Week 6: Sensitive Analysis

1. Sensitive Analysis

Sensitivity Analysis is a systematic study of how, well, sensitive, the solutions of the LP are to small changes in the data. The basic idea is to be able to give answers to questions of the form:

1. If the objective function changes in its parameter c_i, how does the solution change?
2. If the resources available change. how does the solution change?
3. If a new constraint is added to the problem, how does the solution change?

2. Graphical Sensitivity Analysis
This section demonstrates the general idea of sensitivity analysis. Two cases will be considered:
    1. Sensitivity of the optimum solution to changes in the availability of the resources (right-hand side of the constraints).
    2. Sensitivity of the optimum solution to changes in unit profit or unit cost (coefficients of the objective function).

Example : (Changes in the Right-Hand Side)
JOYCO produces two products on two machines. A unit of product 1 requires 2 hours on machine 1 and 1 hour on machine 2. For product 2, a unit requires 1 hour on machine 1 and 3 hours on machine 2. The revenues per unit of products 1 and 2 are $30 and $20, respectively. The total daily processing time available for each machine is 8 hours.

Solution:
Let   x1=number of unit of product 1
           X2=number of unit of product 2

                    Maximize z=30x1+20x2
        Subject to
                    2x1+ x2 ≤ 8   (machine 1)
                    X1+ 3x2≤8   (machine 2)
                    X1,x2 ≥ 0

Solution:

To draw this model following as:

