CRASHING

Academic Script

Introduction

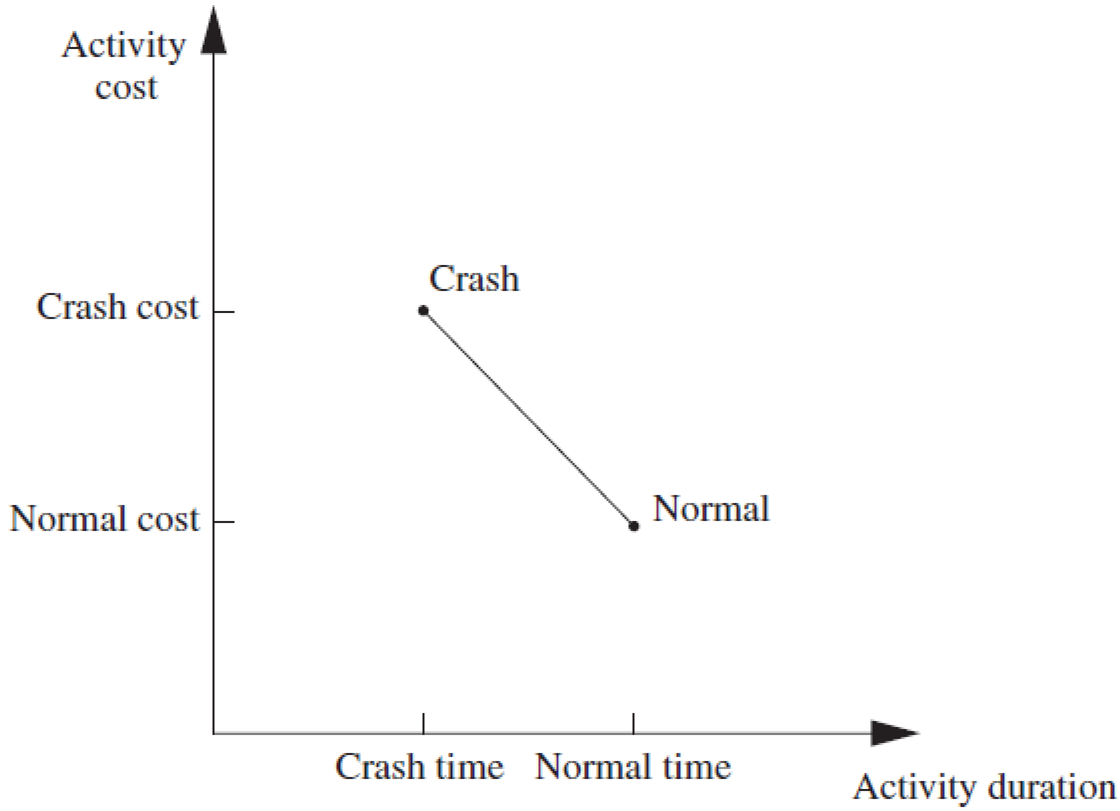
In previous talk, we discussed how to schedule project activities in logical sequence. The cost of resources consumed by activities were not taken into consideration. The project completion time can be reduced by reducing the normal completion time of critical activities. The reduction in normal time of completion will increase the total budget of the project. However, the decision-maker will always look for trade-off between the total cost of project and the total time required to complete it.

Project Crashing

Crashing an activity refers to taking special costly measures to reduce the duration of an activity below its normal value. These special measures might include using overtime, hiring additional temporary help, using special time-saving materials, obtaining special equipment, etc. Crashing the project refers to crashing a number of activities in order to reduce the duration of the project below its normal value.

The CPM method of time-cost trade-offs is concerned with determining how much (if any) to crash each of the activities in order to reduce the anticipated duration of the project to a desired value.

The data necessary for determining how much to crash a particular activity are given by the time-cost graph for the activity. Following Figure shows a typical time-cost graph. Note the two key points on this graph labeled Normal and Crash.



The normal point on the time-cost graph for an activity shows the time (duration) and cost of the activity when it is performed in the normal way. The crash point shows the time and cost when the activity is fully crashed, i.e., it is fully expedited with no cost spared to reduce its duration as much as possible. As an approximation, CPM assumes that these times and costs can be reliably predicted without significant uncertainty.

