MATH3431 Machine Learning and Neural Networks III — Epiphany term 2023

Exercise sheet

Lecturer: Georgios P. Karagiannis — georgios.karagiannis@durham.ac.uk

Part I

Stochastic learning

Exercise 1. ($\star$)Let $f:\mathbb{R}^{d}\rightarrow\mathbb{R}$ such that $f\left(w\right)=g\left(<w,x>+y\right)$ or some $x\in\mathbb{R}^{d}$, $y\in\mathbb{R}$. If $g$is convex function then $f$is convex function.

Exercise 2. ($\star$)Let functions g_{1} be ρ_{1}-Lipschitz and g_{2} be ρ_{2}-Lipschitz. Then f with $f\left(x\right)=g_{1}\left(g_{2}\left(x\right)\right)$ is $\rho_{1}\rho_{2}$-Lipschitz.

Exercise 3. ($\star$)Let $f:\mathbb{R}^{d}\rightarrow\mathbb{R}$ with $f\left(w\right)=g\left(\langle w,x\rangle+y\right)$ $x\in\mathbb{R}^{d}$ and $y\in\mathbb{R}$. Let $g:\mathbb{R}\rightarrow\mathbb{R}$ be a β-smooth function. Then f is a $\left(\beta\left\Vert x\right\Vert ^{2}\right)$-smooth.

Hint: You may use Cauchy-Schwarz inequality $\langle y,x\rangle\leq\left\Vert y\right\Vert \left\Vert x\right\Vert$

Exercise 4. ($\star$)$f:S\rightarrow\mathbb{R}$ is ρ-Lipschitz over an open convex set S if and only if for all $w\in S$ and $v\in\partial f\left(w\right)$ it is $\left\Vert v\right\Vert \leq\rho$.

Hint: You may use Cauchy-Schwarz inequality $\langle y,x\rangle\leq\left\Vert y\right\Vert \left\Vert x\right\Vert$

Exercise 5. ($\star$)Let $g_{1}\left(w\right),...,g_{r}\left(w\right)$ be r convex functions, and let $g\left(\cdot\right)=\max_{\forall j}\left(g_{j}\left(\cdot\right)\right)$. Show that for some w it is $\nabla g_{k}\left(w\right)\in\partial g\left(w\right)$ where $k=\arg\max_{j}\left(g_{j}\left(w\right)\right)$ is the index of function $g_{j}\left(\cdot\right)$ presenting the greatest value at w.
