COMPUTATIONALLY EFFICIENT METHODS FOR UNCERTAINTY QUANTIFICATION IN SEISMIC INVERSION

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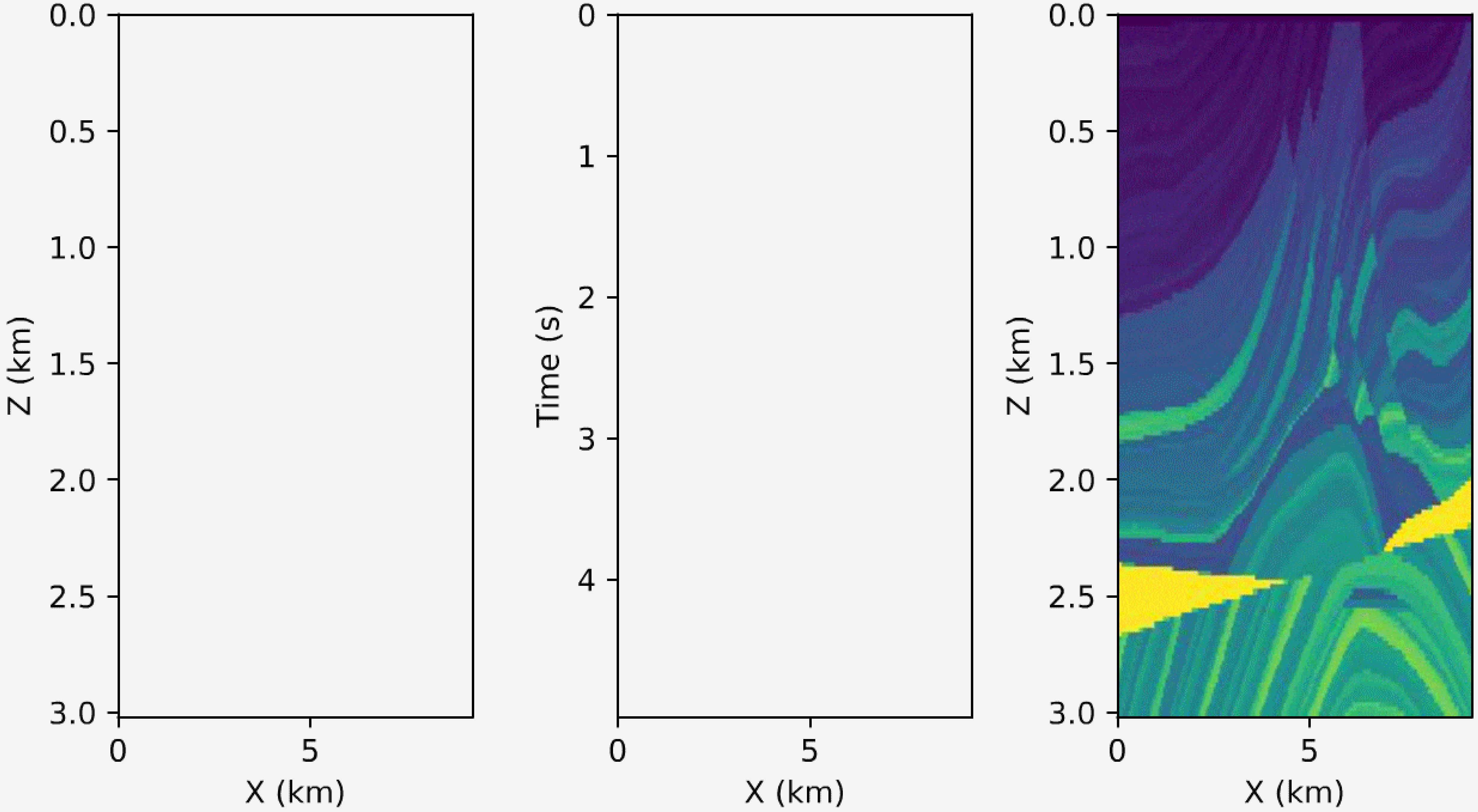
OUTLINE

