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A mathematically assisted methodology for the experimental calculation of the internal gear ratios to extend the lifecycle of a hobbing machine

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Abstract
Manufacturing of gears can be achieved with various methods, including the cutting of gears in a hobbing machine. The function of such a machine tool is based on a combination of timing gears which synchronize all moving components accurately. Variations in the synchronization, which alter the geometric features of the gear being cut, are achieved via external gearboxes with interchangeable gears. Thus, precise knowledge of the change gears that correspond to the desired gear geometry is essential in order to operate the machine tool properly. The hobbing machine examined in this paper had remained unused for years, without a manual to indicate the correct use of the external gearboxes. The task was to render the machine tool functional by devising an experimental procedure for the calculation of the internal gear ratios and the correct combination of change gears, making use only of simple tools. The project was carried out successfully and proved to be an excellent case study to delve into the manufacturing as well as the use of gears. The documentation of the whole procedure and a short explanatory video are integrated within the educational aids of a relative course in the School of Mechanical Engineering of the National Technical University of Athens.

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Introduction

Gears are some of the most widely used machine elements. Their manufacturing processes vary in the geometric accuracy required, the budget and other particularities of the application. Most commonly gears are formed using material removal processes which include hobbing, milling, broaching and the Fellows method among others,[1,2] while other process may also be required for finishing.[3,4] Above all, gear manufacturing processes require great geometric precision and accurate synchronization of all moving components. Hobbing, in particular, is the most commonly used material removal process for the manufacturing of gears, with significant works published on the product quality of process, tool wear and cutting forces.[5–7]

In hobbing, the cutting tool is the hob which is actually a helical rack wrapped around a cylinder. Grooves that run the hob lengthwise serve as chip removers. The hob's shaft rotates at a constant angular velocity regardless of the type of gear being cut. The workpiece is mounted on a separate shaft that is placed at a certain angle to the hob's shaft. This angle depends on the desired helix angle of the gear and takes into consideration the angle of the hob's helix. The workpiece rotates as well and the angular velocity of its shaft determines how many teeth are cut and is accurately set by placing the right combination of gears in the divisor gearbox. A wide range of gears with different number of teeth can be produced using change gears placed appropriately in this double-stage divisor gearbox. In general, the above constitute the internal gearing of a typical hobbing machine such as the one examined in this paper, which is a Pfauter RS00.

The appropriate combination of change gears that need to be placed in the gearboxes can be easily obtained from the manual of the machine tool. The manual provides either tables, in which a large number of commonly appearing combinations are listed, or formulas that can be used to calculate the ratio needed in each gearbox, which take into consideration the internal drivetrain of the machine.

In the relevant literature, works pertaining to geometrical and mathematical modelling of hobbing can be found.[8] The aforementioned works refer to the process and marginally to the machine tool. Although Computer Numerical Control (CNC) hobbing machines are in use in contemporary industry,[9] non-CNC hobbing machines, as the one examined in this paper, are still widely used in workshops. This is the reason for laying down a methodology for the identification of the internal gear ratios of a hobbing machine in order to achieve its restoration. The issue with the specific hobbing machine, which is addressed in this paper, lies in the fact that the manual was not available. Furthermore, it had remained unused for years while the change gears that were installed did not indicate a recognizable pattern. Unfortunately, a manual for this specific machine tool could not be obtained because all the available sources were either unreliable or too expensive. This issue was addressed as part of a project for two courses included in the curriculum of the School of Mechanical Engineering of the National Technical University of Athens, namely a course in material removal processes and a course in machine elements. The idea of using a hands-on training for educational purposes is not new.[10–12] It was actually applauded by both the teachers and the students, offering better understanding of the studied subject.[13,14] In order to restore the machine tool to its full functionality, the calculation of the internal gear ratios of the hobbing machine was essential. Since disassembling the machine tool without proper instructions involved considerable risk, it was decided to evaluate the gear ratios experimentally, with simple tools.

