IHME
Measuring what matters

Modeling Forest Malaria Using a Time-at-Risk Based Approach

Incorporating Human Movement into the Equation

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Background

- What makes forest malaria different?
    - Risk is spread across different “populations” based on time spent in certain locations
    - Forest-going population has different risk than those who stay in village
    - Transmission is occurring away from home
        - People who travel to the same place form a different population

Contrasting Malaria Models

- Ross-Macdonald Model
  - Entire human population exposed to the same mosquito population
  - One transmission location
- Forest Model
  - Transmission occurs at home <u>and</u> away from home
  - Multiple human populations exposed to multiple mosquito populations

Objectives

- Show how reproductive rates and time-at-risk can be used together to model malaria prevalence in a forest system
- Show why time-at-risk and forest reproductive rates have an impact on elimination in villages

Reproductive Rate

Reproductive Rate

$$R = \frac{\boxed{M} a^2 bce^{-gn}}{\boxed{H} gr}$$

Mosquitoes Per Human

Higher values <u>increase</u> R

Reproductive Rate

$$R = \frac{M\boxed{a^2}bce^{-gn}}{Hgr}$$

Mosquito Biting Rate

Higher values <u>increase</u> R

Reproductive Rate

$$R = \frac{M a^2 \boxed{bc} e^{-gn}}{H g r}$$

Proportion of Bites That
Cause Infection

Higher values <u>increase</u> R

Reproductive Rate

$$R = \frac{Ma^2bce^{-gn}}{Hgr}$$

Time for Sporogonic
Cycle

Higher values decrease R

Reproductive Rate

$$R = \frac{Ma^2bce^{-\boxed{g}n}}{H\boxed{g}r}$$

Mosquito Death Rate

Higher values <u>decrease</u> R

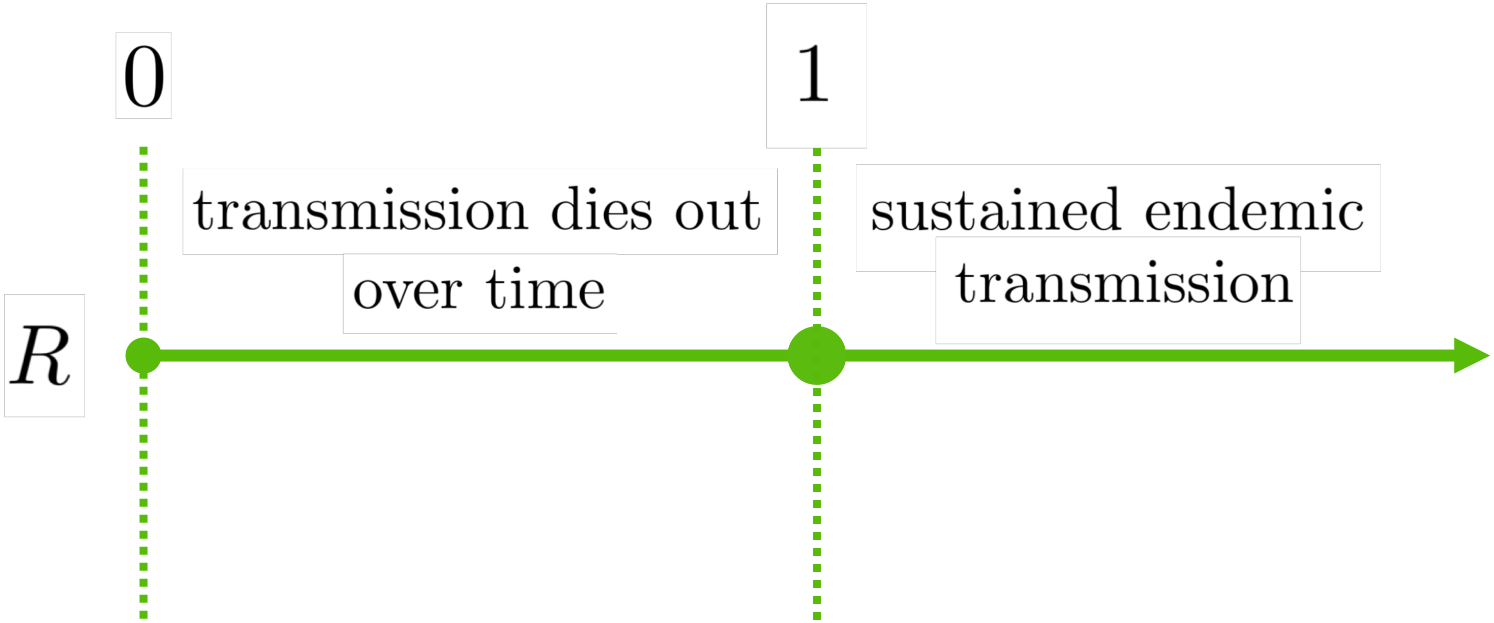
Reproductive Rate

$$R = \frac{M a^2 b c e^{-gn}}{H g r}$$

Human Disease
Recovery

Higher values decrease R

Reproductive Rate



Time-at-Risk

Time-at-Risk (TaR)

$$\mathbf{\Psi} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mn} \end{bmatrix}$$

Rows Sum to 1

Time-at-Risk (TaR)

$$\boldsymbol{\Psi} = \begin{array}{c c} & \begin{array}{cccc} \text{loc } 1 & \text{loc } 2 & \dots & \text{loc } n \end{array} \\ \begin{array}{c} \text{population } 1 \\ \text{population } 2 \\ \vdots \\ \text{population } m \end{array} & \begin{bmatrix} p_{11} & p_{12} & \dots & p_{1n} \\ p_{21} & p_{22} & \dots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & \dots & p_{mn} \end{bmatrix} \end{array}$$

Rows Sum to 1

Time-at-Risk (TaR)

$$
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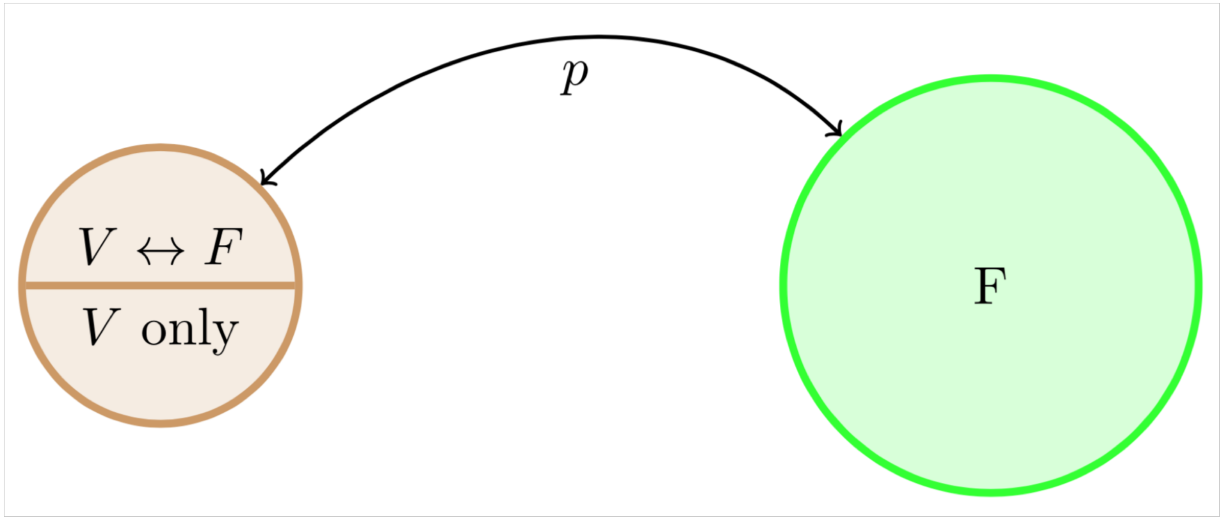
\end{array}
$$

Proportion of time spent by population 1 in location 2

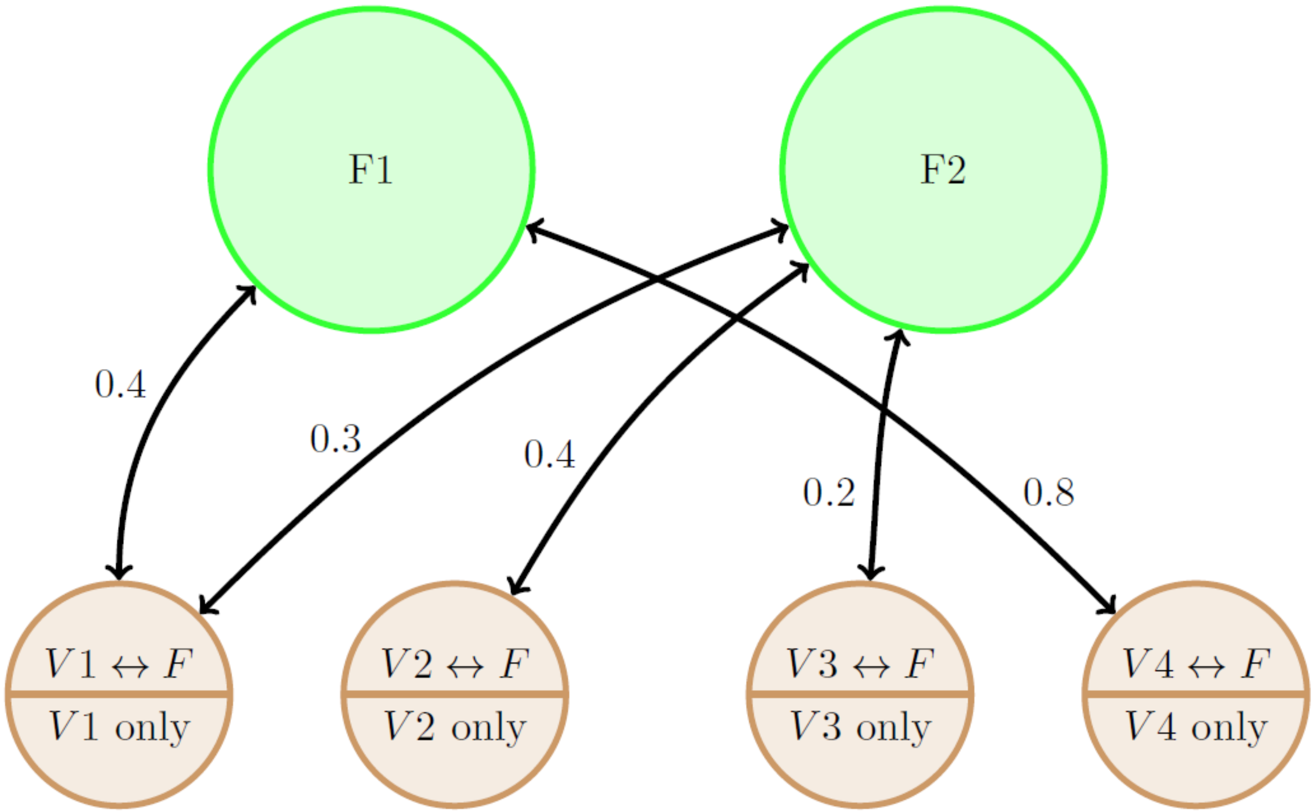
Rows Sum to 1

Time-at-Risk (TaR)



