Hopping only

In[ ]:=
```
H[Nl_] := Table[If[Abs[i - j] == 1, -1, 0], {i, 1, Nl}, {j, 1, Nl}];

H[10] // MatrixForm
```

Out[ ]//MatrixForm=

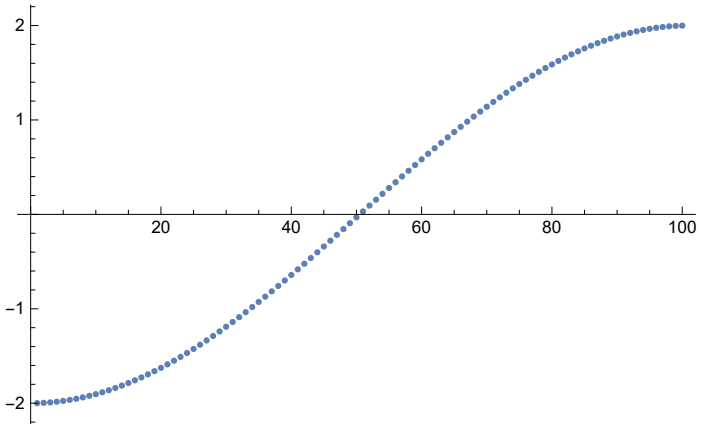
$$\begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

In[ ]:=
```
{EVals, EVecs} = Eigensystem[N[H[100]]];
```

In[ ]:=
```
sortedEVecs = (EVecs)[[Ordering[EVals]]];
sortedEVals = Sort[EVals];
```

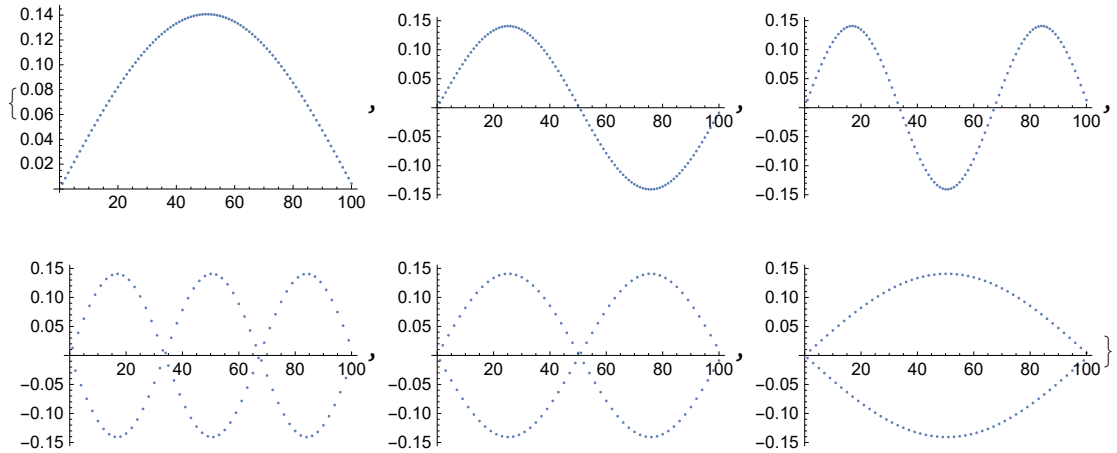
In[ ]:=
```
ListPlot[sortedEVals]
```

Out[ ]=



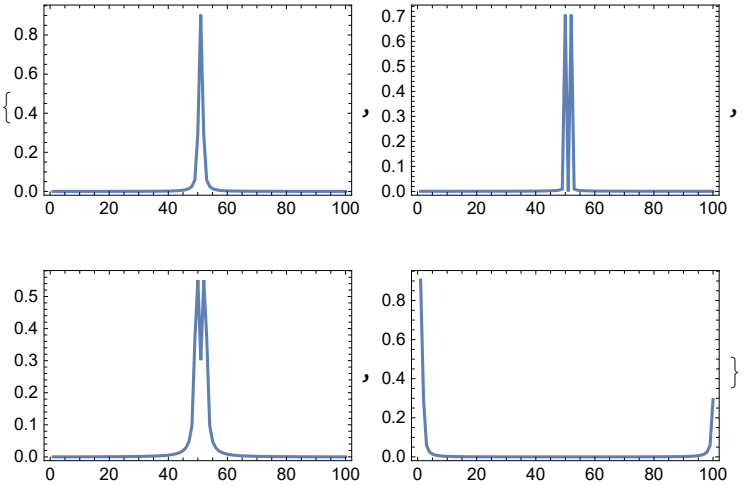
In[ ]:=
```
{ListPlot[sortedEVecs[[1]]], ListPlot[sortedEVecs[[2]]],
 ListPlot[sortedEVecs[[3]]], ListPlot[sortedEVecs[[98]]],
 ListPlot[sortedEVecs[[99]]], ListPlot[sortedEVecs[[100]]]}
```

