SWAYAM NPTEL COURSE ON

MINE AUTOMATION AND DATA ANALYTICS

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Module 7: Probability

Lecture 18A : Introduction to Probability and its associated terms

CONCEPTS COVERED

- Basic Terms of Probability
- Types of Probability
- Conditional Probability
- Marginal Probability
- Joint Probability

Probability

Definition:

- Probability is a measure of the likelihood that a given event will occur.
- To do a certain task, we have certain chances of success. These chances of success increase by increasing the number of attempts

Importance:

Used in various fields, such as statistics, finance, science, and more.

Real-world examples:

Coin toss, weather forecasting.

Probability

Random Experiment:

- **We know possible outcomes but do not know the exact outcomes.**

Example:

i. **Toss a coin: Possible Outcome: {H ,T}, Exact outcome ??**

ii. **Rolling a dice: Possible Outcome: {1,2,3,4,5,6}, Exact outcome ??**

iii. **Box containing R,G,B balls. The possible outcome is {R,G,B}. A ball is picked @ random, Exact outcome ??**

Probability

Sample Space:

Set of all possible outcomes of a random experiment

Toss a coin: Possible Outcome: {H ,T}

Rolling a dice: Possible Outcome: {1,2,3,4,5,6},

Box containing Red, Green, and Blue balls. A ball is picked @ random. The possible outcome is {R, G, B}.

Probability

Event:

- Subset of sample space which is our area of interest

Toss a coin: Possible Outcome: {H,T}
E_1 = {H}
E_2 = {T}
E1 and E2 are sample spaces

Since Event and Sample Space both are sets, so all set operations can be applied to them like Union, Intersection, Set difference, etc.,
Sample space = {1,2,3,4,5,6}
E1 = {1,2}
E2 = {2,3,4}
E1 ∩ E2 = {2}
E1 ∪ E2 = {1,2,3,4}
E1 - E2 = {1}
E1 ∩ E2 = {2}
$E1^C$ = {3,4,5,6}

Probability

Mutually Exclusive Events

- Two events E1 and E2 are said to be mutually exclusive iff $E1 \cap E2 = \{\}$ or $\emptyset$

$E_1 = \{H\}$

$E_2 = \{T\}$

E1 and E2 are Mutually Exclusive Events because $E1 \cap E2 = \{\}$

Basic Probability Terms

Sample Space:

- Definition: The set of all possible outcomes of an experiment.
- Example: Coin toss (Heads, Tails).

Event:

- Definition: A subset of the sample space.
- Example: Getting a Head in a coin toss.

Probability of an Event:

- Definition: The likelihood of an event occurring.
- Formula: P(Event) = Number of favorable outcomes / Total outcomes.

Types of Probability

Classical Probability:

1. Definition: Based on equally likely outcomes.
2. Formula: P(E) = Number of favorable outcomes / Total number of outcomes.
3. Example: Rolling a fair six-sided die.

Empirical Probability:

1. Definition: Based on observed outcomes.
2. Formula: P(E) = Number of times event E occurs / Total number of trials.
3. Example: Probability of rain based on historical data.

Subjective Probability:

1. Definition: Based on personal judgment or opinion.
2. Example: Probability of winning a game based on a person's intuition.

Probability

Probability:

- Probability is a measure of the likelihood that a given event will occur

- Probability = favorable outcomes / total number of outcomes = E / SS

- Ex: Roll a dice , then probability of getting even number?
- E = { 2, 4 ,6} and SS = {1,2 3,4,5,6 }
- Probability = E / SS = 3/6 = 0.5

- Ex: Roll a die; probability of getting a number 5?
- E = { 5 } and SS = {1,2 3,4,5,6 }
- Probability = E / SS = 1/6

- Ex: Toss 2 coins. Probability of getting 2 heads or 2 tails?
- E = { TT,HH } and SS = {TT,HH,TH,HT}
- Probability = E / SS = 2/4 = 0.5

Probability Example

- Suppose you roll a fair six-sided die. What is the probability of rolling a 3?
- Sample Space (S): {1, 2, 3, 4, 5, 6}
- Event (E): Rolling a 3
- Probability (P(E)): 1/6 (because there is 1 favorable outcome out of 6 possible outcomes)

Probability

- Probability of a Sample Space?
- E = SS
- P = E/SS = SS/SS = 1

- $P(E1 \cup E2) = P(E1) + P(E2) – P(E1 \cap E2)$
- $P(E1 \cup E2 \cup E3) = P(E1) + P(E2) + P(E3) – P(E1 \cap E2) - P(E2 \cap E3) - P(E3 \cap E1) + P(E1 \cap E2 \cap E3)$

- E1 and E2 are Mutually Exclusive Events, $P(E1 \cup E2)$ = ? $P(E1 \cup E2 \cup E3)$ = ?
- $P(E1 \cup E2) = P(E1) + P(E2)$ Since $P(E1 \cap E2) = 0$
- $P(E1 \cup E2 \cup E3) = P(E1) + P(E2) + P(E3)$

