Measuring Malaria in Complex Transmission Systems

A Time-at-Risk-Based Approach

Alec Georgoff, IHME

Introduction

UNIVERSITY OF MICHIGAN

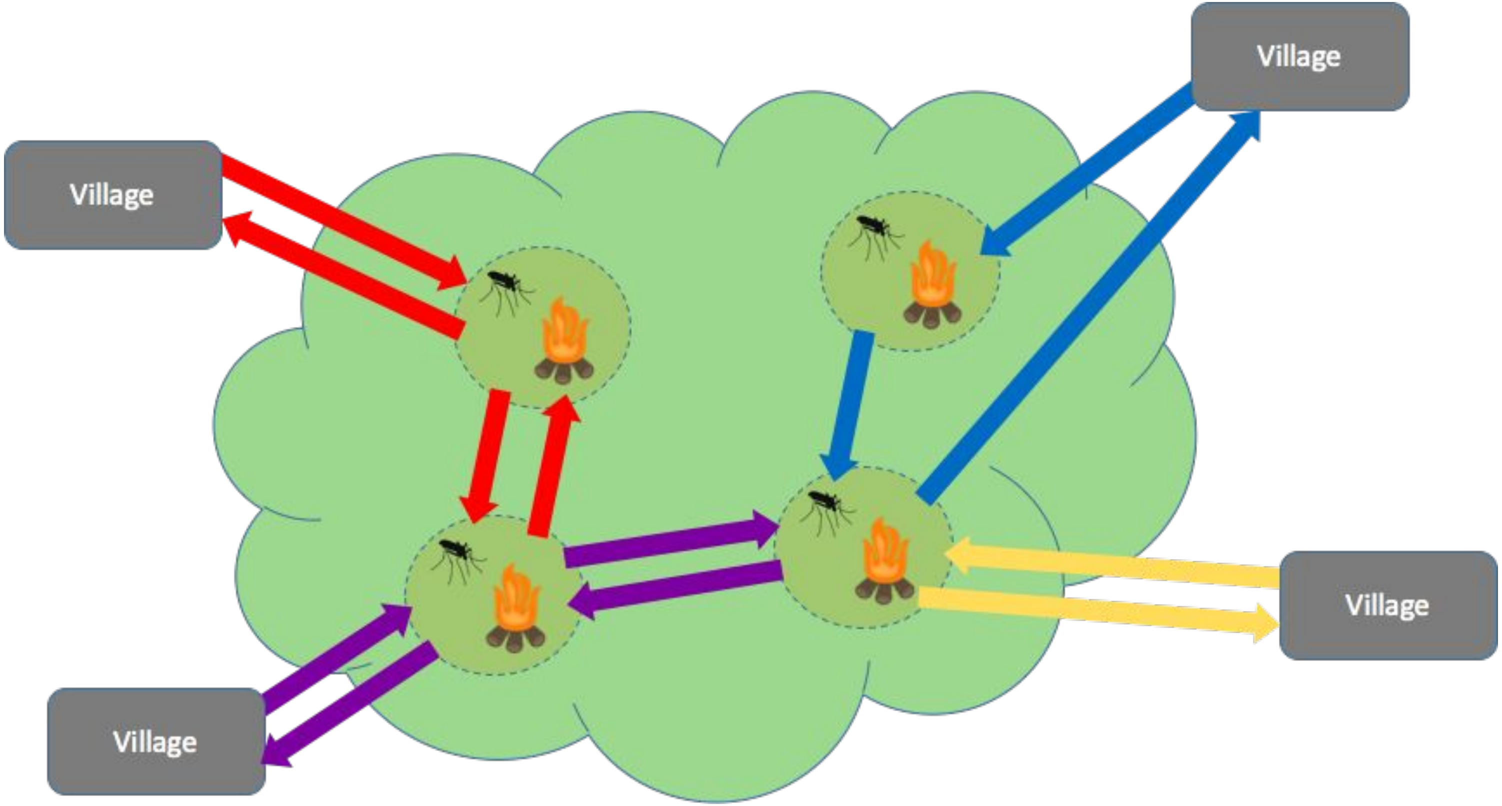
UNIVERSITY of WASHINGTON

Ross-Macdonald Equations

$$\frac{dX}{dt} = abe^{-gn}\frac{Y}{H}(H - X) - rX$$

$$\frac{dY}{dt} = ac\frac{X}{H}(V - Y) - gY$$

Forest Malaria



Time-at-Risk (TaR)

$$\mathbf{\Psi} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{bmatrix}$$

Objective

How does R_0 affect malaria transmission in complex systems with multiple interconnected locations?