Out[ ]=



In[•]:=
```
{ListLinePlot[RotateRight[Abs[Fourier[sortedEVecs[[1]]]], 50],
  PlotRange → All, Frame → True],
 ListLinePlot[RotateRight[Abs[Fourier[sortedEVecs[[2]]]], 50], PlotRange → All,
  Frame → True], ListLinePlot[RotateRight[Abs[Fourier[sortedEVecs[[3]]]], 50],
  PlotRange → All, Frame → True], ListLinePlot[
  RotateRight[Abs[Fourier[sortedEVecs[[100]]]], 50], PlotRange → All, Frame → True]}
```

Out[•]=



With Trap

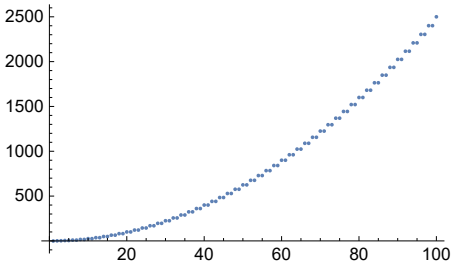
In[•]:=
```
Ht[Nl_, ω_] :=
  Table[If[i == j, (i - Nl/2)^2 ω^2, If[Abs[i - j] == 1, -1, 0]], {i, 1, Nl}, {j, 1, Nl}];
Ht[10, 0.5] // MatrixForm
```

Out[•]//MatrixForm=

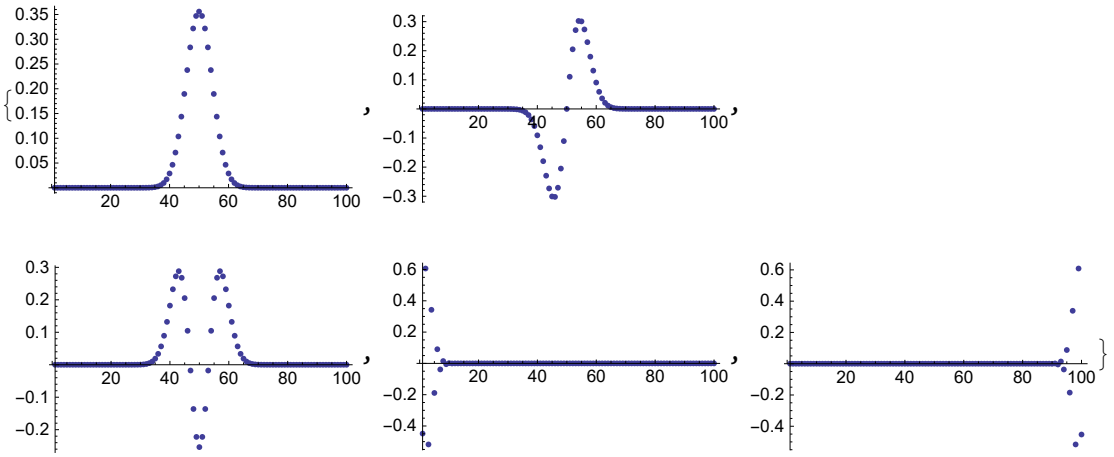
$$\begin{pmatrix} 4. & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2.25 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1. & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0.25 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0. & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0.25 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1. & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2.25 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 4. & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 6.25 \end{pmatrix}$$

In[•]:=
```
{EVals, EVecs} = Eigensystem[N[Ht[100, 1]]];
sortedEVecs = (EVecs)[[Ordering[EVals]]];
sortedEVals = Sort[EVals];
ListPlot[sortedEVals]
```

