

Solution for Homework 1

In [2]: `robot_localization()`

Initial Position:

$\text{bel}(x_0 = p_0) = 0.25$

$\text{bel}(x_0 = p_1) = 0.25$

$\text{bel}(x_0 = p_2) = 0.25$

$\text{bel}(x_0 = p_3) = 0.25$

At step $t=1$, after the control u_1 , the robot returns a measurement of $z_1 = \text{door}$.

State Transition Probability:

$(x_1 = p_0 | u_1, x_0=p_0, x_0=p_1, x_0=p_2, x_0=p_3) = [0.2, 0, 0, 0]$

$(x_1 = p_1 | u_1, x_0=p_0, x_0=p_1, x_0=p_2, x_0=p_3) = [0.7, 0.2, 0, 0]$

$(x_1 = p_2 | u_1, x_0=p_0, x_0=p_1, x_0=p_2, x_0=p_3) = [0.1, 0.7, 0.2, 0]$

$(x_1 = p_3 | u_1, x_0=p_0, x_0=p_1, x_0=p_2, x_0=p_3) = [0, 0.1, 0.7, 0.2]$

Calculations of bel_bar for all potential locations ($p_0 \sim p_3$):

$\text{bel_bar}(x_1=p_0) = 0.05$

$\text{bel_bar}(x_1=p_1) = 0.225$

$\text{bel_bar}(x_1=p_2) = 0.25$

$\text{bel_bar}(x_1=p_3) = 0.25$

Probability of the robot sensing the door at step = 1:

$p(z_1 = \text{door} | x_1=p_0) = 0.3$

$p(z_1 = \text{door} | x_1=p_1) = 0.8$

$p(z_1 = \text{door} | x_1=p_2) = 0.3$

$p(z_1 = \text{door} | x_1=p_3) = 0.8$

Normalization and η calculation:

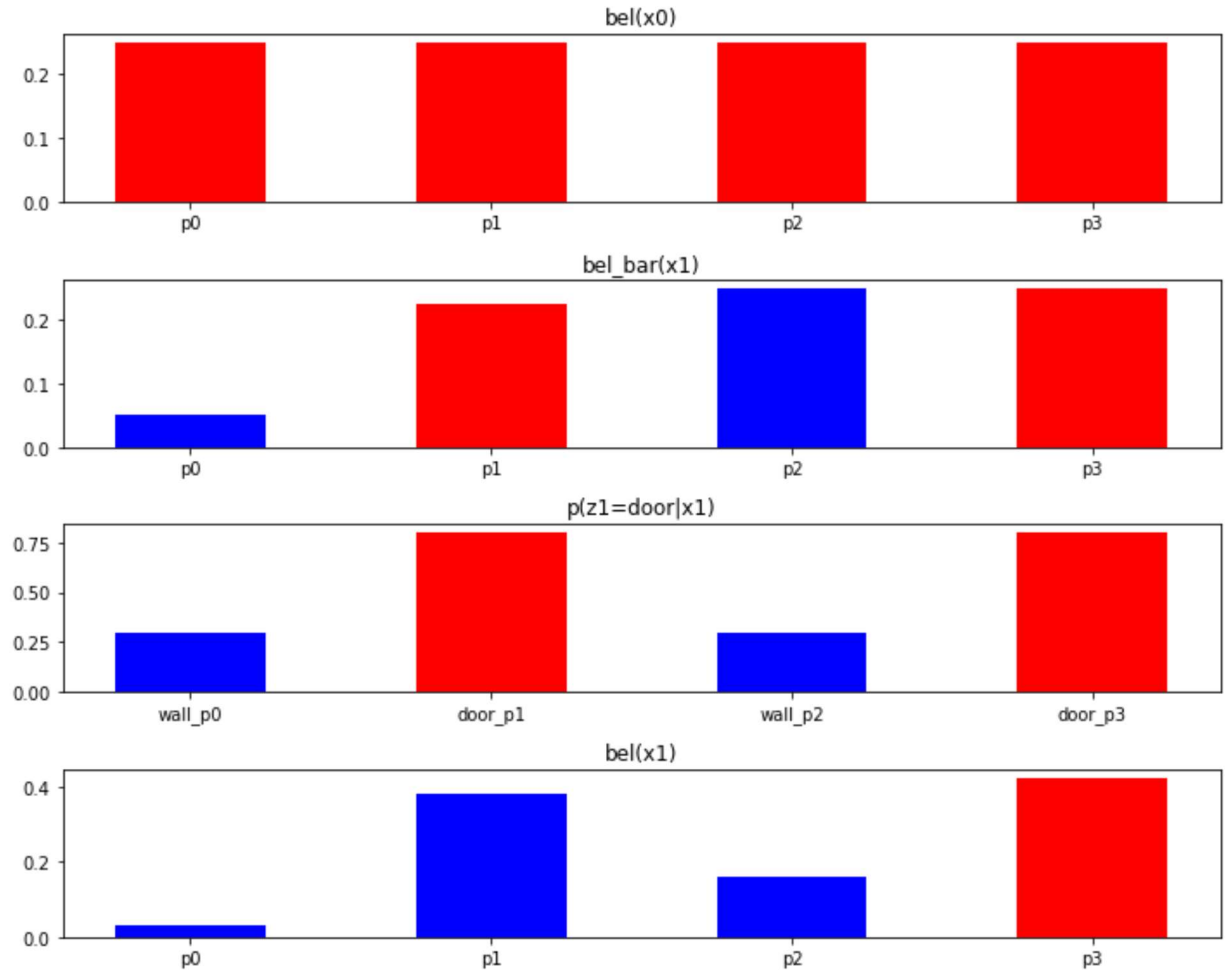
$\text{bel}(x_1 = p_0) = p(z_1 = \text{door} | x_1=p_0) * \text{bel_bar}(x_1 = p_0) * \eta = 0.015 * \eta$