In the following sections, the methodology adopted to experimentally calculate the internal gear ratios and obtain formulas for the calculation of the change gears is analysed, placing emphasis on the accuracy required and the errors that occur in the measurements. The procedure that was followed was documented so that it can be implemented in similar problems in the future. Furthermore, a short video and a presentation of the restoration of the machine tool, the machine tool function, the use of the external gearboxes and the cutting of gears were produced for educational purposes, posted on-line and were included in the educational aids of the two aforementioned courses (http://www.mdlab.mech.ntua.gr/?page_id=414).

Methodology for the calculation of the internal gear ratios

The machine tool's internal gear ratios can be measured and verified in a variety of ways. The simplest and most straightforward is to disassemble the machine tool and count the teeth of all internal gears. However, in doing so without the machine tool's manual, one runs the risk of either damaging some components or not reassembling them properly. Thus, it is safer to measure the ratios externally, by comparing the angular displacement of coupled shafts, which are visible without disassembling the machine tool. This can be done using very simple tools, i.e. a protractor and a suitable ring spanner. In a pair of coupled shafts, at least one of which has a nut on it, the spanner is placed around the nut, and the corresponding shaft is rotated counter clockwise until the spanner touches a specific, nearby, relatively immovable part of the machine tool; see Figure 1(a) and (b) for the hobbing machine and the procedure with the ring spanner, respectively.

At this point, the angular displacement of the other shaft is measured. These steps are then repeated for different positions of the spanner around the nut and all the measurements are combined using least square regression to estimate the value of the gearing ratio.

Before delving into the details of the statistics involved, it is essential to determine the accuracy that is required, since timing gear ratios have to be specified to their exact value. There are two factors that limit the theoretically infinite amount of accuracy required, which are:

1. The nature of gear drivetrains themselves, in particular the fact that all gear ratios are ratios of integer numbers; this is not true, for example, for belt drives.

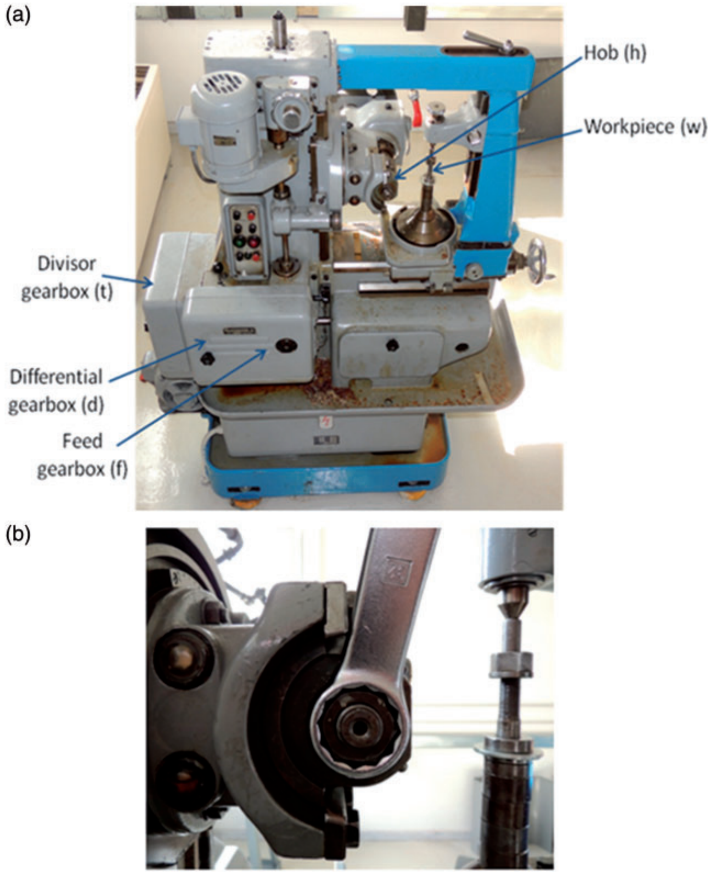

Figure 1. (a) The Pfauter RS00 hobbing machine and (b) locking the hob's shaft with a 32 mm ring spanner. The shaft is rotated counter clockwise so that the nut is not loosened.