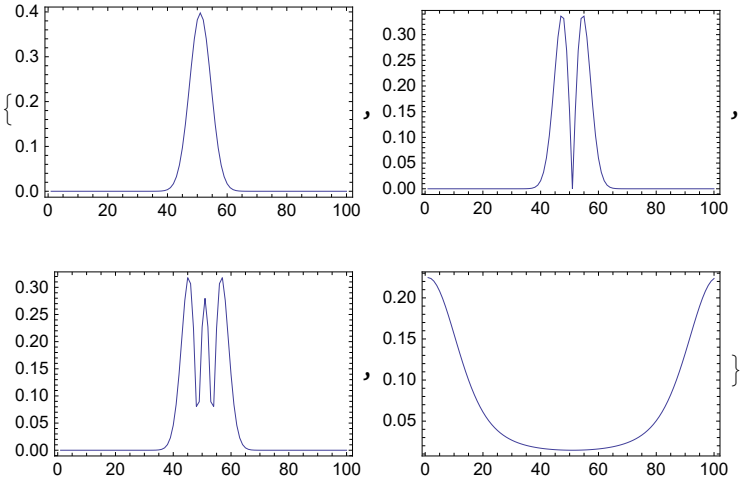
Out[•]=



```
{ListPlot[sortedEVecs[[1]], PlotRange → All],
 ListPlot[sortedEVecs[[2]], PlotRange → All],
 ListPlot[sortedEVecs[[3]], PlotRange → All],
 ListPlot[sortedEVecs[[99]], PlotRange → All],
 ListPlot[sortedEVecs[[100]], PlotRange → All]}
```



```
{ListLinePlot[RotateRight[Abs[Fourier[sortedEVecs[[1]]]], 50],
  PlotRange → All, Frame → True],
 ListLinePlot[RotateRight[Abs[Fourier[sortedEVecs[[2]]]], 50], PlotRange → All,
  Frame → True], ListLinePlot[RotateRight[Abs[Fourier[sortedEVecs[[3]]]], 50],
  PlotRange → All, Frame → True], ListLinePlot[
  RotateRight[Abs[Fourier[sortedEVecs[[100]]]], 50], PlotRange → All, Frame → True]}
```



With force term

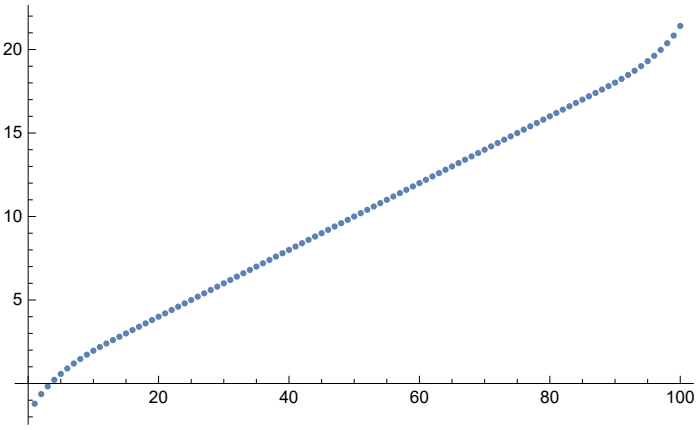
```
Ht[Nl_, ω_, F_] := Table[
   If[i == j, (i - Nl/2)^2 ω^2 + F * i, If[Abs[i - j] == 1, -1, 0]], {i, 1, Nl}, {j, 1, Nl}];
Ht[10, 0.05, 0.05] // MatrixForm
```

Out[•]//MatrixForm=

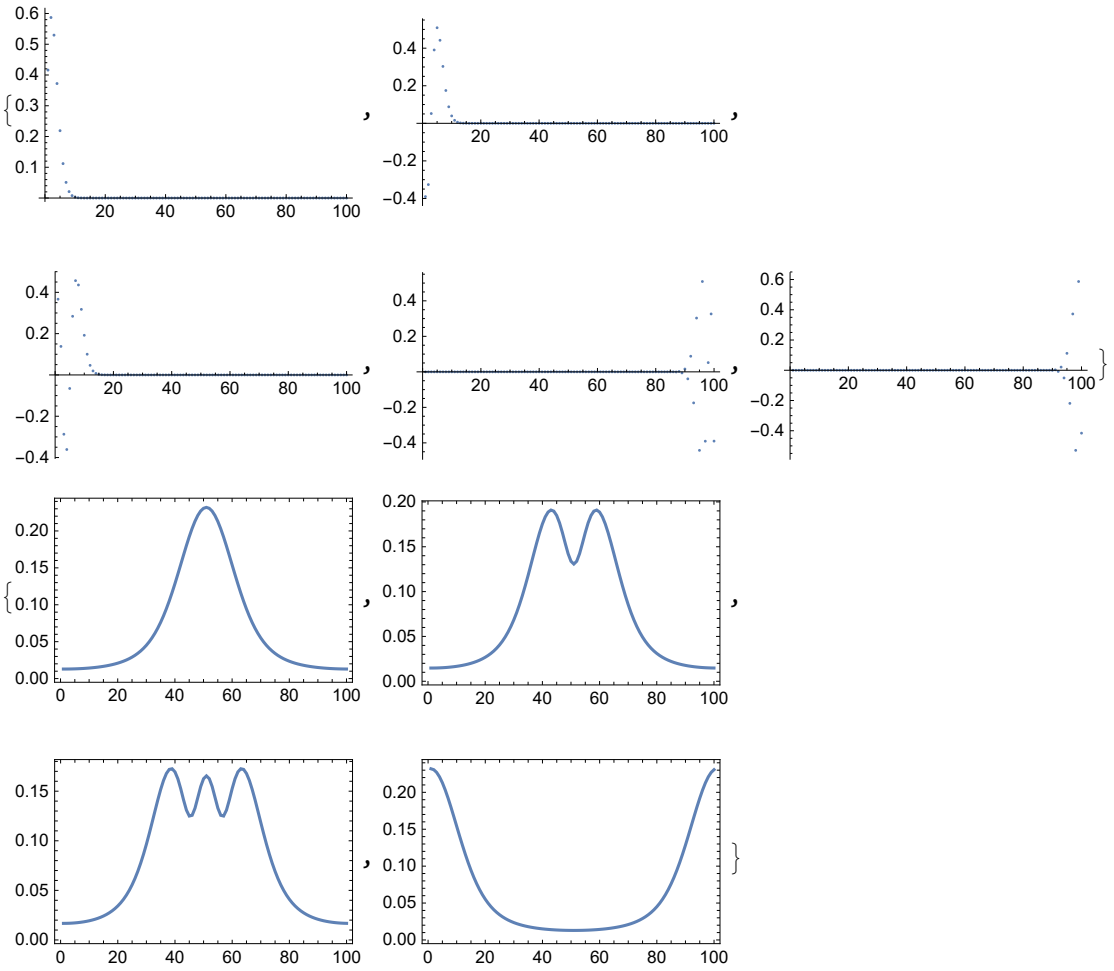
$$\begin{pmatrix} 0.09 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0.1225 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0.16 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0.2025 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0.25 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0.3025 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0.36 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0.4225 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0.49 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0.5625 \end{pmatrix}$$

