MATH3431 Machine Learning and Neural Networks III — Epiphany term 2023

Exercise sheet

Lecturer: Georgios P. Karagiannis — georgios.karagiannis@durham.ac.uk

Part I

Stochastic learning

Exercise 1. ($\star$)Let $f : \mathbb{R}^d \to \mathbb{R}$ such that $f(w) = g(< w, x > +y)$ or some $x \in \mathbb{R}^d$, $y \in \mathbb{R}$. If $g$is convex function then $f$is convex function.

Solution. Let $u, v \in \mathbb{R}^d$ and $a \in [0, 1]$. It is

$$\begin{aligned}
f(\alpha u + (1-\alpha) v) =& g(< \alpha u + (1-\alpha) v, x > +y) \\
=& g(< \alpha u, x > + < (1-\alpha) v, x > +y) \\
=& g(\alpha(< u, x > +y) + (1-\alpha)(< v, x > +y)) \qquad y = \alpha y + (1-\alpha) y \\
\leq& \alpha g(< u, x > +y) + (1-\alpha) g(< v, x > +y) \qquad (g \text{ is convex}) \\
=& \alpha f(u) + (1-\alpha) f(v)
\end{aligned}$$

Exercise 2. ($\star$)Let functions g_1 be ρ_1-Lipschitz and g_2 be ρ_2-Lipschitz. Then f with $f(x) = g_1(g_2(x))$ is $\rho_1\rho_2$-Lipschitz.

Solution.

$$\begin{aligned}
|f(w_1) - f(w_2)| =& |g_1(g_2(w_1)) - g_1(g_2(w_2))| \\
\leq& \rho_1 |g_2(w_1) - g_2(w_2)| \\
\leq& \rho_1\rho_2 |w_1 - w_2|
\end{aligned}$$
