LINEAR PROGRAMMING: THE GRAPHICAL METHOD

- Linear Programming Problem
- Properties of LPs
- LP Solutions
- Graphical Solution
- Introduction to Sensitivity Analysis

Linear Programming (LP) Problem

- A mathematical programming problem is one that seeks to maximize or minimize an objective function subject to constraints.
- If both the objective function and the constraints are linear, the problem is referred to as a linear programming problem.
- Linear functions are functions in which each variable appears in a separate term raised to the first power and is multiplied by a constant (which could be 0).
- Linear constraints are linear functions that are restricted to be "less than or equal to", "equal to", or "greater than or equal to" a constant.

Linear Programming (LP) M

- There are five common types of decisions in which LP may play a role
  - Product mix- blending problem in mining
  - Production plan
  - Ingredient mix-plant concentration
  - Transportation
  - Assignment- workers job distribution

Steps in Developing a Linear Programming (LP) Model

1) Formulation

2) Solution

3) Interpretation and Sensitivity Analysis

Steps in Formulating LP Problems

1. Define the objective. (min or max)
2. Define the decision variables. (positive, binary) ?
3. Write the mathematical function for the objective.
4. Write a 1- or 2-word description of each constraint.
5. Write the right-hand side (RHS) of each constraint.
    5. Also called resources
6. Write $\leq$, =, or $\geq$ for each constraint.
7. Write the decision variables on LHS of each constraint.
8. Write the coefficient for each decision variable in each constraint.

Properties of LP Models

1) Seek to minimize or maximize
2) Include “constraints” or limitations
3) There must be alternatives available
4) All equations are linear

LP Problems in: Product Mix

- <u>Objective</u>

  To select the mix of products or services that results in maximum profits for the planning period

- <u>Decision Variables</u>

  How much to produce and market of each product or service for the planning period

- <u>Constraints</u>

  Maximum amount of each product or service demanded; Minimum amount of product or service policy will allow; Maximum amount of resources available

Example LP Model Formulation: The Product Mix Problem

Decision: How much to make of $\geq$ 2 products?

Objective: Maximize profit

Constraints: Limited resources

Example: Pine Furniture Co.

Two products: Chairs and Tables

Decision: How many of each to make this month?

Objective: Maximize profit

Pine Furniture Data

	Tables (per table)	Chairs (per chair)	Hours Available
Profit Contribution	$7	$5	
Carpentry	3 hrs	4 hrs	2400
Painting	2 hrs	1 hr	1000

Other Limitations:

- Make no more than 450 chairs
- Make at least 100 tables

Constraints:

- Have 2400 hours of carpentry time available

  $3\,T + 4\,C \leq 2400$ (hours)

- Have 1000 hours of painting time available

  $2\,T + 1\,C \leq 1000$ (hours)

More Constraints:

- Make no more than 450 chairs

  $C \le 450$ (num. chairs)

- Make at least 100 tables

  $T \ge 100$ (num. tables)

Non-negativity:

Cannot make a negative number of chairs or tables

$$T \ge 0$$

$$C \ge 0$$

Model Summary

Max $7T + 5C$ (profit)

Subject to the constraints:

$$3T + 4C \le 2400 \quad \text{(carpentry hrs)}$$

$$2T + 1C \le 1000 \quad \text{(painting hrs)}$$

$$C \le 450 \quad \text{(max \# chairs)}$$

$$T \ge 100 \quad \text{(min \# tables)}$$

$$T, C \ge 0 \quad \text{(nonnegativity)}$$

Graphical Solution

- Graphing an LP model helps provide insight into LP models and their solutions.

- While this can only be done in two dimensions, the same properties apply to all LP models and solutions.

LP Characteristics

- **Feasible Region**: The set of points that satisfies all constraints
- **Corner Point Property**: An optimal solution must lie at one or more corner points
- **Optimal Solution**: The corner point with the best objective function value is optimal

