First Order Predicate Logic

Overview

The assignment asks to solve several logic problems using First Order Predicate Logic (FOPL). I have completed all three parts and provided an alternative solution to the last problem as an extension. For all problems, I have, first, defined the functions and objects that will be used. Second, converted the given statements and the required statement to FOPL and conjunctive normal form (CNF). Third, negated the required statement. Finally, used resolution to show that the obtained set of clauses is unsatisfiable, therefore the negation of the required statement is false, and so the required statement is true.

Part 1. Some problems by Lewis Carroll

Ice cream

Definitions

D(X): X is a dessert
N(X): X is nice
IC(X): X is ice cream
H(X): X is healthy

Logic statements

(a) All desserts are nice.

$$\forall X \quad D(X) \rightarrow N(X)$$
$$\forall X \quad \neg D(X) \lor N(X)$$
$$\neg D(X) \lor N(X)$$

(b) Ice cream is a dessert.

$$\forall X \quad IC(X) \rightarrow D(X)$$
$$\forall X \quad \neg IC(X) \lor D(X)$$
$$\neg IC(Y) \lor D(X)$$

(c) No nice things are healthy.

$$\neg \exists X \quad N(X) \land H(X)$$
$$\forall X \quad \neg N(X) \lor \neg H(X)$$
$$\neg N(Z) \lor \neg H(Z)$$

Required statement: Ice cream is not healthy.

$$\forall X \quad IC(X) \rightarrow \neg H(X)$$
$$\forall X \quad \neg IC(X) \vee \neg H(X)$$

Negated (p):

$$\neg(\forall X \quad \neg IC(X) \vee \neg H(X))$$
$$\exists X \quad IC(X) \wedge H(X)$$

Let *skolem* be an object that satisfies the above clauses. We now have:
(p1):

$$IC(skolem)$$

(p2):

$$H(skolem)$$

Proof

(1) Unify (p1) and (b), setting Y to *skolem*.

$$[IC(skolem)] \quad \wedge \quad [\neg IC(skolem) \vee D(skolem)]$$
$$[IC(skolem) \wedge \neg IC(skolem)] \quad \vee \quad [IC(skolem) \wedge D(skolem)]$$
$$[\,] \quad \vee \quad [IC(skolem) \wedge D(skolem)]$$
$$IC(skolem) \wedge D(skolem)$$

Now we know that *skolem* is a dessert.

(2) Unify (1.2) and (a), setting X to *skolem*.

$$[D(skolem)] \quad \wedge \quad [\neg D(skolem) \vee N(skolem)]$$
$$[D(skolem) \wedge \neg D(skolem)] \quad \vee \quad [D(skolem) \wedge N(skolem)]$$
$$[\,] \quad \vee \quad [D(skolem) \wedge N(skolem)]$$
$$D(skolem) \wedge N(skolem)$$

Now we know that *skolem* is nice.

(3) Unify (2.2) and (c), setting Z to *skolem*.

$$[N(skolem)] \quad \wedge \quad [\neg N(skolem) \vee \neg H(skolem)]$$
$$[N(skolem) \wedge \neg N(skolem)] \quad \vee \quad [N(skolem) \wedge \neg H(skolem)]$$
$$[\,] \quad \vee \quad [N(skolem) \wedge \neg H(skolem)]$$
$$N(skolem) \wedge \neg H(skolem)$$

Now we know that *skolem* is not healthy.

(4) Unify (3.2) and (p2) to obtain the empty clause.

$$H(skolem) \wedge \neg H(skolem)$$
$$()$$

Having obtained the empty clause, it means that one of the previous clauses is false. So, our assumption ‘clause (p)’ is false, and thus, the required statement is true. Ice cream is not healthy.

Birds

For this problem, the statements "I have X", "X belongs to me", and "X is mine" are considered the same, as they carry the same real-world meaning. A clarification email was sent to say that statement (a) should end with 'are 9 or more feet high' so that it can be opposed to ending of statement (d) 'less than 9 feet high'.

Definitions

B(X): X is a bird
O(X): X is an ostrich
F(X): X is 9 feet or higher
Z(X): X is in the zoo
M(X): X is mine
L(X): X lives on mince pies

Logic Statements

(a) No birds except ostriches are 9 feet high.

$$\begin{array}{ll} \neg\exists X & [B(X) \wedge \neg O(X) \wedge F(X)] \\ \forall X & \neg B(X) \vee O(X) \vee \neg F(X) \\ & \neg B(X) \vee O(X) \vee \neg F(X) \end{array}$$

(b) There are no birds in this zoo that belong to anyone but me.

$$\begin{array}{ll} \neg\exists X & [B(X) \wedge Z(X) \wedge \neg M(X)] \\ \forall X & \neg B(X) \vee \neg Z(X) \vee M(X) \\ & \neg B(Y) \vee \neg Z(Y) \vee M(Y) \end{array}$$

(c) No ostrich lives on mince pies.

$$\begin{array}{ll} \neg\exists X & [O(X) \wedge L(X)] \\ \forall X & \neg O(X) \vee \neg L(X) \\ & \neg O(Z) \vee \neg L(Z) \end{array}$$

(d) I have no birds less than 9 feet high.

$$\begin{array}{ll} \neg\exists X & [M(X) \wedge B(X) \wedge \neg F(X)] \\ \forall X & \neg M(X) \vee \neg B(X) \vee F(X) \\ & \neg M(W) \vee \neg B(W) \vee F(W) \end{array}$$

Required statement: No birds in this zoo live on mince pies.

$$\neg\exists X \quad [B(X) \wedge Z(X) \wedge L(X)]$$

Negated (p):

$$\neg\neg\exists X \quad [B(X) \ \wedge \ Z(X) \ \wedge \ L(X)]$$
$$\exists X \quad [B(X) \ \wedge \ Z(X) \ \wedge \ L(X)]$$

Let *skolem* be an object that satisfies the above clauses. We now have:

(p1):

$$B(skolem)$$

(p2):

$$Z(skolem)$$

(p3):

$$L(skolem)$$

Proof

(1) Unify (p3) and (c), setting Z to skolem.

$$[L(skolem)] \quad \wedge \quad [\neg O(skolem) \ \vee \ \neg L(skolem)]$$
$$[L(skolem) \ \wedge \ \neg O(skolem)] \quad \vee \quad [L(skolem) \ \wedge \ \neg L(skolem)]$$
$$[L(skolem) \ \wedge \ \neg O(skolem)] \quad \vee \quad [\,]$$
$$L(skolem) \ \wedge \ \neg O(skolem)$$

Now we know that *skolem* is not an ostrich.

(2) Unify (p1) and (a), setting X to skolem.

$$[B(skolem)] \quad \wedge \quad [\neg B(skolem) \ \vee \ O(skolem) \ \vee \ \neg F(skolem)]$$
$$[B(skolem) \wedge \neg B(skolem)] \quad \vee \quad [B(skolem) \wedge (O(skolem) \vee \neg F(skolem))]$$
$$[\,] \quad \vee \quad [B(skolem) \wedge (O(skolem) \vee \neg F(skolem))]$$
$$B(skolem) \ \wedge \ (O(skolem) \ \vee \ \neg F(skolem))$$

Now we know that *skolem* is an ostrich or less than 9 feet high.

(3) Unify (1.2) and (2.2).

$$[\neg O(skolem)] \quad \wedge \quad [O(skolem) \ \vee \ \neg F(skolem)]$$
$$[\neg O(skolem) \ \wedge \ O(skolem)] \quad \vee \quad [\neg O(skolem) \ \wedge \ \neg F(skolem)]$$
$$[\,] \quad \vee \quad [\neg O(skolem) \ \wedge \ \neg F(skolem)]$$
$$\neg O(skolem) \ \wedge \ \neg F(skolem)$$

Now we know that *skolem* is less than 9 feet high.