For most applications, it is assumed that partially crashing the activity at any level will give a combination of time and cost that will lie somewhere on the line segment between these two points. (For example, this assumption says that half of a full crash will give a point on this line segment that is midway between the normal and crash points.) This simplifying approximation reduces the necessary data gathering to estimating the time and cost for just two situations: normal conditions (to obtain the normal point) and a full crash (to obtain the crash point).

Time-Cost Trade-off Procedure

The method of establishing time-cost trade-off for the completion of a project can be summarized as:

Step-1: Determine the normal project completion time and
associated critical path for the two cases

1. When all critical activities are completed with their normal time. This provides the starting point for crashing analysis.
2. When all critical activities are crashed. This provides the stopping point for crashing analysis.

Step-2: Identify critical activities and compute the cost slope for each of these by using the relationship.

$$\textbf{Cost slope=}\frac{Crash\ cost - normal\ cost}{Normal\ time - crash\ time} = \frac{C_c - C_N}{T_N - T_C}$$

The values of cost slope for critical activities indicate the direct extra cost required to execute an activity per unit of time.

Step-3: For reducing the total project completion time, identify and crash an activity time on the critical path with lowest cost slope value to the point where

a. Another path in the network becomes critical, or
b. The activity has been crash to its lowest possible time.

Step-4: if the critical path under crashing is still critical, return to step 3. However, if due to crashing of an activity time in step-3, other path(s) in the network also become critical, then identify and crash the activity(s) on the critical path(s) with the minimum joint cost slope.

Step-5: terminate the procedure when each critical activity has been crashed to its lower possible time. Determine total project cost corresponding to different project duration.

By using this procedure we can solve the examples of crashing.

Example 1:

The following table gives data on normal time, and cost and crash time and cost for a project.

Activity	Normal		Crash	
	Time(weeks)	**Cost(Rs)**	**Time(weeks)**	**Cost(Rs)**
1-2	3	300	2	400
2-3	3	30	3	30
2-4	7	420	5	580
2-5	9	720	7	810
3-5	5	250	4	300
4-5	0	0	0	0
5-6	6	320	4	410
6-7	4	400	3	470
6-8	13	780	10	900
7-8	10	1000	9	1200

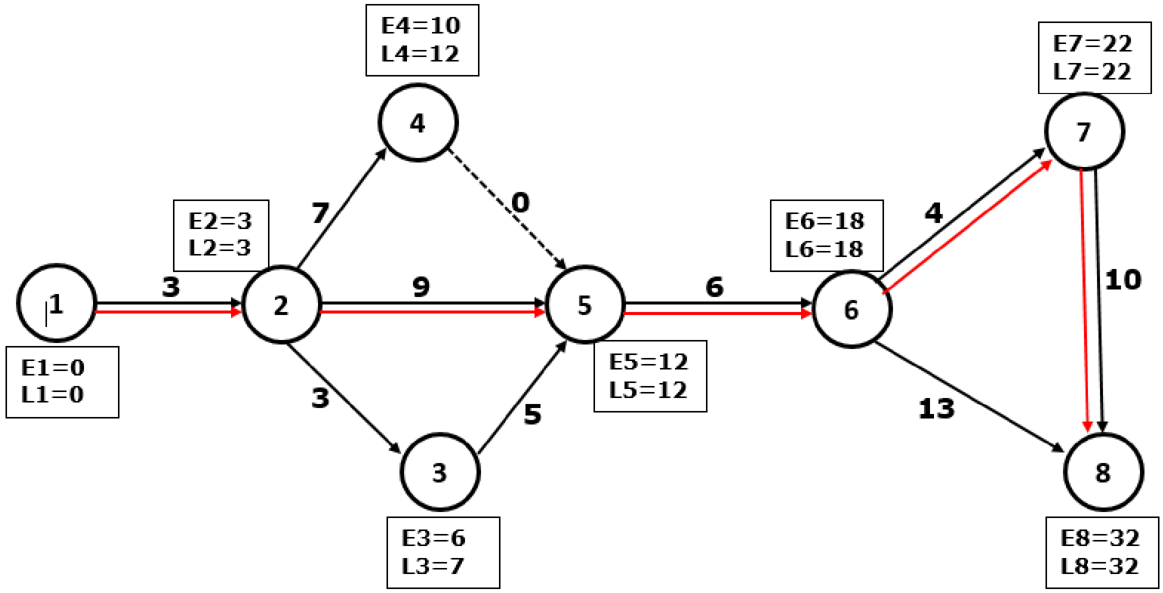
Indirect cost is Rs. 30 per week.