In[•]:=
```
{EVals, EVecs} = Eigensystem[N[Ht[100, 0.00, 0.2]]];
sortedEVecs = (EVecs)[[Ordering[EVals]]];
sortedEVals = Sort[EVals];
ListPlot[sortedEVals]

{ListPlot[sortedEVecs[[1]], PlotRange → All],
 ListPlot[sortedEVecs[[2]], PlotRange → All],
 ListPlot[sortedEVecs[[3]], PlotRange → All],
 ListPlot[sortedEVecs[[99]], PlotRange → All],
 ListPlot[sortedEVecs[[100]], PlotRange → All]}

{ListLinePlot[RotateRight[Abs[Fourier[sortedEVecs[[1]]]], 50],
  PlotRange → All, Frame → True],
 ListLinePlot[RotateRight[Abs[Fourier[sortedEVecs[[2]]]], 50], PlotRange → All,
  Frame → True], ListLinePlot[RotateRight[Abs[Fourier[sortedEVecs[[3]]]], 50],
  PlotRange → All, Frame → True], ListLinePlot[
  RotateRight[Abs[Fourier[sortedEVecs[[100]]]], 50], PlotRange → All, Frame → True]}
```

Out[•]=



Out[ ]=



Out[ ]=

Time dependent force term - NDSolve

In[ ]:=
```
Ht[Nl_, ω_, F_, t_] :=
  Table[If[i == j, F * i * Cos[ω * t], If[Abs[i - j] == 1, -1, 0]], {i, 1, Nl}, {j, 1, Nl}];
Ht[10, 0.05, 0.05, 0] // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0.05 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0.1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0.15 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0.2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0.25 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0.3 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0.35 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0.4 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0.45 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0.5 \end{pmatrix}$$

In[•]:=
```
Nl = 10;
psi0 = Table[If[i == Nl / 2, 1, 0], {i, 1, Nl}];
s =
  NDSolve[{I D[ψ[t], t] == Ht[10, 0.05, 0.05, t] .ψ[t], ψ[0] == psi0}, ψ, {t, 0, 1}];
Table[Plot[{Re[ψ[t][[i]] /. s], Im[ψ[t][[i]] /. s], Norm[ψ[t][[i]] /. s]},
  {t, 0, 1}, PlotLegends → {"Re", "Im", "Norm"}], {i, 1, Nl}]