Carpentry

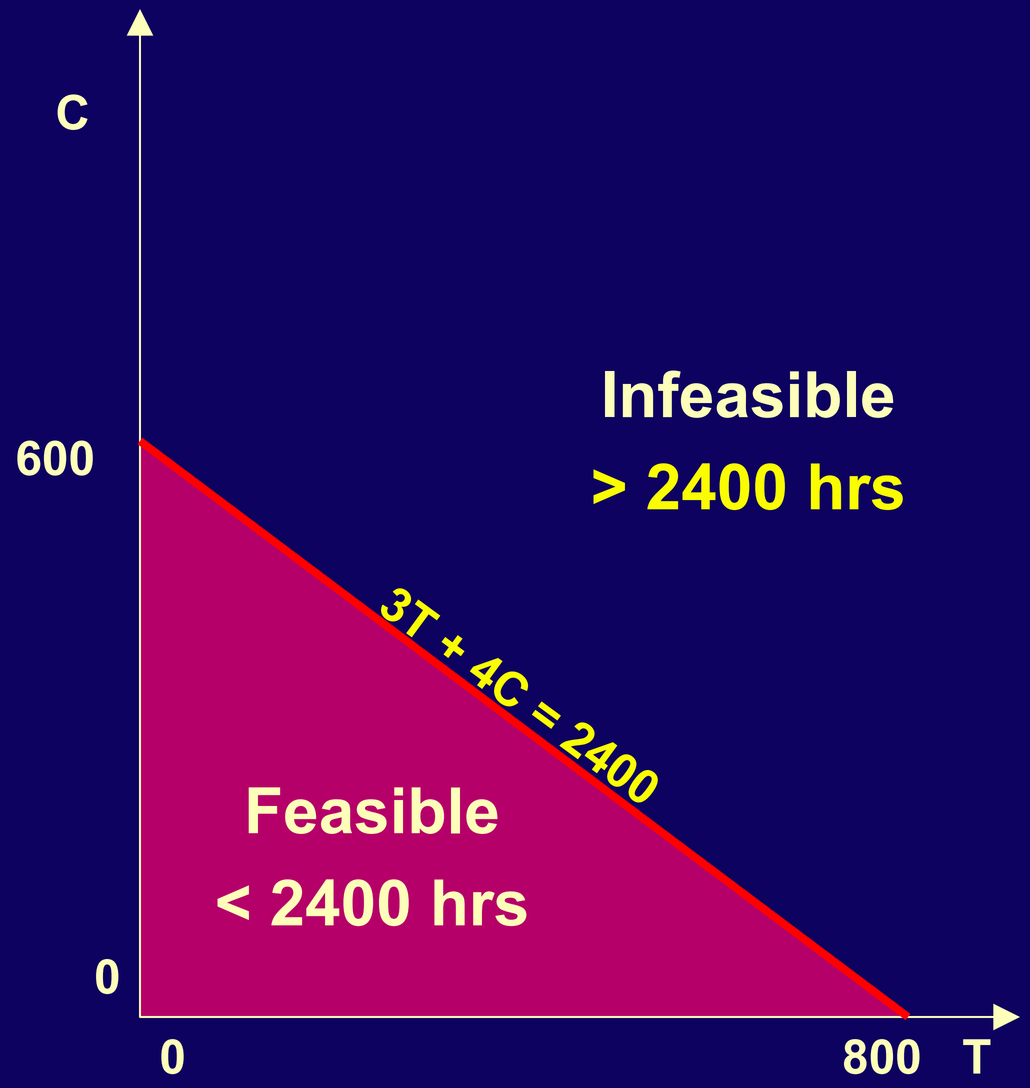
Constraint Line

$3T + 4C = 2400$

Intercepts

$(T = 0, C = 600)$

$(T = 800, C = 0)$



Painting

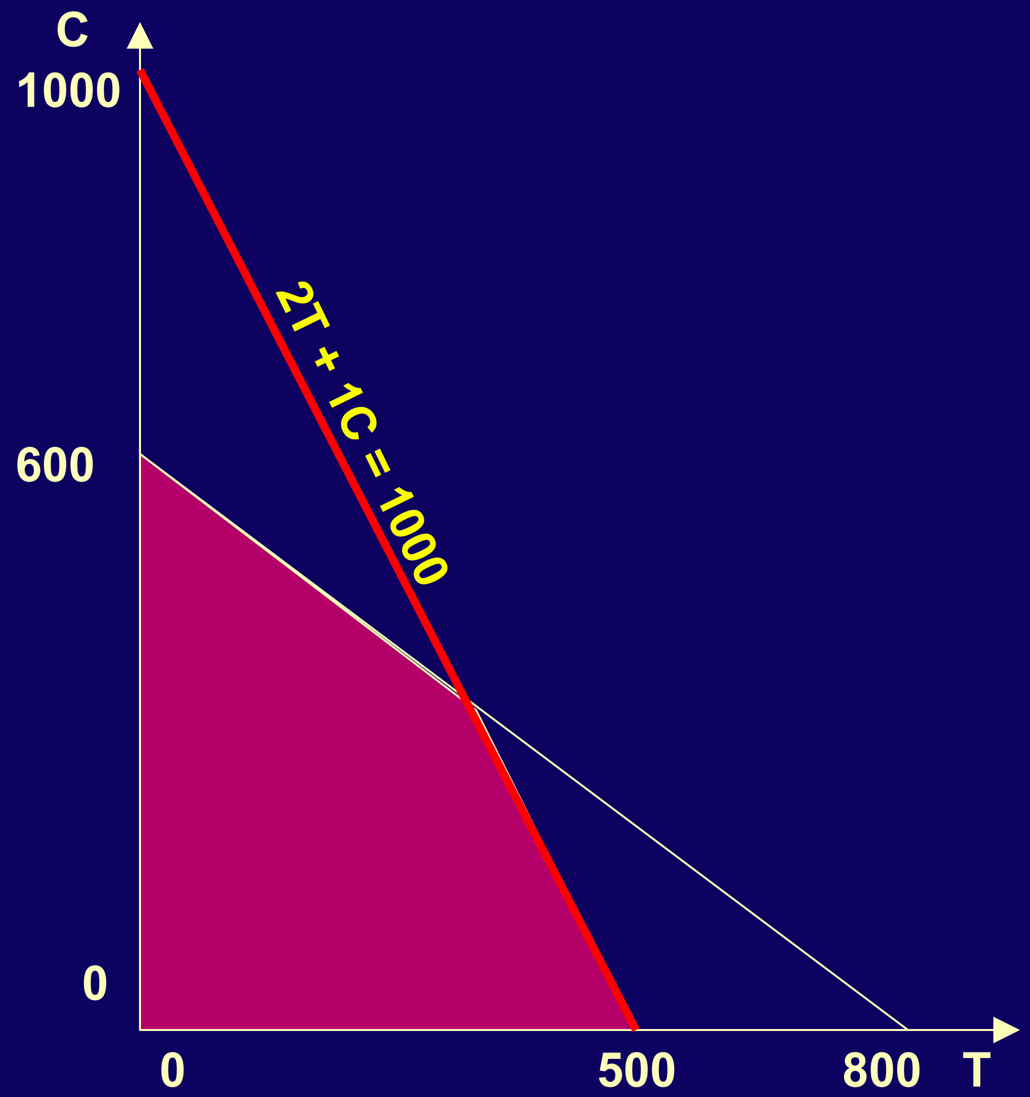
Constraint Line

2T + 1C = 1000

Intercepts

(T = 0, C = 1000)

(T = 500, C = 0)