1. Draw a network diagram for the project and identify the critical path.
2. What are the normal project duration an associated cost?
3. Find out the total float associated with each activity.
4. Crash the relevant activities systematically and determine the optimal project completion time and cost.

Solution

1. Here we draw a network diagram which indicated a project completion time of 32 weeks with the critical path: 1-2-5-6-7-8.

Figure: 1



2. Normal Project duration is 32 weeks and the associated cost is as follow:

Total Cost=Direct Normal cost+ Indirect cost of 32 weeks

= 4220+ 30 x 32

= Rs. 5180

3. Calculations for total float associated with each activity are calculated as follow

Activity	**Total Float**
1-2	(3-0)-3=0
2-3	(7-3)-3=1
2-4	(12-3)-7=2
2-5	(12-3)-9=0
3-5	(12-6)-5=1
4-5	(12-10)-0=2
5-6	(18-12)-6=0
6-7	(22-18)-4=0
6-8	(32-18)-13=1
7-8	(32-22)-10=0

4. For critical activities, crash cost-slope is calculated as

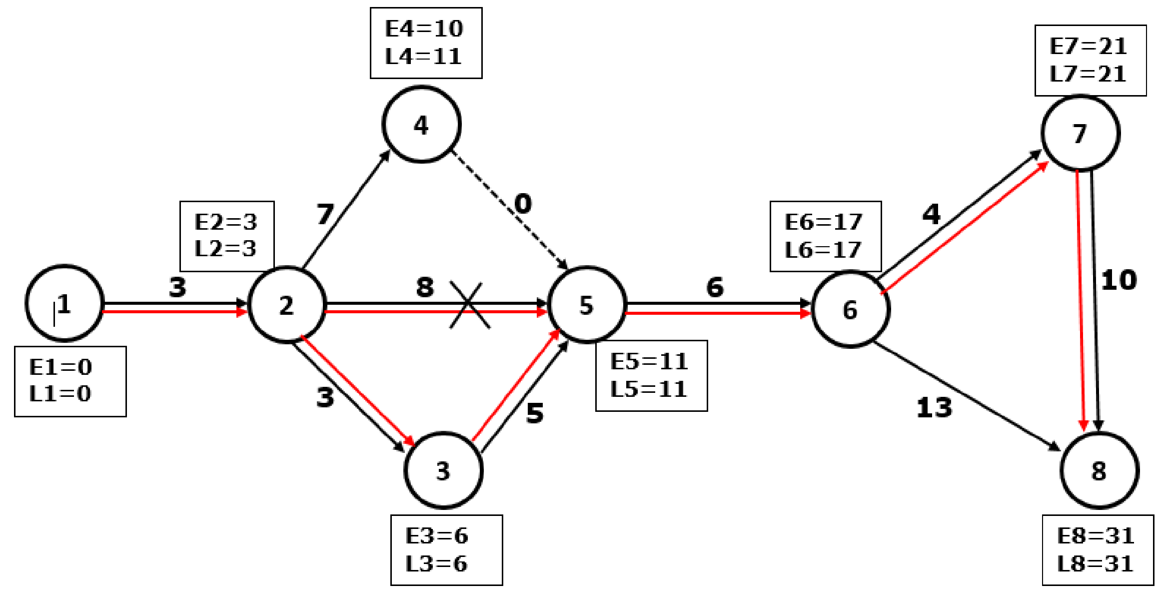
Critical Activity	Crash cost per week (Rs)
1-2	$\frac{400-300}{3-2}=100$
2-5	$\frac{810-720}{9-7}=45$
5-6	$\frac{410-320}{6-4}=45$
6-7	$\frac{470-400}{4-3}=70$
7-8	$\frac{1200-1000}{10-9}=200$

Which Activities Should Be Crashed?

The minimum value of crash cost per week is for activity 2-5 and 5-6. Hence, crashing activity 2-5 by 2 days from 9 weeks to 7 weeks.

