SWAYAM NPTEL COURSE ON

MINE AUTOMATION AND DATA ANALYTICS

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Module 8: Probability

Lecture 18B: Introduction to Probability and its associated terms

CONCEPTS COVERED

- Independent Events
- Multiplication Theorem
- Total Probability
- Bayes Theorem
- Applications of Probability in Mining Industry

Independent Events

Two events are independent if knowledge of the happening of one event does not affect the happening of the other event.

Let A & B are independent events:

$$P(A/B) = P(A)$$
$$P(B/A) = P(B)$$

$P(A/B) = P(A \cap B) / P(B)$

$E1 \cap E2$ $P(B/A) = P(B \cap A) / P(A)$ multiplication theorem for independent events

$P(B \cap A) = P(B).P(A)$

$P(A/B) = P(A \cap B) / P(B)$

$P(A \cap B) = P(A / B) . P(B)$ multiplication theorem for dependent events

$P(B \cap A) = P(B / A) . P(A)$

Independent Events

Definition

Independent events are events where the occurrence of one event does not affect the occurrence of another.

- **Mathematically, two events, A and B, are independent if and only if**

$$\boldsymbol{P(A \cap B) = P(A) \cdot P(B)}$$

Solved Example - 1 of Independent Events

Q) One coin is tossed, and one dice is rolled; E1: appearing head on coin, E2: appearing 3 on dice; Check whether the E1 and E2 are independent events.

$P(E1 \cap E2) = P(E1).P(E2)$
$P(E1 \cap E2) = (1/2).(1/6) = 1/12$

1 coin tossed AND 1 dice rolled:
{(H,1),(H,2),(H,3),(H,4),(H,5),(H,6),(T,1),(T,2),(T,3),(T,4),(T,5),(T,6)}

E1: appearing head on coin ; E2: appearing 3 on dice
$P(E1 \cap E2) = 1/12$

$P(E1 \cap E2) = P(E1).P(E2)$
E1 and E2 are two independent random events

Solved Example - 2 of Independent Events

Person A: The probability of hitting a target = P(A) = 3/4
Person B: The probability of hitting a target = P(B) = 4/5
The probability of both A and B hitting a target $P(E1 \cap E2)$ = ?

A) since both events are independent events
$P(A \cap B) = P(B).P(A)$
$P(A \cap B) = 3/4.4/5 = 12/20 = 3/5$

Multiplication Theorem

$P(A \cap B \cap C) = P(A). P(B/A) . P(C / A \cap B)$

Where in independent events

$P(A \cap B \cap C) = P(A).P(B).P(C)$

So $P(A \cap B \cap C \cap D \cap E \ldots. \cap K) = P(A).P(B).P(C)\ldots P(K)$

Bag A contain 3R & 2G balls, Bag B contain 3R & 5G balls , Bag C contain 1R & 4G balls
If one ball is drawn from each bag, then find the probability of getting Red from Bag A, Green from Bag B, and Red from Bag C. $P(E1 \cap E2 \cap E3) = ?$

A) $P(E1 \cap E2 \cap E3) = P(E1).P(E2).P(E3)$

$P(E1 \cap E2 \cap E3) = 3/5 . 5/8 . 1/5$

$= 15/200$

$= 3/40$

Total Probability

Let A1, A2, A3 ….. be events that form a partition of the sample space s. Let B be any event, then.

$P(B) = P(B \cap A1) + P(B \cap A1) + P(B \cap A1) + \ldots\ldots$

$P(B) = P(A1) \,.\, P(B / A1) + P(A2) \,.\, P(B / A2) + P(A3)\, P(B / A3) + \ldots\ldots$

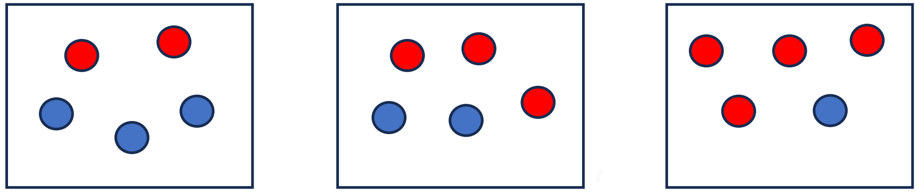
Total Probability

Q) Total Probability theorem is applicable for both dependent and independent events?
A) Yes

Q) The number of subsets of Sample Space that form a partition can be finite or infinite.
A) Yes

Example 1 of Total Probability

There are three boxes. A ball is picked @ random from a box. What is the probability that it is red? Assuming there is an equal likelihood of selecting a box.