Table[Plot[{Re[ψ[t][[i]] /. s], Im[ψ[t][[i]] /. s],
   Abs[ψ[t][[i]] /. s], Norm[ψ[t][[i]] /. s]}, {i, 1, Nl}], {t, 0, 1, 0.1}]
```

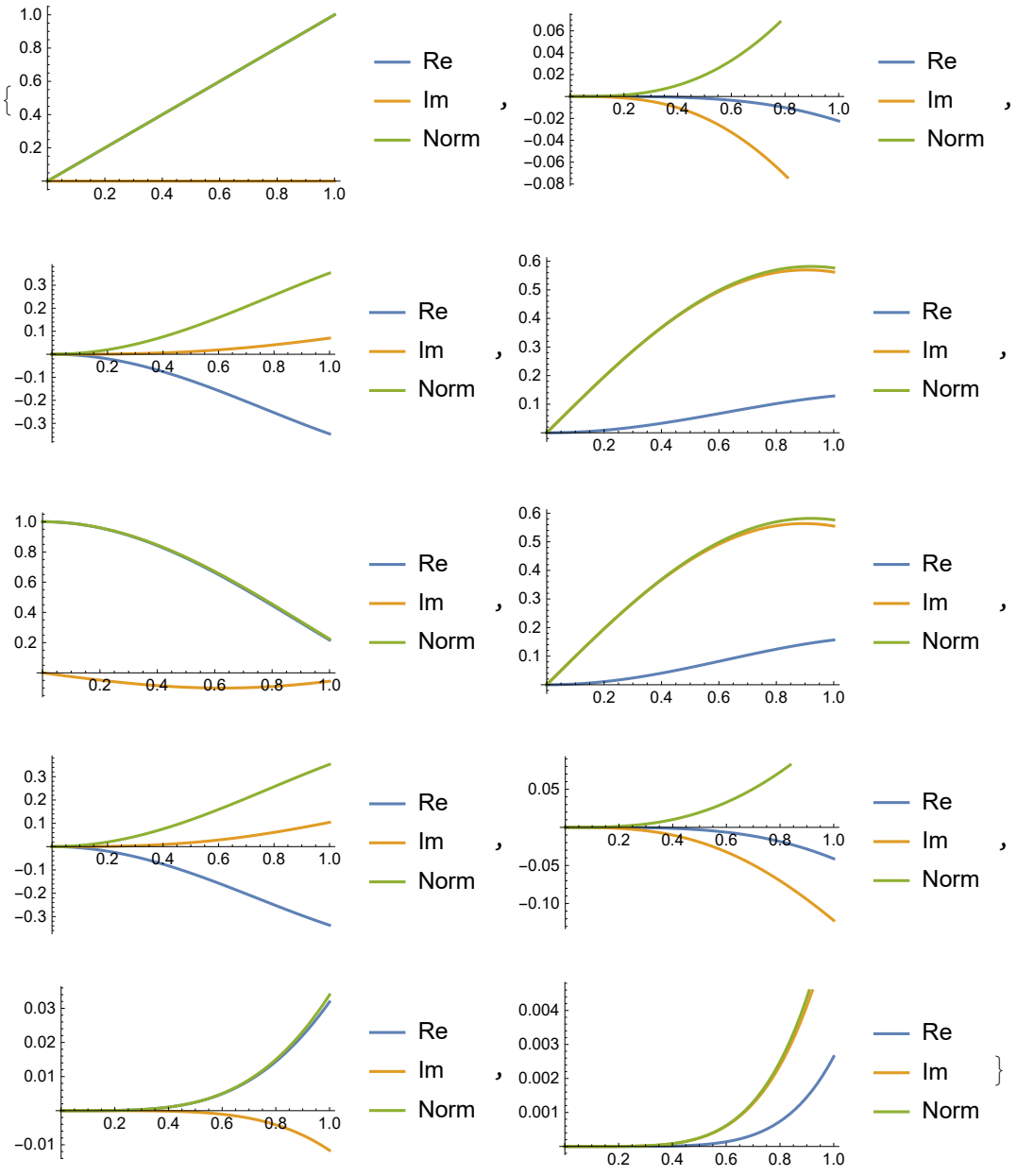
Part: Part 2 of ψ[0.0000204286] does not exist.

Part: Part 2 of ψ[0.0000204286] does not exist.

Part: Part 2 of ψ[0.0000204286] does not exist.

General: Further output of Part::partw will be suppressed during this calculation.

Out[•]=



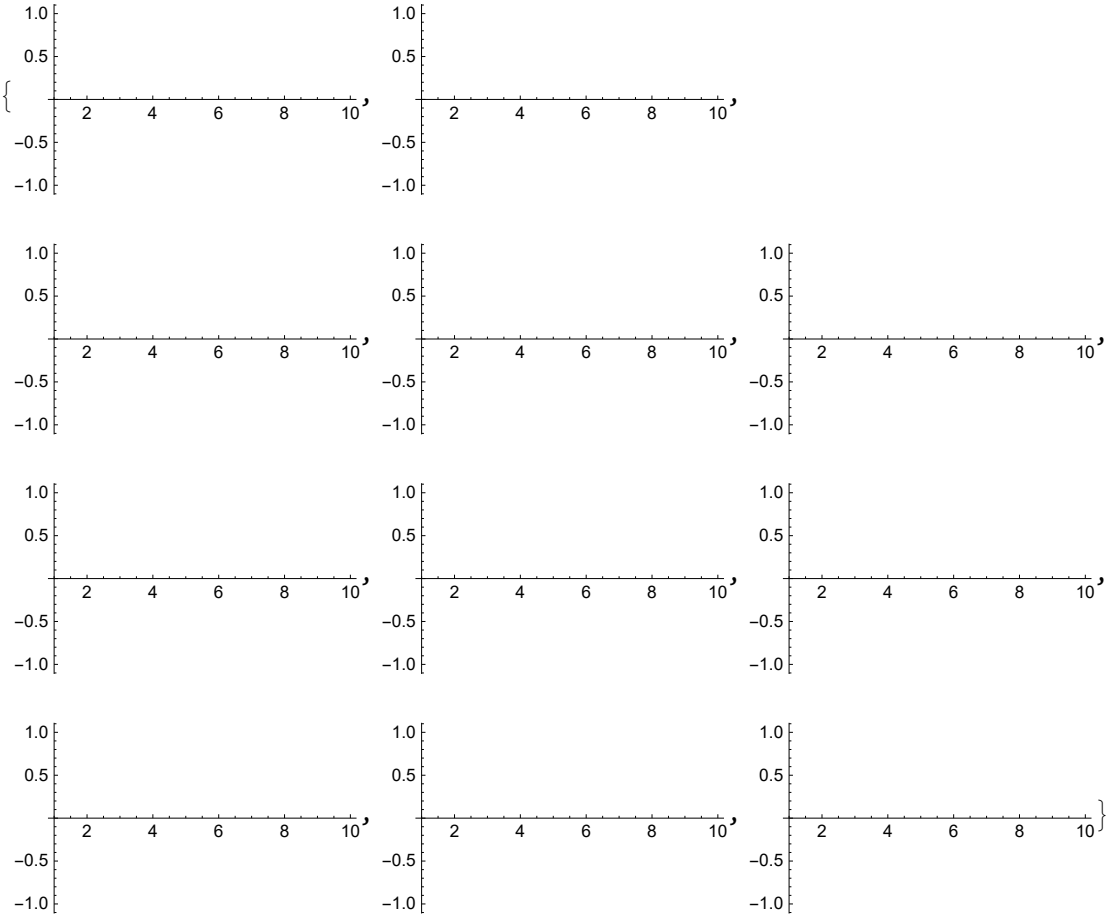
Part: The expression 1.0001838571428572` cannot be used as a part specification.

Part: The expression 1.0001838571428572` cannot be used as a part specification.

Part: The expression 1.1838573265306123` cannot be used as a part specification.

General: Further output of Part::pkspec1 will be suppressed during this calculation.

Out[ ]=



In[•]:=
```
tt = Range[0, 1, 0.1]
ww = Flatten[Evaluate[Abs[ψ[#]] /. s]] & /@ tt
Plot[ tt, ww[[All, 2]] ]
(*ListPlot[Table[Transpose[{tt, ww[[All, i]]}], {i, 3}]]
```

Out[•]=
```
{0., 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.}
```

Out[•]=
```
{{0., 0., 0., 0., 1., 0., 0., 0., 0., 0.},
 {4.15962 × 10^-6, 0.000166245, 0.00498335, 0.0995007, 0.990025,
  0.0995007, 0.00498335, 0.000166245, 4.1582 × 10^-6, 8.34361 × 10^-8},
 {0.0000662139, 0.00132003, 0.0197345, 0.196026, 0.960398, 0.196026,
  0.0197345, 0.00132004, 0.0000661251, 2.65111 × 10^-6},
 {0.000332449, 0.0043995, 0.0436643, 0.286699, 0.912006, 0.286699, 0.0436644,
  0.00439955, 0.000331447, 0.0000199874}, {0.00103847, 0.010246, 0.0758154,
  0.368837, 0.846292, 0.368837, 0.0758154, 0.0102463, 0.0010329, 0.0000833934},
 {0.0024973, 0.0195605, 0.114898, 0.440042, 0.765209, 0.440042, 0.114898,
  0.019562, 0.00247628, 0.000251213}, {0.00508345, 0.0328656, 0.159339,
  0.498277, 0.671155, 0.498277, 0.159339, 0.032871, 0.00502154, 0.000615288},
 {0.00921334, 0.0504753, 0.20734, 0.541935, 0.566894, 0.541935, 0.207338,
  0.0504908, 0.00905977, 0.00130516}, {0.0153233, 0.0724723, 0.256945,
  0.569885, 0.455463, 0.569885, 0.256941, 0.072511, 0.0149876, 0.00248993},