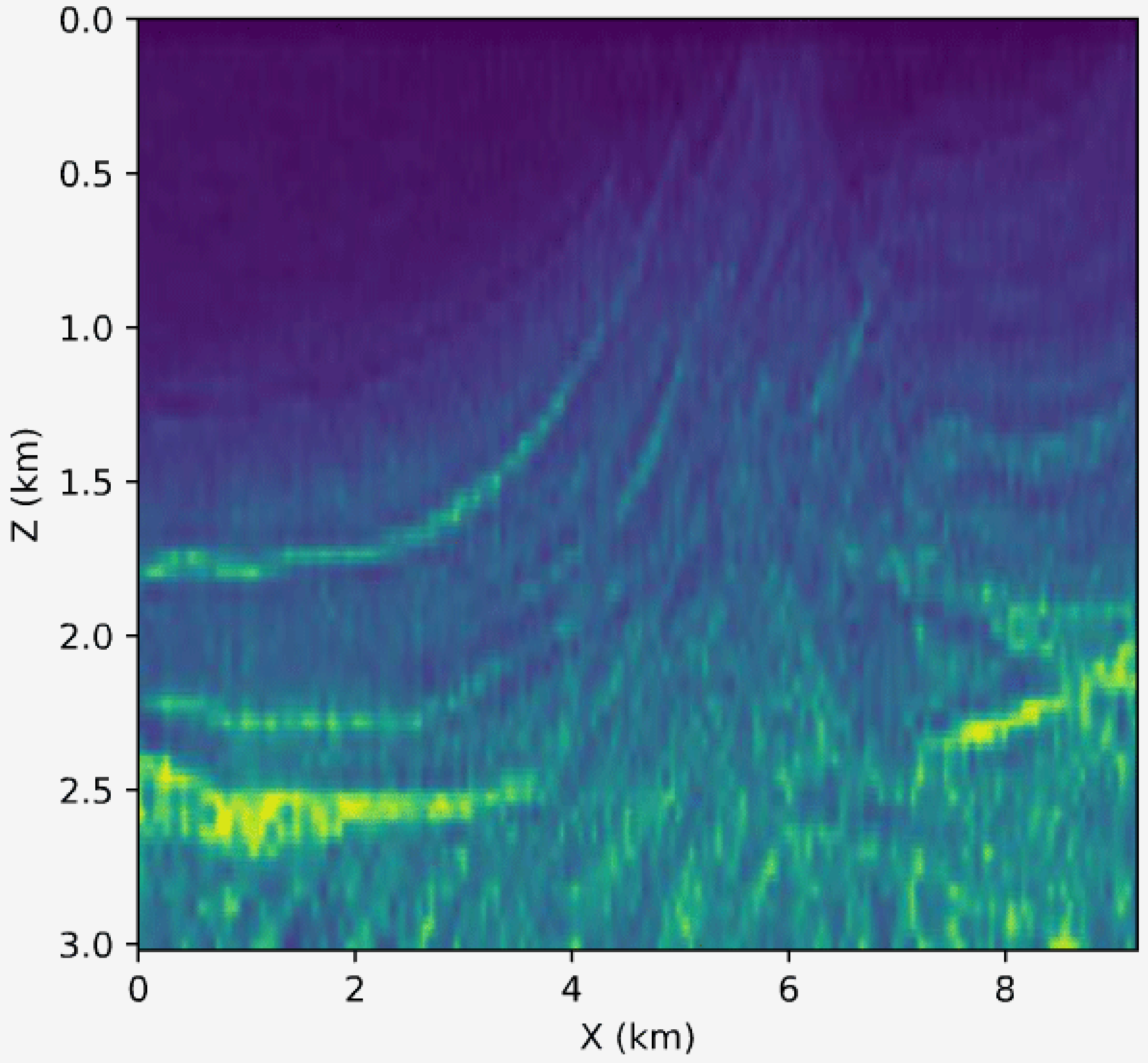
EXPLORATION SEISMOLOGY

WAVE DATA


0.0
0.5
1.0
1.5
2.0
2.5
3.0
Z (km)
0
5
X (km)
0
1
2
3
4
Time (s)
0
5
X (km)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Z (km)
0
5
X (km)

FULL WAVEFORM INVERSION (FWI)

UNCERTAINTY QUANTIFICATION AND FWI



BAYES' RULE

MARKOV CHAIN MONTE CARLO (MCMC)

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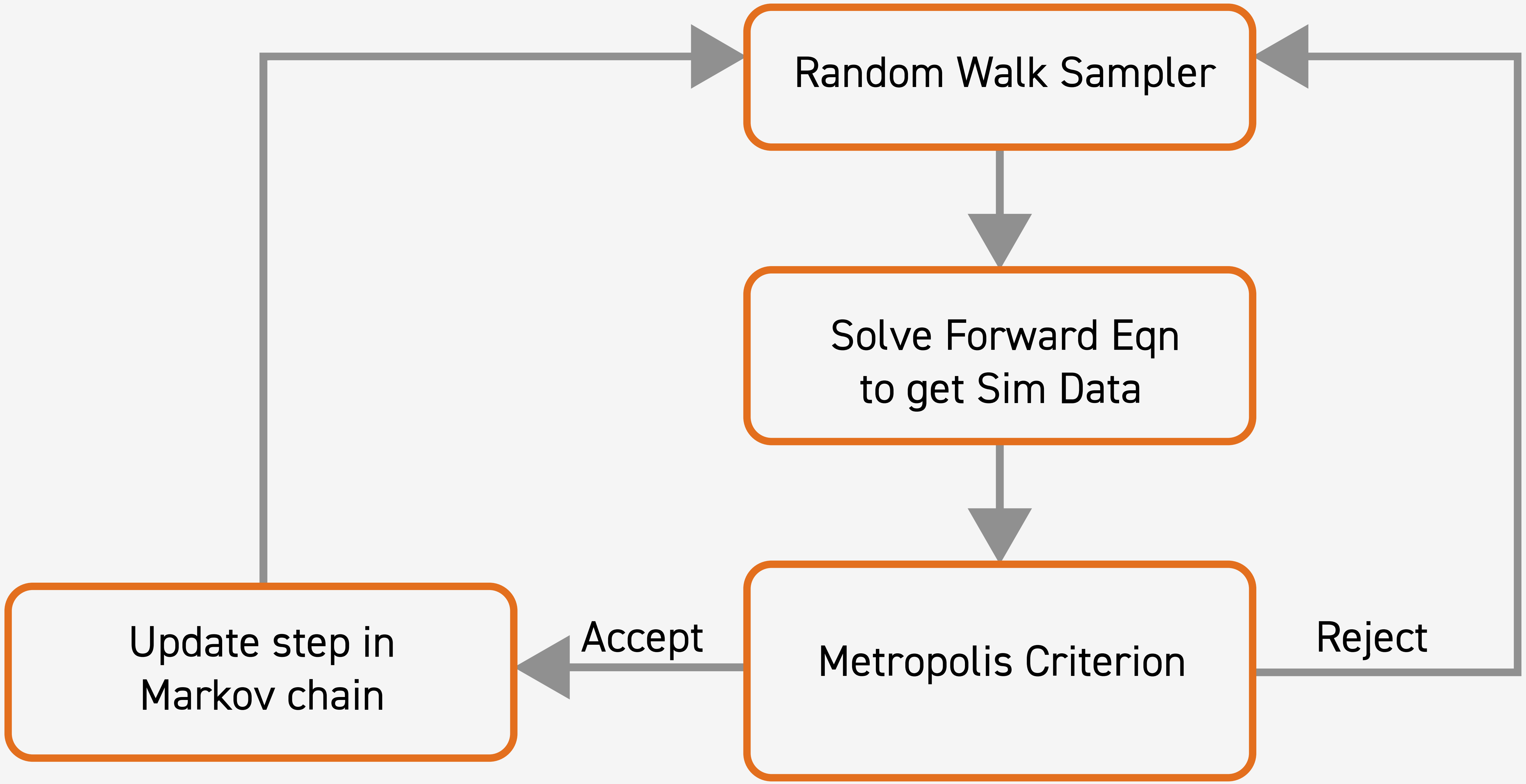


THE PROBLEM WITH MCMC

HOW CAN WE REDUCE THE COMPUTATIONAL COST OF MCMC METHODS FOR FWI?

STRATEGIES

TWO-STAGE MCMC

CHOICE OF FILTERS

OPERATOR UPSCALING

- Modeling wave propagation can be computationally expensive.
- We use the 2D constant-density acoustic wave equation

$$\frac{1}{c^2(x,z)}\frac{\partial^2 p}{\partial t^2} - \nabla p = f$$

- Operator upscaling[1] decomposes the solution into two parts:
  1. Fine grid problem on independent subdomains
  2. Small coarse grid problem over the whole domain
- In this upscaling technique we do NOT upscale the model.

