

**Homework 1: Point referenced data (building concepts)**

Lecturer: Georgios P. Karagiannis

georgios.karagiannis@durham.ac.uk

**Exercise 1.** (★) Let  $Z = (Z(s) : s \in \mathbb{R}^d)$  be an intrinsic random field with  $E(Z(s) - Z(t)) = 0$  and let  $\gamma : \mathbb{R}^d \rightarrow \mathbb{R}$  be its semivariogram.

(1) Let  $a \in \mathbb{R}^n$  be a vector of constants. Consider sites  $\{s_1, \dots, s_n \subseteq \mathbb{R}^d\}$  Show that

$$\text{Var} \left( \sum_{i=1}^n a_i Z(s_i) \right) = \sum_{i=1}^n \sum_{j=1}^n a_i a_j c_I(s_i, s_j)$$

where  $c_I(s, t) = \gamma(s - s_0) + \gamma(t - s_0) - \gamma(s - t)$  at some additional  $s_0 \in \mathbb{R}^d$ .

(2) Show that for all  $n \in \mathbb{N}$ ,  $(a_1, \dots, a_n) \subseteq \mathbb{R}^n$  s.t.  $\sum_{i=1}^n a_i = 0$ , and for all  $(s_1, \dots, s_n) \subseteq S^n$ , it is

$$\sum_{i=1}^n \sum_{j=1}^n a_i a_j \gamma(s_i - s_j) \leq 0$$

**Solution.**

**Exercise 2.** (★) Consider the zero-mean random field  $Z = (Z(s) : s \in \mathbb{R}^d)$  with covariogram function given by

$$c(h) = \begin{cases} \xi^2 (1 + \rho \|h\|) \exp(-\rho \|h\|), & h > 0 \\ \nu^2 + \xi^2, & h = 0 \end{cases}$$

- (1) Compute the semivariogram for the random field  $(Z(s) : s \in \mathbb{R}^d)$
- (2) What are the nugget, sill and partial sill for this covariance model? Justify your answer.
- (3) Would the slightly altered covariance function defined below be a good model for spatial data for  $\phi > 0$ ? Justify your answer.

$$c(h) = \begin{cases} \xi^2 (1 + \rho \|h\|) \exp(-\rho \|h\|) + \phi, & h > 0 \\ \nu^2 + \xi^2 + \phi, & h = 0 \end{cases}$$

**Solution.**