If it is assumed, for arguments sake, that all the gears involved have less than 100 teeth, then for a single-stage drivetrain (or a drivetrain whose ratio can be expressed as a single fraction of integers from 1 to 100, for example, one reduction stage and one reversing (1:1) stage), the two possible ratio values that are closest to each other are $\frac{99}{100}$ and $\frac{98}{99}$, which differ by approximately 10^{-4}. Thus, if the absolute deviation of a single-stage ratio value is bounded by 5×10^{-5}, the ratio value itself can be determined unambiguously. The required accuracy is less in case the ratio value is not close to unity, as the 'density' of fractions decreases.

2. In most cases, hobbing machines are designed to facilitate the calculation of change gears and to increase productivity. Thus, it is often the case that gearbox constants are round numbers, either integers or simple fractions, i.e. fractions with simple decimal representations, e.g. $\frac{73}{8} = 9.125$. Bearing this in mind, it is

possible to specify the internal gear ratios, even if the drivetrain is multi-stage, without measuring the ratio of each separate stage. In this case, however, a gear must be cut and measured to verify the results, which may not be possible in the initial stages of the machine tool's restoration.

With the above-mentioned considerations in mind, accuracy requirements can be translated into the number of measurements (N) that are necessary. The angular displacement of the shaft on which the spanner is placed shall be considered the independent variable (x), while the dependent variable (y) is the angular displacement of the coupled shaft. Measurements are made at a constant step (δx) and combined into a linear, least square regression ($y = a_1 x + a_0$). The slope (a_1), whose value equals the gear ratio, is calculated with the following formula

$$a_1 = \frac{\sum x_i y_i - N\bar{x}\bar{y}}{\sum x_i{}^2 - N\bar{x}^2} \tag{1}$$

while the margin of error can be estimated by assuming that the slope obeys a normal distribution with variance σ_{a1}^2 and a nominal range of $\pm 3\sigma_{a1}$. Although it would be more accurate to assume that the slope obeys a student distribution at this point, the above-described assumption not only alleviates the computational effort required but also enables the error of the slope and that of each individual measurement to be correlated in a simple and elegant way. The variance (σ_{a1}^2) is best approximated with the following formula[15]

$$\sigma_{a1}^2 \approx s_{a1}^2 = \frac{s^2}{\sum x_i{}^2 - N\bar{x}^2} \tag{2}$$

where

$$s^2 = \frac{\sum y_i{}^2 + a_1^2 \sum x_i{}^2 - 2a_1 \sum x_i y_i + 2a_1 a_0 \sum x_i - 2a_0 \sum_i^y + N a_0^2}{N-2} \tag{3}$$

However, the variance could also be estimated in a simpler manner a priori, if some reasonable assumptions are made regarding the error distribution of each individual measurement. For the sake of simplicity, measurements of the independent variable will be considered error-free, while, as far as the dependent variable is concerned, it is assumed that these errors obey a normal distribution with variance σ_y^2 and a nominal range of $\pm 3\sigma_y$, as was the case with a_1. Under these circumstances, σ_y and the standard deviations of the regression's parameters are related in the following way[16]

$$(\boldsymbol{X}^T\boldsymbol{X})^{-1}\sigma_y^2 = \begin{bmatrix} \sigma_{a1}^2 & \sigma_{a1}\sigma_{a0} \\ \sigma_{a1}\sigma_{a0} & \sigma_{a0}^2 \end{bmatrix} \tag{4}$$

where $\boldsymbol{X}$ is an $(N \times 2)$ matrix

$$\boldsymbol{X} = \begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_N & 1 \end{bmatrix} \tag{5}$$

In our case, where measurements are taken at a constant step (δx), thus $x_i = (i-1)\delta x$ and as a result $(\boldsymbol{X}^T\boldsymbol{X})$ is simplified to the following form

$$(\boldsymbol{X}^T\boldsymbol{X}) = \begin{bmatrix} \delta x^2 \sum_1^{N-1} i^2 & \delta x \sum_1^{N-1} i \\ \delta x \sum_1^{N-1} i & N \end{bmatrix} \tag{6}$$

