MATH3431 Machine Learning and Neural Networks III	Epiphany term 2023

Exercise sheet

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Part I

Stochastic learning

Exercise 1. ($\star$)Let $f : \mathbb{R}^d \to \mathbb{R}$ such that $f(w) = g(< w, x > +y)$ or some $x \in \mathbb{R}^d$, $y \in \mathbb{R}$. If $g$is convex function then $f$is convex function.

Exercise 2. ($\star$)Let functions g_1 be ρ_1-Lipschitz and g_2 be ρ_2-Lipschitz. Then f with $f(x) = g_1(g_2(x))$ is $\rho_1\rho_2$-Lipschitz.

Exercise 3. ($\star$)Let $f : \mathbb{R}^d \to \mathbb{R}$ with $f(w) = g(\langle w, x\rangle + y)$ $x \in \mathbb{R}^d$ and $y \in \mathbb{R}$. Let $g : \mathbb{R} \to \mathbb{R}$ be a β-smooth function. Then f is a $\left(\beta \|x\|^2\right)$-smooth.

Hint: You may use Cauchy-Schwarz inequality $\langle y, x\rangle \le \|y\| \|x\|$

Exercise 4. ($\star$)$f : S \to \mathbb{R}$ is ρ-Lipschitz over an open convex set S if and only if for all $w \in S$ and $v \in \partial f(w)$ it is $\|v\| \le \rho$.

Hint: You may use Cauchy-Schwarz inequality $\langle y, x\rangle \le \|y\| \|x\|$

Exercise 5. ($\star$)Let $g_1(w), ..., g_r(w)$ be r convex functions, and let $g(\cdot) = \max_{\forall j}(g_j(\cdot))$. Show that for some w it is $\nabla g_k(w) \in \partial g(w)$ where $k = \arg\max_j(g_j(w))$ is the index of function $g_j(\cdot)$ presenting the greatest value at w.