(1) Vdovina et al. (2005), Korostyshevskaya and Minkoff (2006), Vdovina and Minkoff (2008)

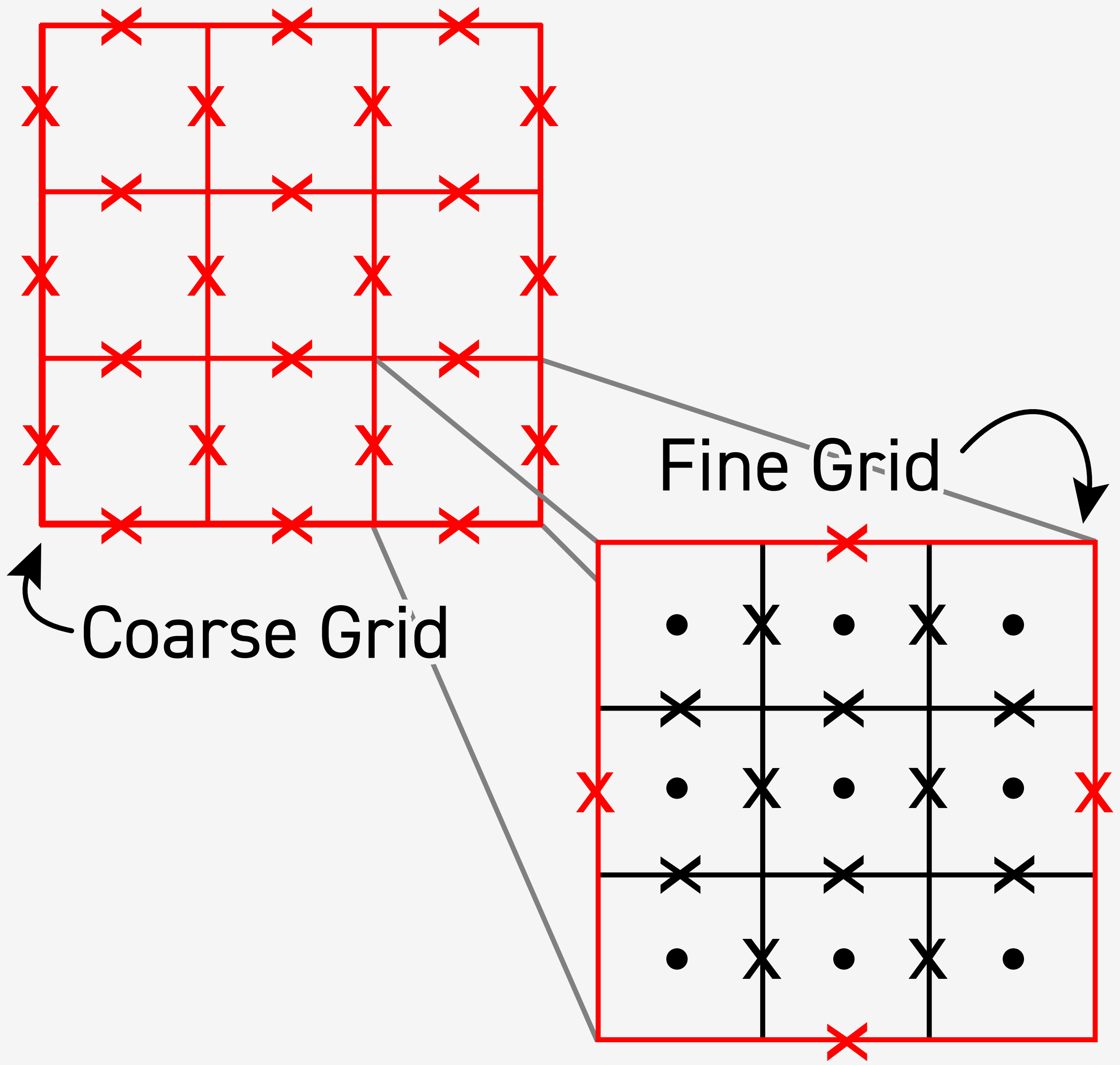
OPERATOR UPSCALING

1. Write the acoustic wave equation as a system in space by introducing acceleration, $\vec{v}$

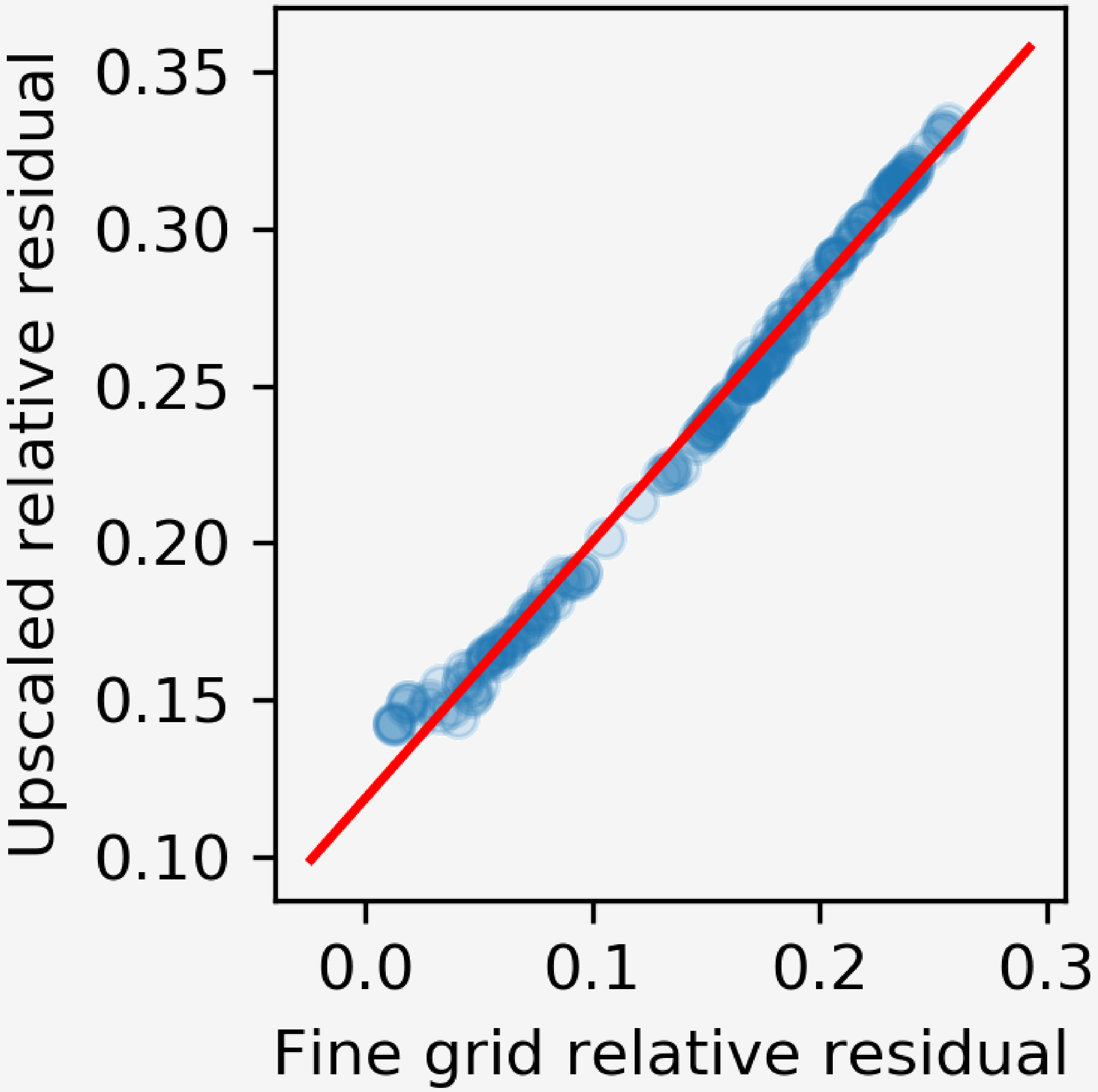
$$\vec{v} = -\nabla p$$
$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = -\nabla \cdot \vec{v} + f$$

2. Solve in parallel for fine grid pressure and acceleration over each independent coarse block. No communication is required at this stage.