After the sums are calculated analytically, the eventual result is

$$(\boldsymbol{X}^T\boldsymbol{X})^{-1} = \begin{bmatrix} \frac{12}{N^3 - N}\frac{1}{\delta x^2} & -\frac{6}{N^2 + N}\frac{1}{\delta x} \\ -\frac{6}{N^2 + N}\frac{1}{\delta x} & \frac{4N - 2}{N^2 + N} \end{bmatrix} \tag{7}$$

and as far as the standard deviation of a_1 is concerned

$$\sigma_{a1} = \frac{\sigma_y}{\delta x}\sqrt{\frac{12}{N^3 - N}} \tag{8}$$

Bearing in mind that both y and a_1 obey normal distributions, equation (8) is also true for the error margins of both variables

$$e_{a1} = \frac{e_y}{\delta x}\sqrt{\frac{12}{N^3 - N}} \tag{9}$$

Equation (9) can be used to estimate the optimum experimental parameters (N, δx) for a specific level of accuracy required. For example, assuming that the angle (y) is measured within a range of $\pm 5^\circ$, the number of measurements required to reach the maximum accuracy needed for a_1 ($\pm 5 \times 10^{-5}$) is approximately $N = 100$, assuming that measurements are made at 360° intervals. Further implications of this equation will be examined in the Results and Discussion section.

The internal gearing layout of a hobbing machine is such that calculating the ratios related to the divisor gearbox first provides useful results for the other gearboxes as well. In most manuals, the change gears that are installed in this gearbox are provided in tables, or the ratio (i_t) can be calculated through a simple formula, i.e.

$$i_t = \frac{C_t g}{z} \tag{10}$$

where z is the number of teeth to be cut, g the number of starts of the hob's thread (in most cases $g = 1$) and C_t a constant related to the machine's internal gearing. It is easy to derive that

$$C_t = \frac{\varphi_h}{\varphi_{t1}} \frac{\varphi_{t2}}{\varphi_w} \tag{11}$$

as $\frac{z}{g} = \frac{\varphi_h}{\varphi_w}$. Thus, in order to estimate the value of C_t, the internal ratios $\frac{\varphi_h}{\varphi_{t1}}$ and $\frac{\varphi_{t2}}{\varphi_w}$ must be calculated experimentally. At this point, it should be noted that C_t has two different values depending on whether the differential is engaged (helical gear hobbing) or disengaged and locked (spur gear hobbing). All values of C_t that are mentioned from now on refer to the former case, which is the most complex of the two, because all the internal gearing is utilized.

In the differential gearbox, the ratio of change gears is calculated via a formula of the following type (if $g = 1$)

$$i_d = \frac{C_d z}{H} \quad \text{or} \quad i_d = \frac{C_d \sin \beta_o}{\pi m_n} \tag{12}$$

where H is the pitch of the helical gear helix, β_o the helix angle on the rolling circle, m_n the normal module and C_d a constant related to the machine tool's internal gearing, which, unlike C_t, which is dimensionless, is measured in mm/rev. Reasoning in the same way as in the divisor gearbox, C_d can be expressed as a product of ratios. Taking into account that $z = \frac{\varphi_h}{\varphi_w}$ and $= \frac{\delta h}{\varphi_w}$, where δh is the hob's vertical displacement, the following equation is derived

$$\begin{gathered} i_d = \frac{C_d \frac{\varphi_h}{\varphi_w}}{\frac{\delta h}{\varphi_w}} = \frac{C_d \frac{\varphi_h}{\varphi_{t1}} \frac{\varphi_{t1}}{\varphi_{t2}} \frac{\varphi_{t2}}{\varphi_w}}{\frac{\delta h}{\varphi_{d1}} \frac{\varphi_{d1}}{\varphi_{d2}} \frac{\varphi_{d2}}{\varphi_{t1}} \frac{\varphi_{t1}}{\varphi_{t2}} \frac{\varphi_{t2}}{\varphi_w}} = \frac{C_d \frac{\varphi_h}{\varphi_{t1}} i_d}{\frac{\delta h}{\varphi_{d1}} \frac{\varphi_{d2}}{\varphi_{t1}}} \Rightarrow \\ C_d = \frac{\frac{\delta h}{\varphi_{d1}} \frac{\varphi_{d2}}{\varphi_{t1}}}{\frac{\varphi_h}{\varphi_{t1}}} \end{gathered} \tag{13}$$