 {0.0238457, 0.098695, 0.306117, 0.581513, 0.340075, 0.581514, 0.306107,
  0.0987815, 0.0231796, 0.00437753}, {0.0351842, 0.128736, 0.352809,
  0.576736, 0.224011, 0.576738, 0.352787, 0.128913, 0.0339609, 0.00721095}}
```

Plot: Range specification {0., 0.000166245, 0.00132003, 0.0043995, 0.010246, 0.0195605, 0.0328656, 0.0504753, 0.0724723, 0.098695, 0.128736} is not of the form {x, xmin, xmax}.

Out[•]=
```
Plot[tt, ww[[All, 2]]]
```

In[•]:=
```
Manipulate[Module[{ψ, sol, tmax = 20},
  sol = First@NDSolve[{I D[ψ[t], t] ==
       Ht[10, 0.05, F, t] .ψ[t], ψ[0] == psi0}, ψ, {t, 0, 1}];
  Plot[Chop[#] &@ (ψ /. sol)[t],
   {t, 0, 1}, PlotRange → All]
 ],
 {{F, 1}, 0, 2}
]
```

Out[ ]=

Dot: Tensors

{{0., −1, 0, 0, 0, 0, 0, 0, 0, 0}, {−1, 0., −1, 0, 0, 0, 0, 0, 0, 0}, {0, −1, 0., −1, 0, 0, 0, 0, 0, 0}, {0, 0, −1, 0., −1, 0, 0, 0, 0, 0}, {0, 0, 0, −1, 0., −1, 0, 0, 0, 0}, {0, 0, 0, 0, −1, 0., −1, 0, 0, 0}, {0, 0, 0, 0, 0, −1, 0., −1, 0, 0}, {0, 0, 0, 0, 0, 0, −1, 0., −1, 0}, {0, 0, 0, 0, 0, 0, 0, −1, 0., −1}, {0, 0, 0, 0, 0, 0, 0, 0, −1, 0.}} and {0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.} have incompatible shapes.

Dot: Tensors

{{0., −1., 0., 0., 0., 0., 0., 0., 0., 0.}, {−1., 0., −1., 0., 0., 0., 0., 0., 0., 0.}, {0., −1., 0., −1., 0., 0., 0., 0., 0., 0.}, {0., 0., −1., 0., −1., 0., 0., 0., 0., 0.}, {0., 0., 0., −1., 0., −1., 0., 0., 0., 0.}, {0., 0., 0., 0., −1., 0., −1., 0., 0., 0.}, {0., 0., 0., 0., 0., −1., 0., −1., 0., 0.}, {0., 0., 0., 0., 0., 0., −1., 0., −1., 0.}, {0., 0., 0., 0., 0., 0., 0., −1., 0., −1.}, {0., 0., 0., 0., 0., 0., 0., 0., −1., 0.}} and {0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.} have incompatible shapes.

NDSolve: Encountered non−numerical value for a derivative at t == 0.`.

ReplaceAll:

{ψ47839′[0.0000204286] == {{0., −1, 0, 0, 0, 0, 0, 0, 0, 0}, {−1, 0., −1, 0, 0, 0, 0, 0, 0, 0}, {0, −1, 0., −1, 0, 0, 0, 0, 0, 0}, {0, 0, −1, 0., −1, 0, 0, 0, 0, 0}, {0, 0, 0, −1, 0., −1, 0, 0, 0, 0}, {0, 0, 0, 0, −1, 0., −1, 0, 0, 0}, {0, 0, 0, 0, 0, −1, 0., −1, 0, 0}, {0, 0, 0, 0, 0, 0, −1, 0., −1, 0}, {0, 0, 0, 0, 0, 0, 0, −1, 0., −1}, {0, 0, 0, 0, 0, 0, 0, 0, −1, 0.}}.ψ47839[0.0000204286], ψ47839[0] == {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