3. Solve for coarse grid acceleration over the whole domain.



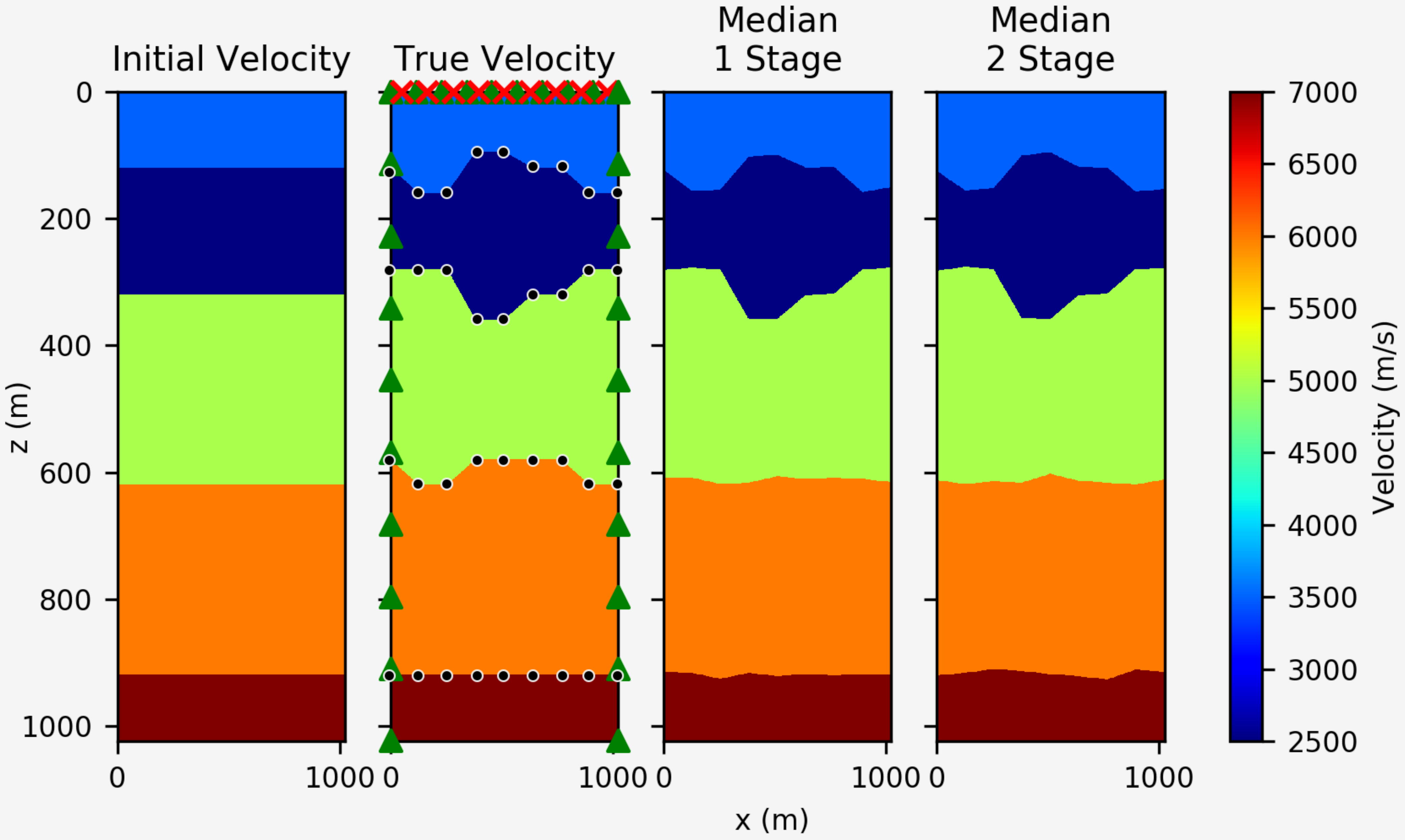
UPSCALING AND FINE GRID CORRELATION



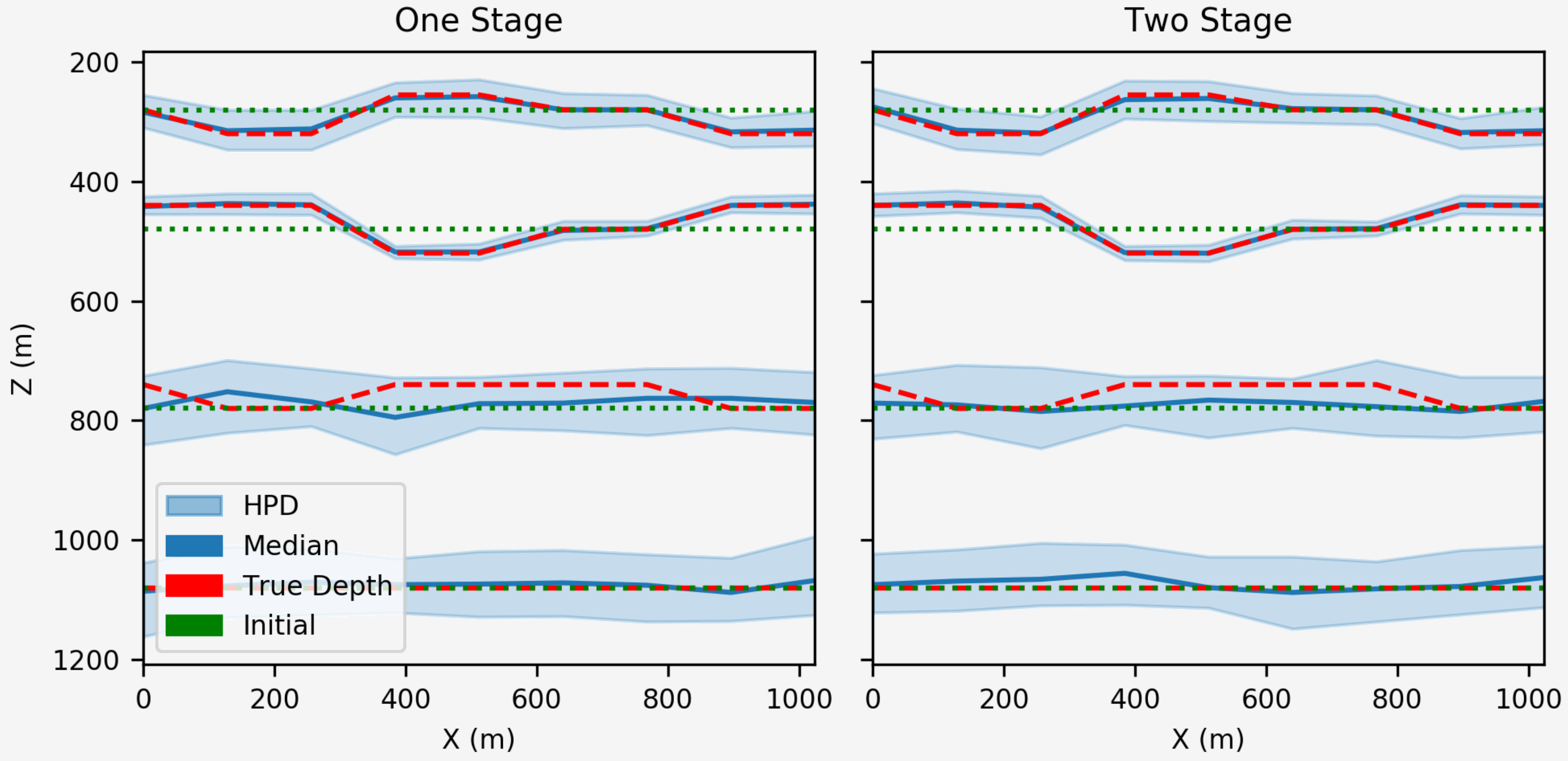
- We see a strong linear relationship between the fine grid relative residuals and the upscaled relative residuals for a layered velocity model.

