}\Sigma_k^{-1} = \Sigma_k^{-1}(x_i-\mu_k)(x_i-\mu_k)^T\Sigma_k^{-1}$$

Multiplying by Σ_k on the left side we get

$$\sum_i r_{ik} I = (x_i-\mu_k)(x_i-\mu_k)^T\Sigma_k^{-1}$$

Now multiplying on the right by Σ_k and dividing by r_{ik} gives us the solution

used solution for this.