(4) Unify (p1) and (b), setting Y to *skolem*.

$$[B(skolem)] \;\wedge\; [\neg B(skolem) \vee \neg Z(skolem) \vee M(skolem)]$$
$$[B(skolem) \wedge \neg B(skolem)] \vee [B(skolem) \wedge (\neg Z(skolem) \vee M(skolem))]$$
$$[\,] \vee [B(skolem) \wedge (\neg Z(skolem) \vee M(skolem))]$$
$$B(skolem) \wedge (\neg Z(skolem) \vee M(skolem))$$

Now we know that *skolem* is not in the zoo or is mine.

(5) Unify (p2) and (4.2).

$$[Z(skolem)] \;\wedge\; [\neg Z(skolem) \vee M(skolem)]$$
$$[Z(skolem) \wedge \neg Z(skolem)] \vee [Z(skolem) \wedge M(skolem)]$$
$$[\,] \vee [Z(skolem) \wedge M(skolem)]$$
$$Z(skolem) \wedge M(skolem)$$

Now we know that *skolem* is mine.

(6) Unify (4.2) and (d), setting W to *skolem*.

$$[M(skolem)] \;\wedge\; [\neg M(skolem) \vee \neg B(skolem) \vee F(skolem)]$$
$$[M(skolem) \wedge \neg M(skolem)] \vee [M(skolem) \wedge (\neg B(skolem) \vee F(skolem))]$$
$$[\,] \vee [M(skolem) \wedge (\neg B(skolem) \vee F(skolem))]$$
$$M(skolem) \wedge (\neg B(skolem) \vee F(skolem))$$

Now we know that skolem is not a bird or is 9 or more feet high.

(7) Unify (3.2) and (6.2).

$$[\neg F(skolem)] \;\wedge\; [\neg B(skolem) \vee F(skolem)]$$
$$[\neg F(skolem) \wedge \neg B(skolem)] \vee [\neg F(skolem) \wedge F(skolem)]$$
$$[\neg F(skolem) \wedge \neg B(skolem)] \vee [\,]$$
$$\neg F(skolem) \wedge \neg B(skolem)$$

Now we know that *skolem* is not a bird.

(8) Unify (8.2) and (p1) to obtain the empty clause.

$$B(skolem) \wedge \neg B(skolem)$$
$$(\,)$$

Having obtained the empty clause, it means that one of the previous clauses is false. So, our assumption ‘clause (p)’ is false, and thus, the required statement is true. No birds in this zoo live on mince pies.

Dreams

A clarification email was sent to state that all dreams are ideas which is added as statement (g).

Definitions

T(X): X can be expressed in a tweet
R(X): X is really ridiculous
I(X): X is an idea of mine
W(X): X is worth writing a blog post about
F(X): X is about fudge doughnuts
C(X): X comes true
H(X): refer X to the Head of School
D(X): X is a dream of mine

Logic Statements

(a) Every idea of mine that cannot be expressed in a tweet, is really ridiculous.

$$\forall X \quad [I(X) \wedge \neg T(X)] \rightarrow R(X)$$
$$\forall X \quad \neg I(X) \vee T(X) \vee R(X)$$
$$\neg I(X1) \vee T(X1) \vee R(X1)$$

(b) None of my ideas about fudge doughnuts are worth writing a blog post about.

$$\neg \exists X \quad [I(X) \wedge F(X) \wedge W(X)]$$
$$\forall X \quad \neg I(X) \vee \neg F(X) \vee \neg W(X)$$
$$\neg I(X2) \vee \neg F(X2) \vee \neg W(X2)$$

(c) No idea of mine that fails to come true, can be expressed in a tweet.

$$\neg \exists X \quad [I(X) \wedge \neg C(X) \wedge T(X)]$$
$$\forall X \quad \neg I(X) \vee C(X) \vee \neg T(X)$$
$$\neg I(X3) \vee C(X3) \vee \neg T(X3)$$

(d) I never have any really ridiculous idea that I do not refer to to my Head of School.

$$\neg \exists X \quad [I(X) \wedge R(X) \wedge \neg H(X)]$$
$$\forall X \quad \neg I(X) \vee \neg R(X) \vee H(X)$$
$$\neg I(X4) \vee \neg R(X4) \vee H(X4)$$

(e) My dreams are all about fudge doughnuts.

$$\forall X \quad D(X) \rightarrow F(X)$$
$$\forall X \quad \neg D(X) \lor F(X)$$
$$\neg D(X5) \lor F(X5)$$

(f) I never refer any idea to the Head of School, unless it is worth writing a blog post about.

$$\forall X \quad I(X) \rightarrow [(\neg W(X) \rightarrow \neg H(X)) \land (W(X) \rightarrow H(X))]$$
$$\forall X \quad I(X) \rightarrow [(W(X) \lor \neg H(X)) \land (\neg W(X) \lor H(X))]$$
$$\forall X \quad \neg I(X) \lor [(W(X) \lor \neg H(X)) \land (\neg W(X) \lor H(X))]$$
$$\forall X \quad [\neg I(X) \lor W(X) \lor \neg H(X)] \land [\neg I(X) \lor \neg W(X) \lor H(X)]$$

This can be expressed in two clauses:

(f1)

$$\forall X \quad \neg I(X) \lor W(X) \lor \neg H(X)$$
$$\neg I(X61) \lor W(X61) \lor \neg H(X61)$$

(f2)

$$\forall X \quad \neg I(X) \lor \neg W(X) \lor H(X)$$
$$\neg I(X62) \lor \neg W(X62) \lor H(X62)$$

(g) All my dreams are ideas of mine.

$$\forall X \quad D(X) \rightarrow I(X)$$
$$\forall X \quad \neg D(X) \lor I(X)$$
$$\neg D(X7) \lor I(X7)$$

Required statement: All my dreams come true.

$$\forall X \quad D(X) \rightarrow C(X)$$
$$\forall X \quad \neg D(X) \lor C(X)$$

Negated (p):

$$\neg(\forall X \quad \neg D(X) \lor C(X))$$
$$\exists X \quad D(X) \land \neg C(X)$$

Let *skolem* be an object that satisfies the above clauses. We now have:

(p1):

$$D(skolem)$$

(p2):

$$\neg C(skolem)$$

Proof

(1) Unify (p1) and (g), setting X7 to *skolem*.

$$[D(skolem)] \quad \wedge \quad [\neg D(skolem) \;\vee\; I(skolem)]$$
$$[D(skolem) \;\wedge\; \neg D(skolem)] \quad \vee \quad [D(skolem) \;\wedge\; I(skolem)]$$
$$[\,] \quad \vee \quad [D(skolem) \;\wedge\; I(skolem)]$$
$$D(skolem) \;\wedge\; I(skolem)$$

We now know that *skolem* is an idea.

(2) Unify (p1) and (e), setting X5 to *skolem*.

$$[D(skolem)] \quad \wedge \quad [\neg D(skolem) \;\vee\; F(skolem)]$$
$$[D(skolem) \;\wedge\; \neg D(skolem)] \quad \vee \quad [D(skolem) \;\wedge\; F(skolem)]$$
$$[\,] \quad \vee \quad [D(skolem) \;\wedge\; F(skolem)]$$
$$D(skolem) \;\wedge\; F(skolem)$$

We now know that *skolem* is about fudge doughnuts.

(3) Unify (1.2) and (b), setting X2 to *skolem*.

$$[I(skolem)] \quad \wedge \quad [\neg I(skolem) \;\vee\; \neg F(skolem) \;\vee\; \neg W(skolem)]$$
$$[I(skolem) \wedge \neg I(skolem)] \;\vee\; [I(skolem) \wedge (\neg F(skolem) \vee \neg W(skolem))]$$
$$[\,] \quad \vee \quad [I(skolem) \;\wedge\; (\neg F(skolem) \;\vee\; \neg W(skolem))]$$
$$I(skolem) \;\wedge\; (\neg F(skolem) \;\vee\; \neg W(skolem))$$

Now we know that skolem is not about fudge doughnuts or not worth writing a blog post about.