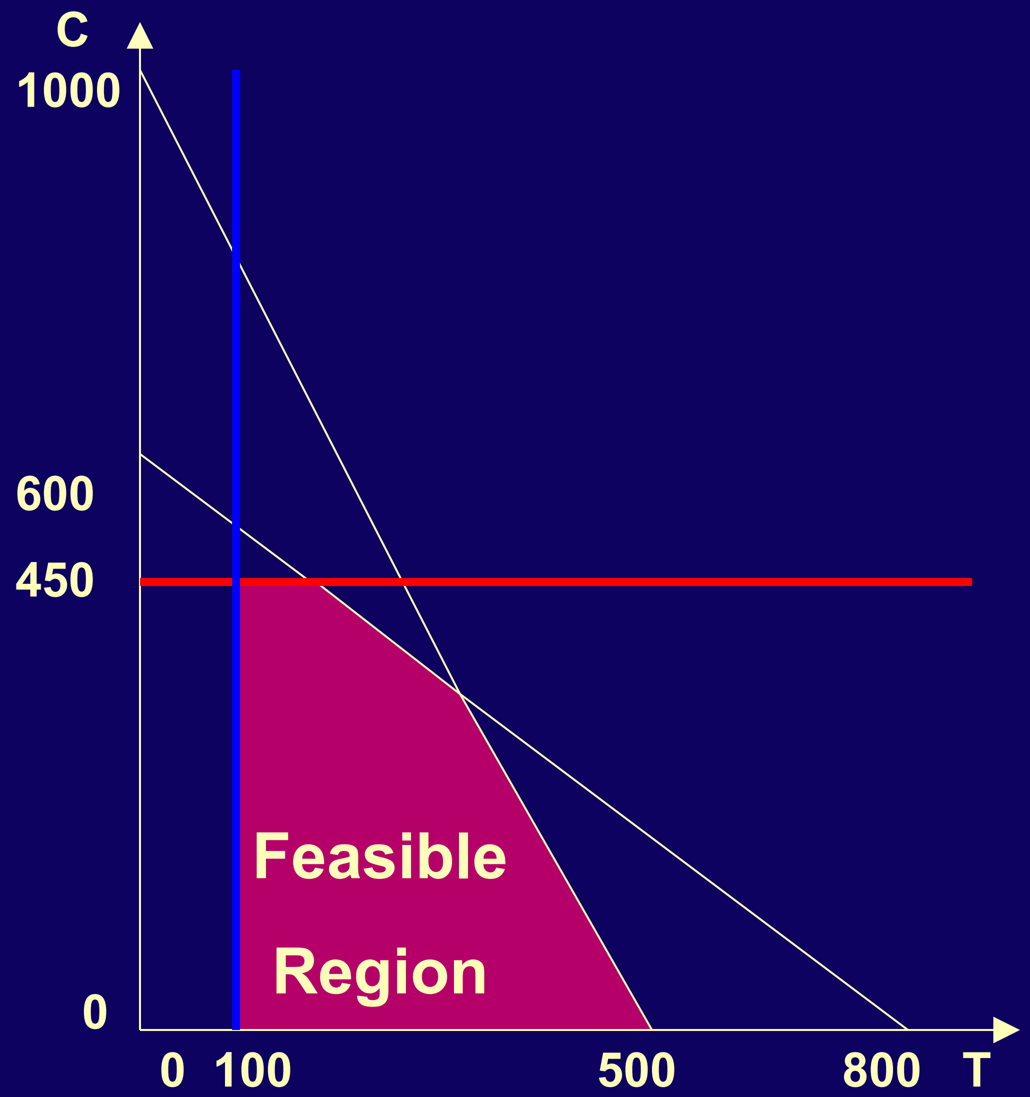


Max Chair Line

C = 450

Min Table Line

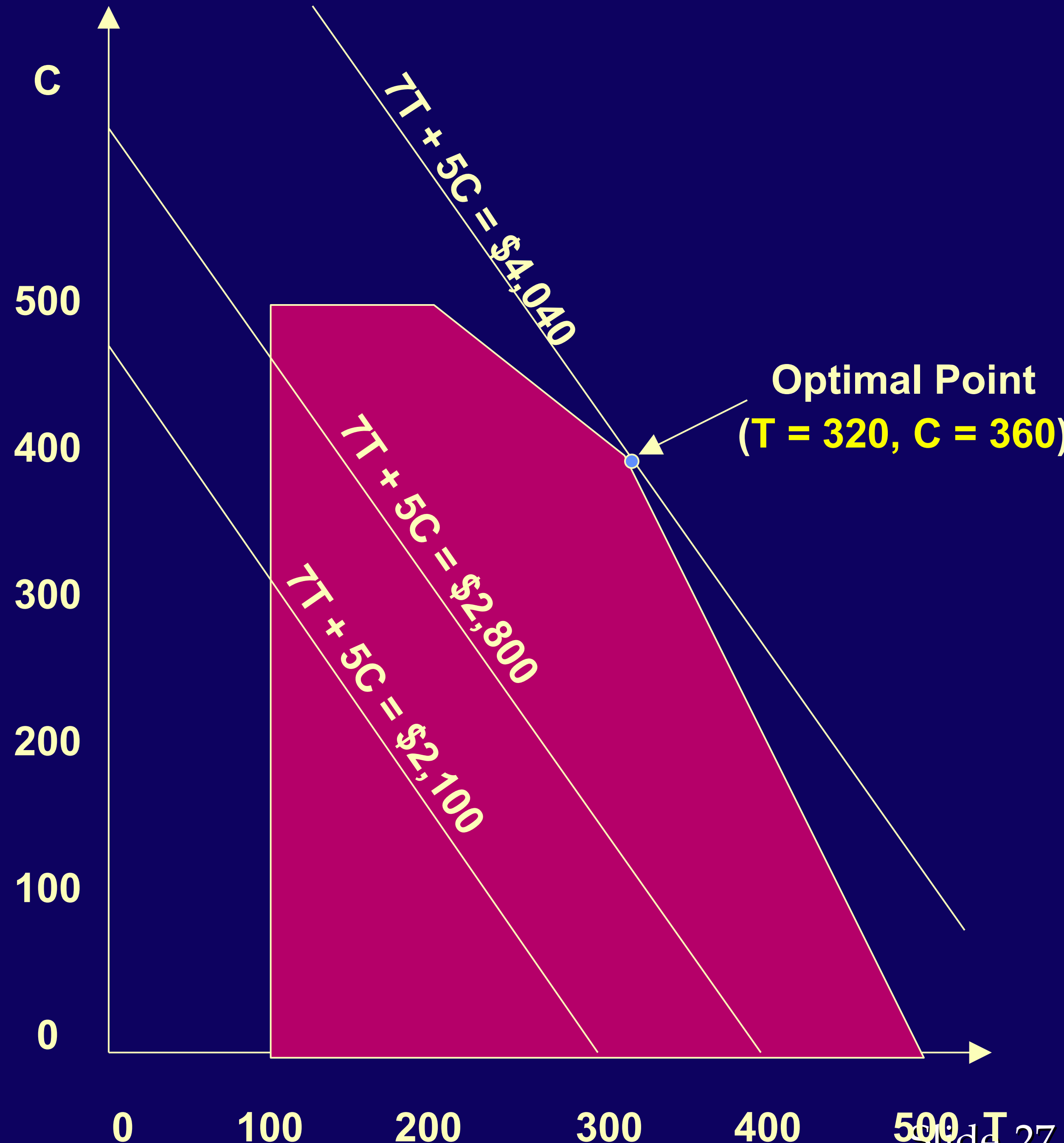
T = 100



Objective

Function Line

7T + 5C = Profit



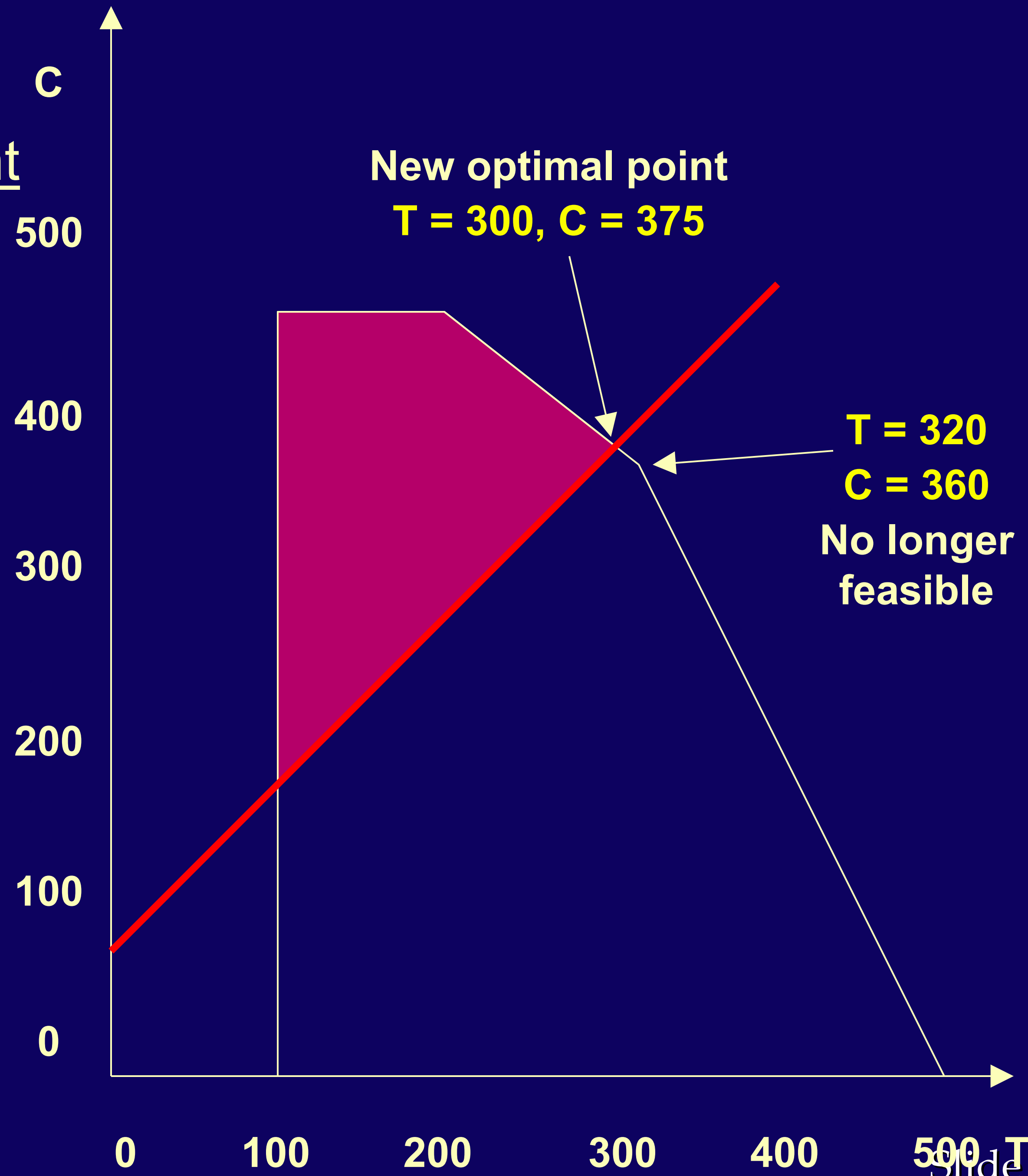
Additional Constraint

Need at least 75 more chairs than tables

$C \geq T + 75$

Or

$C - T \geq 75$



Special Situation in LP

1. **Redundant Constraints** - do not affect the feasible region

Example: $x \leq 10$

$x \leq 12$

The second constraint is redundant because it is *less* restrictive.

Special Situation in LP

2. **Infeasibility** – when no feasible solution exists (there is no feasible region)

Example: $x \leq 10$

$x \geq 15$

Special Situation in LP

3. **Alternate Optimal Solutions** – when there is more than one optimal solution

Max 2T + 2C

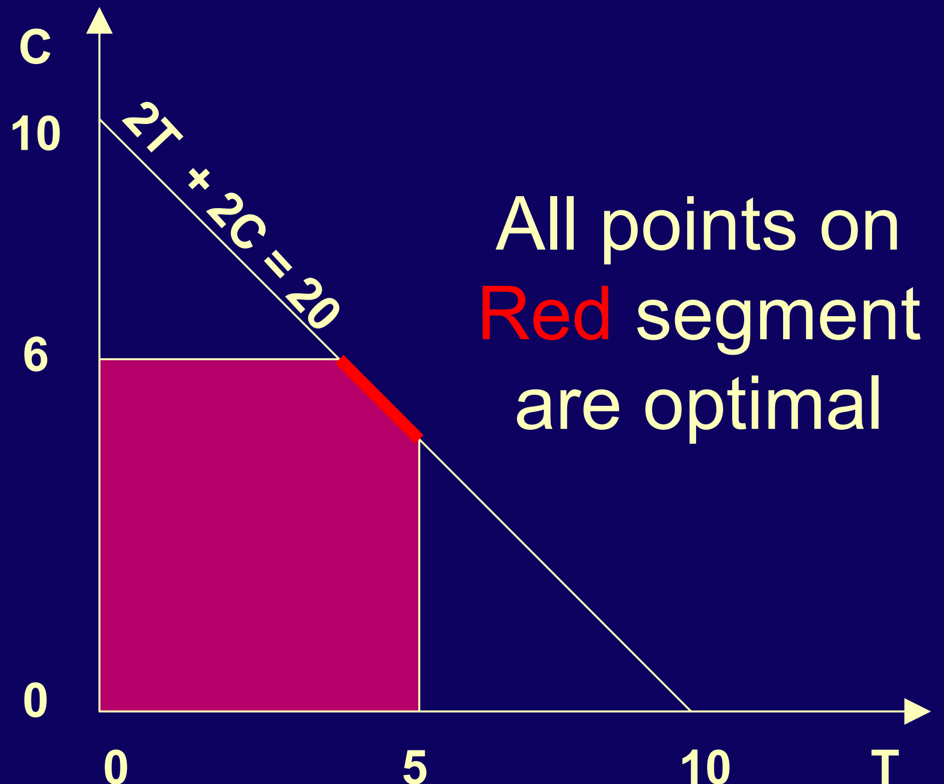
Subject to:

$T + C \leq 10$

$T \leq 5$

$C \leq 6$

$T, C \geq 0$



All points on Red segment are optimal

Special Situation in LP

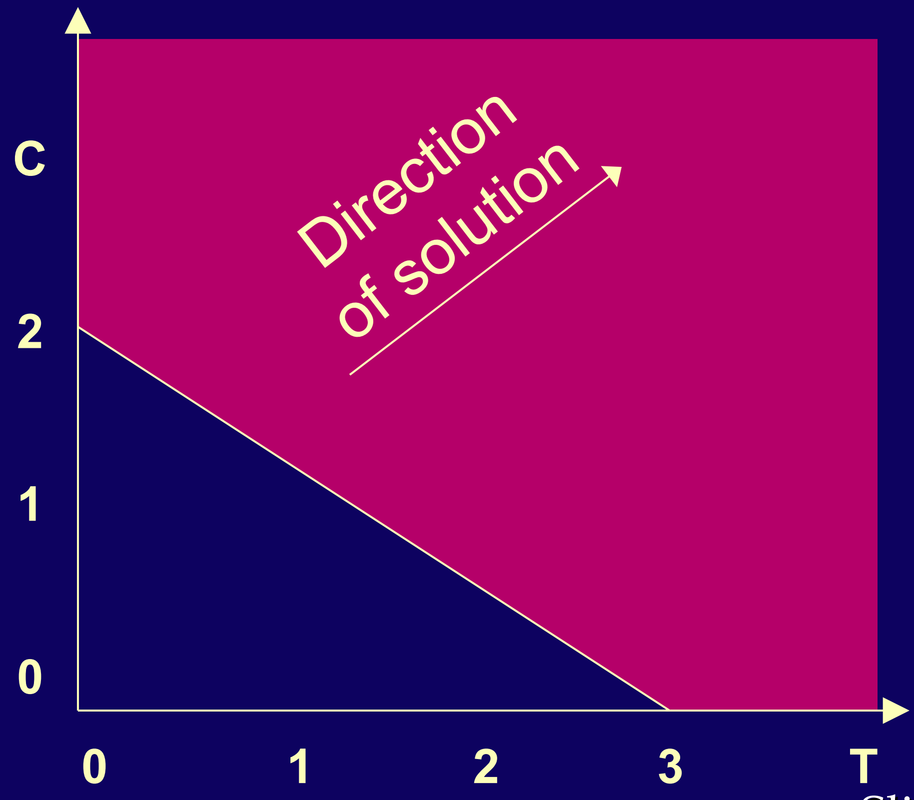
4. **Unbounded Solutions** – when nothing prevents the solution from becoming infinitely large

Max 2T + 2C

Subject to:

$2T + 3C \geq 6$

$T, C \geq 0$



Building Linear Programming Models

- 1. What are you trying to decide - Identify the decision variable to solve the problem and define appropriate variables that represent them. For instance, in a simple maximization problem, RMC, Inc. interested in producing two products: fuel additive and a solvent base. The decision variables will be X1 = tons of fuel additive to produce, and X2 = tons of solvent base to produce.
- 2. What is the objective to be maximized or minimized? Determine the objective and express it as a linear function. When building a linear programming model, only relevant costs should be included, sunk costs are not included. In our example, the objective function is: z = 40X1 + 30X2; where 40 and 30 are the objective function coefficients.

Building Linear Programming Models

- 3. What limitations or requirements restrict the values of the decision variables? Identify and write the constraints as linear functions of the decision variables. Constraints generally fall into one of the following categories:
  - **a. Limitations** - The amount of material used in the production process cannot exceed the amount available in inventory. In our example, the limitations are:
  - Material 1 = 20 tons
  - Material 2 = 5 tons
  - Material 3 = 21 tons available.
  - The material used in the production of X1 and X2 are also known.

Building Linear Programming Models

- To produce one ton of fuel additive uses .4 ton of material 1, and .60 ton of material 3. To produce one ton of solvent base it takes .50 ton of material 1, .20 ton of material 2, and .30 ton of material 3. Therefore, we can set the constraints as follows:

  $.4X1 + .50\ X2 <= 20$

  $.20X2 <= 5$

  $.6X1 + .3X2 <=21$, where

  .4, .50, .20, .6, and .3 are called constraint coefficients. The limitations (20, 5, and 21) are called Right Hand Side (RHS).
- **b. Requirements - specifying a minimum levels of performance. For instance, production must be sufficient to satisfy customers' demand.**

LP Solutions

- The maximization or minimization of some quantity is the objective in all linear programming problems.
- A feasible solution satisfies all the problem's constraints.
- Changes to the objective function coefficients do not affect the feasibility of the problem.
- An optimal solution is a feasible solution that results in the largest possible objective function value, z, when maximizing or smallest z when minimizing.
- In the graphical method, if the objective function line is parallel to a boundary constraint in the direction of optimization, there are alternate optimal solutions, with all points on this line segment being optimal.

LP Solutions

- A graphical solution method can be used to solve a linear program with two variables.
- If a linear program possesses an optimal solution, then an extreme point will be optimal.
- If a constraint can be removed without affecting the shape of the feasible region, the constraint is said to be redundant.
- A nonbinding constraint is one in which there is positive slack or surplus when evaluated at the optimal solution.
- A linear program which is overconstrained so that no point satisfies all the constraints is said to be infeasible.

LP Solutions

- A feasible region may be unbounded and yet there may be optimal solutions. This is common in minimization problems and is possible in maximization problems.
- The feasible region for a two-variable linear programming problem can be nonexistent, a single point, a line, a polygon, or an unbounded area.
- Any linear program falls in one of three categories:
  - is infeasible
  - has a unique optimal solution or alternate optimal solutions
  - has an objective function that can be increased without bound

Example: Graphical Solution

- Solve graphically for the optimal solution:

$$\text{Min} \quad z = 5x_1 + 2x_2$$

$$\begin{aligned} \text{s.t.} \quad 2x_1 + 5x_2 &\geq 10 \\ 4x_1 - x_2 &\geq 12 \\ x_1 + x_2 &\geq 4 \end{aligned}$$

$$x_1, x_2 \geq 0$$

Example: Graphical Solution

- Graph the Constraints

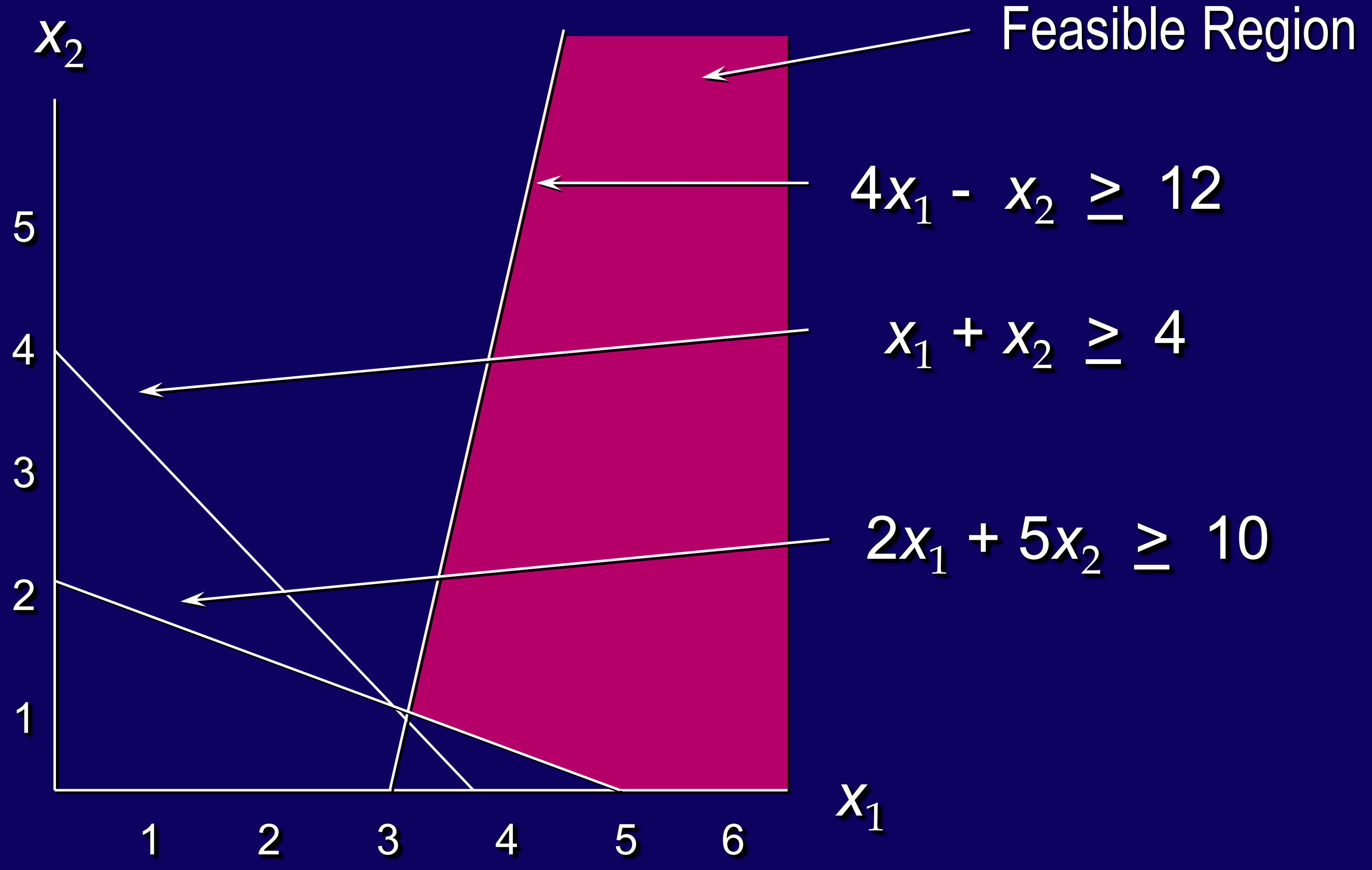
<u>Constraint 1</u>: When $x_1 = 0$, then $x_2 = 2$; when $x_2 = 0$, then $x_1 = 5$. Connect (5,0) and (0,2). The ">" side is above this line.

