

Homework 2: Log-linear models

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Exercise 1

Assume model (X, Y, Z) , under Poisson sampling scheme. Consider corner points constraints.

1. Find the Likelihood equations.
2. Express the Log linear coefficients with respect to the expected counts
3. Find the MLEs of the Log linear coefficients

Exercise 2

Consider the dataset in Table 1.

Gender (Z)	Information Opinion (X)	Health Opinion (Y)	
		Support	Oppose
Male	Support	76	160
	Oppose	6	25
Female	Support	114	181
	Oppose	11	48

Table 1: The 1988 General Social Survey compiled by the National Opinion Research Center asked: “Do you support or oppose the following measures to deal with AIDS? (1) Have the government pay all of the health care costs of AIDS patients; (2) Develop a government information program to promote safe sex practices, such as the use of condoms. This Table summarizes opinions about health care costs (Y) and the information program (X), classified also by the respondent’s gender (Z). [Source: 1988 General Social Survey, National Opinion Research Center.]

Use suitable tools in order to compare the following models:

1. $[Y, X, Z \text{ are independent}]$ vs. $[X \text{ and } Y \text{ are conditionally independent on } Z]$
2. $[X \text{ and } Z \text{ are jointly independent from } Y]$ vs. $[X \text{ and } Y \text{ are jointly independent from } Z]$

Any inference based on hypothesis tests should be performed at sig. level 5%.

Exercise 3

Consider a $I \times J \times K$ table n_{ijk} generated from a Poisson sampling scheme; i.e.

$$n_{ijk} \sim \text{Poi}(\mu_{ijk})$$

1. Show that the rejection area of the Goodness of fit test for model M_0 based on the deviance/likelihood ratio statistic is

$$\text{RA} = \left\{ \sum_{\forall i,j,k} n_{ijk} \log\left(\frac{n_{ijk}}{\hat{\mu}_{ijk}}\right) > q \right\} \quad (1)$$

for some value q , where $\hat{\mu}_{ijk}$ are the MLE of μ_{ijk} under model M_0 .

2. In an exercise/example in the Handout [Handouts: The Log-linear model], you were asked to compute the rejection area of the LR statistic under the Multinomial sampling scheme, and in fact the resulting rejection area was the the same as in (1). State the assumption, we secretly took in order to get (1) under the Poisson sampling scheme in order to make the likelihood ratio under the Poisson sampling scheme to look like that under the Multinomial sampling scheme.