1.to draw 2x1+ x2 ≤ 8   , convert to equality form as

            a- first equality   2x1+ x2 =8

            b-set x1=0   this lead to x2=8

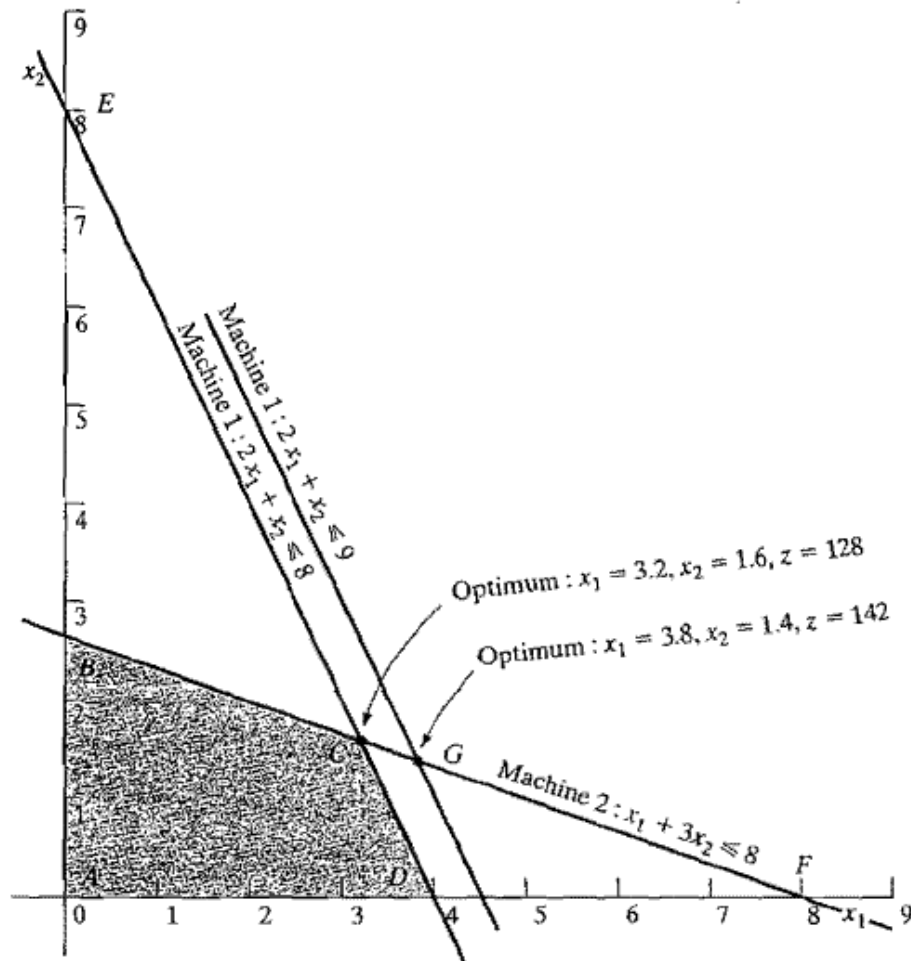   c-set x2=0 this lead to x1=4

1. To draw x1+ 3x2≤8 ,convert t equality form as
            a- First equality x1+ 3x2 = 8
            b- Set x1 =0 , this lead to x2=8/3

c- Set x2=0 , this lead to x1=8

Draw of model show in figure below

If the daily capacity is increased from 8 hours to 9 hours, the new optimum will occur at point G. The rate of change in optimum z resulting from changing machine 1 capacity from 8 hours to 9 hours can be computed as follows:

$$\begin{pmatrix} rate\ of\ revenue\ change \\ resulting\ from\ increaseing \\ mechine\ 1\ capacity \\ by\ 1\ hr\ (point\ C\ to\ point\ G)) \end{pmatrix} = \frac{z_G - z_C}{capacity\ \ change} = \frac{142 - 128}{9 - 8} = 14.00\ \$/hr$$

This means that a unit increase (decrease) in machine 1 capacity will increase (decrease) revenue
by \$14.00. clear that the dual price of \$14.00/hr remains valid for changes (increases or decreases) in machine 1 capacity that move its constraint parallel to itself to any point on the line segment BF.

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Minimum machine 1 capacity [at B=(0,2.67)]= 2*0 +1* 2.67=2.67
Maximum machine 1 capacity[at f=(8,0)]= 2*8+1*0=16

$$2.67 \leq \text{machine 1 capacity} \leq 16$$

Same manner computation for machine 2

Minimum machine 2 capacity [at D=(4,0)]=1*4 +3* 0=4
Maximum machine 2 capacity [at f=(0,8)]= 1*0+3*8=24

The conclusion is that the dual price of $2.00/hr for machine 2 will remain applicable for the range

$$4 \leq \text{machine 2 capacity} \leq 14$$

The dual prices allow making economic decisions about the LP problem, as the following questions demonstrate:

Question 1: If JOBCO can increase the capacity of both machines, which machine should receive higher priority?
The dual prices for machines 1 and 2 are $14.00Ihr and $2.00/hr.ll1is means that each additional hour of machine 1 will increase revenue by $14.00, as opposed to only $2.00 for machine 2. Thus, priority should be given to machine 1.

Question 2. A suggestion is made to increase the capacities of machines 1 and 2 at the additional cost of $10/hr. Is this advisable?
For machine 1, the additional net revenue per hour is 14.00 -10.00 = $4.00 and for machine
2, the net is $2.00 $10.00 -$8.00. Hence, only the capacity of machine 1 should be increased.

Question 3. If the capacity of machine 1 is increased from the present 8 hours to 13 hours, how will this increase impact the optimum revenue?
The dual price for machine 1 is $14.00 and is applicable in the range (2.67, 16) hr. The proposed increase to 13 hours falls within the feasibility range. Hence, the increase in revenue is $14.00(13 - 8) = $70.00, which means that the total revenue will be increased to
(current revenue + change in revenue) 128 + 70 =$198.00.

