Abstract

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1 Motivation

2 Solution on $\Omega \subset \mathbb{R}$

In this Section we will demonstrate the method outlined in Section 1 where the solution is defined on a bounded interval on $\mathbb{R}$. In this case, we have the true solution to the diffusion equation. We will compare the asymptotic expansion to the true solutio.

The PDE we will solve is the following BC/IC problem

$$\frac{\partial}{\partial t} q(x,t) = \frac{1}{2}\sigma^2 \frac{\partial^2}{\partial x^2} q(x,t), \tag{1}$$

$$q(x,0) = \delta_{x_0}(x), \tag{2}$$

$$q(a,t) = q(b,t) = 0. \qquad \text{(i.e. } \Omega = [a,b]\text{)} \tag{3}$$

Problem (1) - (3) can be solved in a variety of ways. We will use the method of images, which repeatedly reflects the fundamental solution

$$q_{fundamental}(x,t) = \frac{1}{\sqrt{2\pi\sigma^2 t}} \exp\left\{ -\frac{1}{2\sigma^2 t}(x - x_0)^2 \right\}$$

about the boundary points a and b. The steps for the full solutions are as follows:

Step 1: Select a kernel for the basis expansion,

Step 2: Perform Gram-Schmidt orthogonalization on the polynomials basis,

Step 3: Compute the weight for each basis element,

Step 4: Profit.

2.1 A suitable kernel for the basis elements

As noted in the motivating Section 1, the kernel we will use must be