$$\boldsymbol{\Psi} = \begin{array}{c} \\ V_{only} \\ V \leftrightarrow F \end{array} \begin{array}{c} \begin{array}{cc} \mathrm{V} & \mathrm{F} \end{array} \\ \begin{bmatrix} 1 & 0 \\ 1-p & p \end{bmatrix} \end{array}$$

Time-at-Risk (TaR)



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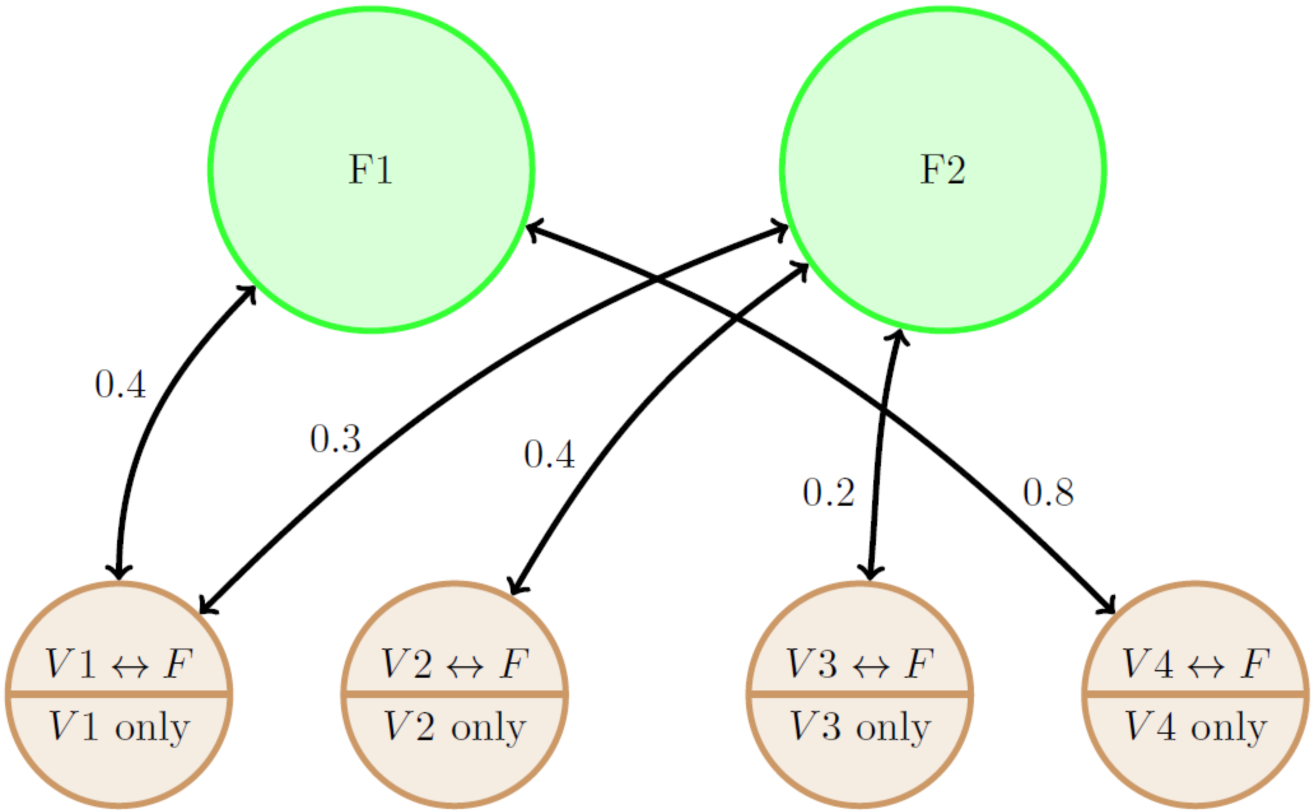
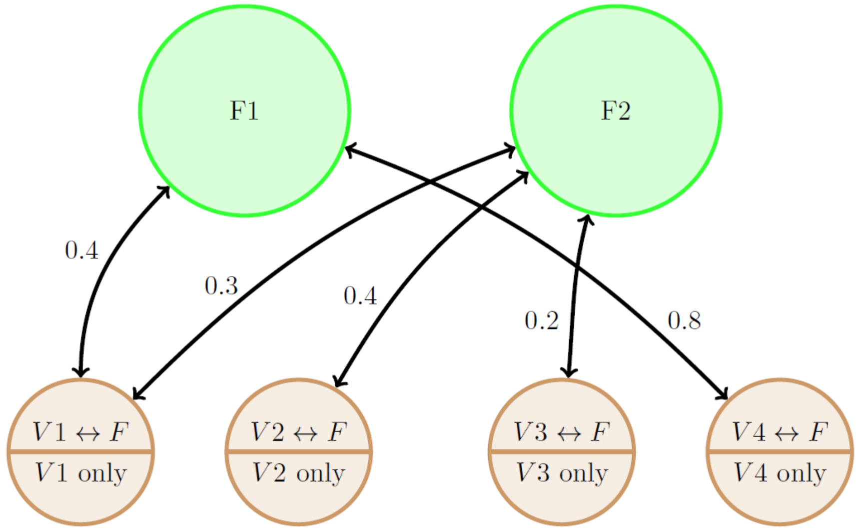
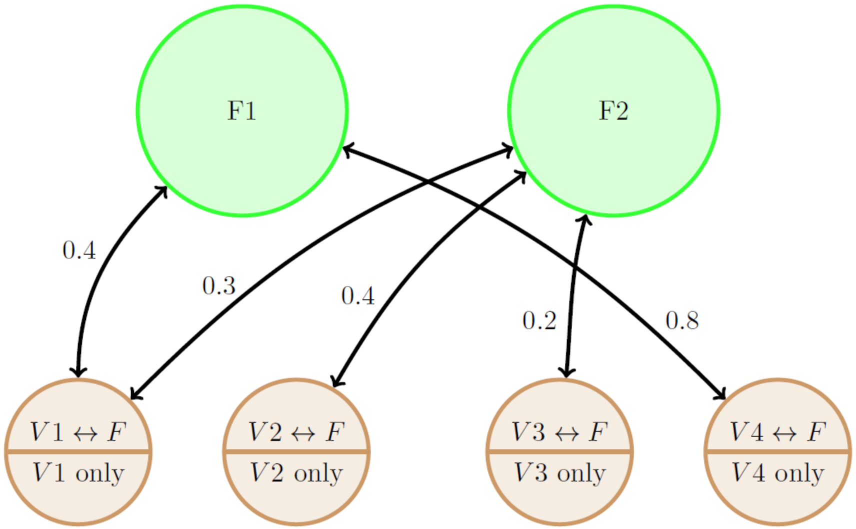


Time-at-Risk (TaR)