Question 4. Suppose that the capacity of machine 1 is increased to 20 hours, hQW will this increase impact the optimum revenue?
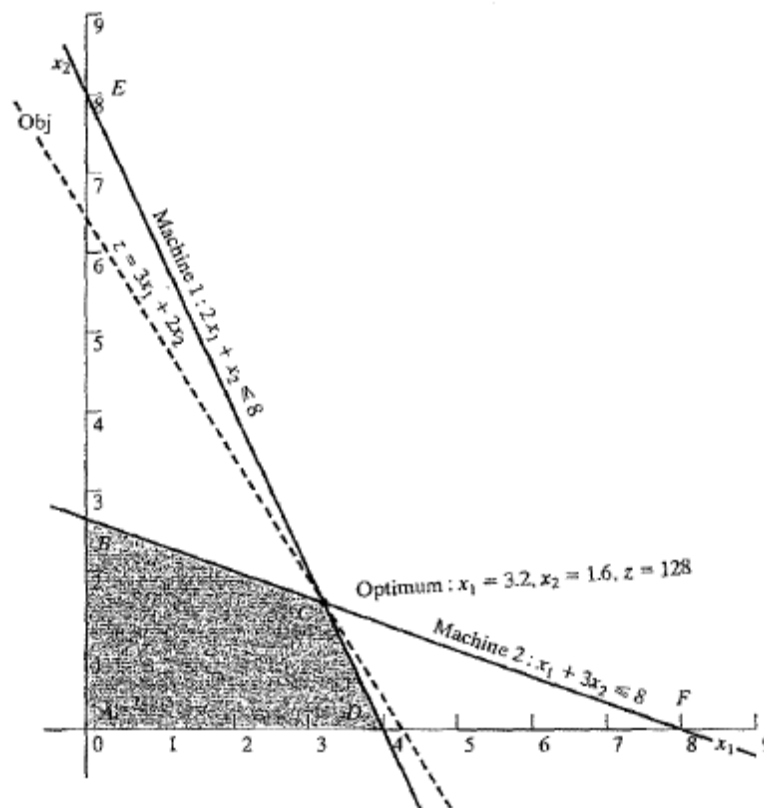The proposed change is outside the range (2.67, 16) hr for which the dual price of $14.00 remains applicable. Thus, we can only make an immediate conclusion regarding an increase up to 16 hours. Beyond that, further calculations are needed to find the answer. Remember that falling outside the feasibility range does *not* mean that the problem has no solution. It only means that we do not have sufficient information to make an *immediate* decision.

Question 5. We know that the change in the optimum objective value equals (dual price x change in resource) so long as the change in the resource is within the feasibility range. What about the associated optimum values of the variables?

The optimum values of the variables will definitely change. However, the level of information we have from the graphical solution is not sufficient to determine the new values.

Example:(Changes in the Objective Coefficients)

Changes in revenue units (i.e., objective-function coefficients) will change the slope of z. However, as can be seen from the figure below, the optimum solution will remain at point C so long as the objective functions between lines BF and DE, the two constraints that define the optimum point. This means that there is a range for the coefficients of the objective function that will keep the optimum solution unchanged at C.

We can write the objective function in the general format

$$\text{Maximize} \quad z = c_1 x_1 + c_2 x_2$$

Imagine now that the line z is pivoted at C and that it can rotate clockwise and counterclockwise. The optimum solution will remain at point C so long as $z = Cx_1 + cx_2$ lies between the two lines $x_1 + 3x_2 = 8$ and $2x_1 + x_2 = 8$. This means that the ratio (c_1/c_2) can vary between 1/3 and 2/1, which yields the following condition:

$$\frac{1}{3} \leq \frac{c1}{c2} \leq \frac{2}{1}$$

This information can provide immediate answers regarding the optimum solution as the following questions demonstrate:

Question 1. Suppose that the unit revenues for products 1 and 2 are changed to $35 and $25, respectively. Will the current optimum remain the same?

The new objective function is
$$\text{Maximize} \quad z = 35x1 + 25x2$$

The solution at C will remain optimal because c1/c2 = 1.4 remains within the optimality range (0.333,2). When the ratio falls outside this range, additional calculations are needed to find the new optimum . Notice that although the values of the variables at the optimum point C remain unchanged, the optimum value of z changes to

$$35 \text{ X } (3.2) + 25 \text{ X } (1.6) = \$152.00.$$

Question 2. Suppose that the unit revenue of product 2 is fixed at its current value of C2 = $20.00. What is the associated range for cj, the unit revenue for product 1 that will keep the optimum unchanged?