Probability

Roll a dice SS = {1,2,3,..,6} , E1 = {1,2,3} , E2 = {4,5} , P(E1 ∪ E2) = ?
P(E1 ∪ E2) = P(E1) + P(E2) Since P(E1 ∩ E2) = 0
P(E1 ∪ E2) = 3/6 + 2/6 = 5/6

Toss 2 coins E1 = {HH}, E2 = {TT} , P(E1 ∪ E2) = ?
P(E1 ∪ E2) = P(E1) + P(E2) Since P(E1 ∩ E2) = 0
P(E1 ∪ E2) = 1/4 + 1/4 = ½

$P(E^{C})$=?
$E \cup E^{C} = SS$
$P(E \cup E^{C}) = P(SS)$
$P(E) + P(E^{C}) = 1$
$P(E^{C}) = 1 - P(E)$

Q: Probability of getting good rank in GATE is 0.9, Probability of not getting a good rank is ??
$P(E^{C}) = 1 - P(E)$
$P(E^{C}) = 1 - 0.9 = 0.1$

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Conditional Probability

- 2 Events A and B are there.
- P(A/B) = Probability of A given B = What is the probability of happening A given B
- P(B/A) = probability of B given A = What is the probability of happening B given A

- A = getting a prime number from a dice = {2,3,5}
- B = getting a even number from a dice = {2,4,6}
- P(A/B) = ?
- B already happened = {2,4,6} , Now sample space would be B = {2,4,6}
- Probability of happening A given B already happened = 1/3

- $P(A/B) = P(A \cap B) / P(B)$
- $P(B/A) = P(B \cap A) / P(A)$

Conditional Probability

- **Definition: Probability of an event given that another event has occurred.**
- **Formula:** $\boldsymbol{P(A|B) = P(A\ and\ B)\ /\ P(B)}$.
- **Example: Probability of getting a Head in the second toss given that the first toss resulted in a Tail.**

Conditional Probability

Steps to solve the questions of Conditional Probability P(A/B)

- Find Sample Space
- Finding Which is Event A
- Finding Which is Event B
- Finding Probabilities P(A), P(B) , P(A ∩ B)
- Calculating required one P(A/B) = P(A ∩ B) / P(B) or P(B/A) = P(B ∩ A) / P(A)

Example of Conditional Probability

- Q) If 2 dice are rolled, and the 1st dice shows 4, then what is the probability that the sum is 6?
- A) |SS| = 36 , 1st dice = {1,2,3,4,5,6} , 2nd dice = {1,2,3,4,5,6}
- A = dice showed 4 = { (4,1),(4,2),(4,3),(4,4),(4,5),(4,6) }
- B = sum is 6 = {(1,5),(2,4),(3,3),(4,2),(5,1)} , |B| = 5
- A ∩ B = {(4,2)}
- P(A) = 6/36 = 1/6 ; P(B) = 5/36 ; P(A ∩ B) = 1/36
- P(A/B) = P(A ∩ B) / P(B) = (1/36) / (5/36) = 1/5
- P(B/A) = P(B ∩ A) / P(A) = (1/36) / (6/36) = 1/6

What is the correct solution to this question = ?
It is P(B/A) = 1/6

What if the question asked: the probability that dice is showing 4 given the dices sum is 6?

= P(A/B)
= 1/5

Example of Conditional Probability

Q) Two dice are rolled, and both show odd numbers. Then the probability of getting the sum 6?

|SS| = 36

A = {(1,1),(3,3),(5,5),(1,3),(1,5),(3,1),(3,5),(5,1),(5,3)} ; |A| = 9

B = {(1,5),(2,4),(3,3),(4,2),(5,1)} ; |B| = 5

A ∩ B = {(1,5), (3,3), (5,1)} ; | A ∩ B | = 3

P(A) = |A| / |SS| = 9/36 ; P(B) = |B| / |SS| = 5/36 ; P(A ∩ B) = | A ∩ B | / |SS| = 3 / 36

- P(A/B) = P(A ∩ B) / P(B) = (3/36) / (5/36) = 9/5
- P(B/A) = P(B ∩ A) / P(A) = (3/36) / (9/36) = 3/9 = 1/3

- P(B/A) is our solution = 1/3

Properties of Conditional Probability

1) $A \subseteq B$

$P(A/B) = P(A \cap B) / P(B) = P(A) / P(B)$

$P(B/A) = P(B \cap A) / P(A) = 1$

2) If A and B are Mutually Exclusive Events

$P(A/B) = P(A \cap B) / P(B) = 0$

$P(B/A) = P(B \cap A) / P(A) = 0$

3) $P(A \cup B / C) = P(A / C) + P(B / C) - P(A \cap B / C)$

4) $P(A^C / B) = 1 - P(A / B)$

Marginal Probability

- To understand marginal probability, let's consider a scenario with two random variables, A and B.
- The joint probability $P(A \cap B)$ represents the probability that events A and B occur together. The marginal probability of A, denoted as $P(A)$, focuses solely on the probability of event A happening, regardless of the occurrence or non-occurrence of event B.
- Mathematically, marginal probability is obtained by summing (or integrating, In the case of continuous random variables) the joint probabilities over all possible values of the other variable.