- This indicates that the upscaling filter is a good surrogate for the fine grid solver.

RESULTS: TWO-STAGE MCMC WITH UPSCALING



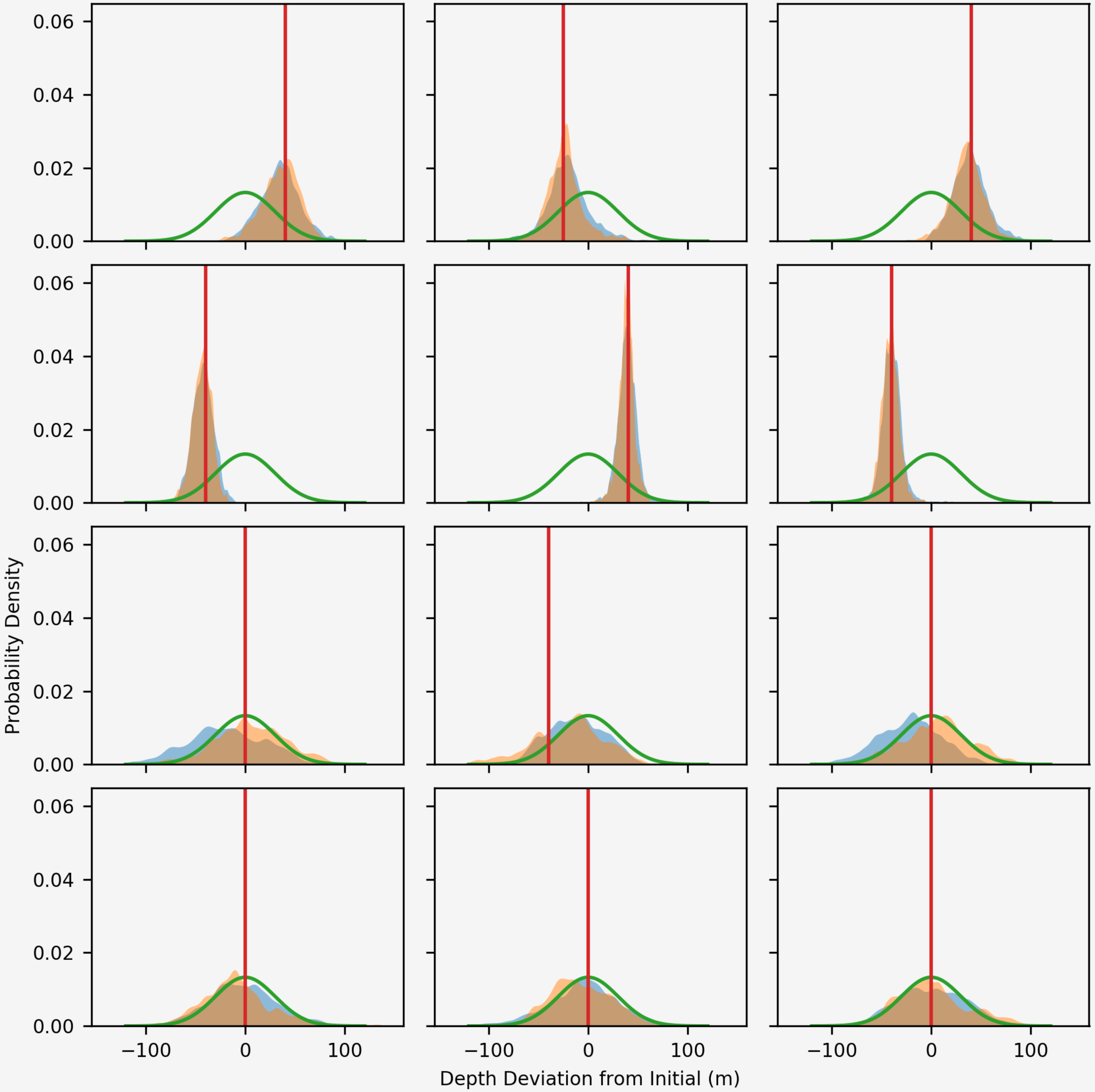
RESULTS: TWO-STAGE MCMC WITH UPSCALING



A comparison between one-stage MCMC highest posterior density (HPD) intervals and two-stage MCMC HPD intervals.