Substitution c2 =20 in the condition

$$\frac{1}{3} \leq \frac{c1}{c2} \leq \frac{2}{1}$$

$$\frac{1}{3} \leq \frac{c1}{20} \leq \frac{2}{1}$$

$$\frac{1}{3} * 20 \leq c1 \leq 2*20$$

$$6.666 \leq c1 \leq 40$$

We can similarly determine the optimality range for Cz by fixing the value of CI at $30.00. Thus,

$$\frac{1}{3} \leq \frac{30}{c2} \leq \frac{2}{1}$$

Reverse

$$\frac{3}{1} \geq \frac{c2}{30} \geq \frac{1}{2}$$

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Equivalent to

$$\frac{1}{2} \leq \frac{c2}{30} \leq \frac{3}{1}$$

$$\frac{1}{2} * 30 \leq c2 \leq \frac{3}{1} * 30$$

$$15 \leq c2 \leq 90$$

2.Algebraic Sensitivity Analysis-Changes in the Right-Hand Side

we used the graphical solution to determine the dual prices (the unit worths of resources) and their feasibility ranges. This section extends the analysis to the general LP model. A numeric example (the TOYCO model) will be used to facilitate the presentation.

Example: (TOYCO Model)

TOYCO assembles three types of toys-trains, trucks, and cars-using three operations. The daily limits on the available times for the three operations are 430,460, and 420 minutes, respectively, and the revenues per unit of toy train, truck, and car are $3, $2, and $5, respectively. The assembly times per train at the three operations are 1, 3, and 1 minutes, respectively. The corresponding times per train and per car are (2,0,4) and (1,2,0) minutes (a zero time indicates that the operation is not used).

Let   x1=daily number of units assembled of trains

x2= daily number of units assembled of trucks

x3= daily number of units assembled of cars

the associated LP model is given as:

maximize z=3x1+2x2+5x3

subject to

x1+2x2+x3≤430 (operation 1)

3x1+   +2x3≤460 (operation 2)

x1+4x2   ≤420 (operation 3)

x1,x2,x3≥0

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Using X4, Xs, and X6 as the slack variables for the constraints of operations 1,2, and 3, respectively, the optimum tableau is

Basic	X1	X2	X3	X4	X5	X6	solution
Z	4	0	0	1	2	0	1350
X2	-1/4	1	0	1/2	-1/4	0	100
X3	3/2	0	1	0	1/2	0	230
X6	2	0	0	-2	1	1	20

The solution recommends manufacturing 100 trucks and 230 cars but no trains. The associated revenue is \$1350.

Determination of Dual Prices: The constraints of the model after adding the slack variables X4, Xs, and X6 can be written as follows:

$$x1 + 2x2 + x3 + x4 = 430 \text{ (Operation 1)}$$

$$3x1 + 2x3 + x5 = 460 \text{ (Operation 2)}$$

$$x1 + 4x2 + x6 = 420 \text{ (Operation 3)}$$

or

$$x1 + 2x2 + x3 = 430 - x4 \text{ (Operation 1)}$$

$$3x1 + 2x3 = 460 - x5 \text{ (Operation 2)}$$

$$x1 + 4x2 = 420 - x6 \text{ (Operation 3)}$$

With this representation, the slack variables have the same units (minutes) as the operation times. Thus, we can say that a one-minute decrease in the slack variable is equivalent to a one-minute increase in the operation time.

can use the information above to determine the dual prices from the z-equation in the optimal tableau:

$$z + 4x1 + x4 + 2x5 + 0x6 = : 1350$$

This equation can be written as

$$z = 1350 - 4x1 - x4 - 2x5 - 0x6$$

$$= 1350 - 4x1 + 1(-x4) + 2(-x5) + 0(-x6)$$

Given that a decrease in the value of a slack variable is equivalent to an increase in its operation time, we get

$$z = 1350 - 4x1 + 1 \text{ x (increase in operation 1 time)}$$

$$+ 2 \text{ x (increase in operation 2 time)}$$

$$+ 0 \text{ x (increase in operation 3 time)}$$

This equation reveals that

(1) a one-minute increase in operation 1 time increases z by \$1,

(2) a one-minute increase in operation 2 time increases z by \$2,

(3) a one-minute increase in operation 3 time does not change z.

