TEMA 5

Determinați inversele următoarelor matrici:

a) $A = \begin{pmatrix} 1 & 3 \\ 4 & 3 \end{pmatrix} \in M_{2 \times 2}(\mathbb{Z}_5)$

b) $B = \begin{pmatrix} 15 & 17 \\ 4 & 9 \end{pmatrix} \in M_{2 \times 2}(\mathbb{Z}_{26})$

c) $C = \begin{pmatrix} 197 & 62 \\ 603 & 271 \end{pmatrix} \in M_{2 \times 2}(\mathbb{Z}_{841})$

a) $A^{-1} = (\det A) \begin{pmatrix} 3 & -3 \\ -4 & 1 \end{pmatrix} = (\det A) \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$

$\det A = 1 \cdot 3 - 4 \cdot 3 = 3 - 12 = 3 - 2 = 1$

b) $B^{-1} = (\det B) \begin{pmatrix} 9 & -17 \\ -4 & 15 \end{pmatrix} = (\det B) \begin{pmatrix} 9 & 9 \\ 22 & 15 \end{pmatrix} = 15 \begin{pmatrix} 9 & 9 \\ 22 & 15 \end{pmatrix} = \begin{pmatrix} 135 & 135 \\ 330 & 225 \end{pmatrix} = $

$\det B = 15 \cdot 9 - 4 \cdot 17 = 135 - 68 = 67 \equiv 15 \pmod{26}$ $= \begin{pmatrix} 5 & 5 \\ 18 & 17 \end{pmatrix}$

c) $C^{-1} = (\det C) \begin{pmatrix} 271 & -62 \\ -603 & 197 \end{pmatrix} = (\det C) \begin{pmatrix} 271 & 780 \\ 239 & 197 \end{pmatrix}$.

$\det C = 197 \cdot 271 - 603 \cdot 62 = 53\,387 - 37\,386 = 16\,001 \equiv 22 \pmod{841}$

$C^{-1} = 22 \begin{pmatrix} 271 & 780 \\ 239 & 197 \end{pmatrix} = \begin{pmatrix} 5962 & 17\,160 \\ 5258 & 4334 \end{pmatrix} = \begin{pmatrix} 75 & 340 \\ 212 & 129 \end{pmatrix} \pmod{841}$