

# 1 Introduction

The estimation and prediction of price volatility from market data is an important problem in econometrics and finance [Abramov and Klebaner, 2007], as well as practical risk management [Brandt and Santa-Clara, 2006]. The literature on the subject of volatility estimation is vast. Model-based approaches for a single observable asset begin with the ARCH and GARCH models of Engle [1982] and Bollerslev [1986], moving on to stochastic volatility models (see Shephard [2005], for example).

Multivariate equivalents for each of these model classes exist (see Bauwens et al. [2006] and Asai et al. [2006] for reviews of multivariate GARCH and for multivariate stochastic volatility, respectively). However, the majority of work on the subject uses opening and closing prices as data. This approach invariably disregards information traditionally contained in financial timeseries: the observed high and low price of an asset over the quoted periods. To our current knowledge, only Horst et al. [2012] use the observed maximum and minimum of prices in a likelihood to estimate volatility. They do so, however, in a univariate setting.

Explicit model-based approaches in the multivariate setting which take into account extrema over observational periods are completely lacking in the literature, because deriving an efficient approximation of the corresponding likelihood function has hereto been an open problem [cite?]. In this paper, we use a result addressing this problem and introduce a novel, *bivariate* stochastic volatility model which takes into account the highest and lowest observed prices of each asset as part of a likelihood-based (Bayesian) estimation procedure.

# 2 Model

The model we will use is a bivariate 1-factor stochastic volatility model with leverage:

$$\begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} x_{t-\Delta} \\ y_{t-\Delta} \end{pmatrix} + \begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix} + \begin{pmatrix} \sqrt{1-\rho_t^2}\sigma_{x,t} & \rho_t \frac{\sigma_{x,t}}{\sigma_{y,t}} \\ 0 & \sigma_{y,t} \end{pmatrix} \begin{pmatrix} \varepsilon_{x,t} \\ \varepsilon_{y,t} \end{pmatrix} \quad (1)$$

$$\log(\sigma_{x,t+\Delta}) = \alpha_x + \theta_x(\log(\sigma_{x,t}) - \alpha_x) + \tau_x \eta_{x,t} \quad (2)$$

$$\log(\sigma_{y,t+\Delta}) = \alpha_y + \theta_y(\log(\sigma_{y,t}) - \alpha_y) + \tau_y \eta_{y,t} \quad (3)$$

$$\text{logit}((\rho_{t+\Delta} + 1)/2) = \text{logit}((\rho_t + 1)/2) + \tau_\rho \eta_{\rho,t} \quad (4)$$

# References

- Vyacheslav M Abramov and Fima C Klebaner. Estimation and prediction of a non-constant volatility. *Asia-Pacific Financial Markets*, 14(1):1–23, 2007.
- Manabu Asai, Michael McAleer, and Jun Yu. Multivariate stochastic volatility: a review. *Econometric Reviews*, 25(2-3):145–175, 2006.
- Luc Bauwens, Sébastien Laurent, and Jeroen VK Rombouts. Multivariate garch models: a survey. *Journal of applied econometrics*, 21(1):79–109, 2006.
- Tim Bollerslev. Generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*, 31(3): 307–327, 1986.
- Michael W Brandt and Pedro Santa-Clara. Dynamic portfolio selection by augmenting the asset space. *The Journal of Finance*, 61(5):2187–2217, 2006.
- Robert F Engle. Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. *Econometrica: Journal of the Econometric Society*, pages 987–1007, 1982.

Enrique Ter Horst, Abel Rodriguez, Henryk Gzyl, and German Molina. Stochastic volatility models including open, close, high and low prices. *Quantitative Finance*, 12(2):199–212, 2012.

Neil Shephard. *Stochastic Volatility: Selected Readings*. Oxford University Press, UK, 2005.