Since the differential has three inputs/outputs, for all the above ratios to be well defined, it should be noted that in case two of these inputs/outputs are involved in a ratio the third one is considered stationary. For example, the notation $H = \frac{\delta h}{\varphi_w}$ implies that the hob, essentially the third input/output, does not rotate. The ratios required to calculate C_d are easy to measure ($\frac{\varphi_h}{\varphi_{t1}}$ has already been measured) with the exception of $\frac{\delta h}{\varphi_{d1}}$ which is the main cause of error in estimating the value of C_d.

Finally, the formula for the change gear ratio in the feed gearbox is

$$i_f = \frac{feed\,(\frac{mm}{rev})}{C_f}, \text{ i.e.} : i_f = \frac{\delta h}{\varphi_w C_f} \tag{14}$$

C_f is measured in mm/rev, similar to C_d, and can be expressed as a product of ratios in the following way

$$\begin{aligned} i_f &= \frac{\delta h}{\varphi_w C_f} = \frac{\delta h}{\varphi_{f2}}\frac{\varphi_{f2}}{\varphi_{f1}}\frac{\varphi_{f1}}{\varphi_{t2}}\frac{\varphi_{t2}}{\varphi_w}\frac{1}{C_f} \Rightarrow \\ C_f &= \frac{\delta h}{\varphi_{f2}}\frac{\varphi_{f1}}{\varphi_{t2}}\frac{\varphi_{t2}}{\varphi_w} \end{aligned} \tag{15}$$

By measuring all the ratios involved, the value of C_f can be estimated.

The estimated values of C_d and C_f can subsequently be verified by utilizing all three gearboxes to cut a spur gear. This technique is used in case the ratios required to cut a particular gear cannot be generated with the available change gears, mostly when the number of teeth is a large prime.[17] It involves installing a slightly offset ratio in the divisor gearbox and then 'correcting' it via the differential and feed gearboxes. Since from equation (13) $\varphi_h = \frac{i_d \delta h}{C_d}$ and from equation (15) $\varphi_w = \frac{\delta h}{C_f i_f}$, this 'correction' is

$$\left(\frac{\varphi_h}{\varphi_w}\right)_{d,f} = \pm \frac{C_f}{C_d} i_d i_f \tag{16}$$

The contribution of the divisor gearbox is of course (see equation (10))

$$\left(\frac{\varphi_h}{\varphi_w}\right)_t = \frac{C_t}{i_t} \tag{17}$$

Thus, adding both contributions together, the following result is obtained

$$\frac{\varphi_h}{\varphi_w} = \frac{C_t}{i_t} \pm \frac{C_f}{C_d} i_d i_f \tag{18}$$

Equation (18) can be utilized to verify the value of the ratio $\frac{C_f}{C_d}$, which in most cases ensures the verification of C_d and C_f as well, since different combinations of candidate values with the same ratio are extremely rare.

Results and discussion

Estimation and verification of gearbox constants

Before exploring the details of experimentally estimating the values of the gearbox constants, a model experiment, demonstrating the various nuances of approximating a single-stage ratio, shall be presented, i.e. the measurement of the ratio between the hob's shaft and the machine tool's flywheel.

The angular displacement of the flywheel's shaft (y) is measured after rotating the hob's shaft at 360° intervals and locking it in position using a ring spanner. The total number of measurements required to rule out all possible ratio values, but the actual ($\frac{27}{8}$), is 7, while a further 13 measurements are conducted for verification purposes. Figure 2 demonstrates the convergence of a_1 towards the actual ratio value and the gradual elimination of the two closest irreducible fractions. Furthermore, Figure 3 depicts the gradual convergence of the estimation for the error of each single measurement according to equation (9), solved for e_y.