$\text{bel}(x_1 = p_1) = p(z_1 = \text{door} | x_1=p_1) * \text{bel_bar}(x_1 = p_1) * \eta = 0.18 * \eta$

$\text{bel}(x_1 = p_2) = p(z_1 = \text{door} | x_1 = p_2) * \text{bel_bar}(x_1 = p_2) * \eta = 0.075 * \eta$
 $\text{bel}(x_1 = p_3) = p(z_1 = \text{door} | x_1 = p_3) * \text{bel_bar}(x_1 = p_3) * \eta = 0.2 * \eta$
 $\eta = 1/0.47 = 2.128$

New updated belief of the robot's localization probability after step 1:

$\text{bel}(x_1 = p_0) = 0.032$
 $\text{bel}(x_1 = p_1) = 0.383$
 $\text{bel}(x_1 = p_2) = 0.16$
 $\text{bel}(x_1 = p_3) = 0.426$



At step $t=2$, after the control u_2 , the robot returns a measurement of $z_2 = \text{wall}$.

State Transition Probability:

$(x_2 = p_0 | u_2, x_1 = p_0, x_1 = p_1, x_1 = p_2, x_1 = p_3) = [0.2, 0, 0, 0]$
 $(x_2 = p_1 | u_2, x_1 = p_0, x_1 = p_1, x_1 = p_2, x_1 = p_3) = [0.7, 0.2, 0, 0]$
 $(x_2 = p_2 | u_2, x_1 = p_0, x_1 = p_1, x_1 = p_2, x_1 = p_3) = [0.1, 0.7, 0.2, 0]$
 $(x_2 = p_3 | u_2, x_1 = p_0, x_1 = p_1, x_1 = p_2, x_1 = p_3) = [0, 0.1, 0.7, 0.2]$

Calculations of bel_bar for all potential locations ($p_0 \sim p_3$):

$\text{bel_bar}(x_2 = p_0) = 0.006$
 $\text{bel_bar}(x_2 = p_1) = 0.099$
 $\text{bel_bar}(x_2 = p_2) = 0.303$
 $\text{bel_bar}(x_2 = p_3) = 0.235$

Probability of the robot sensing the wall at step = 2:

$p(z_2 = \text{wall} | x_2 = p_0) = 0.7$
 $p(z_2 = \text{wall} | x_2 = p_1) = 0.2$
 $p(z_2 = \text{wall} | x_2 = p_2) = 0.7$
 $p(z_2 = \text{wall} | x_2 = p_3) = 0.2$

Normalization and η calculation:

$\text{bel}(x_2 = p_0) = p(z_2 = \text{wall} | x_2 = p_0) * \text{bel_bar}(x_2 = p_0) * \eta = 0.004 * \eta$
 $\text{bel}(x_2 = p_1) = p(z_2 = \text{wall} | x_2 = p_1) * \text{bel_bar}(x_2 = p_1) * \eta = 0.02 * \eta$
 $\text{bel}(x_2 = p_2) = p(z_2 = \text{wall} | x_2 = p_2) * \text{bel_bar}(x_2 = p_2) * \eta = 0.212 * \eta$
 $\text{bel}(x_2 = p_3) = p(z_2 = \text{wall} | x_2 = p_3) * \text{bel_bar}(x_2 = p_3) * \eta = 0.047 * \eta$
 $\eta = 1/0.284 = 3.527$

New updated belief of the robot's localization probability after step 2:

$\text{bel}(x_2 = p_0) = 0.016$
 $\text{bel}(x_2 = p_1) = 0.07$
 $\text{bel}(x_2 = p_2) = 0.749$
 $\text{bel}(x_2 = p_3) = 0.166$

