Spatio-temporal statistics (MATH4341) Michaelmas term, 2024

Homework 1: Point referenced data (building concepts)

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Exercise 1. ($\star$) Let $Z = \left(Z\left(s\right) : s \in \mathbb{R}^d\right)$ be an intrinsic random field with $\mathrm{E}\left(Z\left(s\right) - Z\left(t\right)\right) = 0$ and let $\gamma : \mathbb{R}^d \to \mathbb{R}$ be its semivariogram.

(1) Let $a \in \mathbb{R}^n$ be a vector of constants. Consider sites $\left\{s_1, ..., s_n \subseteq \mathbb{R}^d\right\}$ Show that

$$\mathrm{Var}\left(\sum_{i=1}^{n} a_i Z\left(s_i\right)\right) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j c_I\left(s_i, s_j\right)$$

where $c_I\left(s, t\right) = \gamma\left(s - s_0\right) + \gamma\left(t - s_0\right) - \gamma\left(s - t\right)$ at some additional $s_0 \in \mathbb{R}^d$.

(2) Show that for all $n \in \mathbb{N}$, $\left(a_1, ..., a_n\right) \subseteq \mathbb{R}^n$ s.t. $\sum_{i=1}^{n} a_i = 0$, and for all $\left(s_1, ..., s_n\right) \subseteq S^n$, it is

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j \gamma\left(s_i - s_j\right) \leq 0$$

Solution.

Exercise 2. ($\star$) Consider the zero-mean random field $Z = \left(Z\left(s\right) : s \in \mathbb{R}^d\right)$ with covariogram function given by

$$c\left(h\right) = \begin{cases} \xi^2\left(1 + \rho\left\|h\right\|\right)\exp\left(-\rho\left\|h\right\|\right), & h > 0 \\ \nu^2 + \xi^2, & h = 0 \end{cases}$$

(1) Compute the semivariogram for the random field $\left(Z\left(s\right) : s \in \mathbb{R}^d\right)$

(2) What are the nugget, sill and partial sill for this covariance model? Justify your answer.

(3) Would the slightly altered covariance function defined below be a good model for spatial data for $\phi > 0$? Justify your answer.

$$c\left(h\right) = \begin{cases} \xi^2\left(1 + \rho\left\|h\right\|\right)\exp\left(-\rho\left\|h\right\|\right) + \phi, & h > 0 \\ \nu^2 + \xi^2 + \phi, & h = 0 \end{cases}$$

Solution.