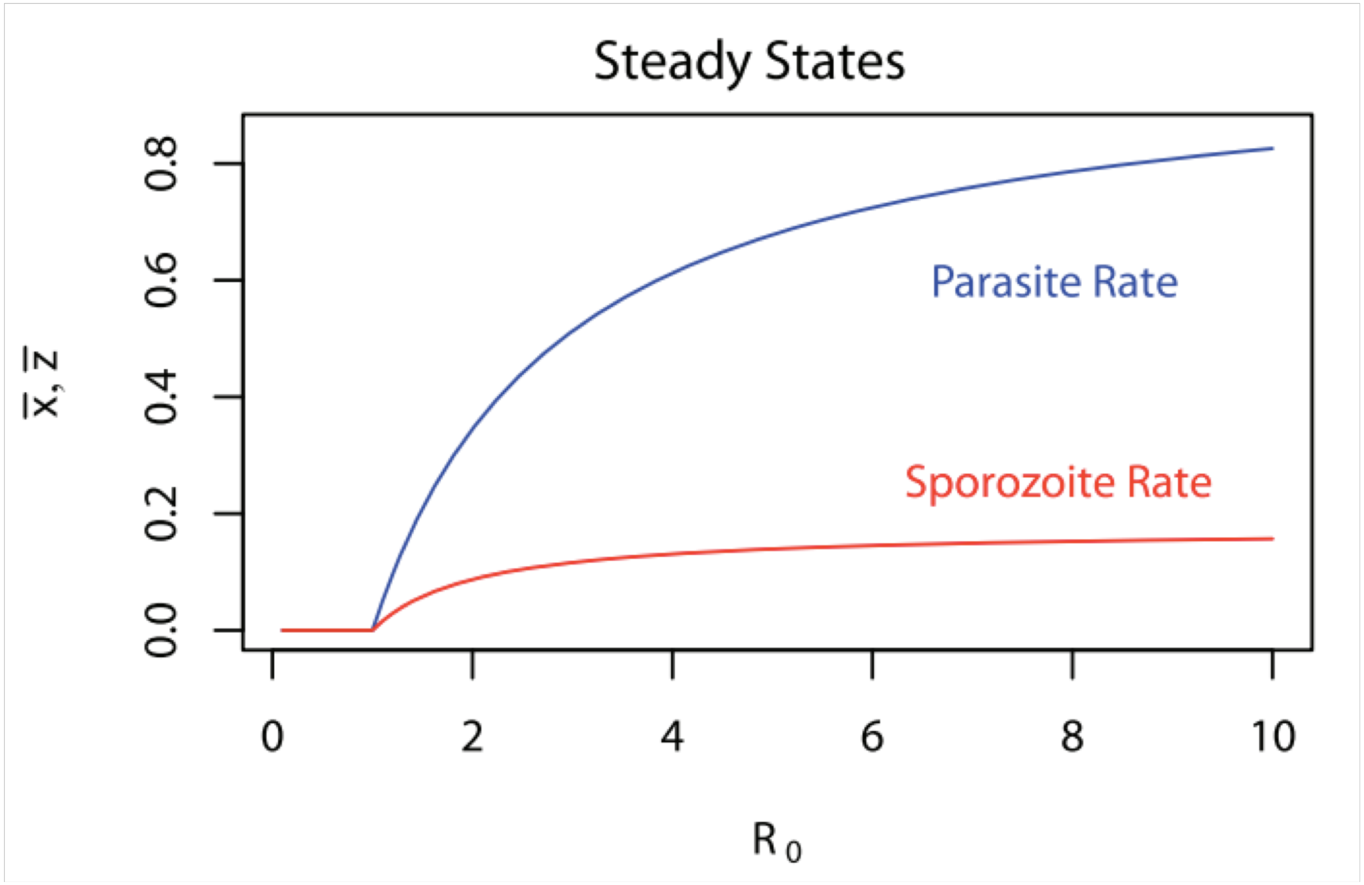
Time-at-Risk (TaR)



$$\Psi = \begin{array}{c} \\ \text{V1} \leftrightarrow \text{F} \\ \text{V1 only} \\ \text{V2} \leftrightarrow \text{F} \\ \text{V2 only} \\ \text{V3} \leftrightarrow \text{F} \\ \text{V3 only} \\ \text{V4} \leftrightarrow \text{F} \\ \text{V4 only} \end{array} \begin{array}{c} \begin{array}{cccccc} \text{V1} & \text{V2} & \text{V3} & \text{V4} & \text{F1} & \text{F2} \end{array} \\ \begin{bmatrix} 0.3 & 0 & 0 & 0 & 0.4 & 0.3 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 & 0 & 0.4 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0 & 0 & 0.2 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0.8 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

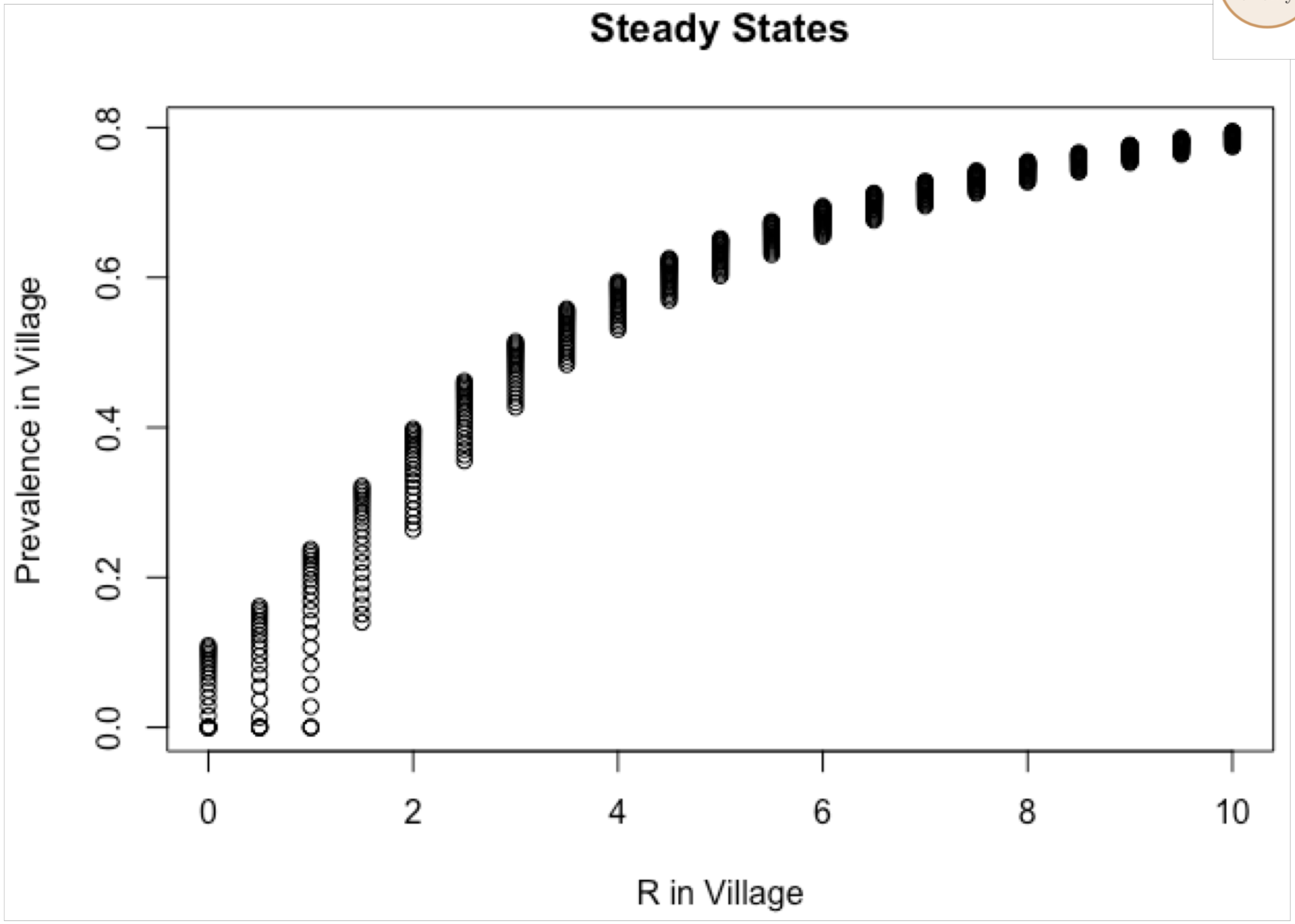
How Forest R and Time-at-Risk Affect Village Transmission

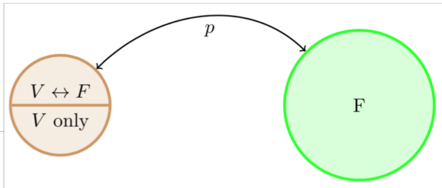
Traditional Model



Every R value has an associated steady state prevalence
(derived from parasite rate)