<u>Constraint 2</u>: When $x_2 = 0$, then $x_1 = 3$. But setting x_1 to 0 will yield $x_2 = -12$, which is not on the graph. Thus, to get a second point on this line, set x_1 to any number larger than 3 and solve for x_2: when $x_1 = 5$, then $x_2 = 8$. Connect (3,0) and (5,8). The ">" side is to the right.

<u>Constraint 3</u>: When $x_1 = 0$, then $x_2 = 4$; when $x_2 = 0$, then $x_1 = 4$. Connect (4,0) and (0,4). The ">" side is above this line.

Example: Graphical Solution

- Constraints Graphed



Example: Graphical Solution

- Graph the Objective Function

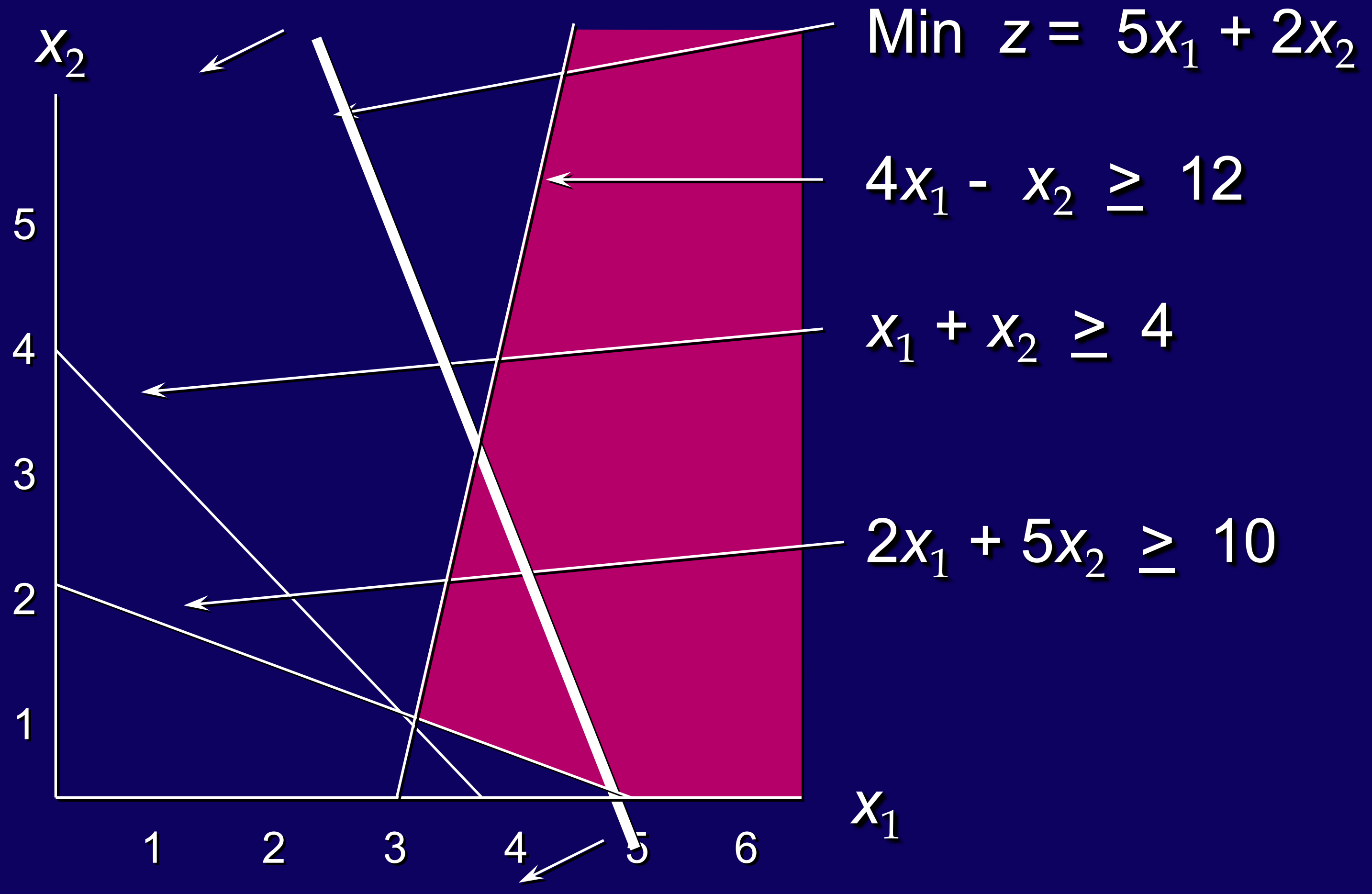
  Set the objective function equal to an arbitrary constant (say 20) and graph it. For $5x_1 + 2x_2 = 20$, when $x_1 = 0$, then $x_2 = 10$; when $x_2 = 0$, then $x_1 = 4$. Connect (4,0) and (0,10).

- Move the Objective Function Line Toward Optimality

  Move it in the direction which lowers its value (down), since we are minimizing, until it touches the last point of the feasible region, determined by the last two constraints. This is called the Iso-Value Line Method.

Example: Graphical Solution

- Objective Function Graphed



Example: Graphical Solution

- Solve for the Extreme Point at the Intersection of the Two Binding Constraints

$$4x_1 - x_2 = 12$$

$$x_1 + x_2 = 4$$

Adding these two equations gives:

$$5x_1 = 16 \text{ or } x_1 = 16/5.$$

Substituting this into $x_1 + x_2 = 4$ gives: $x_2 = 4/5$

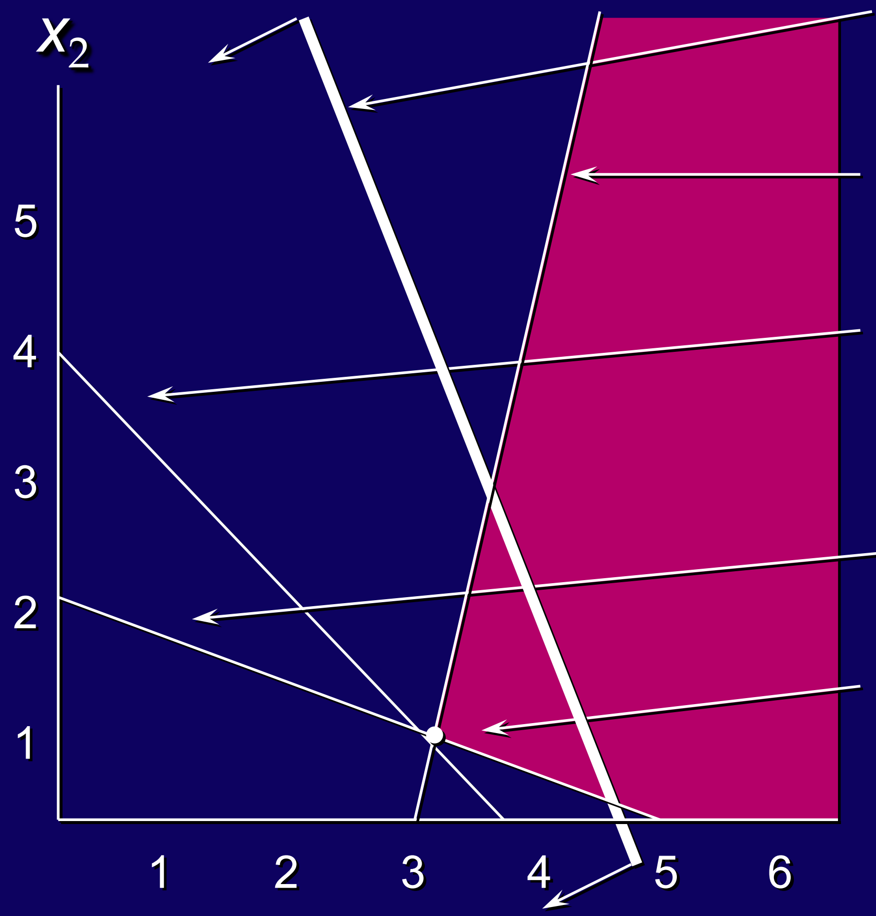
- Solve for the Optimal Value of the Objective Function

Solve for $z = 5x_1 + 2x_2 = 5(16/5) + 2(4/5) = 88/5$.

