

Homework 1: Regional statistical & probabilistic concepts

Lecturer: Georgios P. Karagiannis

georgios.karagiannis@durham.ac.uk

 As formative assessment, submit the solutions to Exercise ...

Exercise 1. (★★) Let $f : \mathbb{R}^d \rightarrow \mathbb{R}$ such that $f(w) = g(\langle w, x \rangle + y)$ or some $x \in \mathbb{R}^d, y \in \mathbb{R}$. Show that: If g is convex function then f is convex function.

Solution. Let $u, v \in \mathbb{R}^d$ and $a \in [0, 1]$. It is

$$\begin{aligned}
 f(\alpha u + (1 - \alpha)v) &= g(\langle \alpha u + (1 - \alpha)v, x \rangle + y) \\
 (1) \qquad &= g(\langle \alpha u, x \rangle + \langle (1 - \alpha)v, x \rangle + y) \\
 &= g(\alpha \langle u, x \rangle + y + (1 - \alpha) \langle v, x \rangle + y) & y = \alpha y + (1 - \alpha)y \\
 &\leq \alpha g(\langle u, x \rangle + y) + (1 - \alpha) g(\langle v, x \rangle + y) & (g \text{ is convex}) \\
 &= \alpha f(u) + (1 - \alpha) f(v)
 \end{aligned}$$