Periodic boundary conditions - matrix eigenvalue solver

```
In[31]:= NN = 8;
Js = Table[-J, NN - 1];
hupper = DiagonalMatrix[Js, 1];
hlower = DiagonalMatrix[Js, -1];
H = hupper + hlower;
H[[1, NN]] = -J;
H[[NN, 1]] = -J;
MatrixForm[H];
Eigenvalues[H];
```

Open boundary conditions - matrix eigenvalue solver

```
In[23]:= NN = 3;
Js = Table[-J, NN - 1];
hupper = DiagonalMatrix[Js, 1];
hlower = DiagonalMatrix[Js, -1];
H = hupper + hlower;
MatrixForm[H];
EE = Eigenvalues[H];
Expand[Apply[Times, Function[x, λ - x] /@ EE]];
(*N[Apply[Times, EE]]*)
```

Open boundary conditions - polynomial solver

```
NN = 3;
matfun[x_] := Factor[λ / J - 1 / x];
diagonalterms = NestList[matfun, λ / J, NN - 1];

diagonaltermsproduct = Apply[Times, diagonalterms] * J^NN;
solutions = Solve[diagonaltermsproduct == 0, λ]
```

Out[•]= $\{\{\lambda \to 0\}, \{\lambda \to -\sqrt{2}\,J\}, \{\lambda \to \sqrt{2}\,J\}\}$

Periodic boundary conditions - polynomial solver

In[55]:=
```
NN = 60;
matfun[x_] := Factor[λ / J - 1 / x];
AList = NestList[matfun, λ / J, NN - 2];
AListAlgebraic = Table[Subscript[A, i], {i, NN - 1}];

(*create end term*)
AProduct = Apply[Times, AList];
tmplist = FoldList[#1 * #2^2 &, 1, AList];
tmplist1 = tmplist[[2 ;; -1]] / AList;
tmplist2 = Map[-1 / ## &, tmplist1][[2 ;;]];
tmplist3 = Apply[Plus, tmplist2];
end = AList[[2]] - (-1)^NN * 2 / AProduct - 1 / AList[[NN - 1]] + tmplist3;

diagonaltermsproduct = AProduct * J^NN * end;
solutions = Solve[diagonaltermsproduct == 0, λ];
sol2 = DeleteDuplicates[solutions]
```

Out[67]= $\left\{\{\lambda\to 0\},\{\lambda\to -2J\},\{\lambda\to -J\},\{\lambda\to J\},\{\lambda\to 2J\},\right.$

$\left\{\lambda\to -\sqrt{3}\,J\right\},\left\{\lambda\to \sqrt{3}\,J\right\},\left\{\lambda\to -\frac{\sqrt{3J^2-\sqrt{5}\,J^2}}{\sqrt{2}}\right\},\left\{\lambda\to \frac{\sqrt{3J^2-\sqrt{5}\,J^2}}{\sqrt{2}}\right\},$

$\left\{\lambda\to -\frac{\sqrt{5J^2-\sqrt{5}\,J^2}}{\sqrt{2}}\right\},\left\{\lambda\to \frac{\sqrt{5J^2-\sqrt{5}\,J^2}}{\sqrt{2}}\right\},\left\{\lambda\to -\frac{\sqrt{3J^2+\sqrt{5}\,J^2}}{\sqrt{2}}\right\},$

$\left\{\lambda\to \frac{\sqrt{3J^2+\sqrt{5}\,J^2}}{\sqrt{2}}\right\},\left\{\lambda\to -\frac{\sqrt{5J^2+\sqrt{5}\,J^2}}{\sqrt{2}}\right\},\left\{\lambda\to \frac{\sqrt{5J^2+\sqrt{5}\,J^2}}{\sqrt{2}}\right\},$

$\left\{\lambda\to -\frac{1}{2}\sqrt{9J^2-\sqrt{5}\,J^2-\sqrt{6\left(5+\sqrt{5}\right)}\,\sqrt{J^4}}\right\},\left\{\lambda\to \frac{1}{2}\sqrt{9J^2-\sqrt{5}\,J^2-\sqrt{6\left(5+\sqrt{5}\right)}\,\sqrt{J^4}}\right\},$

$\left\{\lambda\to -\frac{1}{2}\sqrt{7J^2+\sqrt{5}\,J^2-\sqrt{6\left(5+\sqrt{5}\right)}\,\sqrt{J^4}}\right\},\left\{\lambda\to \frac{1}{2}\sqrt{7J^2+\sqrt{5}\,J^2-\sqrt{6\left(5+\sqrt{5}\right)}\,\sqrt{J^4}}\right\},$

$\left\{\lambda\to -\frac{1}{2}\sqrt{9J^2-\sqrt{5}\,J^2+\sqrt{6\left(5+\sqrt{5}\right)}\,\sqrt{J^4}}\right\},\left\{\lambda\to \frac{1}{2}\sqrt{9J^2-\sqrt{5}\,J^2+\sqrt{6\left(5+\sqrt{5}\right)}\,\sqrt{J^4}}\right\},$

$\left\{\lambda\to -\frac{1}{2}\sqrt{7J^2+\sqrt{5}\,J^2+\sqrt{6\left(5+\sqrt{5}\right)}\,\sqrt{J^4}}\right\},\left\{\lambda\to \frac{1}{2}\sqrt{7J^2+\sqrt{5}\,J^2+\sqrt{6\left(5+\sqrt{5}\right)}\,\sqrt{J^4}}\right\},$

$\left\{\lambda\to -\frac{1}{2}\sqrt{7J^2-\sqrt{5}\,J^2-\sqrt{6}\,\sqrt{-\left(-5+\sqrt{5}\right)J^4}}\right\},$

$\left\{\lambda\to \frac{1}{2}\sqrt{7J^2-\sqrt{5}\,J^2-\sqrt{6}\,\sqrt{-\left(-5+\sqrt{5}\right)J^4}}\right\},$

$\left\{\lambda\to -\frac{1}{2}\sqrt{9J^2+\sqrt{5}\,J^2-\sqrt{6}\,\sqrt{-\left(-5+\sqrt{5}\right)J^4}}\right\},$

$\left\{\lambda\to \frac{1}{2}\sqrt{9J^2+\sqrt{5}\,J^2-\sqrt{6}\,\sqrt{-\left(-5+\sqrt{5}\right)J^4}}\right\},$

$\left\{\lambda\to -\frac{1}{2}\sqrt{7J^2-\sqrt{5}\,J^2+\sqrt{6}\,\sqrt{-\left(-5+\sqrt{5}\right)J^4}}\right\},$

$\left\{\lambda\to \frac{1}{2}\sqrt{7J^2-\sqrt{5}\,J^2+\sqrt{6}\,\sqrt{-\left(-5+\sqrt{5}\right)J^4}}\right\},$

$\left\{\lambda\to -\frac{1}{2}\sqrt{9J^2+\sqrt{5}\,J^2+\sqrt{6}\,\sqrt{-\left(-5+\sqrt{5}\right)J^4}}\right\},$

$\left.\left\{\lambda\to \frac{1}{2}\sqrt{9J^2+\sqrt{5}\,J^2+\sqrt{6}\,\sqrt{-\left(-5+\sqrt{5}\right)J^4}}\right\}\right\}$

In[41]:= **Cos[pi / 5]**

Out[41]= $\mathrm{Cos}\left[\frac{\mathrm{pi}}{5}\right]$