Thus the optimal solution is

$$\boxed{x_1 = 16/5; \; x_2 = 4/5; \; z = 88/5}$$

Example: Graphical Solution



Min $z = 5x_1 + 2x_2$

$4x_1 - x_2 \geq 12$

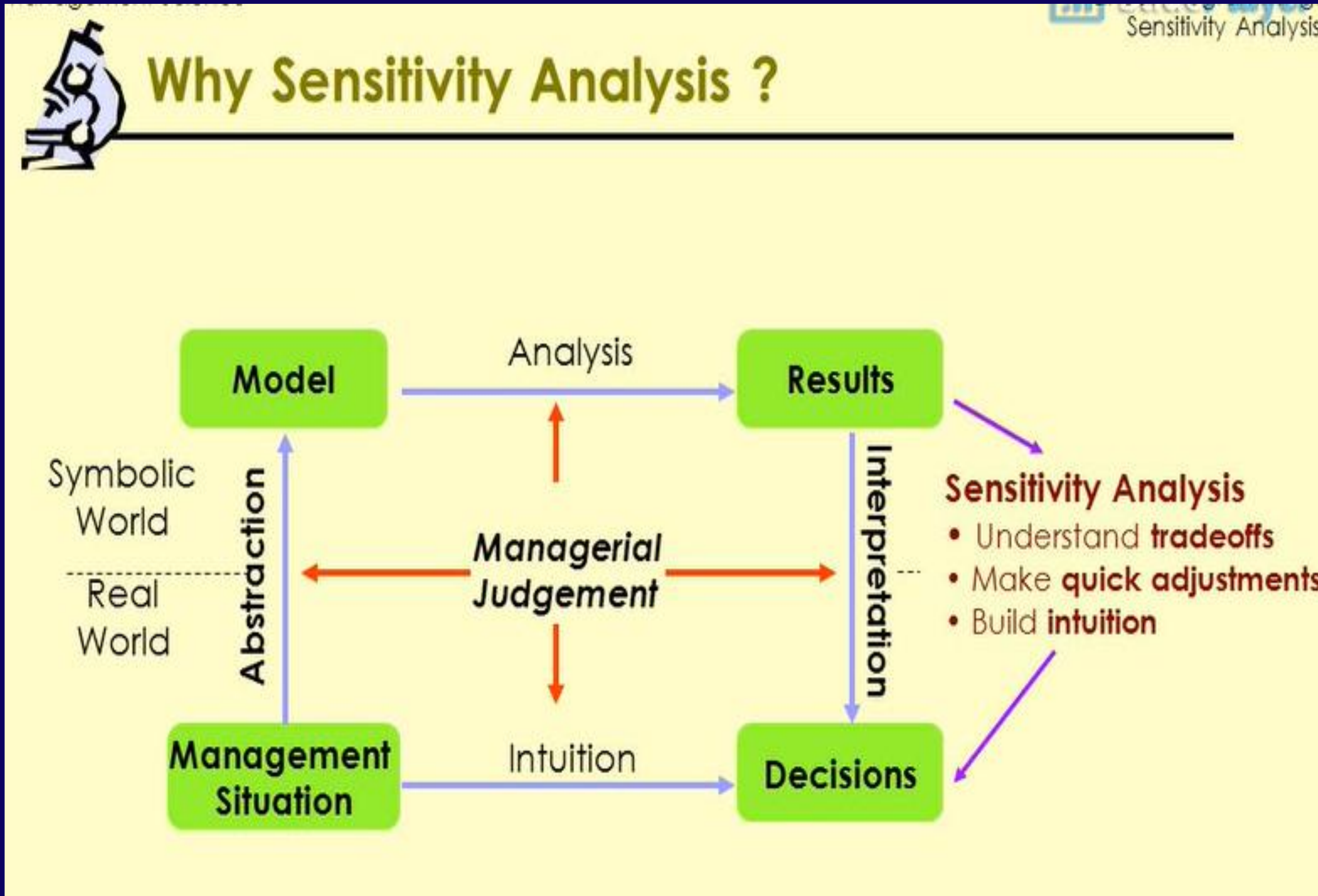
$x_1 + x_2 \geq 4$

$2x_1 + 5x_2 \geq 10$

Optimal: $x_1 = 16/5$
$x_2 = 4/5$

POST OPTIMALITY ANALYSIS

- How sensitive our solution is to the external environmental changes
- Such as
  - Change in market selling price per ton of ore/coal
  - Change in production cost of material
  - Change in size and number of equipment
- How are these changes going to affect our OPTIMAL SOLUTION


Sensitivity Analysis
Why Sensitivity Analysis ?
Model
Analysis
Results
Symbolic World
Real World
Abstraction
Managerial Judgement
Interpretation
Sensitivity Analysis
• Understand tradeoffs
• Make quick adjustments
• Build intuition
Management Situation
Intuition
Decisions

The Role of Sensitivity Analysis of the Optimal Solution

- Is the optimal solution sensitive to changes in input parameters?
- Possible reasons for asking this question:
  - Parameter values used were only best estimates.
  - Dynamic environment may cause changes.
  - "What-if" analysis may provide economical and operational information.

Sensitivity Analysis

- Sensitivity analysis is used to determine effects on the optimal solution within specified ranges for the objective function coefficients, constraint coefficients, and right hand side values.
- Sensitivity analysis provides answers to certain what-if questions.

Sensitivity Analysis

- How will a change in a coefficient of the objective function affect the optimal solution?
- How will a change in the right-hand side value for a constraint affect the optimal solution?

Linear Programming – Sensitivity Analysis

How much can the objective function coefficients change before the values of the variables change?

How much can the right hand side of the constraints change you obtain a different basic solution?

How much value is added/reduced to the objective function if I have a larger/smaller quantity of a scarce resource?

TERMS

Sensitivity Analysis

SENSITIVITY ANALYSIS INFORMATION PROVIDED BY LINEAR PROGRAMMING	
Key Term	Definition
Reduced Cost	How much the objective function coefficient of a decision variable must improve (increase for maximization or decrease for minimization) before the optimal solution changes and the decision variable "enters" the solution with some positive number
Shadow price	The marginal improvement in *Z* (increase for maximization and decrease for minimization) caused by relaxing the constraint by one unit
Range of optimality	The interval (lower and upper bounds) of an objective function coefficient over which the optimal values of the decision variables remain unchanged
Range of feasibility	The interval (lower and upper bounds) over which the right-hand-side parameter can vary while its shadow price remains valid

Range of Optimality

- A range of optimality of an objective function coefficient is found by determining an interval for the objective function coefficient in which the original optimal solution remains optimal while keeping all other data of the problem constant. The value of the objective function may change in this range.
- Graphically, the limits of a range of optimality are found by changing the slope of the objective function line within the limits of the slopes of the binding constraint lines. (This would also apply to simultaneous changes in the objective coefficients.)
- The slope of an objective function line, **Max $c_1x_1 + c_2x_2$, is $-c_1/c_2$, and the slope of a constraint, $a_1x_1 + a_2x_2 = b$, is $-a_1/a_2$.**