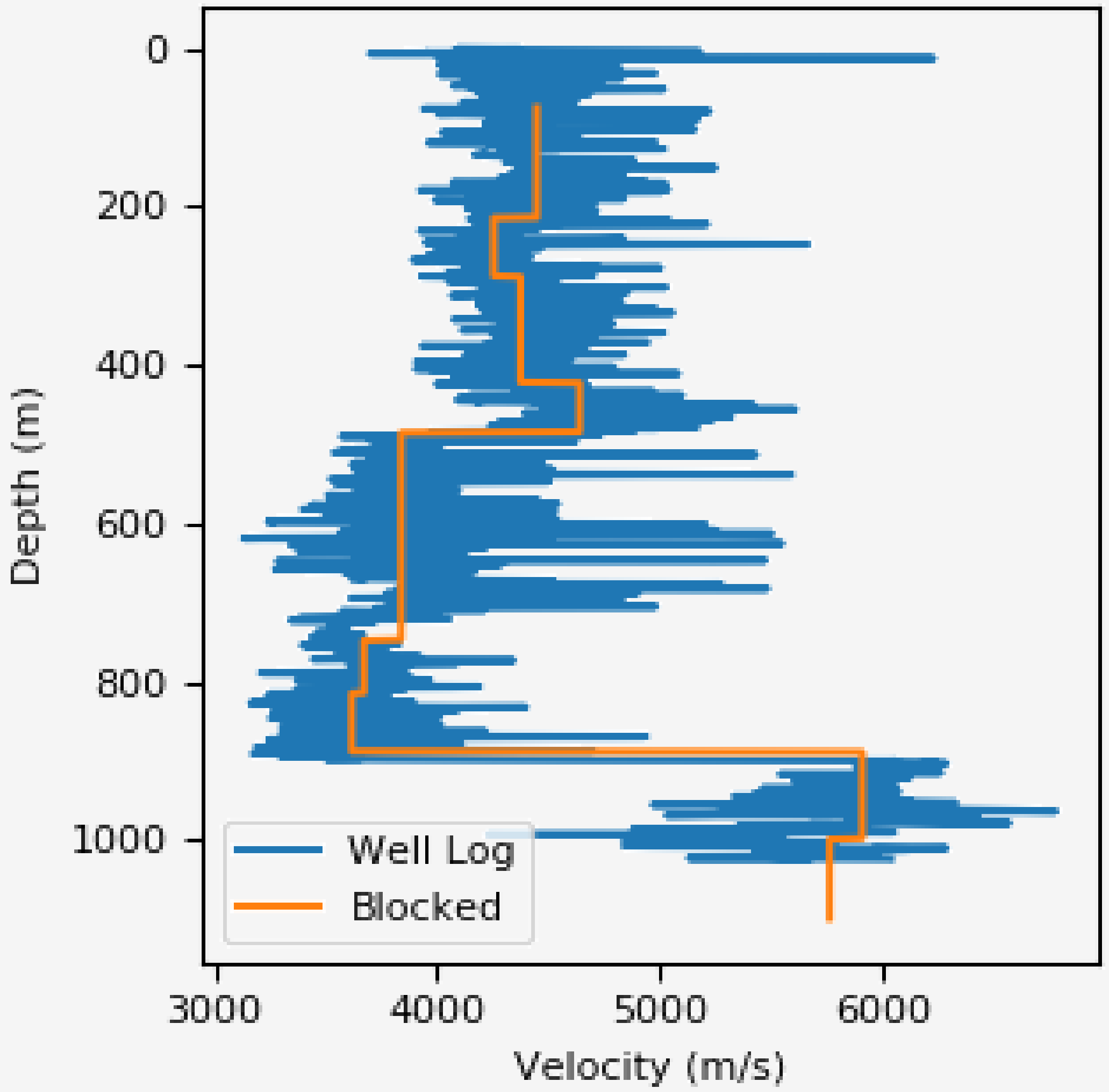
- Acceptance rate increases from 10% to 40%.
- Time per sample decreases by 22% (40% in other experiments).
- Time per rejection decreases by 33%.

RESULTS: TWO-STAGE MCMC WITH UPSCALING

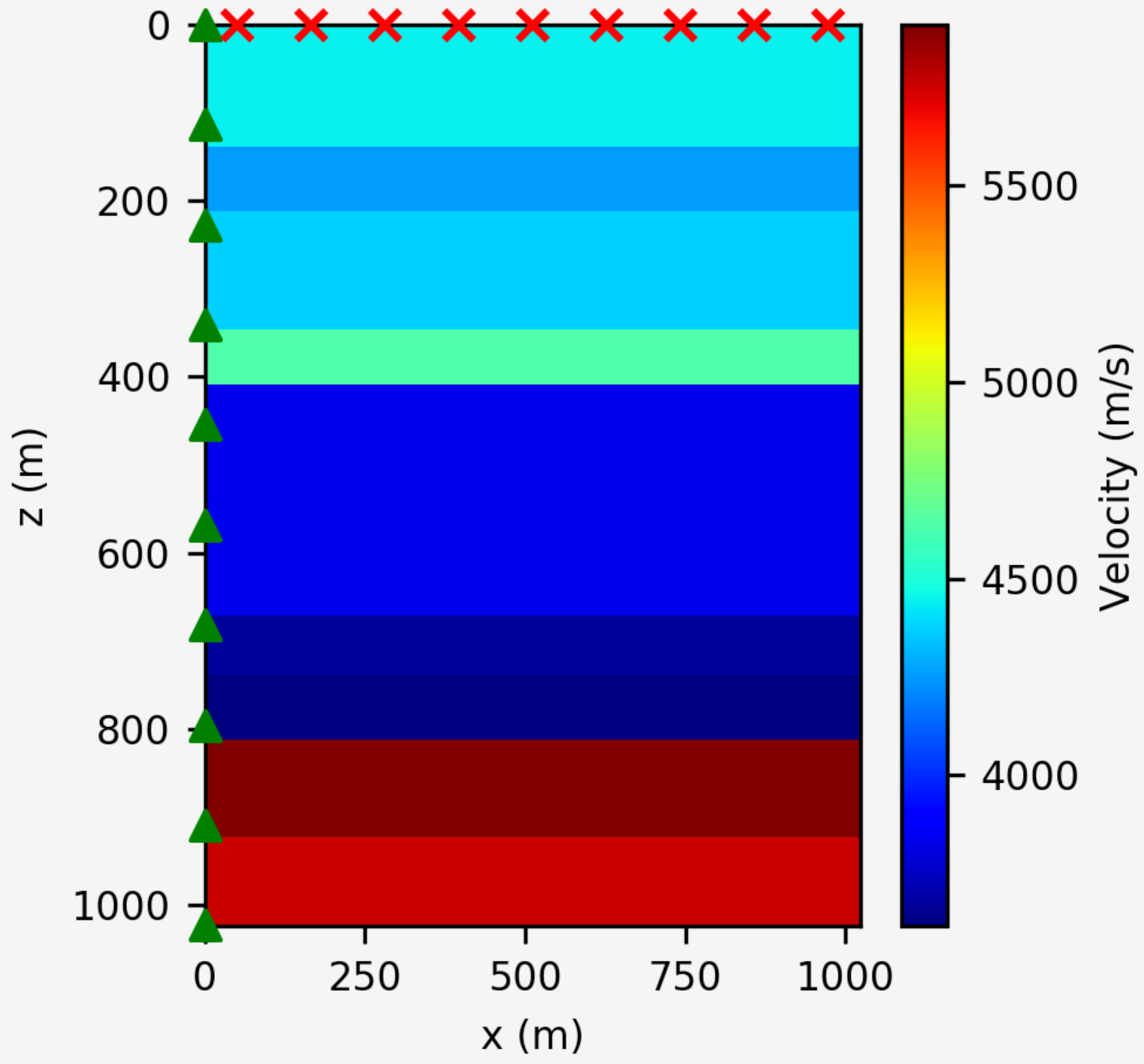


NEURAL NETWORK FILTER

RESULTS: TWO-STAGE MCMC WITH NEURAL NET



Well log from the Midland, TX basin (blue, courtesy of Pioneer Natural Resources and 9-layer block (orange).