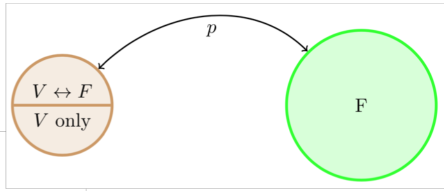
Forest Model – Effect of Forest R

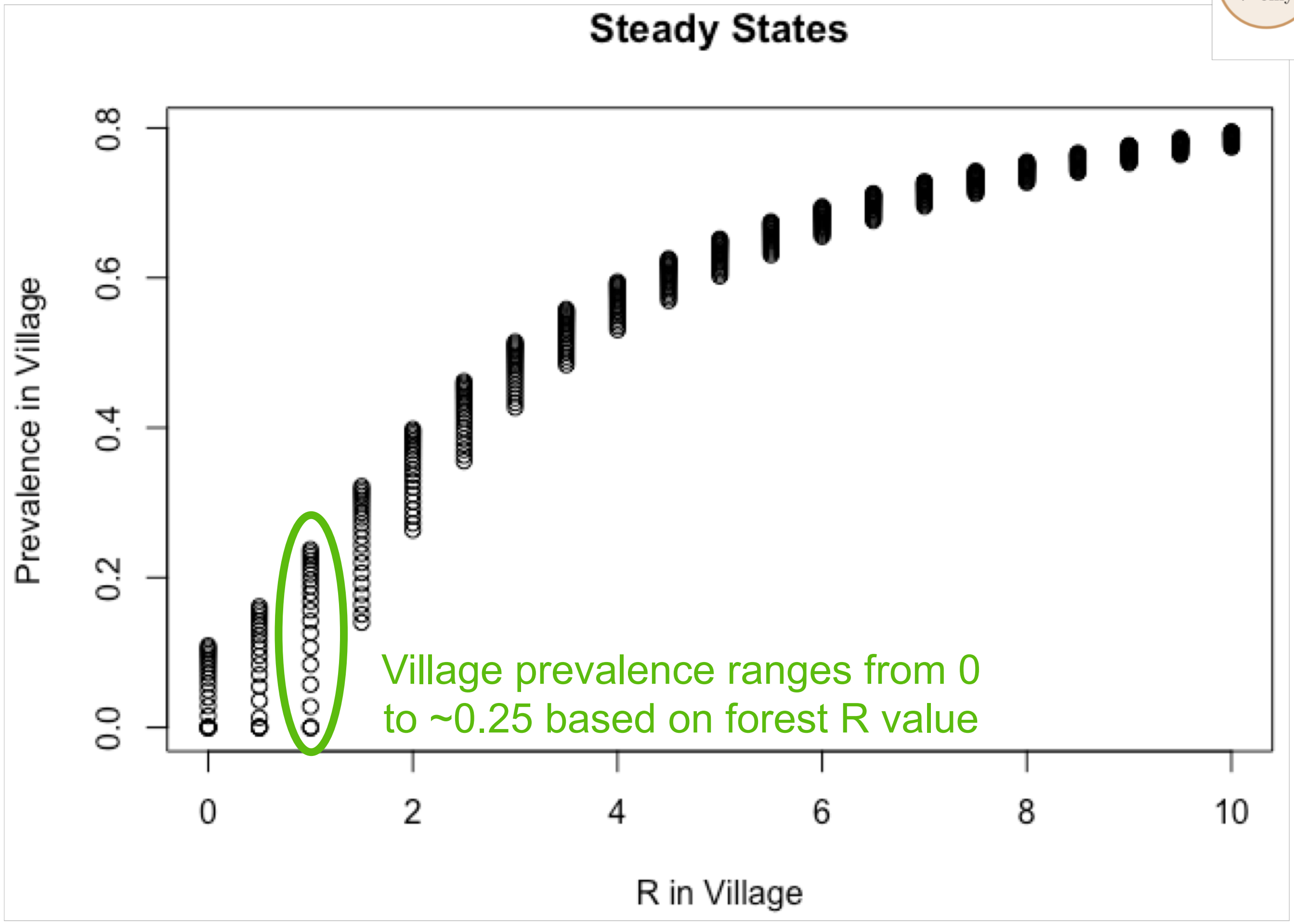




Village prevalence at given R value depends on R value in the forest

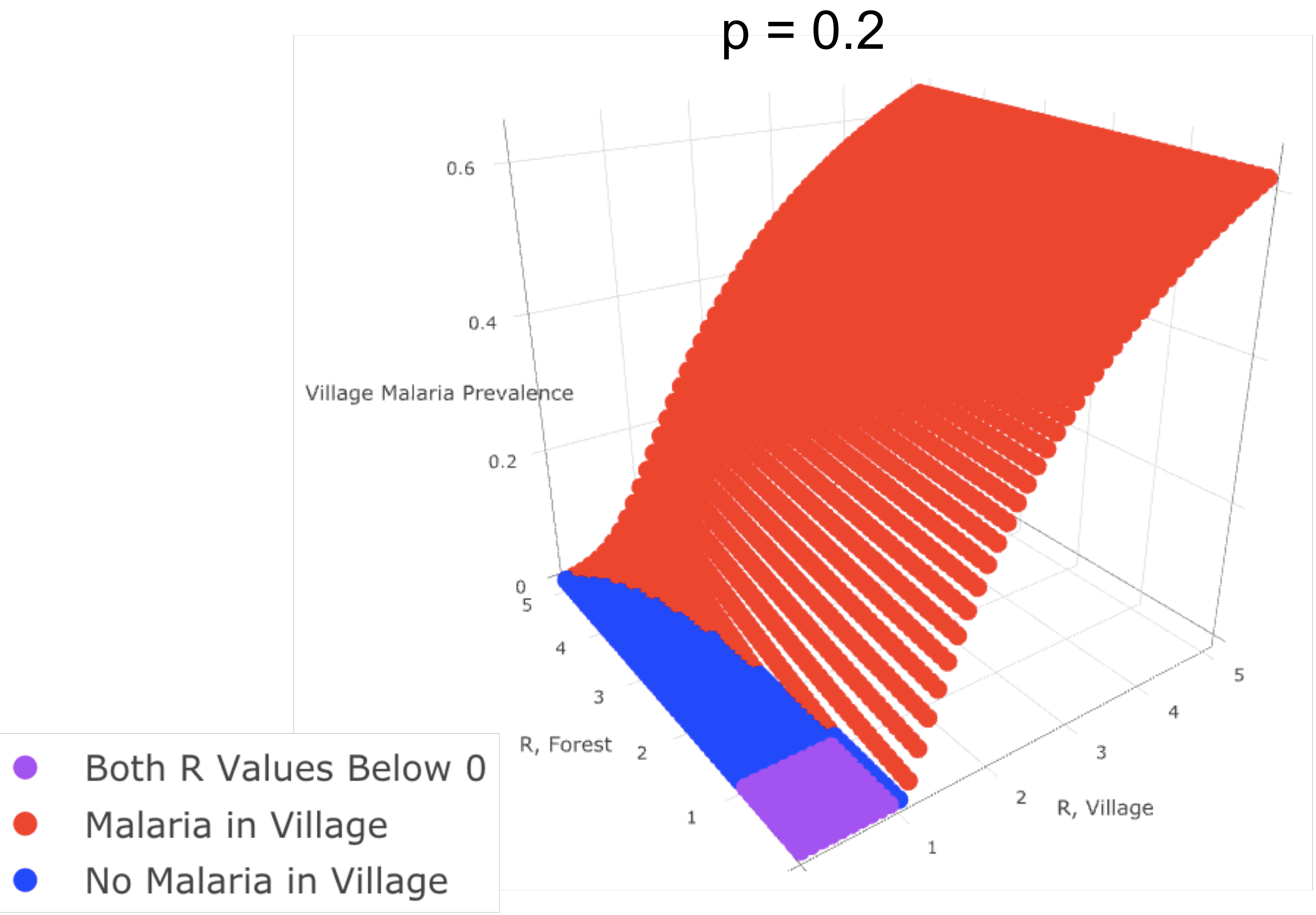
Forest Model – Effect of Forest R





Village prevalence at given R value <u>depends on R value in the forest</u>

Forest Model – Effect of Time-at-Risk



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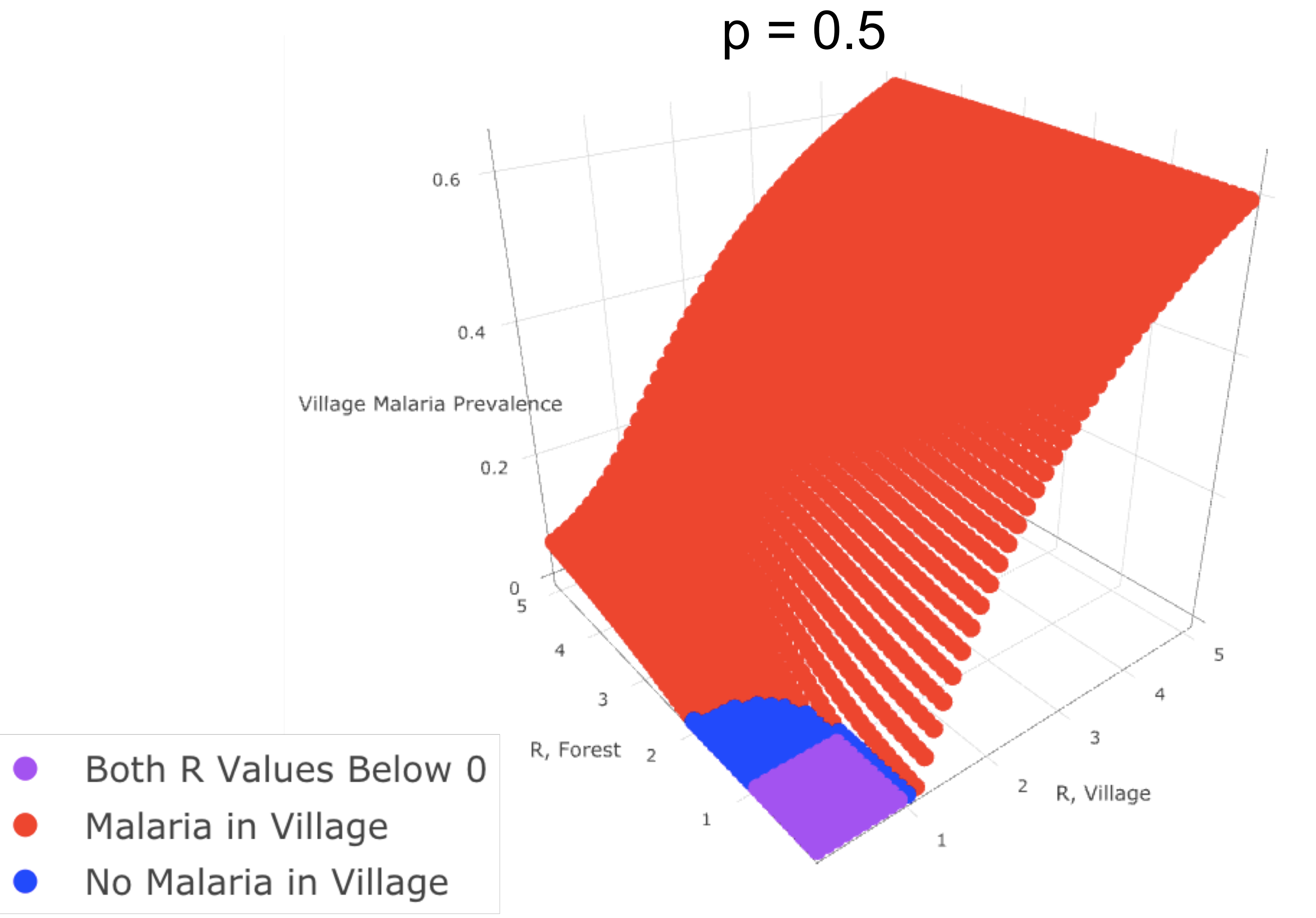