Example: Sensitivity Analysis

- Solve graphically for the optimal solution:

$$\text{Max} \quad z = 5x_1 + 7x_2$$

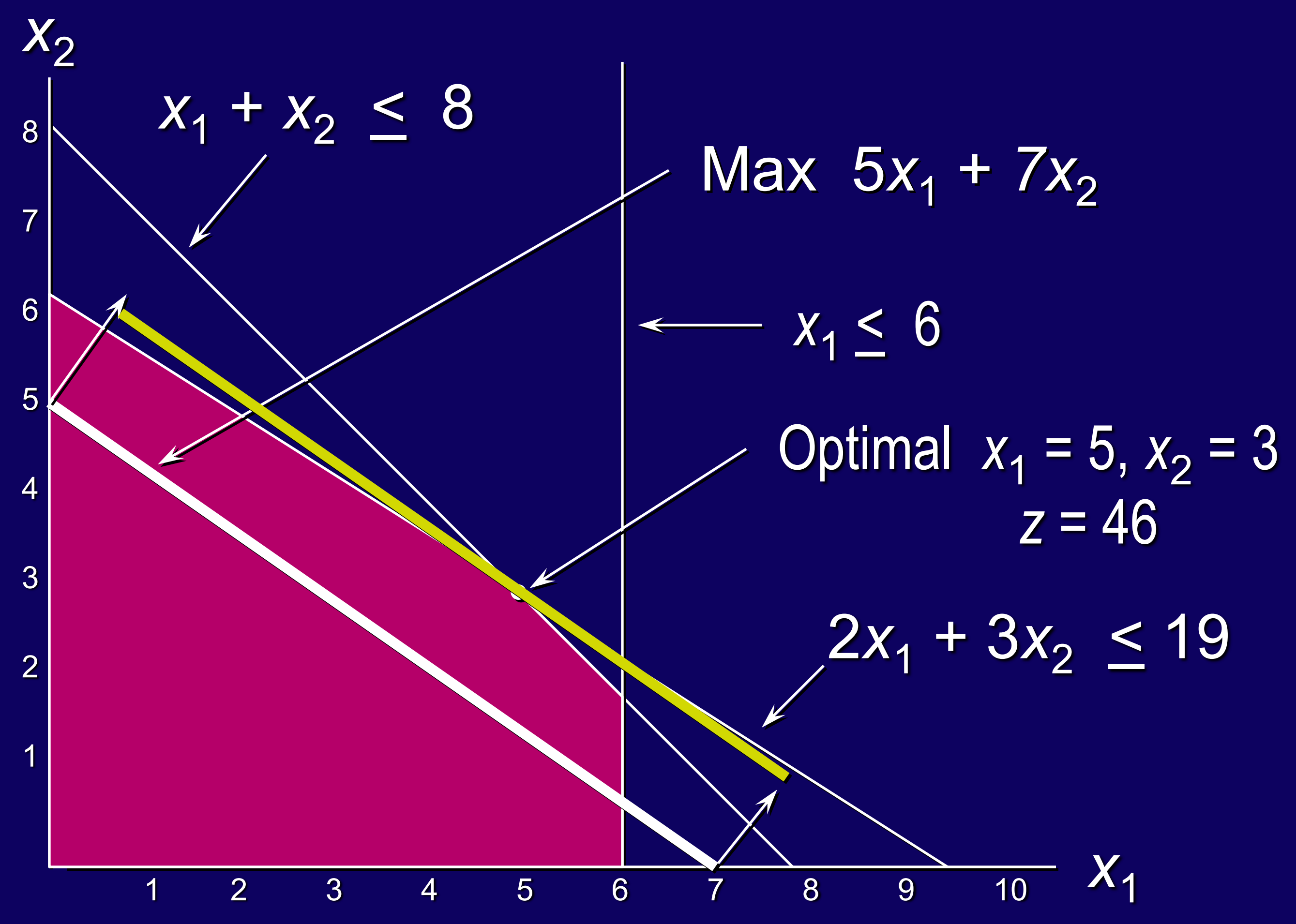
$$\begin{aligned} \text{s.t.} \quad x_1 &\leq 6 \\ 2x_1 + 3x_2 &\leq 19 \\ x_1 + x_2 &\leq 8 \end{aligned}$$

$$x_1, x_2 \geq 0$$

The slope of an objective function line, **Max $c_1x_1 + c_2x_2$, is $-c_1/c_2$, and the slope of a constraint, $a_1x_1 + a_2x_2 = b$, is $-a_1/a_2$.**

Example: Sensitivity Analysis

- Graphical Solution



Example: Sensitivity Analysis

- Range of Optimality for c_1

The slope of the objective function line is $-c_1/c_2$. The slope of the first binding constraint, $x_1 + x_2 = 8$, is -1 and the slope of the second binding constraint, $x_1 + 3x_2 = 19$, is -2/3.

Find the range of values for c_1 (with c_2 staying 7) such that the objective function line slope lies between that of the two binding constraints:

$$-1 < -c_1/7 < -2/3$$

Multiplying through by -7 (and reversing the inequalities):

$$\boxed{14/3 \leq c_1 \leq 7}$$

Example: Sensitivity Analysis

- Range of Optimality for c_2

  Find the range of values for c_2 (with c_1 staying 5) such that the objective function line slope lies between that of the two binding constraints:

$$-1 \leq -5/c_2 \leq -2/3$$

Multiplying by -1: $1 \geq 5/c_2 \geq 2/3$

Inverting, $1 \leq c_2/5 \leq 3/2$

Multiplying by 5: $\boxed{5 \leq c_2 \leq 15/2}$

Example 1

- Range of Optimality for c_1 and c_2

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B8	X1	5.0	0.0	5	2	0.333333333
C8	X2	3.0	0.0	7	0.5	2

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B13	#1	5	0	6	1E+30	1
B14	#2	19	2	19	5	1
B15	#3	8	1	8	0.333333333	1.666666667

RANGE OF OPTIMALITY

Supplies the range of values that will allow the current solution to continue to be optimal

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B13	Solution: Economy	80	0	70	5	22.5
C13	Solution: Standard	120	0	95	45	3.333333333
D13	Solution: Deluxe	0	-10	135	10	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
A18	Constraints	200	45	200	80	40
A19	Constraints	320	25	320	80	120
A20	Constraints	2080	0	2400	1E+30	320

RANGE OF OPTIMALITY

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B13	Solution: Economy	160	0	70	65	10
C13	Solution: Standard	0	-6.666666667	85	6.666666667	1E+30
D13	Solution: Deluxe	40	0	135	145	20

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
A18	Constraints	200	48.33333333	200	93.33333333	120
A19	Constraints	320	21.66666667	320	280	120
A20	Constraints	1840	0	2400	1E+30	560

TERMS

Sensitivity Analysis

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Shadow price	The marginal improvement in *Z* (increase for maximization and decrease for minimization) caused by relaxing the constraint by one unit
Range of optimality	The interval (lower and upper bounds) of an objective function coefficient over which the optimal values of the decision variables remain unchanged
Range of feasibility	The interval (lower and upper bounds) over which the right-hand-side parameter can vary while its shadow price remains valid

Example: Sensitivity Analysis

- Shadow Prices

  Constraint 1: Since $x_1 \leq 6$ is not a binding constraint, its shadow price is 0.

  Constraint 2: Change the RHS value of the second constraint to 20 and resolve for the optimal point determined by the last two constraints:

  $$2x_1 + 3x_2 = 20 \quad \text{and} \quad x_1 + x_2 = 8.$$

  The solution is $x_1 = 4$, $x_2 = 4$, $z = 48$. Hence, the shadow price $= z_{new} - z_{old} = 48 - 46 = 2$.

Example: Sensitivity Analysis

- Shadow Prices (continued)

Constraint 3: Change the RHS value of the third constraint to 9 and resolve for the optimal point determined by the last two constraints:

$$2x_1 + 3x_2 = 19 \quad \text{and} \quad x_1 + x_2 = 9.$$

The solution is: $x_1 = 8$, $x_2 = 1$, $z = 47$. Hence, the shadow price is $z_{new} - z_{old} = 47 - 46 = 1$.

Example 1

■ Dual Prices

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B8	X1	5.0	0.0	5	2	0.33333333
C8	X2	3.0	0.0	7	0.5	2

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B13	#1	5	0	6	1E+30	1
B14	#2	19	2	19	5	1
B15	#3	8	1	8	0.33333333	1.66666667

Dual Value

DUAL VALUE

- Graphically, a dual value is determined by adding one to the right hand side value and then resolving for the optimal solution in terms of the same two binding constraints.
- The dual value is equal to the difference in the value of the objective functions between the new and original problems.
- The dual value for a nonbinding constraint is **0**.
- A **negative** dual value indicates that the objective function will not improve if the right hand side is increased.

Shadow Price	Constraint R.H. Side
31	200
32	320
0	2400

- • The dual value is equal to the difference in the value of the objective functions between the new and original problems.
- • The dual value for a nonbinding constraint is 0.
- • A negative dual value indicates that the objective function will not improve if the right hand side is increased.