To summarize, the z-row in the optimal tableau:

Basic	X1	X2	X3	X4	X5	X6	solution
Z	4	0	0	1	2	0	1350

yields directly the dual prices, as the following table shows:

Resource	Slack variable	Optimal z-equation coefficient of slack variable	Dual price
Operation 1	X4	1	\$l/min
Operation 2	X5	2	\$2/min
Operation 3	X6	0	\$0/min

The zero dual price for operation 3 means that there is no economic advantage in allocating more production time to this operation. The zero dual price for operation 3 means that there is no economic advantage in allocating more production time to this operation. The result makes sense because the resource is already abundant, as is evident by the fact that the slack variable associated with Operation 3 is positive (20) in the optimum solution. As for each of Operations 1 and 2, a one minute increase will improve revenue by \$1 and \$2, respectively.

 The dual prices also indicate that, when allocating additional resources, Operation 2 may be given higher priority because its dual price is twice as much as that of Operation 1.

Determination of the Feasibility Ranges: Let D1,D2 and D3 be the changes (positive or negative) in the daily manufacturing time allocated to operations 1,2, and 3, respectively. The model can be written as follows:

$$\text{Maximize } z = 3x1 + 2xz + 5x3$$

subject to

$$x1 + 2x2 + x3 \leq 430 + d1 \quad \text{(operation 1)}$$

$$3x1 + 2x3 \leq 460 + d2 \quad \text{(operation 2)}$$

$$xi + 4x2 \leq 420 + d3 \quad \text{(operation 3)}$$

$$x1, x2, x3 \geq 0$$

The procedure is based on recomputing the optimum simplex tableau with the modified right-hand side and then deriving the conditions that will keep the solution feasible and The starting tableau will thus appear as:

basic	X1	X2	X3	X4	X5	X6	solution RHS	D1	D2	D3
Z	-3	-2	-5	0	0	0	0	0	0	0
X4	1	2	1	1	0	0	430	1	0	0
X5	3	0	2	0	1	0	460	0	1	0
X6	1	4	0	0	0	1	420	0	0	1

The columns under Dj, D2, and D3 are identical to those under the starting basic columns X4, Xs, and x6- This means that when we carry out the same simplex iterations as in the original model, the columns in the two groups must come out identical as well Effectively, the new optimal tableau will become

basic	X1	X2	X3	X4	X5	X6	solution RHS	D1	D2	D3
Z	4	0	0	1	2	0	1350	1	2	0
X2	-1/4	1	0	1/2	-1/4	0	100	1/2	-1/4	0
X3	3/2	0	1	0	1/2	0	230	0	1/2	0
X6	2	0	0	-2	1	1	20	-2	1	1

The new optimum tableau provides the following optimal solution:

$$z = 1350 + D1 + 2D2$$

$$x2 = 100 + 1/2D1 - 1/4D2$$

$$x3 = 230 + 1/2D2$$

$$x6 = 20 - 2D1 + D2 + D3$$

Interestingly, as shown earlier, the new z-value confirms that the dual prices for operations 1,2, and 3 are 1,2, and 0, respectively. The current solution remains

feasible so long as all the variables are nonnegative, which leads to the following feasibility conditions:

$$x_2 = 100 + 1/2 D_1 - 1/4 D_2 \geq 0$$

$$x_3 = 230 + 1/2 D_2 \geq 0$$

$$x_6 = 20 - 2D_1 + D_2 + D_3 \geq 0$$

Any simultaneous changes D_1, D_2, and D_3 that satisfy these inequalities will keep the solution feasible. If all the conditions are satisfied, then the new optimum solution can be found through direct substitution of D_1, D_2 and D_3 in the equations given above.