But the time should be reduced by 1 week otherwise another path 1-2-3-5-6-7-8 become a parallel path. Network as shown in figure:2 is developed when it is observed that new project time is 31 weeks and the critical path are 1-2-5-6-7-8 and 1-2-3-5-6-7-8.

Figure: 2



With crashing of activity 2-5, the new total cost involved can be calculated as
New Total cost= Total direct Normal Cost + Increased direct cost due to crashing od activity (2-5) + Indirect cost for 31 weeks

= (4220 + 1 x 45) + 31 x 30

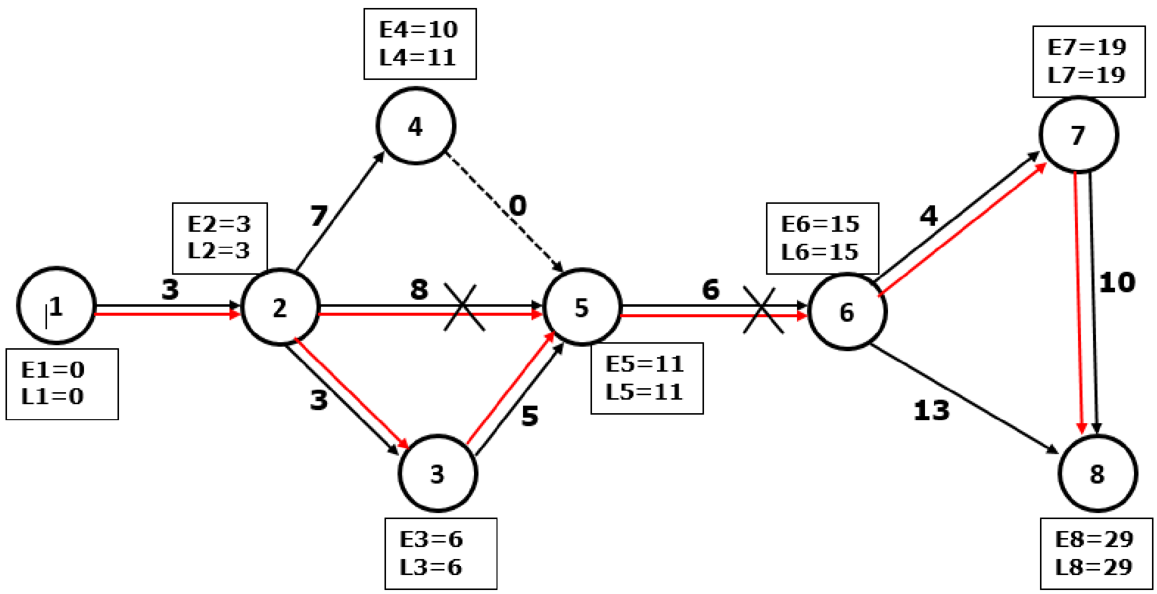
= 4265 + 930 = Rs. 5195

Now the new possibilities for crashing in the critical paths are listed in the following table

Critical Activity	Crashed Cost Per week (Rs.)
1-2	100
2-5	x (Crashed)
2-3	0 (No crashing is needed)
3-5	50
5-6	45
6-7	70
7-8	200

The minimum value of crashed cost slope is 45 for activity 5-6.
Hence, by crashing it by 2 weeks from 6 weeks to 4 weeks, we will get a new network diagram which will now look like figure 3.

Figure: 3



Now one can observed from figure-3, that both the critical paths shown in figure-2 remain unchanged because activity 5-6 is common between critical paths shown in figure-2. But with this crashing of activity 5-6 by 2 weeks, the new cost involved is:

New Total cost = Total direct Normal cost + Increased direct cost due to crashing of 5-6 + Indirect cost for 29 weeks

= (4220 + 1 x 45 + 2 x 45) + 29 x 30

= Rs 5225

With respect to network given in figure-3, the new possibilities for crashing in the critical paths are

Critical Activity	Crashed Cost Per week (Rs.)
1-2	100
2-3	0 (No crashing is needed)
2-5	x (Crashed)
5-6	x (Crashed)
6-7	70
7-8	200

The further crashing of 6-7 activity time from 4 weeks to 3 weeks will result in increased direct cost than the gain due to reduction in project time. Hence, here we must stop further crashing.