Flat layered experimental setup (Stuart et al. 2019a)

RESULTS: TWO-STAGE MCMC WITH NEURAL NET

TROUBLE: THE RANDOM WALK SAMPLER

- In theory, MCMC will converge to the target distribution.
- In practice, methods based on random walk sampling (RWS) can handle a limited number of unknowns (< 100 in our experience)
- RWS produces samples that are highly correlated.

Neal (2011)

THE RANDOM WALK SAMPLER PRACTICALLY LIMITS THE NUMBER OF UNKNOWNS WE CAN USE

HAMILTONIAN MONTE CARLO (HMC)

THE HAMILTONIAN

Potential Energy

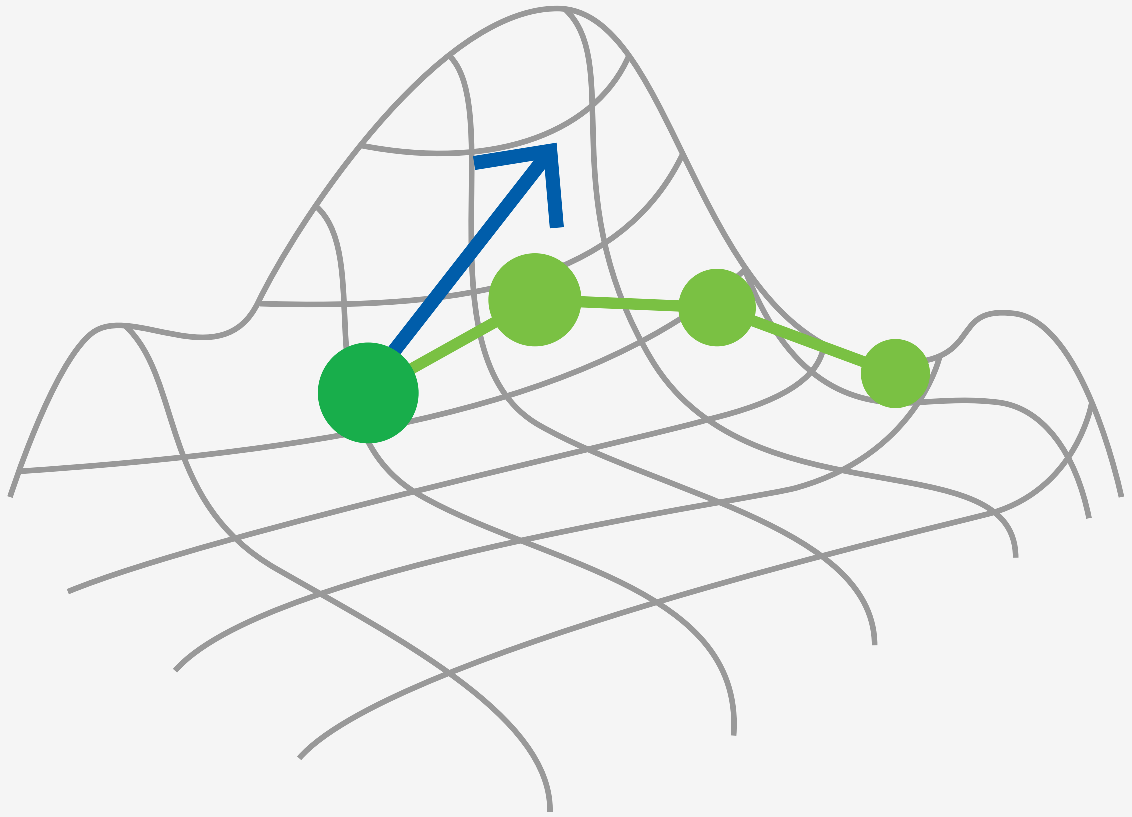
Kinetic Energy

$$H(q, p) = U(q) + K(p)$$

Hamiltonian

Position

Momentum



$$\frac{dq}{dt} = \frac{\partial H(q,p)}{\partial p}$$

$$\frac{dp}{dt} = -\frac{\partial H(q,p)}{\partial q}$$

HMC FLOWCHART

PROBLEM: GRADIENT COMPUTATION IS EXPENSIVE!

TWO-STAGE HAMILTONIAN MONTE CARLO

NEURAL NETWORK-ENHANCED TWO-STAGE HMC (NNHMC)