To illustrate the use of these conditions, suppose that the manufacturing time available for operations 1,2,and 3 are 480,440, and 410 minutes respectively. Then, $D_1 = 480 - 430 = 50$, $D_2 = 440 - 460 = -20$, and $D_3 = 410 - 420 = -10$. Substituting in the feasibility conditions, we get

$$x_2 = 100 + 1/2(50) - 1/4(-20) \ 130 > 0 \qquad \text{(feasible)}$$

$$x_3 = 230 + 1/2(-20) = 220 > 0 \qquad \text{(feasible)}$$

$$x_6 = 20 - 2(50) + (-20) + (-10) = -110 < 0 \qquad \text{(infeasible)}$$

The calculations show that $X_6 < 0$, hence the current solution does not remain feasible Additional calculations will be needed to find the new solution. Alternatively, if the changes in the resources are such that $D_1 = -30$, $D_2 = -12$,and $D_3 = 10$, then

$$x_2 = 100 + 1/2(-30) - 1/4(-12) = 88 > 0 \qquad \text{(feasible)}$$

$$x_3 = 230 + 1/2(-12) = 224 > 0 \qquad \text{(feasible)}$$

$$x_6 = 20 - 2(-30) + (-12) + (10) = 78 > 0 \qquad \text{(feasible)}$$

The given conditions can be specialized to produce the individual feasibility ranges that result from changing the resources one at a time

Case 1. Change in operation 1 lime from 460 to 460 + D_1 minutes. This change is equivalent to setting $D_2 = D_3 = 0$ in the simultaneous conditions, which yields

$$x2 = 100 + \frac{1}{2}D1 \geq 0 \ \rightarrow D1 \geq -200$$
$$\left.\begin{array}{l} x3 = 230 > 0 \\ x6 = 20 - 2D1 \geq 0 \ \rightarrow D1 \leq 10 \end{array}\right\} \rightarrow -200 \leq D1 \leq 10$$

Case 2. Change in operation 2 time from 430 to 430 + ~ minutes. This change is equivalent to setting D1 = D3 = 0 in the simultaneous conditions, which yields

$$\left.\begin{array}{l} x2 = 100 - \frac{1}{4}D2 \geq 0 \rightarrow D2 \leq 400 \\ x3 = 230 + \frac{1}{2D2} \geq 0 \rightarrow D2 \geq -460 \\ x6 = 20 + D2 \geq 0 \ \rightarrow D2 \geq -20 \end{array}\right\} \rightarrow -20 \leq D2 \leq 400$$

Case 3. Change in operation 3 time from 420 to 420 + D3 minutes. This change is equivalent to setting D1 = D2 = 0 in the simultaneous conditions, which yields

$$\left.\begin{array}{l} x2 = 100 > 0 \\ x3 = 230 > 0 \\ x6 = 20 + D3 \geq 0 \end{array}\right\} \rightarrow -20 \leq D3 \leq \infty$$

summarize the dual prices and their feasibility ranges for the

TOYCO model as follows:

Resource	Dual price	Feasible range	Resource amount (minutes)		
			minimum	current	maximum
Operation 1	1	$-200 \leq D1 \leq 10$	230	430	440
Operation2	2	$-20 \leq D1 \leq 400$	440	440	860
Operation 3	0	$-20 \leq D1 \leq \infty$	400	420	∞

It is important to notice that the dual prices will remain applicable for any

simultaneous changes that keep the solution feasible, even if the changes violate the individual ranges. For example, the changes Dl = 30, D2 = -12, and D3 = 100, will keep the solution feasible even though D1 = 30 violates the feasibility range $-200 \leq D1 \leq 10$ as the following computations show

$$x2 = 100 + \frac{1}{2(30)} - \frac{1}{4(-12)} = 118 > 0$$
$$x3 = 23(-12) = 224 > 0$$
$$x6 = 20 - 2(30) + (-12) + (100) > 0$$

This means that the dual prices will remain applicable, and we can compute the new

optimum objective value from the dual prices as z = 1350 + 1(30) + 2(-12) +0(100) = $1356