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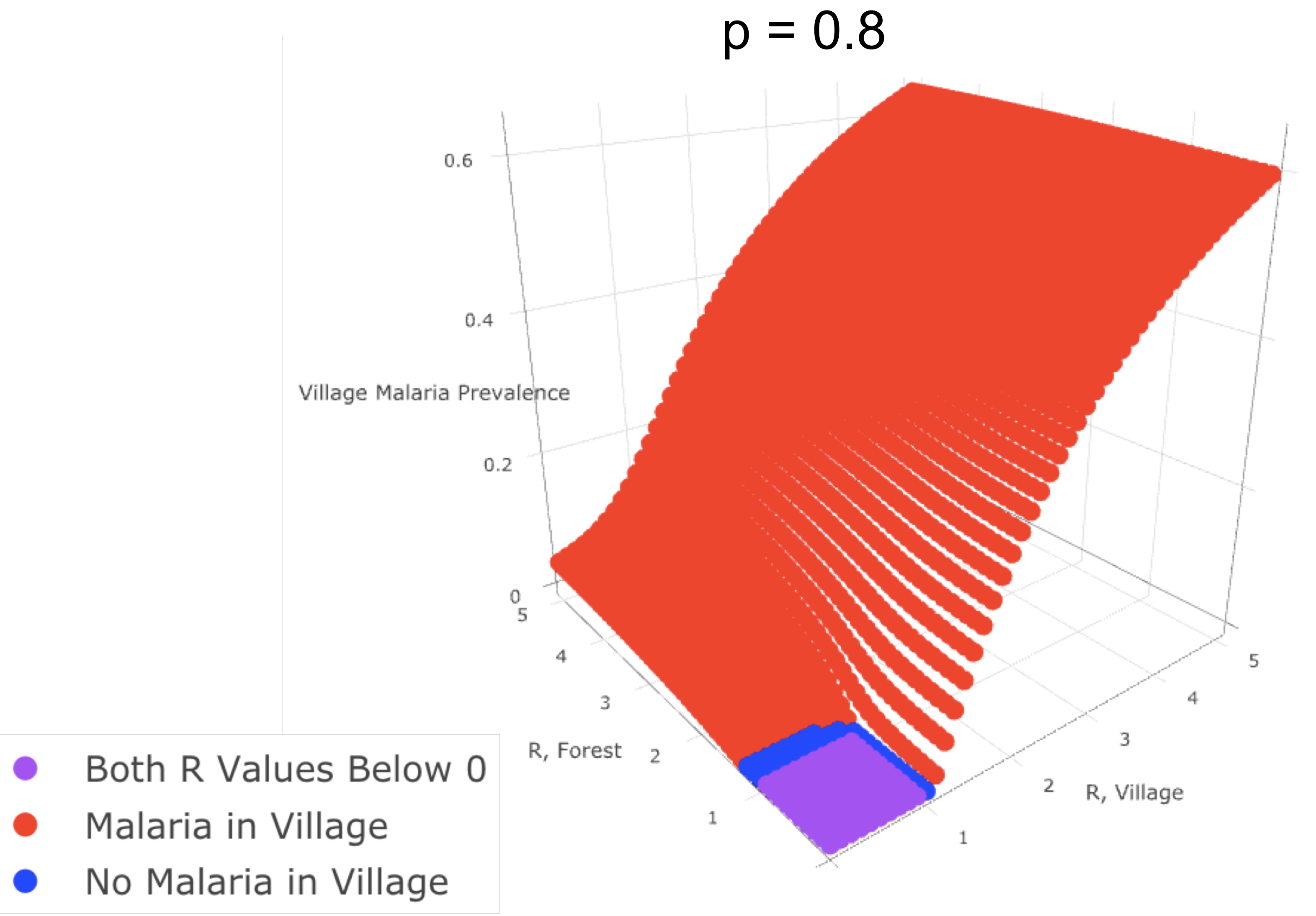
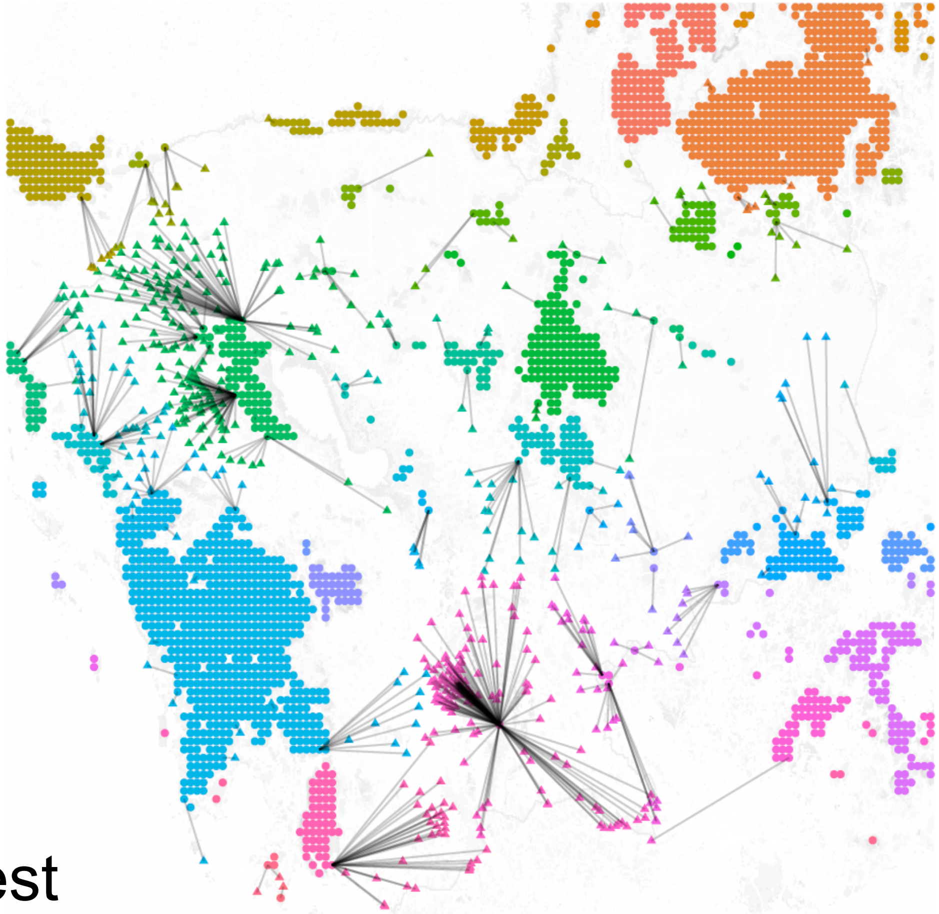


Next Steps

Data, Data, Data



Connecting villages to closest forest by travel time

Sources:
Satellite landcover data,
OSM village locations,
Friction travel surface

Data, Data, Data

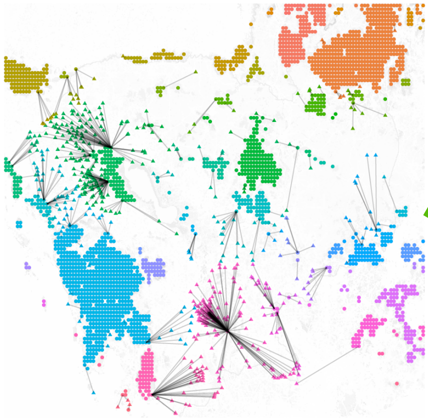
Table 1.4: Details of travellers, risk zones 1-4

	Total		Domain 1		Domain 2	
	N	% [95% CI]	N	% [95% CI]	N	% [95% CI]
	3,906	**100**	**2,013**	**100**	**1,893**	**100**
Last travelled away from home						
Last night	110	3.1 [2.1, 4.6]	47	2.6 [1.4,4.6]	63	3.8 [2.3, 6.2]
<1 week	836	20.3 [17.6, 23.4]	400	18.8 [15.2, 23.1]	436	22.0 [18.1, 26.5]
1-<4 weeks	1,040	25.5 [22.6, 28.6]	582	27.3 [23.4, 31.7]	458	23.4 [19.3, 28.0]
≥4 weeks	1,831	48.6 [44.5, 52.8]	943	49.3 [43.9, 54.8]	888	47.9 [41.6, 54.2]
Not specified	89	2.4 [1.6, 3.6]	41	1.9 [1.1, 3.5]	48	3.0 [1.7, 5.0]
Reasons for travel						
Work in forest	342	8.9 [6.9, 11.4]	130	6.3 [3.8, 10.3]	212	11.7 [8.8, 15.5]
Work on ***chamkar***/plantation	918	22.8 [17.7, 29.0]	291	14.3 [8.8, 22.4]	627	32.4 [24.5, 41.5]
Visit relatives	552	13.1 [10.8, 15.8]	325	14.1 [10.7, 18.4]	227	12.0 [9.3, 15.3]
Other	1,512	40.0 [34.1, 46.2]	805	42.4 [32.8, 52.7]	707	37.3 [31.7, 43.2]
Trips away from home past 3 months						
1-2	2,689	69.5 [66.1, 72.8]	1,500	74.8 [70.5, 78.6]	1,189	63.7 [58.1, 69.0]
3-5	669	17.2 [14.6, 20.0]	267	13.8 [10.4, 18.1]	402	20.9 [17.4, 25.0]
6-10	221	5.2 [4.0, 6.8]	95	4.2 [3.0, 5.9]	126	6.3 [4.3, 9.3]
>10	95	2.4 [1.9, 3.1]	30	1.4 [0.9, 2.2]	65	3.5 [2.5, 4.8]
Not specified	232	5.7 [4.5, 7.0]	121	5.8 [4.4, 7.6]	111	5.6 [3.9, 7.8]
Countries visited in 2013						
Laos, Thailand, Vietnam	811	21.4 [15.7, 28.5]	594	30.2 [19.9, 42.9]	217	11.6 [8.3,15.9]

Travel surveys

Source:
Cambodia Malaria Survey 2013

Data, Data, Data



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Use data to build a realistic TaR matrix

Takeaways

- Standard Ross-Macdonald models are not sufficient for modeling forest malaria
- Time-at-Risk accounts for human travel and its impact on population definitions
- Transmission dynamics in the forest can have a large impact on transmission dynamics in the village
- Data from high and low level sources are needed to accurately represent the TaR matrix

Questions?

