MATH3431 Machine Learning and Neural Networks III Epiphany term 2023

Exercise sheet

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Part I

Stochastic learning

Exercise 1. ($\star$)Let $f : \mathbb{R}^d \to \mathbb{R}$ such that $f(w) = g(< w, x > +y)$ or some $x \in \mathbb{R}^d$, $y \in \mathbb{R}$. If $g$is convex function then $f$is convex function.

Solution. Let $u, v \in \mathbb{R}^d$ and $a \in [0, 1]$. It is

$$\begin{aligned}
f(\alpha u + (1-\alpha) v) =& g(< \alpha u + (1-\alpha) v, x > +y) \\
=& g(< \alpha u, x > + < (1-\alpha) v, x > +y) \\
=& g(\alpha(< u, x > +y) + (1-\alpha)(< v, x > +y)) && y = \alpha y + (1-\alpha) y \\
\leq& \alpha g(< u, x > +y) + (1-\alpha) g(< v, x > +y) && (g \text{ is convex}) \\
=& \alpha f(u) + (1-\alpha) f(v)
\end{aligned}$$