$P(B) = P(A1) \,.\, P(B / A1) + P(A2) \,.\, P(B / A2) + P(A3)\, P(B / A3) + \ldots..$

$P(\text{red}) = P(\text{Box A1}).\, P(\text{ red} / \text{Box A1}) + P(\text{Box A2}) \,.\, P(\text{ red} / \text{Box A2}) + P(\text{Box A3})\, P(\text{ red} / \text{Box A3})$

$P(\text{red}) = (1/3)(2/5) \;+\; (1/3)(3/5) \;+\; (1/3)(4/5) \;=\; 9/15$

Example 2 of Total Probability

A box containing 5 fair coins and 3 unfair coins (P(H) = 1/3 ; P(T) = 2/3). A coin is picked and tossed. Find the probability of getting a head from a picked coin.

$$P(H) = P(F)(PH/F) + P(UF)(H/UF)$$
$$P(H) = (5/8)(1/2) + (3/8)(1/3)$$

Example 3 of Total Probability

Bayes Theorem

P(A/B) = P(A ∩ B) / P(B) = P(A).P(B / A) / P(B)

P(A/B) = P(A).P(B / A) / P(B)

Example of Bayes Theorem

Suppose you are reaching home in three ways: 1) Bus, 2) Car, 3)Scooty
P(late/Bus) = 0.5 , P(Late/Car) = 0.1 , P(Late/Scooty) = 0.2
P(Bus) = 0.2 , P(Car) = 0.7 , P(Scooty) = 0.1
P(Bus / late) = ? , P(Car / late) = ? , P(Scooty / late) = ?

Ans) $P(B/L) = P(B \cap L)/ P(L) = P(L/B).P(B) / P(L) = (0.5)(0.2) / P(L)$

$P(L) = P(B \cap L) + P(C \cap L) + P(S \cap L)$
$= P(B).P(L/B) + P(C).P(L/C) + P(S).P(L/S)$
$= (0.2)(0.5) + (0.7)(0.1) + (0.1)(0.2) = 0.19$

$P(B/L)\ P(B \cap L)/ P(L) = P(L/B).P(B) / P(L) = (0.5)(0.2) / 0.19 = 10/19$
$P(C/L) = P(C \cap L) / P(L) = P(L/C).\ P(C) / P(L) = (0.1)(0.7) / 0.19 = 7/19$
$P(S/L) = P(S \cap L) / P(L) = P(L/S).\ P(S) / P(L) = (0.2)(0.1) / 0.19 = 2/19$

Probability Distributions

- **Definition:**

  Describes the likelihood of different outcomes in a random experiment.

- **Examples:**
  - **Uniform distribution: Equal probability for all outcomes.**
  - **Normal distribution: Bell-shaped curve, common in many natural phenomena.**

Applications of Probability

Probability theory plays a significant role in various aspects of mining engineering. Here are some applications:

Resource Estimation: Probability theory is extensively used in estimating the reserves of minerals or ores in a given area. Techniques such as kriging and geostatistics rely on probabilistic models to interpolate and extrapolate data from sampling points to estimate the quantity and quality of mineral resources.

Risk Assessment: Mining projects involve various risks, including geological uncertainties, market fluctuations, and operational hazards. Probability theory helps in quantifying these risks through techniques like Monte Carlo simulation, which evaluates the potential outcomes of different scenarios based on probabilistic inputs.

Safety Analysis: Probability theory is employed in assessing safety risks associated with mining operations. By analyzing historical data and identifying potential hazards, engineers can calculate probabilities of accidents or failures, allowing them to implement preventive measures and design safety protocols accordingly.

Applications of Probability

Probability theory plays a significant role in various aspects of mining engineering. Here are some applications:

Equipment Reliability: Mining equipment reliability is crucial for maintaining productivity and minimizing downtime. Probability theory is used to model the reliability and availability of equipment, predicting failure rates and optimizing maintenance schedules to ensure continuous operation.

Environmental Impact Assessment: Probability theory assists in assessing the environmental impact of mining activities. By analyzing probabilistic models of pollutant dispersion, groundwater contamination, and ecosystem disruption, engineers can evaluate the potential consequences of mining operations on the environment and devise mitigation strategies.

Exploration Decision Making: Probability theory aids in decision-making during mineral exploration. Through techniques like Bayesian inference, engineers can update their beliefs about the presence and characteristics of mineral deposits based on new data, guiding the allocation of exploration resources more efficiently.

Applications of Probability

Probability theory plays a significant role in various aspects of mining engineering. Here are some applications:

Grade Control: Probability theory is employed in grade control strategies to optimize ore extraction and minimize waste. By incorporating probabilistic models of ore grade variability, mining engineers can design sampling protocols and ore blending strategies to maximize the economic value of extracted material.

Financial Analysis: Probability theory is used in financial modeling and risk analysis of mining projects. By evaluating the probabilistic distribution of costs, revenues, and commodity prices, stakeholders can assess the financial viability of investments, make informed decisions, and manage investment risks effectively.

Overall, probability theory serves as a fundamental tool in various aspects of mining engineering, helping professionals make informed decisions, manage risks, and optimize resource utilization throughout the lifecycle of mining projects.

REFERENCES

- Introduction to Probability and Statistics for Engineers and Scientists, Sixth Edition, Sheldon M. Ross
- Statistical Methods Combined Edition (Volume I& II), N G Das

CONCLUSION

- Discussed Independent Events
- Discussed Multiplication Theorem
- Discussed Total Probability
- Discussed Bayes Theorem
- Discussed the Applications of Probability in Mining Industry


THANK YOU