Examination: VII Sem. B.Tech (Mining Engineering) **Session: 2025-26**

Semester: Monsoon Semester **Max. Marks: 30**

Subject: Mine Systems Engineering MND 404 **Time: 1 Hrs.**

Q1. LINEAR PROGRAMMING GRAPHICAL **(10 points)**

A contractor needs to hire earth-moving equipment and has the option of choosing from two types of machines.

- **Type A machine** costs **£25 per day** to hire, requires **1 man** to operate, and can move **30 tonnes of earth per day**.
- **Type B machine** costs **£10 per day** to hire, requires **4 men** to operate, and can move **70 tonnes of earth per day**.

The contractor faces the following limits:

1. The total daily hiring cost cannot exceed **£500**.
2. The contractor has only **64 workers** available to operate the machines.
3. A maximum of **25 machines** can be used on the site.

The task is to determine: **What is the maximum weight of earth that the contractor can move in one day, and how many machines of each type should be hired?**

Q2. LINEAR PROGRAMMING DUALITY **(10 points)**

$$
\begin{array}{lll}
\text{Maximize} & 2X_1 - 13X_2 - 3X_3 - 2X_4 - 5X_5 & \\
\text{subject to} & X_1 - X_2 - 4X_4 - X_5 & = 5 \\
 & X_1 - 7X_4 - 2X_5 & \geq -1 \\
 & 5X_2 + X_3 + X_4 + 2X_5 & \leq 5 \\
 & 3X_2 + X_3 - X_4 + X_5 & \geq 2 \\
 & \multicolumn{2}{l}{X_j \geq 0 \text{ for all } j=1, 2, 3;\ X_4 \leq 0;\ X_5 \text{ unrestricted in sign}}
\end{array}
$$

a. Is this solution feasible at the primal point X (6,0,1,0,1) ? Why
b. A basic feasible solution (BFS) is a solution to a linear programming problem that satisfies all constraints and has at most as many non-zero variables as there are constraints (often denoted as 'm'). Is this solution basic? Yes or No , Explain why
c. Write out in full a dual problem of the LP above, denoting your dual variables by Y1, Y2, etc.
d. IF X=(6,0,1,0,1) is optimal in the primal problem, then which dual variables (including slack or surplus variables) must be zero in the dual optimal solution, according to the complementary slackness conditions for this primal-dual pair of problems?

Examination: VII Sem. B.Tech (Mining Engineering) **Session: 2025-26**

Semester: Monsoon Semester **Max. Marks: 30**

Subject: Mine Systems Engineering MND 404 **Time: 1 Hrs.**

Q3. Transportation Problem: **(10 points)**

destinations

sources	1	2	3	supply
1	5 $\boxed{9}$	$\boxed{5}$	$\boxed{4}$	5
2	$\boxed{7}$	$\boxed{6}$	$\boxed{12}$	18
3	$\boxed{10}$	$\boxed{9}$	$\boxed{3}$	7
demand	10	5	15	

a. If the ordinary simplex tableau were to be written for this problem, how many rows (including the objective) will it have? __

b. How many variables (excluding the objective value -z) will it have? _

c. Is this transportation problem "balanced?" __

d. How many basic variables will this problem have?

e. An initial basic feasible solution is found using the "Northwest Corner Method"; complete the computation of this solution and write the values of the variables in the tableau.

Examination: VII Sem. B.Tech (Mining Engineering) **Session: 2025-26**

Semester: Monsoon Semester **Max. Marks: 30**

Subject: Mine Systems Engineering MND 404 **Time: 1 Hrs.**

Answer 1

Define

x = number of Type A machines

y = number of Type B machines

Objective (maximize tonnes moved per day):

Maximize $Z = 30x + 70y$

Subject to (from problem statement):

1. Cost: $25x + 10y \leq 500$
2. Labour: $x + 4y \leq 64$
3. Machine cap: $x + y \leq 25$
4. $x, y \geq 0$

Solve by examining the feasible corner points (intersections of the active constraint lines and axes). The feasible corner points are:

- $(0,0) \rightarrow Z = 0$
- $(20,0)$ (from cost & axes) $\rightarrow Z = 30\cdot20 + 70\cdot0 = 600$
- $(0,16)$ (from labour & axes) $\rightarrow Z = 30\cdot0 + 70\cdot16 = 1120$
- $(16\frac{2}{3}, 8\frac{1}{3})$ (intersection of cost & machine-cap) $\rightarrow Z = 30\cdot(50/3) + 70\cdot(25/3) = 1083.33...$
- $(12,13)$ (intersection of labour and machine-cap) $\rightarrow Z = 30\cdot12 + 70\cdot13 = 1270$

Maximum value occurs at $x = 12$, $y = 13$.

Answer: the contractor can move **1270 tonnes of earth** in one day by using **12 Type A** machines and **13 Type B** machines.

Answer 2:

Examination: VII Sem. B.Tech (Mining Engineering) **Session: 2025-26**

Semester: Monsoon Semester **Max. Marks: 30**

Subject: Mine Systems Engineering MND 404 **Time: 1 Hrs.**

Let's check the feasibility of X = (6, 0, 1, 0, 1) with the given constraints.

The first constraint is satisfied: 6 - 0 - 0 - 1 = 5.

The second one is satisfied: 6 - 0 - 2 = 4, which is greater than -1.

The third is satisfied too: 0 + 1 + 0 + 2 = 3, which is less than or equal to 5.

Now for basicness—We have 4 constraints, and we need the number of basic variables to be 4. There are nonzero variables (X1, X3, and X5), but we need to check if the solution involves 4 basic variables, even if some might be zero.To determine if the solution is basic, we need to solve the system of equalities and check if the columns of non-zero variables form an independent set. The solution has 3 non-zero variables, which is less than 4, but we need to verify if the basic variables can be independent and fulfill the constraints. One way is to check the rank of the system.We need to convert the constraints to standard form. For the second and fourth constraints, I'll introduce surplus variables (s2 and s4) to handle the inequalities. For the third, I'll add a slack variable (s3). By introducing these variables, we convert the system into one with 4 potential basic variables.

Primal (given)

Maximize

$$Z = 2X_1 - 13X_2 - 3X_3 - 2X_4 - 5X_5$$

subject to

$$\begin{aligned}
&(1) \quad X_1 - X_2 - 4X_4 - X_5 = 5,\\
&(2) \quad X_1 - 7X_4 - 2X_5 \geq -1,\\
&(3) \quad 5X_2 + X_3 + X_4 + 2X_5 \leq 5,\\
&(4) \quad 3X_2 + X_3 - X_4 + X_5 \geq 2,
\end{aligned}$$

with sign restrictions: $X_1, X_2, X_3 \geq 0,\ X_4 \leq 0,\ X_5$ unrestricted.

We test the primal point $X = (6, 0, 1, 0, 1)$ i.e. $X_1 = 6, X_2 = 0, X_3 = 1, X_4 = 0, X_5 = 1$.

Examination: VII Sem. B.Tech (Mining Engineering) **Session: 2025-26**

Semester: Monsoon Semester **Max. Marks: 30**

Subject: Mine Systems Engineering MND 404 **Time: 1 Hrs.**

(a) Is $X = (6, 0, 1, 0, 1)$ feasible? — Yes.

Check each constraint:

1. $X_1 - X_2 - 4X_4 - X_5 = 6 - 0 - 0 - 1 = 5 \ \Rightarrow$ equals 5 ✓
2. $X_1 - 7X_4 - 2X_5 = 6 - 0 - 2 = 4 \ \Rightarrow \ 4 \geq -1$ ✓
3. $5X_2 + X_3 + X_4 + 2X_5 = 0 + 1 + 0 + 2 = 3 \ \Rightarrow \ 3 \leq 5$ ✓
4. $3X_2 + X_3 - X_4 + X_5 = 0 + 1 - 0 + 1 = 2 \ \Rightarrow \ 2 \geq 2$ ✓

Sign conditions: $X_1, X_2, X_3 \geq 0$ (6,0,1 OK), $X_4 = 0 \leq 0$ OK, $X_5 = 1$ allowed (unrestricted).
Hence the point satisfies all constraints and is feasible.