ReplaceAll: ReplaceAll called with 2 arguments; 1 argument is expected.

ReplaceAll:

{(0. + 1. $\bar{i}$) $\psi$$47839′[0.0000204286] == {{0., −1., 0., 0., 0., 0., 0., 0., 0., 0.}, {−1., 0., −1., 0., 0., 0., 0., 0., 0., 0.}, {0., −1., 0., −1., 0., 0., 0., 0., 0., 0.}, {0., 0., −1., 0., −1., 0., 0., 0., 0., 0.}, {0., 0., 0., −1., 0., −1., 0., 0., 0., 0.}, {0., 0., 0., 0., −1., 0., −1., 0., 0., 0.}, {0., 0., 0., 0., 0., −1., 0., −1., 0., 0.}, {0., 0., 0., 0., 0., 0., −1., 0., −1., 0.}, {0., 0., 0., 0., 0., 0., 0., −1., 0., −1.}, {0., 0., 0., 0., 0., 0., 0., 0., −1., 0.}}.$\psi$$47839[0.0000204286], $\psi$$47839[0.] == {0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.}} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

ReplaceAll: ReplaceAll called with 2 arguments; 1 argument is expected.

ReplaceAll:

{$\bar{i}$ $\psi$$47839′[0.0204286] == {{0., −1, 0, 0, 0, 0, 0, 0, 0, 0}, {−1, 0., −1, 0, 0, 0, 0, 0, 0, 0}, {0, −1, 0., −1, 0, 0, 0, 0, 0, 0}, {0, 0, −1, 0., −1, 0, 0, 0, 0, 0}, {0, 0, 0, −1, 0., −1, 0, 0, 0, 0}, {0, 0, 0, 0, −1, 0., −1, 0, 0, 0}, {0, 0, 0, 0, 0, −1, 0., −1, 0, 0}, {0, 0, 0, 0, 0, 0, −1, 0., −1, 0}, {0, 0, 0, 0, 0, 0, 0, −1, 0., −1}, {0, 0, 0, 0, 0, 0, 0, 0, −1, 0.}}.$\psi$$47839[0.0204286], $\psi$$47839[0] == {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

General: Further output of ReplaceAll::reps will be suppressed during this calculation.

ReplaceAll: ReplaceAll called with 2 arguments; 1 argument is expected.

General: Further output of ReplaceAll::argx will be suppressed during this calculation.

Create U

In[•]:=
```
ClearAll@constructU;
constructU[h_, tinit_, tfinal_, n_] := Module[{dt = N[(tfinal - tinit) / n],
    curVal = IdentityMatrix[Length@h[0]]},
  Do[curVal = MatrixExp[-I * h[t] * dt].curVal, {t, tinit, tfinal - dt, dt}];
  curVal]