The optimal project duration is 29 weeks with associated cost of Rs. 5225 as shown below

Project Duration (weeks)	Crashing activity and weeks	Direct Cost (Rs.)			Indirect Cost (Rs)	Total Cost (Rs.)
		Normal	Crashing	Total		
32	-	4220	-	4220	32x30=960	5180
31	2-5(1)	4220	1x45=45	4265	31x30=930	5195
29	5-6(2)	4220	45+2x45=135	4355	29x30=870	5225
28	6-7(1)	4220	132+1x70=205	4425	28x30=840	5265

Example 2:

The following table gives the activities in construction project and also gives other relevant informationHere activity A to G is given and corresponding normal time and cost and Crash time and Crash cost is given.

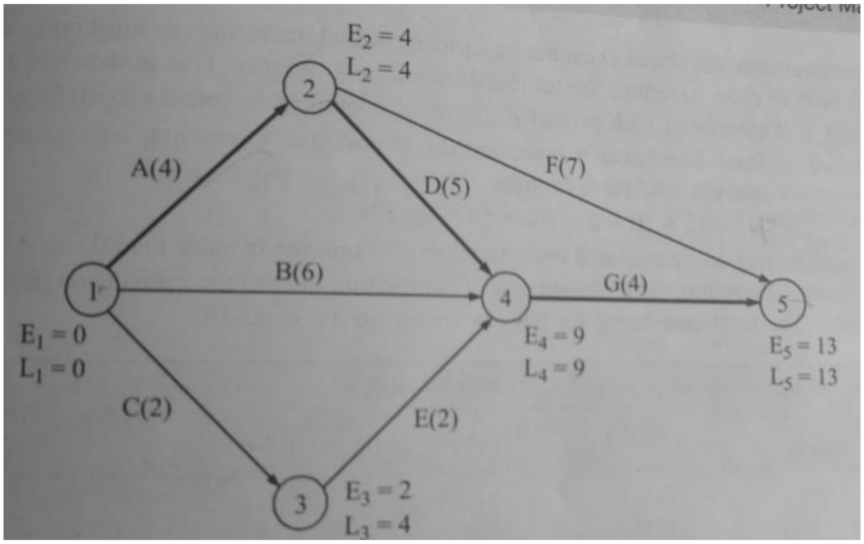
Activity	Immediate Predecessor	Normal		Crash	
		Time(weeks)	Cost(Rs)	Time(weeks)	Cost(Rs)
A	-	4	60	3	90
B	-	6	150	4	250
C	-	2	38	1	60
D	A	5	150	3	250
E	C	2	100	2	100
F	A	7	115	5	175
G	D, B, E	4	100	2	240

Indirect costs vary as follows:

Months	15	14	13	12	11	10	9	8	7	6
Crash cost Slope	600	500	400	250	175	100	75	50	35	25

1. Draw an arrow diagram for the project.
2. Determine the project duration that will result in minimum total project cost.

Solution: The network for normal activity indicates a project duration of 13 months with critical path A-D-G as shown in figure-1. The crash cost slope for various activities of the project is given here



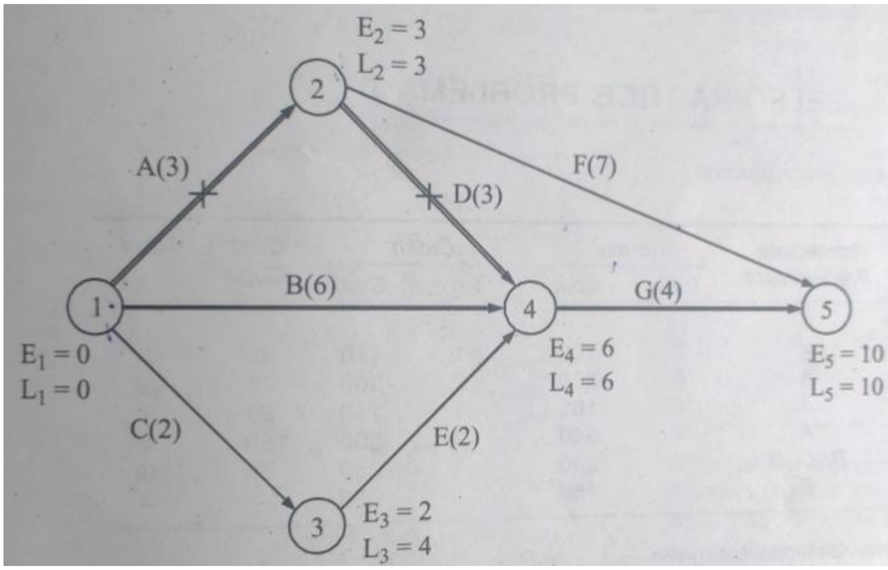
Activity	Crash Cost-Slope
A	30
B	50
C	22
D	50
E	-
F	30
G	70