RANGE OF FEASIBILITY

The range over which the dual value is applicable

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B13	Solution: Economy	160	0	63	12	15.5
C13	Solution: Standard	80	0	95	31	8
D13	Solution: Deluxe	0	-24	135	24	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
A18	Constraints	240	31	240	40	80
A19	Constraints	320	32	320	40	80
A20	Constraints	2240	0	2400	1E+30	160

REDUCED COST

- The reduced cost associated with a variable is equal to the dual value of the non-negativity constrain associated with the variable.
- In general, if a variable has a non-zero value in the optimal solution, then it will have a reduced cost equal to 0.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B13	Solution: Economy	284.4444444	0	63	14.14285714	4.5
C13	Solution: Standard	0	-5	95	5	1E+30
D13	Solution: Deluxe	8.888888889	0	135	117	22.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
A18	Constraints	293.3333333	0	300	1E+30	6.666666667
A19	Constraints	320	11	320	365.7142857	20
A20	Constraints	2400	6.5	2400	40	1280

Example 1

- Refer to the "Woodworking" example of Chapter 2, where X1 = Tables and X2= Chairs. The problem is shown below.

Max. Z= $\$100X1+60X2$

s.t. $12X1+4X2 \leq 60$ (Assembly time in hours)

$4X1+8X2 \leq 40$ (Painting time in hours)

The optimum solution was X1=4, X2=3, and Z=$580.
Answer the following questions regarding this problem.

Answer the following Questions:

1. Compute the range of optimality for the contribution of X1 (Tables)

2. Compute the range of optimality for the contribution of X2 (Chairs)

3. Determine the dual Price (Shadow Price) for the assembly stage.

4. Determine the dual Price (Shadow Price) for the painting stage.

LIMITATIONS OF CLASSICAL SENSITIVITY ANALYSIS

- • Simultaneous Changes - The range analysis for objective function coefficients and the constraint right-hand sides is only applicable for changes in a single coefficient.
- • Changes in Constraint Coefficients – Classical sensitivity analysis provides no information about changes resulting from a change in a coefficient of a variable in a constraint.

2. Connellsville Coal Company (15 points)

Suppose that CCC Inc. (Connellsville Coal Company) sells three types of coal: Type A, Type B, and Type C. Type A coal comes directly from one of its mines and so does Type C. The weekly mining capacity for Type A coal is 1000 tons and for Type C it is 1500 tons. Type B, however, is not mined. Rather it is produced by simply blending Type A and Type C coal.

Type A and Type C coal have the following characteristics:

Type	Coal dust (in lbs per ton)	Steam produced (in lbs per ton)	Mining Cost ($ per ton)
A	3	24,000	$10
C	7	36,000	$15

The blended Type B coal is guaranteed to have no more than 4 lbs of coal dust per ton and no less than 25,000 lbs of steam produced per ton. The company can sell as much of Type A, B and C coal that it can produce. Excluding the cost of coal, based on current prices the contributions to profit for next week are $50 per ton for Type A, $40 per ton for Type B, and $35 per ton for Type C.

The company has a commitment to sell at least 500 tons of Type B coal. Also, because of market constraints it can sell no more than 1000 tons of Type A.

(a) Formulate the problem as an algebraic linear optimization model. Be sure to define your variables. (Handwrite answer. 5 points)

(b) Construct an EXCEL model for this problem.
(Hand in your formula sheet. 5 points)

(c) Optimize your EXCEL model using Solver and give the Answer and Sensitivity Reports. From the Answer Report, what is the optimal solution and the optimal value of the objective? (Handwrite answer. 5 points)

4 Chemco produces three products: 1, 2, and 3. Each pound of raw material costs \$25. It undergoes processing and yields 3 oz of product 1 and 1 oz of product 2. It costs \$1 and takes 2 hours of labor to process each pound of raw material. Each ounce of product 1 can be used in one of three ways.

It can be sold for \$10/oz.

It can be processed into 1 oz of product 2. This requires 2 hours of labor and costs \$1.

It can be processed into 1 oz of product 3. This requires 3 hours of labor and costs \$2.

Each ounce of product 2 can be used in one of two ways.

It can be sold for \$20/oz.

It can be processed into 1 oz of product 3. This requires 1 hour of labor and costs \$6.

Product 3 is sold for \$30/oz. The maximum number of ounces of each product that can be sold is given in Table 23. A maximum of 25,000 hours of labor are available. Determine how Chemco can maximize profit.

TABLE 23

Product	Oz
1	5,000
2	5,000
3	3,000

(From MIT OpenCourseWare) An operations manager is trying to determine a production plan for the next week. There are three products (say, P, Q, and Q) to produce using four machines (say, A and B, C, and D). Each of the four machines performs a unique process. There is one machine of each type, and each machine is available for 2400 minutes per week. The unit processing times for each machine is given in following Table 1:

Table 1: Machine Data

	Unit Processing Time (min)			
Machine	Product P	Product Q	Product R	Availability(min)
A	20	10	10	2400
B	12	28	16	2400
C	15	6	16	2400
D	10	15	0	2400
Total processing time	57	59	42	9600

The unit revenues and maximum sales for the week are indicated in Table 2. Storage from one week to the next is not permitted. The operating expenses associated with the plant are $6000 per week, regardless of how many components and products are made. The $6000 includes all expenses except for material costs.

Table 2: Product Data

Item(per unit)	Product P	Product Q	Product R
Revenue	$90	$100	$70
Material cost	$45	$40	$20
Profit	$45	$60	$50
Maximum sales	100	40	60

Find the "optimal" product mix-- that is, the amount of each product that should be manufactured during the present week in order to maximize profits.

Formulate a linear programming (LP) model for this problem (1 pts).

Example: Infeasible Problem

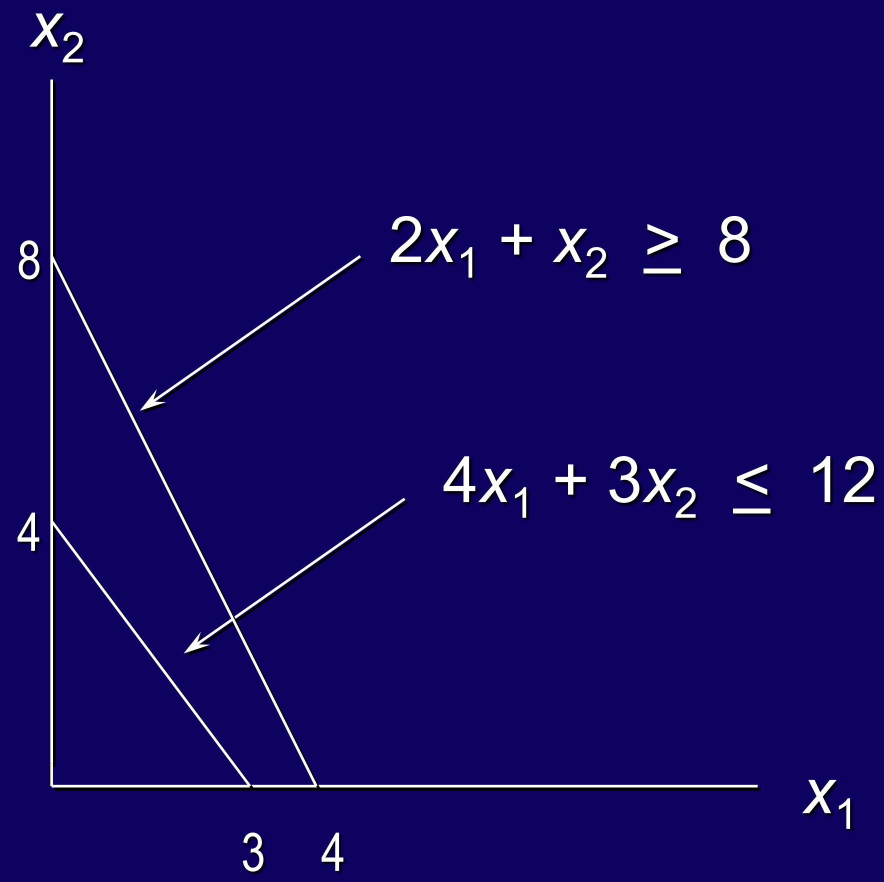
- Solve graphically for the optimal solution:

$$\text{Max} \quad z = 2x_1 + 6x_2$$

$$\begin{aligned} \text{s.t.} \quad & 4x_1 + 3x_2 \leq 12 \\ & 2x_1 + x_2 \geq 8 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Example: Infeasible Problem

- There are no points that satisfy both constraints, hence this problem has no feasible region, and no optimal solution.



Example: Unbounded Problem

- Solve graphically for the optimal solution:

$$\text{Max} \quad z = 3x_1 + 4x_2$$

$$\begin{aligned} \text{s.t.} \qquad x_1 + x_2 &\geq 5 \\ 3x_1 + x_2 &\geq 8 \end{aligned}$$

$$x_1, x_2 \geq 0$$

Example: Unbounded Problem

- The feasible region is unbounded and the objective function line can be moved parallel to itself without bound so that z can be increased infinitely.