Nl = 10;
psi0 = Table[If[i == Nl / 2, 1, 0], {i, 1, Nl}];

constructU[Ht[Nl, 0.01, 0. × 10], 0, 0.1, 10].psi0
(*ListLinePlot[
 Chop[constructU[Ht[Nl,0.01, 0, 10], 0, 0.1, 10]].psi0, PlotRange→All]*)
(*ListPlot[sortedEVecs[[1]],PlotRange→All],*)
```

In[•]:=
```
ListPlot[
 Table[Chop[constructU[Ht[Nl, 0.01, 0.01, #].psi0 &, 0, upt, 100]],
  {upt, .1, 1, .1}
 ],
 Joined → True,
 PlotRange → {-1, 1}
]
```

```
ListPlot[
 Table[
  Chop[#] &@(constructU[
      Ht[Nl, 0.05, F, #] &, 0, upt, 100].psi0),
  {upt, .01, 20, .1}
 ],
 Joined → True,
 PlotRange → {-1, 1}
]
```

```
ham[e1_, e2_, b_, omega_, t_] := {{e1, b * Cos[omega * t]}, {b * Cos[omega * t], e2}};
Module[{ψ, sol, tmax = 20},
 sol = NDSolve[
    {I D[ψ[t], t] == Ht[10, 0.05, 0.05, t] .ψ[t], ψ[0] == psi0}, ψ, {t, 0, tMax}];
```

```
Module[{ψ, sol, tmax = 20},
 sol = First@NDSolve[{I D[ψ[t], t] ==
      Ht[10, 0.05, 0.05, t] .ψ[t], ψ[0] == psi0}, ψ, {t, 0, tMax}];
 Plot[Chop[#*.PauliMatrix[3].#] &@(ψ /. sol)[t],
  {t, 0, tMax}, PlotRange → {-1, 1}]
]
```

Create U (2x2)

```
ham[e1_, e2_, b_, omega_, t_] := {{e1, b * Cos[omega * t]}, {b * Cos[omega * t], e2}}
```

```
ClearAll@constructU;
constructU[h_, tinit_, tfinal_, n_] := Module[{dt = N[(tfinal - tinit) / n],
    curVal = IdentityMatrix[Length@h[0]]},
  Do[curVal = MatrixExp[-I * h[t] * dt].curVal, {t, tinit, tfinal - dt, dt}];
  curVal]
```

```
ClearAll[cU, psi0];
psi0 = {1., 0};
Manipulate[
 ListPlot[
  Table[
   Chop[#*.PauliMatrix[3].#] &@(constructU[
       ham[-1., 1., b, 1., #] &, 0, upt, 100].psi0),
   {upt, .01, 20, .1}
  ],
  Joined → True,
  PlotRange → {-1, 1}
 ],
 {b, 0, 2}
]
```

NDsolve (2x2)

In[ ]:=
```
Manipulate[Module[{ψ, sol, tmax = 20},
  sol = First@NDSolve[{I D[ψ[t], t] ==
       ham[-1, 1, b, 1, t] .ψ[t], ψ[0] == {1, 0}}, ψ, {t, 0, 1}];
  Plot[Chop[#*.PauliMatrix[3].#] &@(ψ /. sol)[t],
   {t, 0, 1}, PlotRange → {-1, 1}]
 ],
 {{b, 1}, 0, 2}
]
```

Out[ ]=