This error in this case converges to the value of ±3.9° which is noticeably smaller than the one assumed in the methodology section (±5°), a fact which could be attributed to a number of favourable conditions during these measurements, which do not exist when the critical ratios $\frac{\varphi_h}{\varphi_{t1}}$ and $\frac{\varphi_{t2}}{\varphi_w}$ are estimated experimentally. Since in this case the drivetrain is obviously multi-stage, there is no simple criterion for the

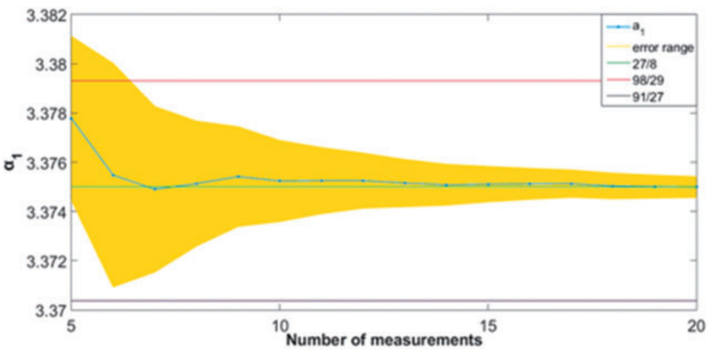

Figure 2. The number of measurements versus a_1.

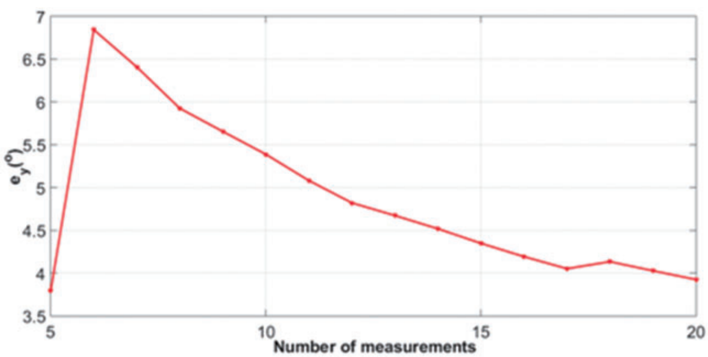

Figure 3. The number of measurements versus e_y.

convergence of measurements and the pursuit of a single fraction is substituted by the convergence of e_y, the estimation of which is necessary in evaluating the accuracy of the measurements. After 20 measurements, the slope was estimated at: $\frac{\varphi_{t1}}{\varphi_h} = 2.1561 \pm 0.0005$. Note that, since the independent variable is φ_h, the slope equals the inverse of the desired ratio and thus the final result is: $\frac{\varphi_h}{\varphi_{t1}} = (0.4638) \pm (0.00011)$, where the error of the inverse ratio $(\frac{1}{r})$ is calculated using the following formula

$$e_{1/r} = \left|\frac{d(\frac{1}{r})}{dr}\right| e_r = \frac{e_r}{r^2} \tag{19}$$

Furthermore, equation (9) is once again used to calculate the error of each single measurement, which converges to $\pm 5^\circ$.

In case of the ratio $\frac{\varphi_{t2}}{\varphi_w}$, elaborate measurements are unnecessary, since in a simple worm drive the ratio is expected to be an integer, for a single start worm screw. However, simple observations indicate that the ratio value is $\frac{\varphi_{t2}}{\varphi_w} = 34.5$, which suggests that the worm screw is double start. This result was confirmed with a series of measurements in which the convergence of e_y was once again the convergence criterion. In particular, after 25 measurements the estimated value of the ratio was $\frac{\varphi_{t2}}{\varphi_w} = 34.50 \pm 0.016$, while the elimination of all candidate fractions except $\frac{69}{2}$ is achieved after very few measurements, since the 'density' of fractions in the area of such high ratio values is very low; the closest candidates are $\frac{34}{1}$ and $\frac{35}{1}$. In this case, the workpiece shaft was locked with the spanner at 30° intervals and angular displacements were measured on the output shaft of the divisor gearbox. The large gearing ratio between the flywheel, the point on which manual rotary input is given, and the workpiece shaft makes the evaluation of the locking force and the induced displacement highly inaccurate and subjective. This unfavourable situation influences the error of each single measurement which converges to $\pm 16.9^\circ$.