(4) Unify (2.2) and (3.2).

$$[F(skolem)] \quad \wedge \quad [\neg F(skolem) \;\vee\; \neg W(skolem)]$$
$$[F(skolem) \;\wedge\; \neg F(skolem)] \quad \vee \quad [F(skolem) \;\wedge\; \neg W(skolem)]$$
$$[\,] \quad \vee \quad [F(skolem) \;\wedge\; \neg W(skolem)]$$
$$F(skolem) \;\wedge\; \neg W(skolem)$$

Now we know that *skolem* is not worth writing a blog post about.

(5) Unify (1.2) and (f1), setting X61 to *skolem*.

$$[I(skolem)] \quad \wedge \quad [\neg I(skolem) \;\vee\; W(skolem) \;\vee\; \neg H(skolem)]$$
$$[I(skolem) \wedge \neg I(skolem)] \quad \vee \quad [I(skolem) \;\wedge\; (W(skolem) \vee \neg H(skolem))]$$
$$[\,] \quad \vee \quad [I(skolem) \quad \wedge \quad (W(skolem) \;\vee\; \neg H(skolem))]$$
$$I(skolem) \quad \wedge \quad (W(skolem) \;\vee\; \neg H(skolem))$$

Now we know that *skolem* is worth writing a blog post about or I will not refer X to the Head of School.

(6) Unify (4.2) and (5.2).

$$[\neg W(skolem)] \quad \wedge \quad [W(skolem) \ \vee \ \neg H(skolem)]$$
$$[\neg W(skolem) \ \wedge \ W(skolem)] \quad \vee \quad [\neg W(skolem) \ \wedge \ \neg H(skolem)]$$
$$[\,] \quad \vee \quad [\neg W(skolem) \ \wedge \ \neg H(skolem)]$$
$$\neg W(skolem) \ \wedge \ \neg H(skolem)$$

Now we know that I will not refer *skolem* to the Head of School.

(7) Unify (1.2) and (d), setting X4 to *skolem*.

$$[I(skolem)] \quad \wedge \quad [\neg I(skolem) \ \vee \ \neg R(skolem) \ \vee \ H(skolem)]$$
$$[I(skolem) \wedge \neg I(skolem)] \quad \vee \quad [I(skolem) \ \wedge \ (\neg R(skolem) \vee H(skolem))]$$
$$[\,] \quad \vee \quad [I(skolem) \quad \wedge \quad (\neg R(skolem) \ \vee \ H(skolem))]$$
$$I(skolem) \quad \wedge \quad (\neg R(skolem) \ \vee \ H(skolem))$$

Now we know that *skolem* is not really ridiculous or I will refer *skolem* to the Head of School.

(8) Unify (6.2) and (7.2).

$$[\neg H(skolem)] \quad \wedge \quad [\neg R(skolem) \ \vee \ H(skolem)]$$
$$[\neg H(skolem) \ \wedge \ \neg R(skolem)] \quad \vee \quad [\neg H(skolem) \ \wedge \ H(skolem)]$$
$$[\neg H(skolem) \ \wedge \ \neg R(skolem)] \quad \vee \quad [\,]$$
$$\neg H(skolem) \ \wedge \ \neg R(skolem)$$

Now we know that *skolem* is not really ridiculous.

(9) Unify (1.2) and (a), setting X1 to *skolem*.

$$[I(skolem)] \quad \wedge \quad [\neg I(skolem) \ \vee \ T(skolem) \ \vee \ R(skolem)]$$
$$[I(skolem) \wedge \neg I(skolem)] \quad \vee \quad [I(skolem) \ \wedge \ (T(skolem) \vee R(skolem))]$$
$$[\,] \quad \vee \quad [I(skolem) \quad \wedge \quad (T(skolem) \ \vee \ R(skolem))]$$
$$I(skolem) \quad \wedge \quad (T(skolem) \ \vee \ R(skolem))$$

Now we know that *skolem* can be expressed in a tweet or *skolem* is really ridiculous.

(10) Unify (8.2) and (9.2).

$$[\neg R(skolem)] \quad \wedge \quad [T(skolem) \vee R(skolem)]$$
$$[\neg R(skolem) \wedge T(skolem)] \quad \vee \quad [\neg R(skolem) \wedge R(skolem)]$$
$$[\neg R(skolem) \wedge T(skolem)] \quad \vee \quad [\,]$$
$$\neg R(skolem) \wedge T(skolem)$$

Now we know that *skolem* can be expressed in a tweet.

(11) Unify (1.2) and (c), setting X3 to *skolem*.

$$[I(skolem)] \quad \wedge \quad [\neg I(skolem) \vee C(skolem) \vee \neg T(skolem)]$$
$$[I(skolem) \wedge \neg I(skolem)] \quad \vee \quad [I(skolem) \wedge (C(skolem) \vee \neg T(skolem))]$$
$$[\,] \quad \vee \quad [I(skolem) \quad \wedge \quad (C(skolem) \vee \neg T(skolem))]$$
$$I(skolem) \quad \wedge \quad (C(skolem) \vee \neg T(skolem))$$

Now we know that *skolem* comes true or it cannot be expressed in a tweet.

(12) Unify (10.2) and (11.2).

$$[T(skolem)] \quad \wedge \quad [C(skolem) \vee \neg T(skolem)]$$
$$[T(skolem) \wedge C(skolem)] \quad \vee \quad [T(skolem) \wedge \neg T(skolem)]$$
$$[T(skolem) \wedge C(skolem)] \quad \vee \quad [\,]$$
$$T(skolem) \wedge C(skolem)$$

Now we know that *skolem* comes true.

(13) Unify (12.2) and (p2) to obtain the empty clause.

$$C(skolem) \wedge \neg C(skolem)$$
$$(\,)$$

Having obtained the empty clause, it means that one of the previous clauses is false. So, our assumption ‘clause (p)’ is false, and thus, the required statement is true. All my dreams come true.

Part 2. A Smashing Story

For this problem we need to add some real-world knowledge. It will be noted with (RW) whenever necessary. This problem also has 2 cases. If a person can hate themselves, than Ian smashed the calculator. If a person cannot hate themselves, then Kate smashed the calculator.

Definitions

a: Alex
i: Ian
k: Kate
S(X): X smashed the calculator
H(X,Y): X hates Y
O(X,Y): X is older than Y

RW – it does not make sense to say ‘I am older than myself’. Therefore, O(X,X) is always false. X is not older than X.

For the purpose of this problem, the word ‘everyone’ in the statements below encompasses the three individuals that we have, because we only need to know the relationships between them.

Logic Statements

(1) Somebody smashed the calculator. It might be Ian, Alex, or Kate.
In the context of this problem, I assume that ‘might be’ means ‘is’. In other words, it **is** one or more of Ian, Alex, or Kate that smashed the calculator and not someone else.
RW – more than one person can smash the calculator. For example, Kate holds the calculator like a brick and Alex performs a ‘karate-chop’ on it.

$$\exists X \quad S(X)$$
$$S(i) \lor S(a) \lor S(k)$$

(2) The person who smashed the calculator hates Ian, and is not older than Ian.

$$\forall X \quad S(X) \rightarrow [H(X,i) \land \neg O(X,i)]$$
$$\forall X \quad \neg S(X) \lor [H(X,i) \land \neg O(X,i)]$$
$$\forall X \quad [\neg S(X) \lor H(X,i)] \land [\neg S(X) \lor \neg O(X,i)]$$

This can be expressed in two clauses:
(2.1)

$$\forall X \quad \neg S(X) \lor H(X,i)$$
$$\neg S(X1) \lor H(X1,i)$$

(2.2)

$$\forall X \quad \neg S(X) \lor \neg O(X,i)$$
$$\neg S(X2) \lor \neg O(X2,i)$$

(3) Kate hates no one that Ian hates.

$$\forall X \quad H(i,X) \rightarrow \neg H(k,X)$$
$$\forall X \quad \neg H(i,X) \lor \neg H(k,X)$$
$$\neg H(i,X3) \lor \neg H(k,X3)$$

(4) Ian hates everyone except for Alex.
Because 'everyone' in this problem encompasses Alex, Ian, and Kate, we can express the statement above with separate clauses for each person. So we have two:
(4.1) Ian hates Ian (himself). This clause is ignored in one of the proofs below.