3. Algebraic Sensitivity Analysis-Objective Function

definition of reduced cost. to facilitate the explanation of the objective function sensitivity analysis, first we need to define reduced costs. in the toyco model previous example , the objective z-equation in the optimal tableau is

basic	x1	x2	x3	x4	x5	x6	solution
z	4	0	0	1	2	0	1350
x2	-1/4	1	0	1/2	-1/4	0	100
x3	3/2	0	1	0	1/2	0	230
x6	2	0	0	-2	1	1	20

z+4x1+x4+2x5=1350

or

z=1350-4x1-x4-2x5

the optimal solution does not recommend the production of toy trains (x1= 0). this recommendation is confirmed by the information in the z-equation because each unit increase in (xi) above its current zero level will decrease the value of z by $4 - namely, z = 1350 - 4 x (1) - 1 x (0) 2 x (0) = $1346.

we can think of the coefficient of xl in the z-equation (= 4) as a unit cost because it causes a reduction in the revenue z. but we know that x i has a unit revenue of $3 in the original model**.** this relationship is formalized in the lp literature by defining the reduced cost as

$$\begin{pmatrix} \textbf{reduce cost} \\ \textbf{per unit} \end{pmatrix} = \begin{pmatrix} \textbf{cost consumed} \\ \textbf{resource per unit} \end{pmatrix} - \begin{pmatrix} \textbf{revenue} \\ \textbf{per unit} \end{pmatrix}$$

in the original toyco model the revenue per unit for toy trucks $2) is less than that for toy trains (= $3). yet the optimal solution elects to manufacture toy trucks (x2 = 100 units) and no toy trains (x1= 0). the reason for this (seemingly nonintuitive) result is that the unit cost of the resources used by toy trucks (le., operations time) is smaller than its unit price. the opposite   applies in the case of toy trains. with the given definition of reduced cost we can now see that an unprofitable variable (such as x1) can be made profitable in two ways:

1. by increasing the unit revenue.
2. by decreasing the unit cost of consumed resources.

most real-life situations, the price per unit may not be a viable option because its value is dictated by market conditions. the real option then is to reduce the consumption of resources, perhaps by making the production process more efficient. determination of the optimality ranges: we now turn our attention to determining the conditions that will keep an optimal solution unchanged. the presentation is based on the definition of reduced cost.

in the toyco model, let d_1, d_2, and d_3 represent the change in unit revenues for toy trucks, trains, and cars, respectively. the objective function then becomes

$$\text{maximize} \quad z = (3+d_1)x_1 + (2+d_2)x_2 + (5+d_3)x_3$$

z-row in starting tableau appear as

basic	x1	x2	x3	x4	x5	x6	solution
z	-3-d1	-2-d2	-5-d3	0	0	0	0

when we generate the simplex tableaus using the same sequence of entering and leaving variables in the original model (before the changes d_j are introduced), the optimal iteration will appear as:

basic	x1	x2	x3	x4	x5	x6	solution
z	4-0.25d2+1.5d3-d1	0	0	1+0.5d2	2-0.25d2+0.5d3	0	1350+100d2+230d3
x2	-1/4	1	0	1/2	-1/4	0	100
x3	3/2	0	1	0	1/2	0	230
x6	2	0	0	-2	1	1	20

a convenient way for computing the new reduced cost is **to add a new top row and a new leftmost column to the optimum tableau, as shown by the shaded areas below.**

		d1	d2	d3	0	0	0	
	basic	x1	x2	x3	x4	x5	x6	solution
1	z	4	0	0	1	2	0	1350
d2	x2	-1/4	1	0	1/2	-1/4	0	100
d3	x3	3/2	0	1	0	1/2	0	230
0	x6	2	0	0	-2	1	1	20

to explain to appear z-coefficient as: the new reduced cost for any variable (or the value of z), multiply the elements of its column by the corresponding elements in the leftmost column, add them up, and subtract the top-row element from the sum.

	d1	
left column	x1	(x1-column*left-column)
1	4	4*1
d2	-1/4	-1/4*d2
d3	3/2	3/2*d3
0	2	2*0
reduced cost for x1=4-1/4d2+3/2d3-d1		

applying the same rule to the solution column produces $z = 1350 + 100d_2 + 230d_3$.