Visualizations

Heatmap

3D Surface

Objectives

Objectives

Mathematical Underpinnings	*The basics of mosquito math*
Vector-Based Approach	*Multiple locations in one equation*
Simple Example	*One village, one forest*
Expanded Example	*More villages, more forests!!*
R Implementation	*Making computers do the hard math*

Objectives

Visualizations

Seeing is believing

Future Directions

Our plans moving forward

Mathematical Underpinnings

Ross-Macdonald Equations

Infected Humans

Infected Mosquitoes

$$\frac{dX}{dt} = abe^{-gn}\frac{Y}{H}(H - X) - rX$$

$$\frac{dY}{dt} = ac\frac{X}{H}(V - Y) - gY$$

Equilibrium Assumptions

$$\boxed{\frac{dY}{dt} = 0} = ac\frac{X}{H}(V - Y) - gY$$

$$Y = \frac{acVX}{Hg + acX}$$

$$\boxed{\frac{dX}{dt} = 0} = abe^{-gn} \cdot \frac{1}{H} \cdot \frac{acVX}{Hg + acX}(H - X) - rX$$

$$0 = \frac{Va^2bce^{-gn}}{Hgr} \cdot \frac{X}{H + \frac{a}{g}cX}(H - X) - X$$

$$S = \frac{a}{g}$$

$$R = \frac{Va^2bce^{-gn}}{Hgr}$$

$$0 = R\frac{X}{H + ScX}(H - X) - X$$

Equilibrium Assumptions

$$\boxed{\frac{dY}{dt} = 0} = ac\frac{X}{H}(V - Y) - gY$$

$$Y = \frac{acVX}{Hg + acX}$$

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$$0 = R\frac{X}{H + ScX}(H - X) - X$$

Equilibrium Assumptions

$$0 = \boxed{R}\frac{X}{H + ScX}(H - X) - X$$

Reproductive Rate

$$R = \frac{Va^2bce^{-gn}}{Hgr}$$

$$R \begin{cases} < 1 & \text{transmission dies out over time} \\ > 1 & \text{sustained endemic transmission} \end{cases}$$

Multi-Site Approach

Multi-Site Variables

For m unique human populations,

$$\mathbf{H} = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_m \end{bmatrix} \qquad \mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_m \end{bmatrix}$$

Time-at-Risk (TaR)

$$\boldsymbol{\Psi} = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{m1} & p_{m2} & \cdots & p_{mn} \end{bmatrix}$$

$$\sum_{j=1}^{n} p_{ij} = 1 \quad \text{for } i \in [1, m]$$

Scaling by Time-at-Risk

$$\mathbf{X}_{\Psi} = \mathbf{\Psi}^{\mathrm{T}}\mathbf{X}$$

$$\mathbf{H}_{\Psi} = \mathbf{\Psi}^{\mathrm{T}}\mathbf{H}$$

Ross-Macdonald Equations

$$\frac{dX}{dt} = abe^{-gn}\frac{Y}{H}(H - X) - rX$$

$$\frac{dY}{dt} = ac\frac{X}{H}(V - Y) - gY$$

Ross-Macdonald Equations

$$\frac{d\mathbf{X}}{dt} = abe^{-gn}\left(\mathbf{\Psi}\frac{\mathbf{Y}}{\mathbf{H}_{\Psi}}\right) \circ (\mathbf{H} - \mathbf{X}) - r\mathbf{X}$$

$$\frac{d\mathbf{Y}}{dt} = ac\frac{\mathbf{X}_{\Psi}}{\mathbf{H}_{\Psi}} \circ (\mathbf{V} - \mathbf{Y}) - g\mathbf{Y}$$

Equilibrium Assumptions

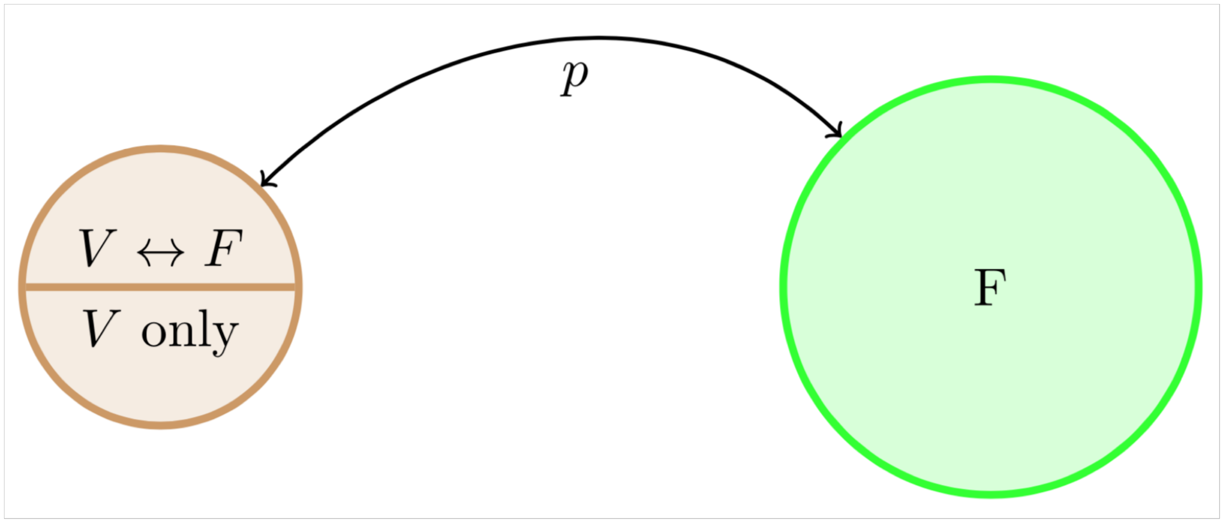
$$0 = \left(\boldsymbol{\Psi} \left(\mathbf{R}^* \circ \frac{\boldsymbol{\Theta}^*}{cS\boldsymbol{\Theta}^* + 1} \right) \right) \circ (\mathbf{H} - \mathbf{X}) - \mathbf{X}$$

$$\mathbf{R}^* = \frac{\mathbf{V} a^2 bce^{-gn}}{\mathbf{H}_\Psi gr}$$

$$\boldsymbol{\Theta}^* = \frac{\mathbf{X}_\Psi}{\mathbf{H}_\Psi}$$

(Not) Simple Example

Simple Example: One Village, One Forest



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$$

Simple Example: One Village, One Forest

$$\mathbf{H} = \begin{bmatrix} H_{V_{\text{only}}} \\ H_{V \leftrightarrow F} \end{bmatrix} \qquad \mathbf{X} = \begin{bmatrix} X_{V_{\text{only}}} \\ X_{V \leftrightarrow F} \end{bmatrix} \qquad \mathbf{R}^* = \begin{bmatrix} R_V^* \\ R_F^* \end{bmatrix}$$

Simple Example: One Village, One Forest

$$\mathbf{H}_{\Psi} = \mathbf{\Psi}^{\mathbf{T}}\mathbf{H} = \begin{bmatrix} 1 & 1-p \\ 0 & p \end{bmatrix} \begin{bmatrix} H_{V_{\text{only}}} \\ H_{V\leftrightarrow F} \end{bmatrix} = \begin{bmatrix} H_{V_{\text{only}}} + (1-p)H_{V\leftrightarrow F} \\ pH_{V\leftrightarrow F} \end{bmatrix}$$

$$\mathbf{X}_{\Psi} = \mathbf{\Psi}^{\mathbf{T}}\mathbf{X} = \begin{bmatrix} 1 & 1-p \\ 0 & p \end{bmatrix} \begin{bmatrix} X_{V_{\text{only}}} \\ X_{V\leftrightarrow F} \end{bmatrix} = \begin{bmatrix} X_{V_{\text{only}}} + (1-p)X_{V\leftrightarrow F} \\ pX_{V\leftrightarrow F} \end{bmatrix}$$

$$\mathbf{\Theta}^{*} = \frac{\mathbf{X}_{\Psi}}{\mathbf{H}_{\Psi}} = \begin{bmatrix} \Theta^{*}_{V_{\text{only}}} \\ \Theta^{*}_{V\leftrightarrow F} \end{bmatrix} = \begin{bmatrix} \dfrac{X_{V_{\text{only}}} + (1-p)X_{V\leftrightarrow F}}{H_{V_{\text{only}}} + (1-p)H_{V\leftrightarrow F}} \\ \dfrac{X_{V\leftrightarrow F}}{H_{V\leftrightarrow F}} \end{bmatrix}$$

Simple Example: One Village, One Forest

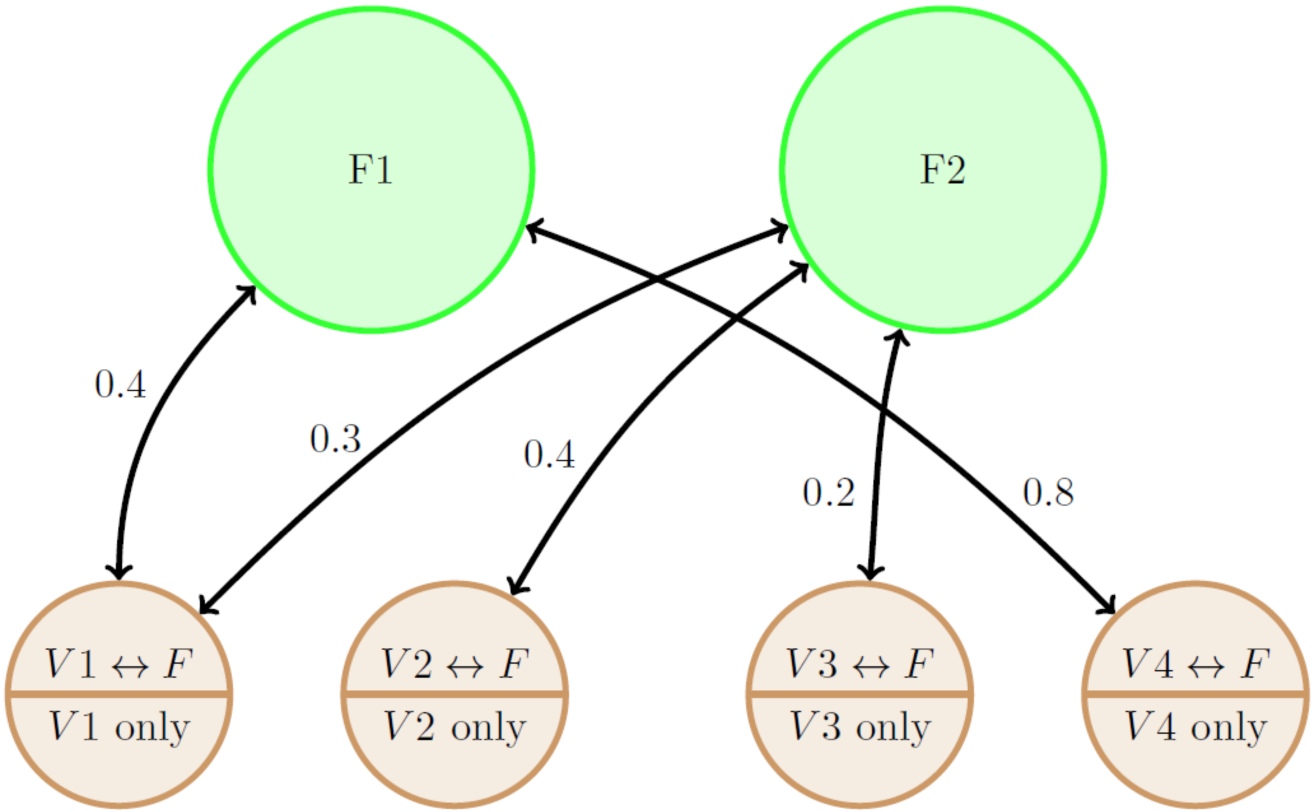
$$0 = \left(\boldsymbol{\Psi}\left(\mathbf{R}^* \circ \frac{\boldsymbol{\Theta}^*}{cS\boldsymbol{\Theta}^* + 1}\right)\right) \circ (\mathbf{H} - \mathbf{X}) - \mathbf{X}$$