(b) Is the solution basic (a BFS)? — No.

Reasoning (clear and direct):

- The LP has $m = 4$ constraints. In standard form a basic feasible solution corresponds to choosing **exactly** m basic variables (the remaining variables set to zero) that give a unique solution of the m equalities.
- To check, convert inequalities to equalities by adding slack/surplus variables (one for each inequality):

  For convenience define surplus/slack variables so the constraints become equalities:

$$
\begin{aligned}
&(1)\ X_1 - X_2 - 4X_4 - X_5 \ = \ 5 \quad (\text{equality already})\\
&(2)\ X_1 - 7X_4 - 2X_5 \ - \ s_2 \ = \ -1 \quad (s_2 \geq 0 \text{ is surplus})\\
&(3)\ 5X_2 + X_3 + X_4 + 2X_5 \ + \ s_3 \ = \ 5 \quad (s_3 \geq 0 \text{ slack})\\
&(4)\ 3X_2 + X_3 - X_4 + X_5 \ - \ s_4 \ = \ 2 \quad (s_4 \geq 0 \text{ surplus})
\end{aligned}
$$

- Evaluate those slack/surplus values at the point $X = (6, 0, 1, 0, 1)$:
  - From (2): $6 - 0 - 2 - s_2 = -1 \Rightarrow s_2 = 5$ (positive)
  - From (3): $0 + 1 + 0 + 2 + s_3 = 5 \Rightarrow s_3 = 2$ (positive)
  - From (4): $0 + 1 - 0 + 1 - s_4 = 2 \Rightarrow s_4 = 0$ (zero)
  - Constraint (1) has no slack (it is equality).

- So the variables that are nonzero at this solution are: X1=6, X3=1, X5=1, s2=5, s3=2X_1=6,\;X_3=1,\;X_5=1,\;s_2=5,\;s_3=2X1=6,X3=1,X5=1,s2=5,s3=2 — that is **five** positive variables (≥0). But a BFS in standard form for 4 constraints should have at most (and typically exactly) 4 basic variables. Because more than mmm variables are positive here, this point cannot correspond to a basis of size mmm in the standard-form system.

Therefore the given feasible point is **not** a basic feasible solution.

So the full dual is:

$$\text{Minimize } W = 5Y_1 - Y_2 + 5Y_3 + 2Y_4$$

$$\text{subject to } \begin{cases} Y_1 + Y_2 \ \geq \ 2, \\ -Y_1 + 5Y_3 + 3Y_4 \ \geq \ -13, \\ Y_3 + Y_4 \ \geq \ -3, \\ -4Y_1 - 7Y_2 + Y_3 - Y_4 \ \leq \ -2, \\ -Y_1 - 2Y_2 + 2Y_3 + Y_4 \ = \ -5, \end{cases}$$

$$Y_1 \text{ free}, \ Y_2 \leq 0, \ Y_3 \geq 0, \ Y_4 \leq 0.$$

Now form one dual constraint per primal variable X_j. Use columns of the primal constraint matrix and primal objective coefficients c_j. The rules:

- if $X_j \geq 0$ then dual constraint is: $(A^T y)_j \geq c_j$.
- if $X_j \leq 0$ then $(A^T y)_j \leq c_j$.
- if X_j free then $(A^T y)_j = c_j$.

(d) If $X = (6, 0, 1, 0, 1)$ is optimal in the primal, which dual variables must be zero? (Complementary slackness)

Use the complementary slackness (CS) facts in simple form:

- **For each primal constraint** i: if that primal constraint is **not tight** (i.e. strict inequality), then the corresponding dual variable Y_i **must be zero**. If the primal constraint is tight (equality) then Y_i may be nonzero.
- **For each primal variable** X_j: if $X_j > 0$ then the corresponding dual constraint must hold with **equality**; if $X_j = 0$ then that dual constraint may be slack.