left column	solution	(x1-column*left-column)
1	1350	1350
d2	100	100*d2
d3	230	230*d3
0	20	20*0
reduced cost for z=1350+100d2+230d3		

note: because we are dealing with a maximization problem, the current solution remains optimal so long as the new reduced costs (z-equation coefficients) remain nonnegative for all the nonbasic variables. we thus have the following **optimality conditions** corresponding to nonbasic x1, x4, and x5:

$$4 - \frac{1}{4}d2 + \frac{3}{2}d3 - d1 \geq 0$$
$$1 + \frac{1}{2}d2 \geq 0$$
$$2 - \frac{1}{4}d2 + \frac{1}{2}d3 \geq 0$$

these conditions must be satisfied simultaneously to maintain the optimality of the current optimum.
to illustrate the use of these conditions, suppose that the objective function of toyco is changed from

$$\text{maximize } z\ 3x1 + 2x2 + 5x3$$

to

$$\text{maximize } z = 2xl + xz + 6x3$$

then, d1 = 2 - 3 =-$1, d2 = 1 - 2 =$1, and d3 = 6 − 5= $1. substitution in the given conditions yields

$$4 - \frac{1}{4}d2 + \frac{3}{2}d3 - d1 = 4 - \frac{1}{4} * 1 + \frac{3}{2} * 1 - (-1) \geq 0 \quad \text{satisfied}$$
$$1 + \frac{1}{2}d2 = 1 + \frac{1}{2} * 1 \geq 0 \qquad \text{satisfied}$$
$$2 - \frac{1}{4}d2 + \frac{1}{2}d3 = 2 - \frac{1}{4} * 1 + \frac{1}{2} * 1 \geq 0 \quad \text{satisfied}$$

the results show that the proposed changes will keep the current solution (xl = 0, x2 = 100, x3 = 230) optimal. hence no further calculations are needed, except that the objective value will change to z= 1350 + 100d2 + 230d3 == 1350 + 100 x -1 + 230 x 1 = $1480

the general optimality conditions can be used to determine the special case where the changes dj occur one at a time instead of simultaneously. this analysis is equivalent to considering the following three cases:

1. maximize z = (3 + d1)xl +2x2 + 5x3
2. maximize z = 3xl + (2 + d 2)x2 + 5x3
3. maximize z = 3xl + 2x2 + (5 + d3)x3

the individual conditions can be accounted for as special cases of the simultaneous case.5

case 1. set d2 = d3 = 0 in the simultaneous conditions, which gives

$$4 - d1 \geq 0 \rightarrow -\infty < d1 \leq 4$$

case 2. set d1 = d3 =0 in the simultaneous conditions, which gives

$$4 - \frac{1}{4}d2 \geq 0 \rightarrow d2 \leq 16$$
$$1 + \frac{1}{2}d2 \geq 0 \rightarrow d2 \geq -2$$
$$2 - \frac{1}{4}d2 \geq 0 \rightarrow d2 \leq 8$$
$$-2 \leq d2 \leq 8$$

case 3. set d1 = d2= 0 in the simultaneous conditions, which gives

$$4 + \frac{3}{2}d3 \geq 0 \rightarrow d3 \geq -8/3$$
$$2 + \frac{1}{2}d3 \geq 0 \rightarrow d3 \geq 4$$
$$-8/3 \leq d3 \leq \infty$$

the given individual conditions can be translated in terms of the total unit revenue. for example, for toy trucks (variable x2), the total unit revenue is 2 + d2 and the associated condition $-2 \leq d3 \leq 8$ translates to

$$-2 + 2 \leq 2 + d2 \leq 8 + 2$$
$$0\$ \leq \text{uint revenue of toy truck} \leq 10\$$$

this condition assumes that the unit revenues for toy trains and toy cars remain fixed at $3 and $5, respectively.

the allowable range ($0, $10) indicates that the unit revenue of toy trucks (variable x2) can be as low as $0 or as high as $10 without changing the current optimum, xl= 0, xz = 100, x3 = 230. the total revenue will change to 1350 + 100d2, however

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