

1 Introduction

We consider the solution of the Fokker-Planck equation for bivariate Brownian motion with absorbing boundaries and correlated increments. In particular, we find the transition density function for the system

$$X_i(t) = x_i + \mu_i + \sigma_i W_i(t), \quad i = 1, 2, \quad 0 < t \leq T, \quad (1)$$

$$a_i \leq X_i(t) \leq b_i, \quad (2)$$

where W_i are standard Brownian motions with $\text{Cov}(W_1(t), W_2(t)) = \rho t$. Our solution is obtained by combining an analytic small-time solution with an approximate finite element method.

Closed-form solutions to (??) - (??) are available for different parameter regimes. For example, when $\rho = 0$, the transition density of the process can be obtained with a Fourier expansion. When $a_1 = -\infty$ and $b_1 = \infty$, the method of images can be used to enforce the remaining boundaries. For either $a_1, a_2 = -\infty$ or $b_1, b_2 = \infty$, an analytic solution exists for a set of countably many values for ρ . However, to the best of our knowledge, there is no closed-form solution to the general problem in (??) - (??). The transition density for this problem is of interest particularly in problems dealing with first passage times (??), structural models in credit risk and default correlations (??), and pricing of double lookback options (?).