These three experiments reveal that in an average situation, such as the calculation of $\frac{\varphi_h}{\varphi_{t1}}$, each single measurement is expected to be accurate within a range of $\pm 5^\circ$, an assumption which was made when calculating an upper bound for the number of measurements required. This assumption is not accurate when high ratio values are involved, as is seen in the third experiment; however, since the distribution of fractions is less 'dense' for such values, this is not expected to affect the upper bound of 100 measurements that was calculated.

Having estimated the ratios $\frac{\varphi_h}{\varphi_{t1}}$ and $\frac{\varphi_{t2}}{\varphi_w}$, the constant of the divisor gearbox (C_t) can be calculated from equation (11), as their product. Since $\frac{\varphi_{t2}}{\varphi_w}$ could be considered error-free with a value of $\frac{\varphi_{t2}}{\varphi_w} = \frac{69}{2}$, the relative errors of C_t and $\frac{\varphi_h}{\varphi_{t1}}$ coincide and thus

$$\frac{e_{C_t}}{C_t} = \frac{e_{\ \varphi_h/\varphi_{t1}}}{\varphi_h/\varphi_{t1}} \Rightarrow e_{C_t} = C_t \frac{e_{\ \varphi_h/\varphi_{t1}}}{\varphi_h/\varphi_{t1}} \tag{20}$$

After the completion of the required measurements, the final result was $C_t = (16.001) \pm (0.004)$, which points to the most probable value of $C_t = 16$. This assumption was verified by cutting a 34-tooth spur gear with a divisor gearbox ratio of $\frac{16}{34}$, as calculated in equation (10). In the absence of a manual, the actual change gears were calculated with a simple computer program which examines all possible combinations of the available gears. During the cutting process, the feed was given manually and the hob was allowed to operate at a particular cutting depth for a considerable amount of time and a large number of full rotations. In this way, a false synchronization would reveal itself through minor cuts after the first rotation and all their side effects, i.e. chip production, noise and vibrations. No such phenomena were observed and thus 16 was confirmed to be the true value of C_t. As a result of this, the value of $\frac{\varphi_h}{\varphi_{t1}} = \frac{32}{69}$ could be considered error-free, based on equation (11), since both $\frac{\varphi_{t2}}{\varphi_w}$ and C_t are considered error-free by this stage.

Having completed the calculations related to the divisor gearbox, estimating the values of the other two machine tool constants becomes a much easier task. As far as the differential gearbox is concerned, calculating C_d requires a further two ratio values, namely $\frac{\varphi_{d2}}{\varphi_{t1}}$ and $\frac{\delta h}{\varphi_{d1}}$. The former ratio is produced by a simple worm gear drivetrain and one can easily observe that $\frac{\varphi_{d2}}{\varphi_{t1}} = \frac{24}{1}$, without conducting any precise measurements. On the contrary, $\frac{\delta h}{\varphi_{d1}}$ cannot be evaluated precisely and after a series of measurements, the most reliable one revealed that the tool holder had moved downwards by approximately 21 mm after 60 complete revolutions of the input shaft of the differential gearbox, hence $\frac{\delta h}{\varphi_{d1}} \approx \frac{7}{20}$ mm/rev. Taking into account that the value of $\frac{\varphi_h}{\varphi_{t1}}$ is $\frac{32}{69} = 0.463768$ an initial approximation of C_d is derived using equation (13): $C_d \approx \frac{\frac{7}{20} \cdot \frac{24}{1}}{\frac{32}{69}} = 18.1125$ mm/rev. This result indicates that the most likely values of C_d are the numbers 18, 18.125 and 18.25 which will have to be tested and verified.

A similar procedure leads to an estimation of the feed gearbox constant (C_f), in which case the ratios that need to be measured are $\frac{\varphi_{f1}}{\varphi_{t2}}$ and $\frac{\delta h}{\varphi_{f2}}$. Thus, the candidate values that will have to be tested are numbers such as 2.75, 2.875 or 3.