  The formulas are as follows,

$$P(A) = \sum_{all\ possible\ B} P(A \cap B)$$

$$P(A) = \int_{all\ possible\ B} P(A \cap B)\, dB$$

- In simpler terms, you "marginalize" or "sum out" the unwanted variable (in this case, variable B) to obtain the probability distribution for the variable of interest (variable A).

Marginal probability

- **Marginal probability refers to the probability of a single event or outcome occurring without considering the occurrence of other events.**
- **It is derived from a joint probability distribution, which describes the probabilities of combinations of events.**
- **Marginal probability is a fundamental concept in probability theory and is used in various statistical analyses and machine learning algorithms, especially when dealing with multiple variables and their interactions.**

Joint Probability

- Joint probability is a concept in probability theory that describes the likelihood of two or more events occurring simultaneously.
- It is denoted as $P(A \cap B)$, where A and B are events. Joint probability is used to quantify the probability of the intersection of events.
- Mathematically, the joint probability of events A and B is calculated as follows: $P(A \cap B)$
- For independent events, where the occurrence of one event does not affect the occurrence of the other, the joint probability simplifies to the product of the individual probabilities:

$$P(A \cap B) = P(A) \cdot P(B)$$

Joint Probability

- However, when events are dependent, meaning the occurrence of one event affects the occurrence of the other, the joint probability is calculated using the conditional probability formula:

$$P(A \cap B) = P(A \mid B) \cdot P(B)$$

- Or, equivalently:

$$P(A \cap B) = P(B \mid A) \cdot P(A)$$

The terms involved:

P(*A*∩*B*): Joint probability of events P(*A)* and P(*B)*.

P(*A*) and *P*(*B*): Marginal probabilities of events P(*A)* and P(*B)*, respectively.

P(*A*|*B*): Conditional probability of event P(*A)* given that event P(*B)* has occurred.

P(*B*|*A*): Conditional probability of event P(*B)* given that event P(*A)* has occurred

Solved Example

Suppose you have two fair coins, one is red (R) and the other is blue (B). We want to calculate the conditional probability, joint probability, and marginal probability for the outcomes.

Conditional Probability:

Let's say we're interested in the probability of getting a red coin (R) given that we flipped two heads (HH).

P(R|HH) = (Number of outcomes where both coins are heads and one is red) / (Total number of outcomes where both coins are heads)

Number of outcomes where both coins are heads and one is red = 1 (RR)

Total number of outcomes where both coins are heads = 1 (HH)

- **So, P(R|HH) = 1/1 = 1**

Solved Example - 1

Suppose you have two fair coins, one is red (R) and the other is blue (B). We want to calculate the conditional probability, joint probability, and marginal probability for the outcomes.

Joint Probability:

The joint probability is the probability of two events happening together.

Let A be the event of getting a red coin (R) and B be the event of getting two heads (HH).

$P(A \cap B) = P(R \text{ and } HH)$

Number of outcomes where both coins are heads, and one is red = 1 (RR)

So, $P(A \cap B) = 1/4$

Solved Example - 1

Suppose you have two fair coins, one is red (R) and the other is blue (B). We want to calculate the conditional probability, joint probability, and marginal probability for the outcomes.

Marginal Probability:

The marginal probability is the probability of a single event occurring without reference to any other event.

Let's calculate the marginal probability of getting a red coin (R).

P(R) = (Number of outcomes where one coin is red) / (Total number of outcomes)

Number of outcomes where one coin is red = 2 (RR, RB)

Total number of outcomes = 4 (RR, RB, BR, BB)

So, P(R) = 2/4 = 1/2

Solved Example - 1

Suppose you have two fair coins, one is red (R) and the other is blue (B). We want to calculate the conditional probability, joint probability, and marginal probability for the outcomes.

In summary:

Conditional Probability: P(R|HH) = 1

Joint Probability: P(R and HH) = 1/4

Marginal Probability: P(R) = 1/2

REFERENCES

- Introduction to Probability and Statistics for Engineers and Scientists, Sixth Edition, Sheldon M. Ross
- Statistical Methods Combined Edition (Volume I& II), N G Das

CONCLUSION

1. Basic Terms of Probability like Random Experiment, Sample Space, Event, and Mutually Exclusive events are being discussed
2. Types of Probability.
3. Discussed Conditional Probability with examples.
4. Discussed Marginal Probability.
5. Discussed Joint Probability.
6. Solved example involving Conditional , Marginal , and Joint Probability.


THANK YOU