

## Homework 1: Convex learning problems

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**Exercise 1.** Let  $f : \mathbb{R}^d \rightarrow \mathbb{R}$  be a convex and  $\beta$ -smooth function.

1. Show that for  $v, w \in \mathbb{R}^d$

$$f(v) - f(w) \in \left( \langle \nabla f(w), v - w \rangle, \langle \nabla f(w), v - w \rangle + \frac{\beta}{2} \|v - w\|^2 \right)$$

2. Show that for  $v, w \in \mathbb{R}^d$  such that  $v = w - \frac{1}{\beta} \nabla f(w)$ , it is

$$\frac{1}{2\beta} \|\nabla f(w)\|^2 \leq f(w) - f(v)$$

3. Additionally assume that  $f(x) > 0$  for all  $x \in \mathbb{R}^d$ . Show that for  $w \in \mathbb{R}^d$ ,

$$\|\nabla f(w)\| \leq \sqrt{2\beta f(w)}$$

**Exercise 2.** Let  $f : \mathbb{R}^d \rightarrow \mathbb{R}$  be a  $\lambda$ -strongly convex function. Assume that  $w^*$  is a minimizer of  $f$  i.e.

$$w^* = \arg \min_w \{f(w)\}$$

Show that for any  $w \in \mathbb{R}^d$  it holds

$$f(w) - f(w^*) \geq \frac{\lambda}{2} \|w - w^*\|^2$$

**Hint** Use the definition of  $\lambda$ -strongly convex function, properly rearrange it, and ...