Subject: Mine System Engineering 22/08/2025

Maximize $z = 3x_1 + 2x_2 + 5x_3$

subject to

$$x_1 + 2x_2 + x_3 \leq 430$$

$$3x_1 + 0 + 2x_3 \leq 460$$

$$x_1 + 4x_2 + 0 \leq 420$$

$$x_1, x_2, x_3 \geq 0$$

Basic	x_1	x_2	x_3	x_4	x_5	x_6	b	
x_4	1	2	1	1	0	0	430	$430/1 = 430$
x_5	3	0	2	0	1	0	460	$460/2 = 230$
x_6	1	4	0	0	0	1	420	
z	-3	-2	-5	0	0	0	0	

Departing Variable

entering variable

Pivot = 2

Basic	x_1	x_2	x_3	x_4	x_5	x_6	b	
x_3	-1/2	2	0	1	-1/2	0	200	$200/2 = 100$
x_4	3/2	0	1	0	1/2	0	230	
x_6	1	4	0	0	0	1	420	$420/4 = 105$
z	9/2	-2	0	0	5/2	0	1150	

Basic	x_1	x_2	x_3	x_4	x_5	x_6	b
x_2	-1/4	1	0	1/2	-1/4	0	100
x_3	3/2	0	1	0	1/2	0	230
x_6	2	0	0	-4	2	1	20
z	4	0	0	1	2	0	1350

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Now, no negative value in the lowest row. Hence, z-eq^n. is an optimal tableau.

$$z = 1350 - 4x_1 - x_4 - 2x_5 - 0\,x_6$$

1 min increase in operation 1 increases z by \$1
1 min increase in operation 2 increases z by \$2
1 min increase in operation 3 does not change z

(-ve) sign indicates decrease in value of slack variable is equivalent to an increase in its operation time.

0 dual price for operation 3 means there is no economic advantage in allocating more production time to it.

Determining Feasible Ranges:

Maximize $z = 3x_1 + 2x_2 + 5x_3$

subject to

$$x_1 + 2x_2 + x_3 \leq 430 + d_1$$
$$3x_1 + 2x_3 \leq 460 + d_2$$
$$x_1 + 4x_2 \leq 420 + d_3$$
$$x_1, x_2, x_3 \geq 0$$

Basic	x_1	x_2	x_3	x_4	x_5	x_6	b	d_1	d_2	d_3
x_4	1	2	1	1	0	0	430	1	0	0
x_5	3	0	2	0	1	0	460	0	1	0
x_6	1	4	0	0	0	1	420	0	0	1
z	-3	-2	-5	0	0	0	0	0	0	0

Optimal tableau:

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Basic	x_1	x_2	x_3	x_4	x_5	x_6	b	d_1	d_2	d_3
x_2	-1/4	1	0	1/2	-1/4	0	100	1/2	-1/4	0
x_3	3/2	0	1	0	1/2	0	230	0	1/2	0
x_6	2	0	0	-2	1	1	20	-2	1	1
z	4	0	0	1	2	0	1350	1	2	0

optimal solution :

$$z = 1350 + D_1 + 2D_2$$
$$x_2 = 100 + \tfrac{1}{2} D_1 - \tfrac{1}{4} D_2$$
$$x_3 = 230 + \tfrac{1}{2} D_2$$
$$x_6 = 20 - 2D_1 + D_2 + D_3$$
$$x_2, x_3, x_6 \geq 0$$

Case-I : change in operation 1 time from 460 to 460 + ~ min.

$$D_2 = D_3 = 0$$

$x_2 = 100 + \tfrac{1}{2} D_1 \geq 0$ $\quad\quad$ $D_1 \geq -200$

$x_3 = 230 \geq 0$ $\quad\quad$ $-200 \leq D_1 \leq 10$

$x_6 = 20 - D_1 \geq 0$ $\quad\quad$ $D_1 \leq 10$

Case-II : change in operation 2 time from 430 to 430 + ~ minutes.

$$D_1 = D_3 = 0$$

$x_2 = 100 - \tfrac{1}{4} D_2 \geq 0$ $\quad\quad$ $D_2 \leq 400$

$x_3 = 230 + \tfrac{1}{2} D_2 \geq 0$ $\quad\quad$ $D_2 \geq -460$

$-20 \leq D_2 \leq 400$

$x_6 = 20 + D_2 \geq 0$ $\quad\quad$ $D_2 \geq -20$

Case-III : change in operation 3 time from 420 to 420+~ minutes $D_1 = D_2 = 0$

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$x_2 = 100 \geq 0$ $\qquad -20 \leq D_3 < \infty$

$x_3 = 230 \geq 0$

$x_6 = 20 + D_3 \geq 0$ $\qquad D_3 \geq -20$

TOYCO model resource allocation table.