$$H(i,i)$$

(4.2) Ian hates Kate.

$$H(i,k)$$

(5) Alex hates everyone not older than Ian.

$$\forall X \quad \neg O(X,i) \rightarrow H(a,X)$$
$$\forall X \quad O(X,i) \lor H(a,X)$$
$$O(X4,i) \lor H(a,X4)$$

(6) Alex is the oldest of the three.
RW – this means that Alex is older than Kate and Alex is older than Ian. We have two statements:
(6.1) Alex is older than Kate.

$$O(a,k)$$

(6.2) Alex is older than Ian.

$$O(a,i)$$

(7) Alex hates everyone Ian hates.

$$\forall X \quad H(i,X) \rightarrow H(a,X)$$
$$\forall X \quad \neg H(i,X) \lor H(a,X)$$
$$\neg H(i,X5) \lor H(a,X5)$$

(8) No one hates all three of Ian, Alex, and Kate.

$$\neg\exists X \quad [H(X,a) \wedge H(X,i) \wedge H(X,k)]$$
$$\forall X \quad \neg H(X,a) \vee \neg H(X,i) \vee \neg H(X,k)$$
$$\neg H(X6,a) \vee \neg H(X6,i) \vee \neg H(X6,k)$$

Proof

With resolution proof, we need to start from an assumption and unify clauses until we get an empty clause. However, in this case, the assumption 'Ian didn't smash the calculator' makes most of the clauses true and leaves us with "Kate smashed the calculator or Alex smashed the calculator". After which, we need to analyze the cases when that statement above is true with further assumptions. So we get resolution within the resolution. Furthermore, two proofs are presented here, as one says that people cannot hate themselves (H(X,X) is always false) and the other leaves that possibility open (H(X,X) could be either true or false).

Proof 1

When people cannot hate themselves. H(X,X) is always false. In this case, Kate smashed the calculator. We assume the opposite:

(10) Kate did not smash the calculator.

$$\neg S(k)$$

(11) Unify (10) and (1).

$$[\neg S(k)] \wedge [S(i) \vee S(a) \vee S(k)]$$
$$[\neg S(k) \wedge (S(i) \vee S(a))] \vee [\neg S(k) \wedge S(k)]$$
$$[\neg S(k) \wedge (S(i) \vee S(a))] \vee []$$
$$\neg S(k) \wedge (S(i) \vee S(a))$$

Now we know that Ian smashed that calculator or Alex smashed the calculator.

Let's first assume Alex smashed the calculator.

(a) Alex smashed the calculator.

$$S(a)$$

(b) Unify (a) and (2.2), setting X2 to Alex.

$$[S(a)] \wedge [\neg S(a) \vee \neg O(a,i)]$$
$$[S(a) \wedge \neg S(a)] \vee [S(a) \wedge \neg O(a,i)]$$
$$[] \vee [S(a) \wedge \neg O(a,i)]$$
$$S(a) \wedge \neg O(a,i)$$

Now we know that Alex is not older than Ian.

(c) Unify (b) and (6.2) to obtain the empty clause.

$$\neg O(a,i) \ \wedge \ O(a,i)$$
$$()$$

Having obtained the empty clause, it means that one of the previous clauses is false. So, our assumption ‘Alex smashed the calculator.’ is false.

The assumption was wrong, but the clause still needs to be satisfied, so let’s assume Ian smashed the calculator.

(a) Ian smashed the calculator.

$$S(i)$$

(b) Unify (a) and (2.1), setting X1 to Ian.

$$[S(i)] \ \wedge \ [\neg S(i) \ \vee \ H(i,i)]$$
$$[S(i) \ \wedge \ \neg S(i)] \ \vee \ [S(i) \ \wedge \ H(i,i)]$$
$$[\,] \ \vee \ [S(a) \ \wedge \ H(i,i)]$$
$$S(a) \ \wedge \ H(i,i)$$

Now we know that Ian hates Ian (himself).
However, we are currently examining the case when people cannot hate themselves.
Therefore this is immediately an empty clause. So our assumption ‘Ian smashed the calculator.’ is false.

This assumption was also wrong, so the clause

$$S(i) \ \vee \ S(a)$$

is unsatisfiable and is therefore an empty clause.

Having obtained the empty clause, it means that one of the previous clauses is false. So, our assumption ‘Kate did not smash the calculator.’ is false. So, to answer the question, Kate smashed the calculator.

Proof 2

When people can hate themselves. H(X,X) is can be true or false. In this case, Ian smashed the calculator. We assume the opposite:

(10) Ian did not smash the calculator.

$$\neg S(i)$$

(11) Unify (10) and (1).

$$[\neg S(i)] \wedge [S(i) \vee S(a) \vee S(k)]$$
$$[\neg S(i) \wedge S(i)] \vee [\neg S(i) \wedge (S(a) \vee S(k))]$$
$$[] \vee [\neg S(i) \wedge (S(a) \vee S(k))]$$
$$\neg S(i) \wedge (S(a) \vee S(k))$$

Now we know that Alex smashed the calculator or Kate smashed the calculator.

Let’s first assume Alex smashed the calculator.

(a) Alex smashed the calculator.

$$S(a)$$

(b) Unify (a) and (2.2), setting X2 to Alex.

$$[S(a)] \wedge [\neg S(a) \vee \neg O(a,i)]$$
$$[S(a) \wedge \neg S(a)] \vee [S(a) \wedge \neg O(a,i)]$$
$$[] \vee [S(a) \wedge \neg O(a,i)]$$
$$S(a) \wedge \neg O(a,i)$$

Now we know that Alex is not older than Ian.

(c) Unify (b) and (6.2) to obtain the empty clause.

$$\neg O(a,i) \wedge O(a,i)$$
$$()$$

Having obtained the empty clause, it means that one of the previous clauses is false. So, our assumption ‘Alex smashed the calculator.’ is false.

The assumption was wrong, but the clause still needs to be satisfied, so let’s assume Kate smashed the calculator.

(a) Kate smashed the calculator.

$$S(k)$$

(b) Unify (a) and (2.1), setting X1 to Kate.

$$[S(k)] \wedge [\neg S(k) \vee H(k,i)]$$
$$[S(k) \wedge \neg S(k)] \vee [S(k) \wedge H(k,i)]$$
$$[] \vee [S(k) \wedge H(k,i)]$$
$$S(a) \wedge H(k,i)$$

Now we know that Kate hates Ian.

(c) Unify (b) and (3), setting X3 to Ian.

$$[H(k,i)] \quad \wedge \quad [\neg H(i,i) \ \vee \ \neg H(k,i)]$$
$$[H(k,i) \ \wedge \ \neg H(i,i)] \quad \vee \quad [H(k,i) \ \wedge \ \neg H(k,i)]$$
$$[H(k,i) \ \wedge \ \neg H(i,i)] \quad \vee \quad [\,]$$
$$H(k,i) \ \wedge \ \neg H(i,i)$$

Now we know that Ian does not hate himself.

(d) Unify (c) and (4.1) to obtain the empty clause.

$$\neg H(i,i) \ \wedge \ H(i,i)$$
$$(\,)$$

Having obtained the empty clause, it means that one of the previous clauses is false. So, our assumption ‘Ian did not smash the calculator.’ is false. So, to answer the question, Ian smashed the calculator.

Part 3. Schubert's Steamroller

Discussion

I will first write why this is a hard problem and prove the required statement in English. I will then prove it with FOPL. There are several points that are somewhat ambiguous.

The first issue comes from the exclusive or statement about the diet. (X) "Every animal either likes to eat all plants or all animals much smaller than itself that like to eat some plants." This is tricky as it implies that these are all the things an animal could eat. I assume that the diet is not limited. There could be an animal that likes to eat all plants and some (but not all) plant-eating animals smaller than itself. This is the bird and here's why.

The birds likes to eat caterpillars but not snails. Snails eat some plants, so from the (X) statement it follows that birds like to eat all plants. The bird can now be the 'grain-eating animal' in the statement that we are trying to prove.