$$0 = \begin{bmatrix} R_V^* \cdot \dfrac{\Theta_V^*}{cS\Theta_V^* + 1} \\ (1-p) \cdot R_V^* \cdot \dfrac{\Theta_V^*}{cS\Theta_V^* + 1} + p \cdot R_F^* \cdot \dfrac{\Theta_F^*}{cS\Theta_F^* + 1} \end{bmatrix} \circ \begin{bmatrix} H_{V_{\text{only}}} - X_{V_{\text{only}}} \\ H_{V \leftrightarrow F} - X_{V \leftrightarrow F} \end{bmatrix} - \begin{bmatrix} X_{V_{\text{only}}} \\ X_{V \leftrightarrow F} \end{bmatrix}$$

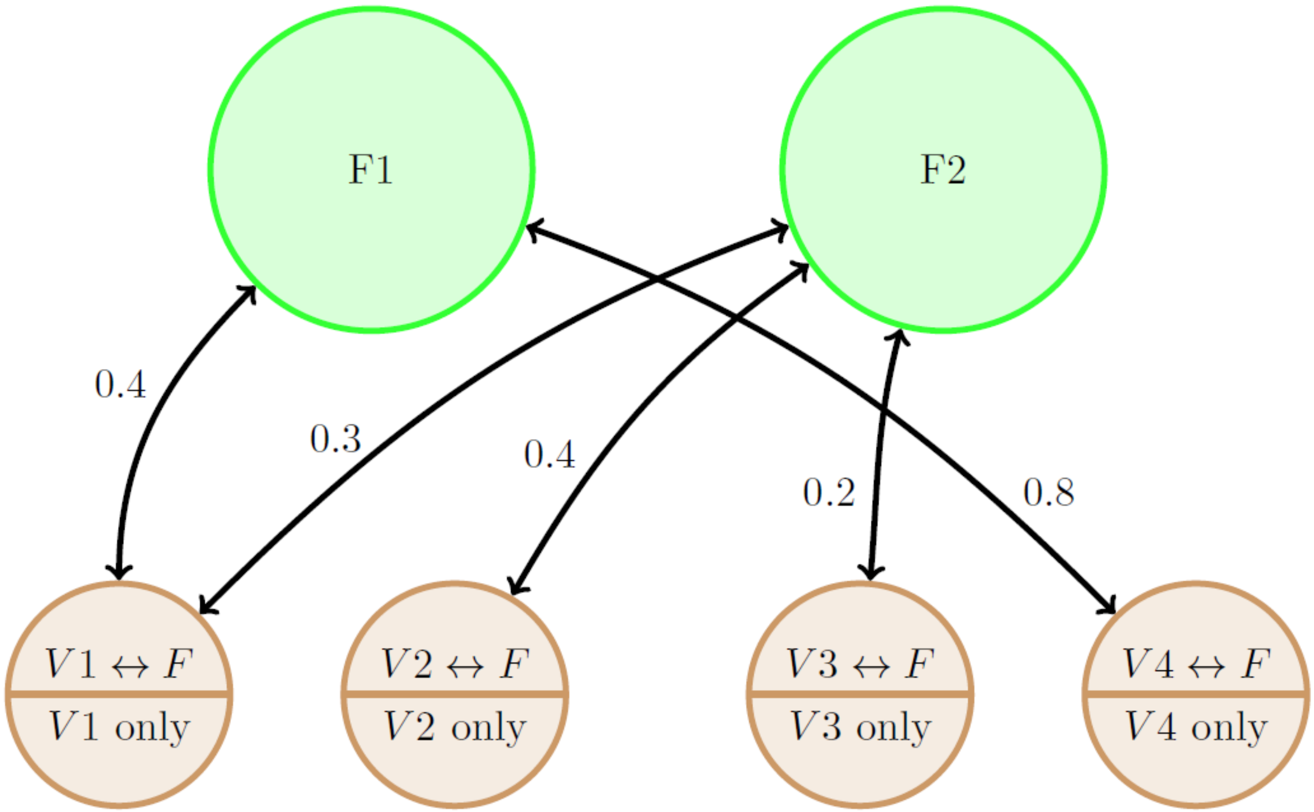
$$0 = \begin{bmatrix} R_V^* \cdot \dfrac{\Theta_V^*}{cS\Theta_V^* + 1} \cdot (H_{V_{\text{only}}} - X_{V_{\text{only}}}) - X_{V_{\text{only}}} \\ \left((1-p) \cdot R_V^* \cdot \dfrac{\Theta_V^*}{cS\Theta_V^* + 1} + p \cdot R_F^* \cdot \dfrac{\Theta_F^*}{cS\Theta_F^* + 1}\right) \cdot (H_{V \leftrightarrow F} - X_{V \leftrightarrow F}) - X_{V \leftrightarrow F} \end{bmatrix}$$

Expanded Example

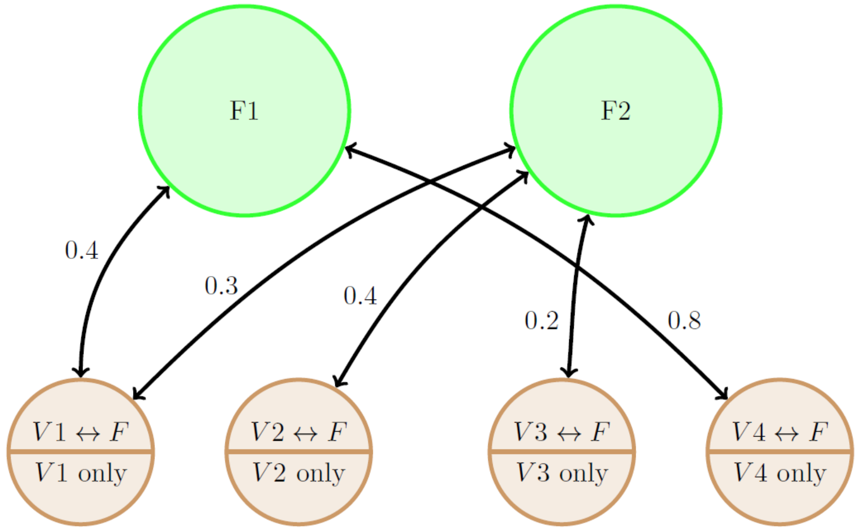
Expand to More Villages and Forests



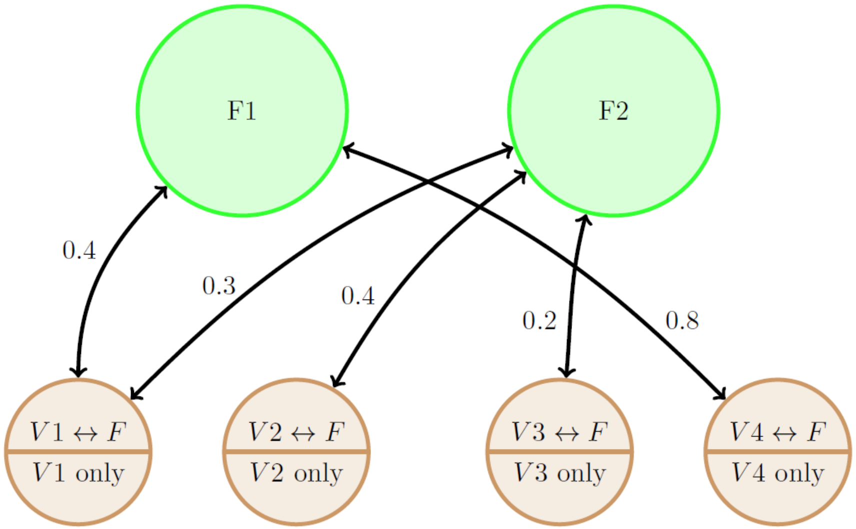
Expand to More Villages and Forests



Expand to More Villages and Forests



Expand to More Villages and Forests



$$\boldsymbol{\Psi} = \begin{array}{r} \\ \text{V1} \leftrightarrow \text{F} \\ \text{V1 only} \\ \text{V2} \leftrightarrow \text{F} \\ \text{V2 only} \\ \text{V3} \leftrightarrow \text{F} \\ \text{V3 only} \\ \text{V4} \leftrightarrow \text{F} \\ \text{V4 only} \end{array} \begin{array}{c} \begin{array}{cccccc} \text{V1} & \text{V2} & \text{V3} & \text{V4} & \text{F1} & \text{F2} \end{array} \\ \begin{bmatrix} 0.3 & 0 & 0 & 0 & 0.4 & 0.3 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 & 0 & 0.4 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.8 & 0 & 0 & 0.2 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0.8 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Implementation In R