NUMERICAL EXPERIMENTS: NNHMC

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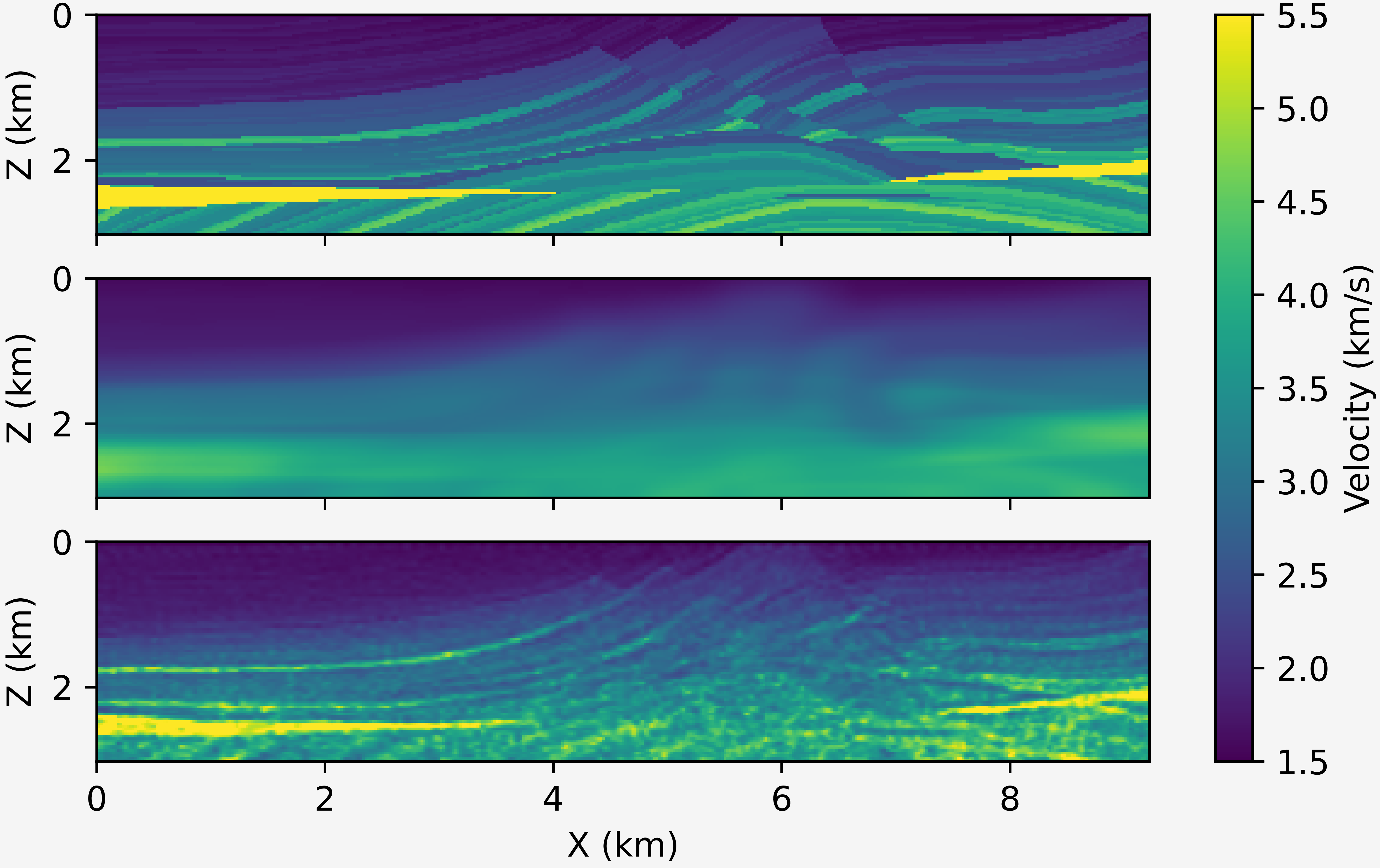
HMC REQUIRES USER-SPECIFIED PARAMETERS TO DISCRETIZE THE HAMILTONIAN DYNAMICS

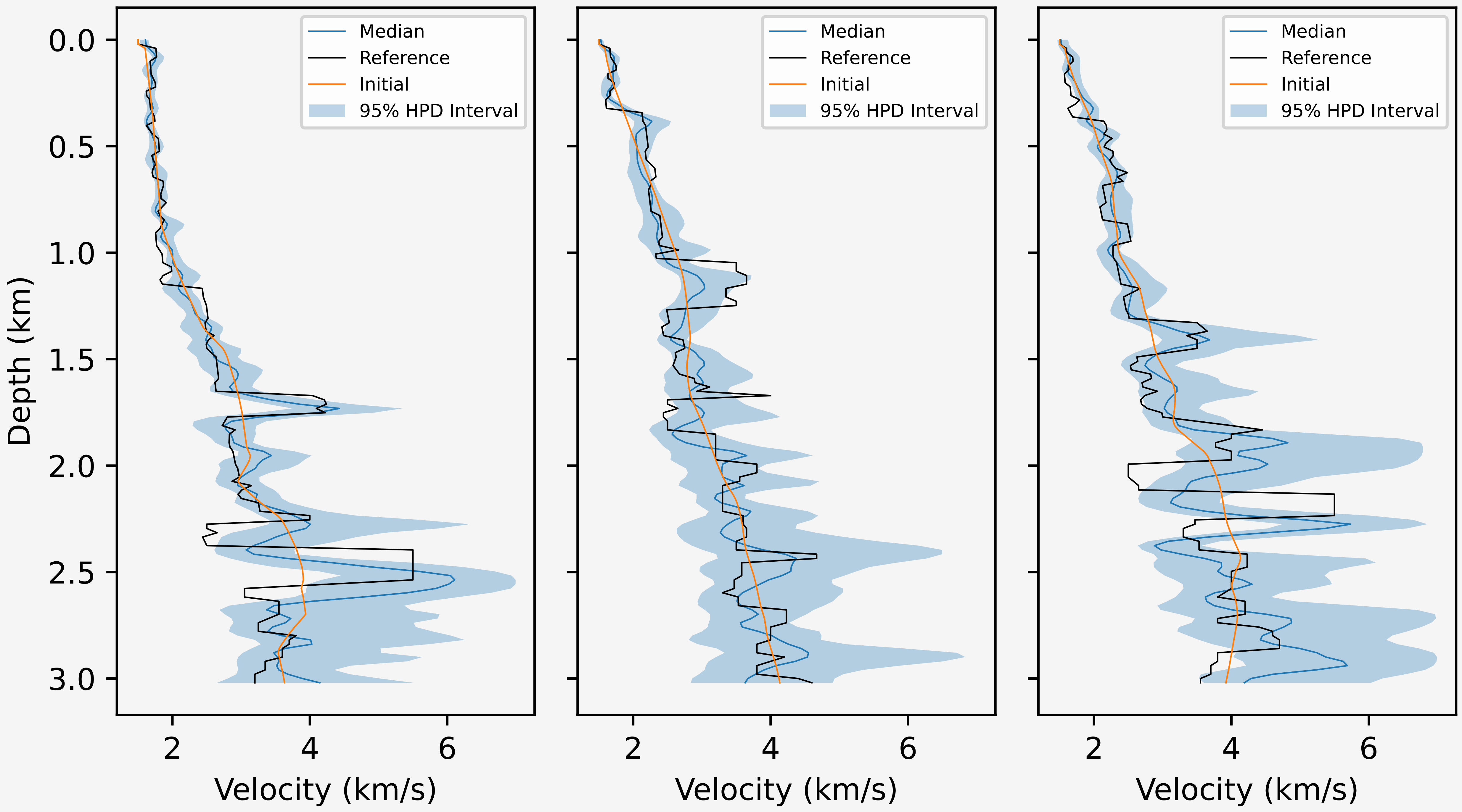
THE NO-U-TURN SAMPLER (NUTS)

THE NO-U-TURN SAMPLER

NUMERICAL EXPERIMENT: NUTS



NUMERICAL EXPERIMENT: NUTS



Vertical slices with HPD intervals

NUMERICAL EXPERIMENT: NUTS

Posterior Distributions

CONCLUSIONS

FUTURE WORK

ACKNOWLEDGEMENTS