The second issue comes from the 'or' in the statement (W) "Wolves do not like to eat foxes or grain." This could mean neither foxes nor grains are on a wolf's diet. But it could also mean that only one of them is off the menu. For FOPL, I assume it means "neither".

Wolves do not like to eat grains, therefore they do not like to eat all plants. So wolves eat all much smaller animals that like to eat some plants. However, wolves also do not like to eat foxes. From these two it follows that foxes do not eat any plants. Therefore they must eat all animals smaller than them that like to eat some plants. The bird fits that role (smaller than fox, eats all plants).

Finally, we have the fox eats the bird which eats all plants (including grain). The third issue is that of "grain-eating animal". I assume it means "an animal that eats *some* grain" and not "only grain".

The fourth issues is what happens to the smallest animal. Is it obliged to eat all plants or is it allowed to eat smaller animals (an empty set)? I assume the smallest animals automatically satisfy the (X) statement because they "eat smaller animals" which just happen to be an empty set.

In reality, birds and caterpillars also fit the role but we cannot prove that with logic, unless we assume that 'some plants' is 'all plants' or go against the previous assumption.

Furthermore, applying real-world sense, birds are smaller than foxes which are smaller than wolves. Thus, birds are smaller than wolves. Wolves eat all smaller plant-eating animals. So, wolves eat birds which are a grain-eating animal. This works even if wolves do eat foxes.

Definitions

Because we have (at least one of wolves, foxes, birds, caterpillars, snails, and grains) and (all statements generalise for all representatives of the species), I have decided to have the following objects:
w: wolf
f: fox

b: birds
c: caterpillar
s: snail
g: grain.

For the function definitions, I assume that ‘eats’ and ‘likes to eat’ are the same.

A(X): X is animal
P(X): X is plant
E(X, Y): X (likes to) eats Y
S(X, Y): X is much smaller than Y

Logic Statements

A lot of the sentences can be split into 2 or 3 separate clauses that are true. I will go sentence by sentence and create multiple clauses from each. Starting from the last statement and working my way up:

(a) Caterpillars and snails like to eat some plants.

(1) There is a plant *skolem1* that caterpillars eat.

$$\exists X \quad P(X) \wedge E(c, X)$$
$$P(skolem1) \wedge E(c, skolem1)$$

(2) There is a plant *skolem2* that snails eat.

$$\exists X \quad P(X) \wedge E(s, X)$$
$$P(skolem2) \wedge E(s, skolem2)$$

(b) Birds like to eat caterpillars but not snails.

(1) $E(b, c)$

(2) $\neg E(b, s)$

(c) Wolves do not like to eat foxes or grains.

(1) $\neg E(w, f)$

(2) $\neg E(w, g)$

(d) Caterpillars and snails are much smaller than birds, which are much smaller than foxes, which in turn are much smaller than wolves.

(1) $S(c, b)$

(2) $S(s, b)$

(3) $S(b,f)$

(4) $S(f,w)$

(e) Every animal either likes to eat all plants or all animals much smaller than itself that like to eat some plants.
I will encode the parts of this separately.

(1) Every animal

$$\forall X \quad A(X)$$

(2) likes to eat all plants

$$\forall Y \quad P(Y) \rightarrow E(X,Y)$$
$$\forall Y \quad \neg P(Y) \vee E(X,Y)$$

(3) likes to eat all animals much smaller than itself that like to eat some plants.

$$\forall Z \exists W \quad [A(Z) \wedge S(Z,X) \wedge E(Z,W) \wedge P(W)] \rightarrow E(X,Z)$$
$$\forall Z \exists W \quad \neg[A(Z) \wedge S(Z,X) \wedge E(Z,W) \wedge P(W)] \vee E(X,Z)$$
$$\forall Z \exists W \quad \neg A(Z) \vee \neg S(Z,X) \vee \neg E(Z,W) \vee \neg P(W) \vee E(X,Z)$$

(4) Now we have combine them to (1) implies either (2) or (3).

$$(1) \rightarrow [(2) \oplus (3)]$$
$$\neg(1) \vee [((2) \vee (3)) \wedge (\neg(2) \vee \neg(3))]$$
$$[\neg(1) \vee (2) \vee (3)] \wedge [\neg(1) \vee \neg(2) \vee \neg(3)]$$

We now have two clauses

(i) The left side expresses the idea that any animal X eats at least one of (all plants Y) or (all animals Z that are smaller than X and eat some plants W). We set exists W to skf3(X,Y,Z).

$$\neg(1) \ \vee \ (2) \ \vee \ (3)$$
$$\forall X \ \ \forall Y \ \ \forall Z \ \exists W \quad [\neg A(X)] \ \vee \ [\neg P(Y) \vee E(X,Y)] \ \vee$$
$$[\neg A(Z) \vee \neg S(Z,X) \vee \neg E(Z,W) \vee \neg P(W) \vee E(X,Z)]$$

$$\forall X \ \ \forall Y \ \ \forall Z \quad [\neg A(X)] \ \vee \ [\neg P(Y) \vee E(X,Y)] \ \vee$$
$$[\neg A(Z) \vee \neg S(Z,X) \vee \neg E(Z, skf3(X,Y,Z)) \vee$$
$$\neg P(skf3(X,Y,Z)) \vee E(X,Z)]$$

$$\neg A(X1) \vee \neg P(Y1) \vee E(X1,Y1) \vee$$
$$\neg A(Z1) \vee \neg S(Z1,X1) \vee \neg E(Z1, skf3(X1,Y1,Z1)) \vee$$
$$\neg P(skf3(X1,Y1,Z1)) \vee E(X1,Z1)$$

(ii) The right side expresses the idea that any animal X eats at most one of (all plants Y) or (all animals Z that are smaller than X and eat some plants W). We set exists Y to skf1(X).

$$\neg(1) \ \vee \ \neg(2) \ \vee \ \neg(3)$$
$$\forall X \ \ \neg A(X) \ \vee \ \neg[\forall Y \ \ \neg P(Y) \vee E(X,Y)] \ \vee \ \neg(3)$$
$$\forall X \ \ \neg A(X) \ \vee \ [\exists Y \ \ P(Y) \wedge \neg E(X,Y)] \ \vee \ \neg(3)$$
$$\forall X \ \ \neg A(X) \ \vee \ [P(skf1(X)) \wedge \neg E(X, skf1(X))] \ \vee \ \neg(3)$$
$$\forall X \ \ \neg A(X) \ \vee \ \neg(3) \ \vee \ [P(skf1(X)) \wedge \neg E(X, skf1(X))]$$
$$\forall X [\neg A(X) \vee \neg(3) \vee P(skf1(X))] \wedge [\neg A(X) \vee \neg(3) \vee \neg E(X, skf1(X))]$$

Again, we have two clauses

(A) Left side, setting exists Z to skf2(X):

$$\forall X \quad \neg A(X) \;\vee\; P(skf1(X)) \;\vee\; \neg(3)$$

$$\forall X \quad \neg A(X) \vee P(skf1(X)) \vee$$
$$\neg[\forall Z \;\; \exists W \;\; \neg A(Z) \vee \neg S(Z,X) \vee \neg E(Z,W) \vee \neg P(W) \vee E(X,Z)]$$

$$\forall X \quad \neg A(X) \vee P(skf1(X)) \vee$$
$$\exists Z \;\; \neg[\exists W \;\; \neg A(Z) \vee \neg S(Z,X) \vee \neg E(Z,W) \vee \neg P(W) \vee E(X,Z)]$$

$$\forall X \quad \neg A(X) \vee P(skf1(X)) \vee$$
$$\exists Z \;\; \forall W \;\; \neg[\neg A(Z) \vee \neg S(Z,X) \vee \neg E(Z,W) \vee \neg P(W) \vee E(X,Z)]$$

$$\forall X \;\; \exists Z \;\; \forall W \quad \neg A(X) \vee P(skf1(X)) \vee$$
$$[A(Z) \wedge S(Z,X) \wedge E(Z,W) \wedge P(W) \wedge \neg E(X,Z)]$$

$$\forall X \;\; \forall W \quad \neg A(X) \vee P(skf1(X)) \vee$$
$$[A(skf2(X)) \wedge S(skf2(X),X) \wedge E(skf2(X),W) \wedge P(W) \wedge \neg E(X,skf2(X))]$$