We already evaluated the primal constraint values at the point:

1. Constraint (1): left = 5 → equality (tight). ⇒ Y_1 may be nonzero.
2. Constraint (2): left = 4 while RHS = −1 → **strictly greater** (not tight). ⇒ Y_2 **must be 0**.
3. Constraint (3): left = 3 while RHS = 5 → **strictly less** (not tight). ⇒ Y_3 **must be 0**.
4. Constraint (4): left = 2 → equality (tight). ⇒ Y_4 may be nonzero.

So by complementary slackness the dual variables that **must be zero** are:

$$\boxed{Y_2 = 0 \quad \text{and} \quad Y_3 = 0.}$$

We can go a step further: since primal $X_1 > 0, X_3 > 0, X_5 > 0$, their corresponding dual constraints must be equalities. Substitute $Y_2 = 0, Y_3 = 0$ into those dual constraints:

- From $X_1 > 0$: $Y_1 + Y_2 = 2 \Rightarrow Y_1 + 0 = 2 \Rightarrow Y_1 = 2.$
- From $X_3 > 0$: $Y_3 + Y_4 = -3 \Rightarrow 0 + Y_4 = -3 \Rightarrow Y_4 = -3.$
- The X_5 dual equation (equality by nature of X_5 being free) becomes: $-Y_1 - 2Y_2 + 2Y_3 + Y_4 = -5.$ Substituting $Y_1 = 2, Y_2 = 0, Y_3 = 0$ gives $-2 + Y_4 = -5 \Rightarrow Y_4 = -3$, consistent.

Thus one consistent dual optimal candidate (satisfying CS) is:

$$(Y_1, Y_2, Y_3, Y_4) = (2,\ 0,\ 0,\ -3),$$

which indeed satisfies the equality for the free-variable dual row and the sign restrictions ($Y_2 \leq 0$ and $Y_4 \leq 0$, $Y_3 \geq 0$).

Examination: VII Sem. B.Tech (Mining Engineering) **Session: 2025-26**

Semester: Monsoon Semester **Max. Marks: 30**

Subject: Mine Systems Engineering MND 404 **Time: 1 Hrs.**

Answer3 :

a. **Number of rows in simplex tableau (including the objective)**

- Transportation problem is a special form of LP.
- General simplex tableau has **m + n constraints + 1 (objective)**.
  Here:
- m=3m = 3m=3 (sources)
- n=3n = 3n=3 (destinations)
- Total constraints = m+n=6m + n = 6m+n=6.
- Add 1 for the objective row.

Answer: 7 rows

b. Number of variables (excluding the objective value z)

Each source–destination allocation is a variable.

- m×n=3×3=9m \times n = 3 \times 3 = 9m×n=3×3=9.

Answer: 9 variables

---

c. Balanced or not?

Check total supply vs. demand.

- Supply = 5+18+7=305 + 18 + 7 = 305+18+7=30
- Demand = 10+5+15=3010 + 5 + 15 = 3010+5+15=30
  Since they match, the problem is balanced.

Answer: Yes, balanced

Examination: VII Sem. B.Tech (Mining Engineering) **Session: 2025-26**

Semester: Monsoon Semester **Max. Marks: 30**

Subject: Mine Systems Engineering MND 404 **Time: 1 Hrs.**

d. Number of basic variables

In a balanced transportation problem, the number of **basic variables = m + n – 1**.

- m+n−1=3+3−1=5m + n - 1 = 3 + 3 - 1 = 5m+n−1=3+3−1=5.

Answer: 5 basic variables

Sources \ Destinations	**D1**	**D2**	**D3**	**Supply**
S1 (5)	5	0	0	5
S2 (18)	5	5	8	18
S3 (7)	0	0	7	7
Demand	10	5	15	30

Initial BFS allocations: X11=5, X21=5, X22=5, X23=8, X33=7

Add them up: 25+35+30+96+21=207

Total transportation cost (initial NW solution) = 207.