1 Introduction

We consider the solution of the Fokker-Planck equation for bivariate Brownian motion with absorbing boundaries and correlated increments. In particular, we find the transition density function for the system

$$X_i(t) = x_i + \mu_i + \sigma_i W_i(t), \qquad i = 1,2, \qquad 0 < t \leq T, \tag{1}$$

$$a_i \leq X_i(t) \leq b_i, \qquad 0 < t \leq T, \tag{2}$$

where W_i are standard Brownian motions with $\mathrm{Cov}(W_1(t), W_2(t)) = \rho t$. Our solution is obtained by combining an analytic small-time solution with an approximate finite element method.

Closed-form solutions to (1) - (2) are available for different parameter regimes. For example, when $\rho = 0$, the transition density of the process can be obtained with a Fourier expansion. When $a_1 = -\infty$ and $b_1 = \infty$, the method of images can be used to enforce the remaining boundaries. For either $a_1, a_2 = -\infty$ or $b_1, b_2 = \infty$, an analytic solution exists for a set of countably many values for ρ. However, to the best of our knowledge, there is no closed-form solution to the general problem in (1) - (2). The transition density for this problem is of interest particularly in problems dealing with first passage times [Kou et al., 2016, Sacerdote et al., 2012], structural models in credit risk and default correlations [Haworth et al., 2008, Ching et al., 2014], and pricing of double lookback options [He et al., 1998].

References

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