Applying the distribution rule we get the following set of clauses:

(I)
$$\forall X \quad \neg A(X) \;\vee\; P(skf1(X)) \;\vee\; A(skf2(X))$$
$$\neg A(X2) \;\vee\; P(skf1(X2)) \;\vee\; A(skf2(X2))$$

(II)
$$\forall X \quad \neg A(X) \;\vee\; P(skf1(X)) \;\vee\; S(skf2(X),X)$$
$$\neg A(X3) \;\vee\; P(skf1(X3)) \;\vee\; S(skf2(X3),X3)$$

(III)
$$\forall X \forall W \quad \neg A(X) \vee P(skf1(X)) \;\vee\; E(skf2(X),W)$$
$$\neg A(X4) \vee P(skf1(X4)) \;\vee\; E(skf2(X4),W1)$$

(IV)
$$\forall X \forall W \quad \neg A(X) \;\vee\; P(skf1(X)) \;\vee\; P(W)$$
$$\neg A(X5) \;\vee\; P(skf1(X5)) \;\vee\; P(W2)$$

(V)
$$\forall X \quad \neg A(X) \;\vee\; P(skf1(X)) \;\vee\; \neg E(X,skf2(X))$$
$$\neg A(X6) \;\vee\; P(skf1(X6)) \;\vee\; \neg E(X,skf2(X6))$$

(B) Right side is similar as they only differ in one function. The five clauses are:

(I)
$$\forall X \quad \neg A(X) \vee \neg E(X, skf1(X)) \vee A(skf3(X))$$
$$\neg A(X7) \vee \neg E(X7, skf1(X7)) \vee A(skf3(X7))$$

(II)
$$\forall X \quad \neg A(X) \vee \neg E(X, skf1(X)) \vee S(skf2(X), X)$$
$$\neg A(X8) \vee \neg E(X8, skf1(X8)) \vee S(skf2(X8), X8)$$

(III)
$$\forall X \forall W \neg A(X) \vee \neg E(X, skf1(X)) \vee E(skf2(X), W)$$
$$\neg A(X9) \vee \neg E(X9, skf1(X9)) \vee E(skf2(X9), W3)$$

(IV)
$$\forall X \forall W \quad \neg A(X) \vee \neg E(X, skf1(X)) \vee P(W)$$
$$\neg A(X10) \vee \neg E(X10, skf1(X10)) \vee P(W4)$$

(V)
$$\forall X \quad \neg A(X) \vee \neg E(X, skf1(X)) \vee \neg E(X, skf2(X))$$
$$\neg A(X11) \vee \neg E(X11, skf1(X11)) \vee \neg E(X11, skf2(X11))$$

(f) All grains are plants.

$$P(g)$$

(g) Wolves, foxes, birds, caterpillars, and snails are animals.

$$(1) \quad A(w)$$
$$(2) \quad A(f)$$
$$(3) \quad A(b)$$
$$(4) \quad A(c)$$
$$(5) \quad A(s)$$

(h) There is an animal that likes to eat a grain-eating animal.

$$\exists X \exists Y \quad A(X) \wedge A(Y) \wedge E(X, Y) \wedge E(Y, g)$$

The statement that I will prove is: Foxes eat birds and birds eat grain. From that, it directly follows that there is an animal that likes to eat a grain-eating animal (the required statement).

(p) Negated statement, with X set to f (fox) and Y to b (bird) at the lest lines:

$$\neg \exists X \{\exists Y \quad A(X) \wedge A(Y) \wedge E(X, Y) \wedge E(Y, g)\}$$
$$\forall X \quad \neg \exists Y \{A(X) \wedge A(Y) \wedge E(X, Y) \wedge E(Y, g)\}$$
$$\forall X \quad \forall Y \quad \neg \{A(X) \wedge A(Y) \wedge E(X, Y) \wedge E(Y, g)\}$$
$$\forall X \quad \forall Y \quad \neg A(X) \vee \neg A(Y) \vee \neg E(X, Y) \vee \neg E(Y, g)$$
$$\neg A(f) \vee \neg A(b) \vee \neg E(f, b) \vee \neg E(b, g)$$

Proof

To prove by resolution that the negated statement is false, we need to consecutively make and disprove two assumptions. First “Birds do not like to eat grain.” and second “Foxes do not like to eat birds.”.
Note the first two clauses of (p) [-A(f) v -A(b)] are immediately false from (g.2) and (g.3). We only need to assume that the others are true.

(q) Birds do not like to eat grain.

$$\neg E(b,g)$$

(1) In (e.4.i), set X1 to b (bird), Y1 to g (grain), Z1 to s (snail). (e.4.i) says that every animal eats at least one of (all plants) or (all smaller plant-eating animals). The following will prove that since it is not the former, then it must be the latter.

$$[\neg A(b)] \lor [\neg P(g) \lor E(b,g)] \lor$$
$$[\neg A(s) \lor \neg S(s,b) \lor \neg E(s, skf3(b,g,s)) \lor \neg P(skf3(b,g,s)) \lor E(b,s)]$$

(2) Unify (g.3) and (1).

$$A(b) \land \{\neg A(b) \lor \neg P(g) \lor E(b,g) \lor$$
$$[\neg A(s) \lor \neg S(s,b) \lor \neg E(s, skf3(b,g,s)) \lor \neg P(skf3(b,g,s)) \lor E(b,s)]\}$$

$$[A(b) \land \neg A(b)] \lor \{A(b) \land [\neg P(g) \lor E(b,g) \lor$$
$$\neg A(s) \lor \neg S(s,b) \lor \neg E(s, skf3(b,g,s)) \lor \neg P(skf3(b,g,s)) \lor E(b,s)]\}$$

$$[\,] \lor \{A(b) \land [\neg P(g) \lor E(b,g) \lor$$
$$\neg A(s) \lor \neg S(s,b) \lor \neg E(s, skf3(b,g,s)) \lor \neg P(skf3(b,g,s)) \lor E(b,s)]\}$$

$$A(b) \land [\neg P(g) \lor E(b,g) \lor$$
$$\neg A(s) \lor \neg S(s,b) \lor \neg E(s, skf3(b,g,s)) \lor \neg P(skf3(b,g,s)) \lor E(b,s)]$$

(3) Unify (2.2) and (f)

$$P(g) \quad \wedge \quad \{\neg P(g) \vee E(b,g) \ \vee$$
$$[\neg A(s) \vee \neg S(s,b) \vee \neg E(s, skf3(b,g,s)) \vee \neg P(skf3(b,g,s)) \vee E(b,s)]\}$$

$$[P(g) \wedge \neg P(g)] \quad \vee \quad \{P(g) \ \wedge \ [E(b,g) \vee \neg A(s) \vee \neg S(s,b) \vee$$
$$\neg E(s, skf3(b,g,s)) \vee \neg P(skf3(b,g,s)) \vee E(b,s)]\}$$

$$[\,] \quad \vee \quad \{P(g) \ \wedge \ [E(b,g) \vee \neg A(s) \vee \neg S(s,b) \vee$$
$$\neg E(s, skf3(b,g,s)) \vee \neg P(skf3(b,g,s)) \vee E(b,s)]\}$$

$$P(g) \ \wedge \ [E(b,g) \vee \neg A(s) \vee \neg S(s,b) \vee$$
$$\neg E(s, skf3(b,g,s)) \vee \neg P(skf3(b,g,s)) \vee E(b,s)]$$

