

## Homework 1: Geostatistics (building concepts)

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As formative assessment 1, submit the solutions to all the Exercises

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**Exercise 1.** (★) Let  $Z = (Z_s)_{s \in \mathbb{R}^d}$  be an intrinsically stationary stochastic process, and let  $\gamma : \mathbb{R}^d \rightarrow \mathbb{R}$  be its semivariogram. Assume  $a \in \mathbb{R}^n$  s.t.  $\sum_{i=1}^n a_i = 0$ .

1. Let  $a \in \mathbb{R}^n$  be a vector of constants. Show that

$$\text{Var} \left( \sum_{i=1}^n a_i Z(s_i) \right) = \sum_{i=1}^n \sum_{j=1}^n a_i a_j c_Y(s_i, s_j)$$

where  $c_Y(s, t) = E(Y(s)Y(t))$ , and  $Y_s = Z_s - Z_0$ .

2. Show that

$$c_Y(s, t) = \gamma(s) + \gamma(t) - \gamma(s - t)$$

3. Show that for all  $\forall \{s_1, \dots, s_n\} \subseteq S$  it is

$$\sum_{i=1}^n \sum_{j=1}^n a_i a_j \gamma(s_i - s_j) \leq 0$$

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**Exercise 2.** (★) Consider the zero-mean geostatistical process  $Z = (Z_s)_{s \in \mathbb{R}^d}$  with a weakly stationary and isotropic covariance function given by

$$c(h) = \begin{cases} \xi^2 (1 + \rho \|h\|) \exp(-\rho \|h\|), & h > 0 \\ \nu^2 + \xi^2, & h = 0 \end{cases}$$

1. Compute the semi-variogram for the geostatistical process  $(Z_s)$
2. What are the nugget, sill and partial sill for this covariance model? Justify your answer.

3. Would the slightly altered covariance function defined below be a good model for spatial data for  $\phi > 0$ ? Justify your answer.

$$c(h) = \begin{cases} \xi^2 (1 + \rho \|h\|) \exp(-\rho \|h\|) + \phi, & h > 0 \\ \nu^2 + \xi^2 + \phi, & h = 0 \end{cases}$$