The verification process for both C_d and C_f is based on equation (18) and involves cutting a spur gear with a prime number of teeth to verify the ratio of the two constants, $\frac{C_f}{C_d}$; non-prime numbers could be selected as well but in this case the ratios required are often impossible to generate with the available change gears. Installing the wrong change gears based on a slightly inaccurate ratio of $\frac{C_f}{C_d}$ would lead to a noticeable lack of synchronization which would result to slightly helical teeth which are thinner than normal, an effect which can be observed on the finished gear.

In the case described in this paper, there are a number of possible ratios of $\frac{C_f}{C_d}$ that could be formed from the candidate values, the most obvious of which is the simplest, $\frac{C_f}{C_d} = \frac{3}{18} = \frac{1}{6}$. This ratio was verified by cutting a 29-tooth spur gear. The ratios of change gears used in the gearboxes were: $i_t = \frac{C_t}{29 - \frac{1}{4}} = 0.5565217$, $i_f = \frac{1}{C_f} = \frac{1}{3}$, for an arbitrary feed of 1 mm/rev and $i_d = \frac{\frac{1}{4}}{\frac{C_f}{C_d} \cdot i_f} = \frac{\frac{1}{4}}{\frac{1}{6} \cdot \frac{1}{3}} = 4.5$, in order to correct the slightly offset divisor gearbox ratio according to equation (18). The change gear used to construct these ratios were: $i_t = \frac{80}{75} \cdot \frac{36}{69}$, $i_f = \frac{24}{72}$ and $i_d = \frac{78}{52} \cdot \frac{75}{25}$ plus an idling gear that was used in the feed gearbox. No significant geometric irregularities were observed on the finished gear and thus the true values of C_d and C_f were confirmed to be 18 and 3 mm/rev, respectively.

Impact of the methodology on the course audience

The aforementioned methodology was presented in front of the students of the third year of studies of the School of Mechanical Engineering, in the National Technical University of Athens. The course pertains to the material removal manufacturing processes; a part of the course is dedicated to machining of gears. In the same course, the students visit the workshop of the Section of Manufacturing Technology for training in cutting of metals with a lathe, a milling machine and a CNC machining centre; these are mandatory classes for students in the workshop. However, gear cutting requires a more complicated procedure and besides the theory performed in the class, the students required extra hands-on training.

The initial approach was to involve the students in the restoration of the hobbing machine. The most effective methodology that was suggested and worked was the one described in the previous sections. The hours spent in the workshop for the application of the methodology, optional for the students, were helpful for the deeper understanding of how gears work and are manufactured, as well as how the machine tool works. All the students involved declared that it was a useful experience that significantly helped them in the understanding of gear manufacturing, machine tools and metal cutting in general.

In order to reach a greater audience and not only the students who volunteered to work in the workshop on the hobbing machine, the methodology was filmed and documented in detail and then presented in class. It was also uploaded to the online learning platform of the course so that all the students would have access to it or review it. After the presentation, a small survey was conducted to assess the impact of the methodology on the audience. The majority of the students, almost 98% of the present students, declared that they had a very good understanding on gear manufacturing. A similar percentage had a very good understanding on hobbing, gear and gearboxes function and declared that improved their knowledge on metal cutting; a percentage just above 85% thought that the procedure followed was performed with simple tools and the mathematical tools employed are within the knowledge they have acquired in the previous years of their studies. Most of the students would like to be involved in a future similar project. Finally, there were some suggestions for enhancement of the methodology that will be implemented in future classes.

Conclusions

The above-analysed results demonstrate that it is possible to calculate all the necessary internal gear ratios in a hobbing machine and thus to obtain the formulas for calculating change gears, using only simple tools, which are available in any workshop, and basic statistical analysis. The methodology described in this paper could be applied in more complicated drivetrains, while the use of more sophisticated measuring devices, such as tachogenerators or hall sensors, could enhance it further. Shortcuts to the complete measuring procedure, such as the ones described in the Results and Discussion section, could be taken in most cases, depending on the extent of knowledge about the internal drivetrain. Furthermore, the simplicity and versatility of this methodology renders it easy to comprehend, even for the uninitiated. In conclusion, it could be argued that the methods described in this paper constitute an effective and at the same time easy to implement approach to calculating timing gear ratios experimentally in case they are not known, for example, if the machine tool's manual is missing.

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