(4) Unify (3.2) and (q)

$$\neg E(b,g) \ \wedge \ [E(b,g) \vee \neg A(s) \vee \neg S(s,b) \vee \neg E(s, skf3(b,g,s)) \vee$$
$$\neg P(skf3(b,g,s)) \vee E(b,s)]$$

$$[\neg E(b,g) \wedge E(b,g)] \quad \vee \quad \{\neg E(b,g) \wedge [\neg A(s) \vee \neg S(s,b) \vee$$
$$\neg E(s, skf3(b,g,s)) \vee \neg P(skf3(b,g,s)) \vee E(b,s)]\}$$

$$[\,] \quad \vee \quad \{\neg E(b,g) \ \wedge \ [\neg A(s) \vee \neg S(s,b) \vee$$
$$\neg E(s, skf3(b,g,s)) \vee \neg P(skf3(b,g,s)) \vee E(b,s)]\}$$

$$\neg E(b,g) \wedge [\neg A(s) \vee \neg S(s,b) \vee \neg E(s, skf3(b,g,s)) \vee \neg P(skf3(b,g,s)) \vee E(b,s)]$$

(5) Now we have that birds eat all smaller grain-eating animals (which includes snails). Unify (4.2) and (g.5).

$$A(s) \ \wedge \ [\neg A(s) \vee \neg S(s,b) \vee \neg E(s, skf3(b,g,s)) \vee \neg P(skf3(b,g,s)) \vee E(b,s)]$$

$$[A(s) \wedge \neg A(s)] \quad \vee \quad \{A(s) \ \wedge \ [\neg S(s,b) \vee \neg E(s, skf3(b,g,s)) \vee \neg P(skf3(b,g,s)) \vee E(b,s)]\}$$

$$[\,] \quad \vee \quad \{A(s) \ \wedge \ [\neg S(s,b) \vee \neg E(s, skf3(b,g,s)) \vee \neg P(skf3(b,g,s)) \vee E(b,s)]\}$$
$$A(s) \ \wedge \ [\neg S(s,b) \vee \neg E(s, skf3(b,g,s)) \vee \neg P(skf3(b,g,s)) \vee E(b,s)]$$

(6) Unify (5.2) and (d.2).

$$S(s,b) \ \wedge \ [\neg S(s,b) \vee \neg E(s, skf3(b,g,s)) \vee \neg P(skf3(b,g,s)) \vee E(b,s)]$$

$$[S(s,b) \wedge \neg S(s,b)] \quad \vee \quad \{S(s,b) \ \wedge \ [\neg E(s, skf3(b,g,s)) \vee \neg P(skf3(b,g,s)) \vee E(b,s)]\}$$

$$[\,] \quad \vee \quad \{S(s,b) \ \wedge \ [\neg E(s, skf3(b,g,s)) \vee \neg P(skf3(b,g,s)) \vee E(b,s)]\}$$
$$S(s,b) \ \wedge \ [\neg E(s, skf3(b,g,s)) \vee \ \neg P(skf3(b,g,s)) \vee E(b,s)]$$

(7) Unify (6.2) and (b.2).

$$[\neg E(s, skf3(b,g,s)) \vee \ \neg P(skf3(b,g,s)) \vee E(b,s)] \ \wedge \ \neg E(b,s)$$
$$\{[\neg E(s, skf3(b,g,s)) \vee \neg P(skf3(b,g,s))] \wedge \neg E(b,s)\} \vee [E(b,s) \wedge \neg E(b,s)]$$
$$\{[\neg E(s, skf3(b,g,s)) \vee \neg P(skf3(b,g,s))] \wedge \neg E(b,s)\} \vee [\,]$$
$$[\neg E(s, skf3(b,g,s)) \vee \neg P(skf3(b,g,s))] \wedge \neg E(b,s)$$

Given the definition of the skf3 function, the first clause above essentially says that there is no plant that snails would eat.

$$\neg[E(s, skf3(b,g,s)) \wedge P(skf3(b,g,s))]$$
$$\neg \exists R \quad [E(s,R) \ \wedge \ P(R)]$$

(8) Unify (7) and (a.2) to obtain the empty clause.

$$\{\neg \exists R \ \ [E(s,R) \ \wedge \ P(R)]\} \quad \wedge \quad \{\exists X \ \ [P(X) \ \wedge \ E(s,R)]\}$$
$$[\,]$$

Having obtained the empty clause, it means that one of the previous clauses is false. So, our assumption (q) is wrong. The statement “Birds like to eat grain.” is true.

(r) Foxes do not like to eat birds.

$$\neg E(f,b)$$

(1) In (e.4.i), set X1 to f (fox), Z1 to b (bird). We know a value of skf3 for the case of birds eating plants. That value is g (grain). (e.4.i) says that every animal eats at least one of (all plants) or (all smaller plant-eating animals). The following will prove that since it is not the latter, then it must be the former.

$$\neg A(f)\vee\neg P(Y1)\vee E(f,Y1)\vee\neg A(b)\vee\neg S(b,f)\vee\neg E(b,g)\vee\neg P(g)\vee E(f,b)$$

(2) Unify (g.2) and (1).

$$A(f) \quad \wedge \quad \{\neg A(f)\vee\neg P(Y1)\vee E(f,Y1)\vee \neg A(b)\vee\neg S(b,f)\vee\neg E(b,g)\vee\neg P(g)\vee E(f,b)\}$$

$$[A(f)\wedge\neg A(f)] \quad \vee \quad \{A(f) \;\wedge\; [\neg P(Y1)\vee E(f,Y1)\vee \neg A(b)\vee\neg S(b,f)\vee\neg E(b,g)\vee\neg P(g)\vee E(f,b)]\}$$

$$[\,] \quad \vee \quad \{A(f) \;\wedge\; [\neg P(Y1)\vee E(f,Y1)\vee \neg A(b)\vee\neg S(b,f)\vee\neg E(b,g)\vee\neg P(g)\vee E(f,b)]\}$$

$$A(f) \;\wedge\; [\neg P(Y1)\vee E(f,Y1)\vee\neg A(b)\vee\neg S(b,f)\vee\neg E(b,g)\vee\neg P(g)\vee E(f,b)]$$

(3) Unify (2.2) and (g.3)

$$A(b) \;\wedge\; [\neg P(Y1)\vee E(f,Y1)\vee\neg A(b)\vee\neg S(b,f)\vee\neg E(b,g)\vee\neg P(g)\vee E(f,b)]$$

$$[A(b)\wedge\neg A(b)] \;\vee\; \{A(b) \;\wedge\; [\neg P(Y1)\vee E(f,Y1) \;\vee\; \neg S(b,f)\vee\neg E(b,g)\vee\neg P(g)\vee E(f,b)]\}$$

$$[\,] \;\vee\; \{A(b) \;\wedge\; [\neg P(Y1)\vee E(f,Y1)\vee\neg S(b,f)\vee\neg E(b,g)\vee\neg P(g)\vee E(f,b)]\}$$
$$A(b) \;\wedge\; [\neg P(Y1)\vee E(f,Y1)\vee\neg S(b,f)\vee\neg E(b,g)\vee\neg P(g)\vee E(f,b)]$$

(4) Unify (3.2) and (f).

$$P(g) \quad \wedge \quad [\neg P(Y1)\vee E(f,Y1)\vee\neg S(b,f)\vee\neg E(b,g)\vee\neg P(g)\vee E(f,b)]$$
$$[P(g)\wedge\neg P(g)] \;\vee\; \{P(g)\wedge[\neg P(Y1)\vee E(f,Y1)\vee\neg S(b,f)\vee\neg E(b,g)\vee E(f,b)]\}$$
$$[\,] \quad \vee \quad \{P(g) \;\wedge\; [\neg P(Y1)\vee E(f,Y1) \;\vee\; \neg S(b,f)\vee\neg E(b,g)\vee E(f,b)]\}$$
$$P(g) \;\wedge\; [\neg P(Y1)\vee E(f,Y1) \;\vee\; \neg S(b,f)\vee\neg E(b,g)\vee E(f,b)]$$

(5) Unify (4.2) and (d.3).

$$S(b,f) \quad \wedge \quad [\neg P(Y1) \vee E(f,Y1) \ \vee \ \neg S(b,f) \vee \neg E(b,g) \vee E(f,b)]$$
$$[S(b,f) \wedge \neg S(b,f)] \quad \vee \quad \{S(b,f) \ \wedge \ [\neg P(Y1) \vee E(f,Y1) \ \vee \ \neg E(b,g) \vee E(f,b)]\}$$
$$[\,] \quad \vee \quad \{S(b,f) \ \wedge \ [\neg P(Y1) \vee E(f,Y1) \vee \neg E(b,g) \vee E(f,b)]\}$$
$$S(b,f) \ \wedge \ [\neg P(Y1) \vee E(f,Y1) \vee \neg E(b,g) \vee E(f,b)]$$