Among the critical activities A, D and G the least expensive activity is A. This means that this activity should be the first to be crashed in order to reduce project duration.

Crashing activity A duration by maximum possible extent of one month, the project duration reduces to 12 months and new total project cost is as

Total Cost= Total direct cost+ indirect cost due to cost of activity+ Indirect cost for 12 months =713 + 1 x30 + 250 = 743 + 250 = Rs 993

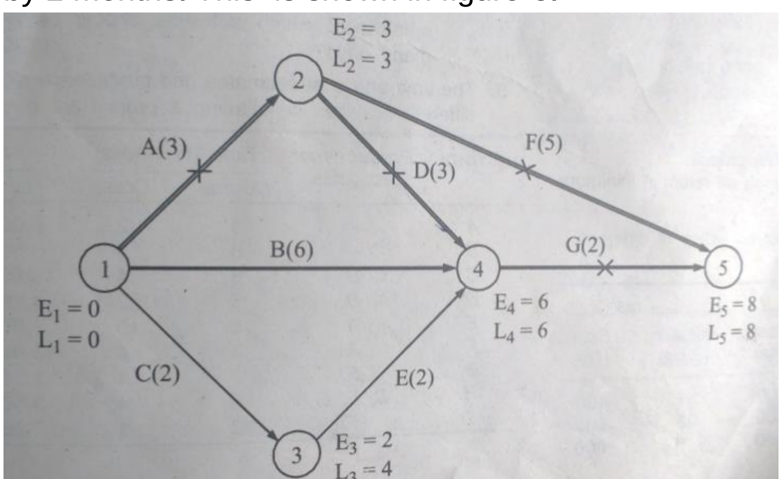
Since the crashing does not result in making any other path critical, therefore we should further crash activity D, that has the next minimum cost slope, by two months. Thus, project duration reduces to 10 months as shown in figure-2



In figure-2 there are three critical paths: 1-2-5, 1-2-4-5 and 1-4-5, each having a duration of 10 months. The new total cost of project is calculated as:

Total Cost = Total direct cost + increased direct cost due to crashing of activity D+ Indirect cost for 10 months =(743+2 x 50)+100=Rs.943

Since the critical activities A and D cannot be further crashed, therefore we must crash activities F and G by 2 months each in order to reduce the project duration by 2 months. This is shown in figure-3.



The new total project cost associated with project duration of 8 months is calculated as

Total cost = Total direct cost+ increased direct cost due to crashing of activities F and G + indirect cost for 8 months = 743 + (2x30+2x70)+50=Rs.1093

Since the total project cost associated with duration of 8 months is more than the cost associated with duration of 10 months, therefore the optimum of completion time of the project and corresponding total cost is 10 months.

The total cost being Rs 993 as shown here

Project Duration (weeks)	Crashing activity and weeks	Direct Cost (Rs.)			Indirect Cost (rs)	Total Cost (Rs.)
		Normal	Crashing	Total		
13	-	713	-	713	400	1113
12	1-2 (1)	713	30	743	250	993
10	2-4(2)	713	30+2x50=130	843	100	943
8	2-5(2)	713	130+2x30+2x70= 330	1043	50	1093
	4-5(2)					

Summary

Continuing with the previous talk of PERT and CPM, today we learn crashing of an activity. Crashing an activity means performing it in the shortest possible time by allocating to it necessary resources.

The time-cost trade-offs approach in CPM enables the project manager to investigate the effect on total cost of changing the estimated duration of the project to various alternatives values.

Thank you