0.1 A small-parameter analytic solution

In the previous chapter we introduced a numerical solution that works for relatively benign parameter ranges. However, the numerical solution fails for small parameters of $(\tilde{\sigma}, \tilde{t})$. Here, we introduce an analytic solution that works for certain small-$\tilde{\sigma}$.

0.1.1 Illustration with $\rho =$

The approach we use to motivate the calculations for the small-parameter solution is considering the problem with $\rho = 0$. Here we consider the normalized problme such that $\tilde{\sigma} \leq 1$ and the diffusion problem at hand is given by

$$\frac{\partial q}{\partial t} = \frac{1}{2}\frac{\partial^2 q}{\partial x^2} + \frac{1}{2}\tilde{\sigma}^2\frac{\partial^2 q}{\partial y^2}$$