(6) Unify (5.2) and -(q). -(q) is the previously proved statement that birds eat grain.

$$E(b,g) \quad \wedge \quad [\neg P(Y1) \vee E(f,Y1) \vee \neg E(b,g) \vee E(f,b)]$$
$$[E(b,g) \wedge \neg E(b,g)] \quad \vee \quad \{E(b,g) \ \wedge \ [\neg P(Y1) \vee E(f,Y1) \vee E(f,b)]\}$$
$$[\,] \quad \vee \quad \{E(b,g) \ \wedge \ [\neg P(Y1) \vee E(f,Y1) \vee E(f,b)]\}$$
$$E(b,g) \ \wedge \ [\neg P(Y1) \vee E(f,Y1) \vee E(f,b)]$$

(7) Unify (6.2) and (r).

$$\neg E(f,b) \quad \wedge \quad [\neg P(Y1) \vee E(f,Y1) \vee E(f,b)]$$
$$[\neg E(f,b) \wedge E(f,b)] \quad \vee \quad \{\neg E(f,b) \ \wedge \ [\neg P(Y1) \vee E(f,Y1)]\}$$
$$[\,] \quad \vee \quad \{\neg E(f,b) \ \wedge \ [\neg P(Y1) \vee E(f,Y1)]\}$$
$$\neg E(f,b) \ \wedge \ [\neg P(Y1) \vee E(f,Y1)]$$

Now we have:

$$\neg E(f,b) \quad \wedge \quad [\neg P(Y1) \vee E(f,Y1) \vee E(f,b)]$$
$$[\neg E(f,b) \wedge E(f,b)] \quad \vee \quad \{\neg E(f,b) \ \wedge \ [\neg P(Y1) \vee E(f,Y1)]\}$$
$$[\,] \quad \vee \quad \{\neg E(f,b) \ \wedge \ [\neg P(Y1) \vee E(f,Y1)]\}$$
$$\neg E(f,b) \ \wedge \ [\neg P(Y1) \vee E(f,Y1)]$$

Foxes eat all plants. By extension, foxes eat some plants (grains).

(8) Now we shall inspect the diet of the wolf. In (e.4.i) set X1 to w (wolf), Y1 to g (grain), Z1 to f (fox). We can also set W (the skolem function) to g (grain) because the previous statement proved that it satisfies the requirements (a plant that is eaten by the fox).

$$\neg A(w) \vee \neg P(g) \vee E(w,g) \vee \neg A(f) \vee \neg S(f,w) \vee \neg E(f,g) \vee \neg P(g) \vee E(w,f)$$
$$\neg A(w) \vee \neg P(g) \vee E(w,g) \vee \neg A(f) \vee \neg S(f,w) \vee \neg E(f,g) \vee E(w,f)$$

The -P(g) appears twice in the first line above and the statement is reduced in the second.

(9) Unify (8.2) and (g.1).

$$A(w) \quad \wedge \quad \{[\neg A(w)] \ \vee \ [\neg P(g) \vee E(w,g)] \ \vee$$
$$[\neg A(f) \vee \neg S(f,w) \vee \neg E(f,g) \vee E(w,f)]\}$$

$$[A(w) \wedge \neg A(w)] \quad \vee \quad \{A(w) \ \wedge [\neg P(g) \vee E(w,g) \ \vee$$
$$\neg A(f) \vee \neg S(f,w) \vee \neg E(f,g) \vee E(w,f)]\}$$

$$[\,] \ \vee \ \{A(w) \wedge [\neg P(g) \vee E(w,g) \vee \neg A(f) \vee \neg S(f,w) \vee \neg E(f,g) \vee E(w,f)]\}$$
$$A(w) \ \wedge [\neg P(g) \vee E(w,g) \vee \neg A(f) \vee \neg S(f,w) \vee \neg E(f,g) \vee E(w,f)]$$

(10) Unify (9.2) and (f).

$$P(g) \ \wedge \ [\neg P(g) \vee E(w,g) \vee \neg A(f) \vee \neg S(f,w) \vee \neg E(f,g) \vee E(w,f)]$$
$$[P(g) \wedge \neg P(g)] \vee \{P(g) \wedge [E(w,g) \vee \neg A(f) \vee \neg S(f,w) \vee \neg E(f,g) \vee E(w,f)]\}$$
$$[\,] \ \vee \ \{P(g) \wedge [E(w,g) \vee \neg A(f) \vee \neg S(f,w) \vee \neg E(f,g) \vee E(w,f)]\}$$
$$P(g) \wedge [E(w,g) \vee \neg A(f) \vee \neg S(f,w) \vee \neg E(f,g) \vee E(w,f)]$$

(11) Unify (10.2) and (c.2).

$$\neg E(w,g) \ \wedge \ \{[E(w,g) \vee \neg A(f) \vee \neg S(f,w) \vee \neg E(f,g) \vee E(w,f)]\}$$
$$[\neg E(w,g) \wedge E(w,g)] \ \vee \ \{\neg E(w,g) \wedge [\neg A(f) \vee \neg S(f,w) \vee \neg E(f,g) \vee E(w,f)]\}$$
$$[\,] \quad \vee \quad \{\neg E(w,g) \ \wedge \ [\neg A(f) \vee \neg S(f,w) \vee \neg E(f,g) \vee E(w,f)]\}$$
$$\neg E(w,g) \ \wedge \ [\neg A(f) \vee \neg S(f,w) \vee \neg E(f,g) \vee E(w,f)]$$

(12) Unify (11.2) and (g.2).

$$A(f) \ \wedge \ [\neg A(f) \vee \neg S(f,w) \vee \neg E(f,g) \vee E(w,f)]$$
$$[A(f) \wedge \neg A(f)] \quad \vee \quad \{A(f) \wedge [\neg S(f,w) \vee \neg E(f,g) \vee E(w,f)]\}$$
$$[\,] \quad \vee \quad \{A(f) \ \wedge \ [\neg S(f,w) \vee \neg E(f,g) \vee E(w,f)]\}$$
$$A(f) \quad \wedge \quad [\neg S(f,w) \ \vee \ \neg E(f,g) \ \vee \ E(w,f)]$$

(13) Unify (12.2) and (d.4).

$$S(f,w) \quad \wedge \quad [\neg S(f,w) \ \vee \ \neg E(f,g) \ \vee \ E(w,f)]$$
$$[S(f,w) \ \wedge \ \neg S(f,w)] \quad \vee \quad \{S(f,w) \ \wedge \ [\neg E(f,g) \ \vee \ E(w,f)]\}$$
$$[\,] \quad \vee \quad \{S(f,w) \ \wedge \ [\neg E(f,g) \ \vee \ E(w,f)]\}$$
$$S(f,w) \ \wedge \ [\neg E(f,g) \ \vee \ E(w,f)]$$

(14) Unify (13.2) and (r).

$$E(f,g) \;\wedge\; [\neg E(f,g) \;\vee\; E(w,f)]$$
$$[E(f,g) \;\wedge\; \neg E(f,g)] \;\vee\; [E(f,g) \;\wedge\; E(w,f)]$$
$$[\,] \;\vee\; [E(f,g) \;\wedge\; E(w,f)]$$
$$E(f,g) \;\wedge\; E(w,f)$$

From here, it follows that wolves eat foxes.

(15) Unify (14.2) and (c.1) to obtain the empty clause.

$$E(w,f) \;\wedge\; \neg E(w,f)$$
$$(\,)$$

Having obtained the empty clause, it means that one of the previous clauses is false. So, our assumption (r) is wrong. The statement “Foxes like to eat birds.” is true.

From here, we know that foxes like to eat birds and birds eat grain. So they are the objects that satisfy the statement “There is an animal that likes to eat a grain-eating animal.”