RM Equations at Equilibrium

$$0 = R_0 \frac{X}{H + ScX} (H - X) - X$$

R_0 as a Threshold

$$R_0 \begin{cases} < 1 & \text{transmission dies out over time} \\ > 1 & \text{sustained endemic transmission} \end{cases}$$

RM Equations in Matrix Form

$$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} \qquad \mathbf{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} \qquad \mathbf{H} = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_n \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}$$

Scaling by Time-at-Risk

$$\mathbf{X}_{\Psi} = \mathbf{\Psi}^{\mathbf{T}}\mathbf{X}$$

$$\mathbf{H}_{\Psi} = \mathbf{\Psi}^{\mathbf{T}}\mathbf{H}$$

RM Equations in Matrix Form

$$\frac{d\mathbf{Y}}{dt} = ac\frac{\mathbf{X}_\Psi}{\mathbf{H}_\Psi} \circ (\mathbf{V} - \mathbf{Y}) - g\mathbf{Y}$$

$$\frac{d\mathbf{X}}{dt} = abe^{-gn}\left(\mathbf{\Psi}\frac{\mathbf{Y}}{\mathbf{H}_\Psi}\right) \circ (\mathbf{H} - \mathbf{X}) - r\mathbf{X}$$

RM Equations in Matrix Form

$$0 = \left(\mathbf{\Psi} \left(\mathbf{R}_\Psi \circ \frac{\mathbf{X}_\Psi}{cS\mathbf{X}_\Psi + \mathbf{H}_\Psi} \right) \right) \circ (\mathbf{H} - \mathbf{X}) - \mathbf{X}$$

$$0 = \left(\mathbf{\Psi} \left(\mathbf{R}_\Psi \circ \frac{\mathbf{\Theta}_\Psi}{cS\mathbf{\Theta}_\Psi + 1} \right) \right) \circ (\mathbf{H} - \mathbf{X}) - \mathbf{X}$$

Example: One Village, One Forest

$$\mathbf{X} = \begin{bmatrix} X_V \\ X_F \end{bmatrix} \qquad \mathbf{H} = \begin{bmatrix} H_V \\ H_F \end{bmatrix} \qquad \mathbf{R}_\Psi = \begin{bmatrix} R_{\Psi,V} \\ R_{\Psi,F} \end{bmatrix}$$

$$\mathbf{\Psi} = \begin{array}{c c} & \begin{array}{cc} \mathrm{V} & \mathrm{F} \end{array} \\ \begin{array}{c} \mathrm{V} \\ \mathrm{F} \end{array} & \begin{pmatrix} 1 & 0 \\ 1-p & p \end{pmatrix} \end{array}$$

Example: One Village, One Forest

$$\mathbf{X}_\Psi = \mathbf{\Psi}^\mathbf{T}\mathbf{X} = \begin{bmatrix} 1 & 1-p \\ 0 & p \end{bmatrix} \begin{bmatrix} X_V \\ X_F \end{bmatrix} = \begin{bmatrix} X_V + (1-p)X_F \\ pX_F \end{bmatrix}$$

$$\mathbf{H}_\Psi = \mathbf{\Psi}^\mathbf{T}\mathbf{H} = \begin{bmatrix} 1 & 1-p \\ 0 & p \end{bmatrix} \begin{bmatrix} H_V \\ H_F \end{bmatrix} = \begin{bmatrix} H_V + (1-p)H_F \\ pH_F \end{bmatrix}$$

$$\mathbf{\Theta}_\Psi = \frac{\mathbf{X}_\Psi}{\mathbf{H}_\Psi} = \begin{bmatrix} \Theta_{\Psi,V} \\ \Theta_{\Psi,F} \end{bmatrix} = \begin{bmatrix} \dfrac{X_V + (1-p)X_F}{H_V + (1-p)H_F} \\ \dfrac{X_F}{H_F} \end{bmatrix}$$

Example: One Village, One Forest

$$0 = \begin{bmatrix} R_{\Psi,V} \cdot \dfrac{\Theta_{\Psi,V}}{cS\Theta_{\Psi,V} + 1} \cdot (H_V - X_V) - X_V \\ \left(R_{\Psi,F} \cdot (1-p) \cdot \dfrac{\Theta_{\Psi,F}}{cS\Theta_{\Psi,F} + 1} + R_{\Psi,F} \cdot p \cdot \dfrac{\Theta_{\Psi,F}}{cS\Theta_{\Psi,F} + 1} \right) \cdot (H_F - X_F) - X_F \end{bmatrix}$$

Example: One Village, One Forest

[Heatmap]

[3D Surface]