Homework 3: Delta method

Lecturer: Georgios Karagiannis georgios.karagiannis@durham.ac.uk

Exercise 1

($\star\star\star$) Consider an M-way contingency table and consider the quantities obs. cell counts, cell probabilities, cell proportions in their vectorised forms as

$$\underset{\sim}{n} = (n_1, ..., n_N)^T \; ; \qquad \underset{\sim}{\pi} = (\pi_1, ..., \pi_N)^T \; ; \qquad \underset{\sim}{p} = (p_1, ..., p_N)^T$$

where $n = \sum_{j=1}^{N} n_j$, and $p_j = n_j/n$.

1. Consider a constant matrix $C \in \mathbb{R}^{k\times N}$, and show that

$$\sqrt{n}(C\log(\underset{\sim}{p}) - C\log(\underset{\sim}{\pi})) \xrightarrow{D} \mathrm{N}(0, C\mathrm{diag}(\pi)^{-1}C^T - C11^TC^T) \tag{1}$$

2. Consider a 3×3 contingency table with probabilities $(\pi_{i,j})$. Find the joint asymptotic distribution of the vector of different log odd ratios

$$\log(\underset{\sim}{\theta}^C) = \begin{bmatrix} \log(\frac{\pi_{11}\pi_{22}}{\pi_{21}\pi_{12}}) \\ \log(\frac{\pi_{22}\pi_{33}}{\pi_{23}\pi_{32}}) \end{bmatrix}$$

Exercise 2

($\star\star\star$) Consider an IID sample $X, X_1, X_2, ...$ with $EX = 0$, $EX^4 < \infty$. Consider that

$$\sqrt{n}\frac{S_n^2 - \sigma^2}{\sqrt{EX^4 - \sigma^4}} \xrightarrow{D} \mathrm{N}(0,1) \tag{2}$$

where $S_n^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2$.

1. Find the asymptotic distribution of $\log(S_n^2)$.

2. Produce the $1-a$ asymptotic confidence interval for $\log(\sigma_n^2)$; by performing suitable calculations, so that the boundaries of the confidence interval do not depend on any unknown moments of the real distribution.

Exercise 3

($\star\star\star$) Consider random sample $X, X_1, X_2, \ldots$ IID from a Bernoulli distribution with probability of success p. Find the variance stabilization transformation for the estimator average $\hat{p}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$.