Resource	Dual Price	Feasible Range	Resource amount (min)		
			Min.	Current	Max.
Operation 1	1	$-200 \leq D_1 \leq 10$	230	430	440
Operation 2	2	$-20 \leq D_2 \leq 400$	440	460	860
Operation 3	0	$-20 \leq D_3 < \infty$	400	420	∞

Reduce cost per unit = cost consumed resource per unit − revenue per unit

Objective function

$$\text{Maximize } z = (3+d_1)x_1 + (2+d_2)x_2 + (5+d_3)x_3$$

$$x_1 = 4(1) - \frac{1}{4}(d_2) + \frac{3}{2}(d_3) - d_1$$

Optimality conditions

Non-basic variables $x_1, x_4, x_5 \geq 0$

$$x_1 = 4 - \tfrac{1}{4}d_2 + \tfrac{3}{2}d_3 - d_1$$

$$x_4 = 1 + \tfrac{1}{2}d_2$$

$$x_5 = 2 - \tfrac{1}{4}d_2 + \tfrac{1}{2}d_3$$

Case I : $d_2 = d_3 = 0$

$x_1 = 4 - d_1 \geq 0$ $\qquad d_1 \leq 4$

$\infty > d_1 \geq 4$

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Case - II :

$d_1 = d_3 = 0$

$x_1 = 4 - 1/4 d_2 \geq 0 \qquad d_2 \leq 16$

$x_4 = 1 + 1/2 \; 2 \geq 0 \qquad d_2 \geq -2 \qquad 8 \geq d_2 \geq -2$

$x_5 = 2 - 1/4 d_2 \geq 0 \qquad d_2 \leq 8$

Case - III :

$d_1 = d_2 = 0$

$x_1 = 4 + 3/2 \, d_3 \geq 0 \qquad d_3 \geq -8/3$

$x_4 = 1 \geq 0$

$x_5 = 2 + 1/2 \, d_3 \geq 0 \qquad d_3 \geq -4$

(a) For operation 1, dual price (D_1) = \$1/min

Given overtime basis \$50/hr

$= 50/60 = \$0.83/\text{min}$

since overtime basis of \$0.83 < dual price of \$1, it is advantageous to use overtime with operation 1.

(b) For operation 2, dual price (D_2) = \$2/min

Revenue Increment in 2hrs = \$2 × 120

= \$240

Overtime Labour work operation = \$45 × 2

= \$90

Overtime operation cost = \$10 × 2

= \$20

Net effect on daily revenue = \$240 − \$90 − \$20

= \$130

(c) Dual price for operation 3 is 0 and an unused 20 min time for operation 3 is non binding

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constant. so no overtime is needed nor is it of an economic perspective.

(d) Optimal analysis for operation 1

$D_1 = 440 - 430 = 10$ min

Shadow price → Feasibility analysis

Increase in revenue $= D_1 \times d_1$

$= 10 \times \$1 = \$10$

Overtime cost $= \$40/60 = \$6.67$

New production plan

$D_2 = \$10$

$x_2 = 100 + 1/2\,(10) = 105$

$x_3 = 230 + 1/2\,(0) = 230$

$z = 1350 + D_1 + 2D_2$

$z = 1350 + 10 = 1360$

New net Revenue $= \$1360 - \$6.67$

$= \$1353.33$

(e) $D_2 = \$2$

Revenue loss of $\$2 \times 15 = \$30$

Hourly cost $\$30/hr = \$30/60 = \$0.50$

Regular cost $= \$0.50 \times 15 = \$7.5$

whereas Revenue loss of \$30. No advantages of decreasing availability of operation 2.