Spatio-temporal statistics (MATH4341) Michaelmas term, 2023

Homework 1: Geostatistics (building concepts)

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As formative assessment 1, submit the solutions to all the Exercises

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Exercise 1. ($\star$) Let $Z = (Z_s)_{s\in\mathbb{R}^d}$ be an intrinsically stationary stochastic process, and let $\gamma : \mathbb{R}^d \to \mathbb{R}$ be its semivariogram. Assume $a \in \mathbb{R}^n$s.t. $\sum_{i=1}^n a_i = 0$.

1. Let $a \in \mathbb{R}^n$ be a vector of constants. Show that

$$\mathrm{Var}\left(\sum_{i=1}^{n} a_i Z(s_i)\right) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j c_Y(s_i, s_j)$$

   where $c_Y(s,t) = \mathrm{E}(Y(s)Y(t))$, and $Y_s = Z_s - Z_0$.

2. Show that

$$c_Y(s,t) = \gamma(s) + \gamma(t) - \gamma(s-t)$$

3. Show that for all $\forall \{s_1, ..., s_n\} \subseteq S$ it is

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j \gamma(s_i - s_j) \le 0$$

Exercise 2. ($\star$) Consider the zero-mean geostatistical process $Z = (Z_s)_{s\in\mathbb{R}^d}$ with a weakly stationary and isotropic covariance function given by

$$c(h) = \begin{cases} \xi^2 (1 + \rho \|h\|) \exp(-\rho \|h\|), & h > 0 \\ \nu^2 + \xi^2, & h = 0 \end{cases}$$

1. Compute the semi-variogram for the geostatistical process (Z_s)

2. What are the nugget, sill and partial sill for this covariance model? Justify your answer.

3. Would the slightly altered covariance function defined below be a good model for spatial data for $\phi > 0$? Justify your answer.

$$c\left(h\right) = \begin{cases} \xi^2 \left(1 + \rho \left\|h\right\|\right) \exp\left(-\rho \left\|h\right\|\right) + \phi, & h > 0 \\ \nu^2 + \xi^2 + \phi, & h = 0 \end{cases}$$