R Code

```
########################
# TaR Malaria
#
# Author: Alec Georgoff
#
# Purpose: Solve for equilibrium prevalence values given R values in a complex system
########################

rm(list = ls())

require(rootSolve, lib.loc = "/ihme/malaria_modeling/georgoff/Rlibs/")
require(data.table, lib.loc = "/ihme/malaria_modeling/georgoff/Rlibs/")
require(plotly, lib.loc = "/ihme/malaria_modeling/georgoff/Rlibs/")
require(ggplot2, lib.loc = "/ihme/malaria_modeling/georgoff/Rlibs/")
require(gridExtra, lib.loc = "/ihme/malaria_modeling/georgoff/Rlibs/")

###################################
#
# Set parameters
#
###################################

# specify filepaths for parameter .csv files:

params_path <- "/homes/georgoff/georgoff.github.io/forest_malaria/example1/params.csv"
psi_path <- "/homes/georgoff/georgoff.github.io/forest_malaria/psi_files/psi_1.csv"
r_values_path <- "/homes/georgoff/georgoff.github.io/forest_malaria/example1/r_values.csv"
```

R Code

```r
###################################
#
# Set parameters
#
###################################

# specify filepaths for parameter .csv files:

params_path <- "/homes/georgoff/georgoff.github.io/forest_malaria/example1/params.csv"
psi_path <- "/homes/georgoff/georgoff.github.io/forest_malaria/psi_files/psi_1.csv"
r_values_path <- "/homes/georgoff/georgoff.github.io/forest_malaria/example1/r_values.csv"
```

R Code – Parameters

id	H	
V1-V	47	v_only
V1-F	11	v <--> f
V2-V	43	v_only
V2-F	13	v <--> f
V3-V	41	v_only
V3-F	17	v <--> f
V4-V	37	v_only
V4-F	19	v <--> f
V5-V	31	v_only
V5-F	23	v <--> f

R Code – TaR Matrix

id	V1	V2	V3	V4	F1	F2
V1-F	0.3	0	0	0	0.4	0.3
V1-V	1	0	0	0	0	0
V2-F	0	0.6	0	0	0	0.4
V2-V	0	1	0	0	0	0
V3-F	0	0	0.8	0	0	0.2
V3-V	0	0	1	0	0	0
V4-F	0	0	0	0.2	0.8	0
V4-V	0	0	0	1	0	0

R Code – R Values

id	R_min	R_max	R_step
V1	0	0.5	0.1
V2	0	0.5	0.1
V3	0	0.5	0.1
V4	0	0.5	0.1
V5	0	0.5	0.1
F1	3	3	1
F2	3	3	1
F3	3	3	1
F4	3	3	1
F5	3	3	1
F6	3	3	1
F7	3	3	1

R Code – Results

V1	V2	V3	V4	V5	F1	F2	F3	F4	F5	F6	F7	theta_V1	theta_V2	theta_V3	theta_V4	theta_V5	theta_F1	theta_F2	theta_F3	theta_F4	theta_F5	theta_F6	theta_F7
0	0	0	0	0	3	3	3	3	3	3	3	0.016828	0.021458	0.028819	0.035052	0.048629	0.376344	0.376344	0.376344	0.376344	0.376344	0.376344	0.376344
0.1	0	0	0	0	3	3	3	3	3	3	3	0.018563	0.021459	0.02882	0.035053	0.048631	0.376379	0.376361	0.376418	0.376369	0.376363	0.376357	0.376356
0.2	0	0	0	0	3	3	3	3	3	3	3	0.020673	0.02146	0.028822	0.035054	0.048633	0.376422	0.376381	0.376507	0.376399	0.376386	0.376372	0.376371
0.3	0	0	0	0	3	3	3	3	3	3	3	0.023285	0.021461	0.028823	0.035056	0.048635	0.376475	0.376407	0.376618	0.376437	0.376414	0.376391	0.376389
0.4	0	0	0	0	3	3	3	3	3	3	3	0.026581	0.021462	0.028825	0.035058	0.048639	0.376543	0.376439	0.376759	0.376485	0.376449	0.376415	0.376412
0.5	0	0	0	0	3	3	3	3	3	3	3	0.030828	0.021464	0.028828	0.035061	0.048643	0.37663	0.37648	0.376942	0.376547	0.376496	0.376447	0.376442
0	0.1	0	0	0	3	3	3	3	3	3	3	0.016829	0.023625	0.028821	0.035054	0.048631	0.376384	0.376418	0.376381	0.37642	0.376399	0.376475	0.376522
0.1	0.1	0	0	0	3	3	3	3	3	3	3	0.018564	0.023626	0.028822	0.035055	0.048633	0.376419	0.376434	0.376454	0.376445	0.376418	0.376487	0.376535
0.2	0.1	0	0	0	3	3	3	3	3	3	3	0.020674	0.023627	0.028823	0.035056	0.048635	0.376462	0.376455	0.376543	0.376475	0.376441	0.376503	0.376549
0.3	0.1	0	0	0	3	3	3	3	3	3	3	0.023286	0.023628	0.028824	0.035058	0.048638	0.376515	0.37648	0.376655	0.376513	0.376469	0.376522	0.376567
0.4	0.1	0	0	0	3	3	3	3	3	3	3	0.026582	0.023629	0.028827	0.03506	0.048641	0.376583	0.376512	0.376795	0.376561	0.376504	0.376546	0.37659
0.5	0.1	0	0	0	3	3	3	3	3	3	3	0.030829	0.023631	0.028829	0.035063	0.048646	0.37667	0.376554	0.376978	0.376623	0.376551	0.376577	0.37662
0	0.2	0	0	0	3	3	3	3	3	3	3	0.01683	0.026243	0.028822	0.035056	0.048634	0.376432	0.376507	0.376425	0.376513	0.376466	0.376633	0.376739
0.1	0.2	0	0	0	3	3	3	3	3	3	3	0.018565	0.026244	0.028823	0.035057	0.048636	0.376468	0.376523	0.376498	0.376537	0.376485	0.376645	0.376751
0.2	0.2	0	0	0	3	3	3	3	3	3	3	0.020675	0.026245	0.028825	0.035058	0.048638	0.37651	0.376544	0.376588	0.376568	0.376508	0.376661	0.376765
0.3	0.2	0	0	0	3	3	3	3	3	3	3	0.023287	0.026246	0.028826	0.03506	0.048641	0.376564	0.376569	0.376699	0.376605	0.376536	0.37668	0.376784
0.4	0.2	0	0	0	3	3	3	3	3	3	3	0.026583	0.026248	0.028828	0.035063	0.048644	0.376631	0.376601	0.37684	0.376653	0.376571	0.376704	0.376807
0.5	0.2	0	0	0	3	3	3	3	3	3	3	0.030831	0.02625	0.028831	0.035065	0.048649	0.376719	0.376643	0.377022	0.376715	0.376618	0.376735	0.376837
0	0.3	0	0	0	3	3	3	3	3	3	3	0.016831	0.029454	0.028824	0.035059	0.048638	0.376492	0.376617	0.376479	0.376626	0.376549	0.376828	0.377005
0.1	0.3	0	0	0	3	3	3	3	3	3	3	0.018566	0.029455	0.028825	0.03506	0.04864	0.376527	0.376633	0.376553	0.376651	0.376567	0.37684	0.377017
0.2	0.3	0	0	0	3	3	3	3	3	3	3	0.020677	0.029456	0.028827	0.035061	0.048642	0.37657	0.376654	0.376642	0.376681	0.37659	0.376856	0.377032
0.3	0.3	0	0	0	3	3	3	3	3	3	3	0.023289	0.029457	0.028828	0.035063	0.048645	0.376623	0.376679	0.376753	0.376719	0.376618	0.376875	0.37705
0.4	0.3	0	0	0	3	3	3	3	3	3	3	0.026585	0.029459	0.02883	0.035065	0.048648	0.376691	0.376711	0.376894	0.376767	0.376654	0.376899	0.377073
0.5	0.3	0	0	0	3	3	3	3	3	3	3	0.030833	0.029461	0.028833	0.035068	0.048653	0.376778	0.376752	0.377077	0.376829	0.3767	0.37693	0.377103
0	0.4	0	0	0	3	3	3	3	3	3	3	0.016833	0.033455	0.028827	0.035062	0.048643	0.376567	0.376754	0.376548	0.376769	0.376652	0.377072	0.377339
0.1	0.4	0	0	0	3	3	3	3	3	3	3	0.018568	0.033456	0.028828	0.035063	0.048645	0.376602	0.376771	0.376621	0.376794	0.37667	0.377085	0.377351
0.2	0.4	0	0	0	3	3	3	3	3	3	3	0.020679	0.033457	0.028829	0.035065	0.048647	0.376645	0.376791	0.37671	0.376824	0.376693	0.3771	0.377366
0.3	0.4	0	0	0	3	3	3	3	3	3	3	0.023291	0.033459	0.028831	0.035067	0.048649	0.376698	0.376816	0.376821	0.376861	0.376721	0.377119	0.377384
0.4	0.4	0	0	0	3	3	3	3	3	3	3	0.026587	0.033461	0.028833	0.035069	0.048653	0.376765	0.376848	0.376962	0.376909	0.376757	0.377143	0.377407
0.5	0.4	0	0	0	3	3	3	3	3	3	3	0.030835	0.033463	0.028836	0.035072	0.048657	0.376853	0.37689	0.377145	0.376971	0.376803	0.377175	0.377437

7776 Rows

Visualizations

Visualizations

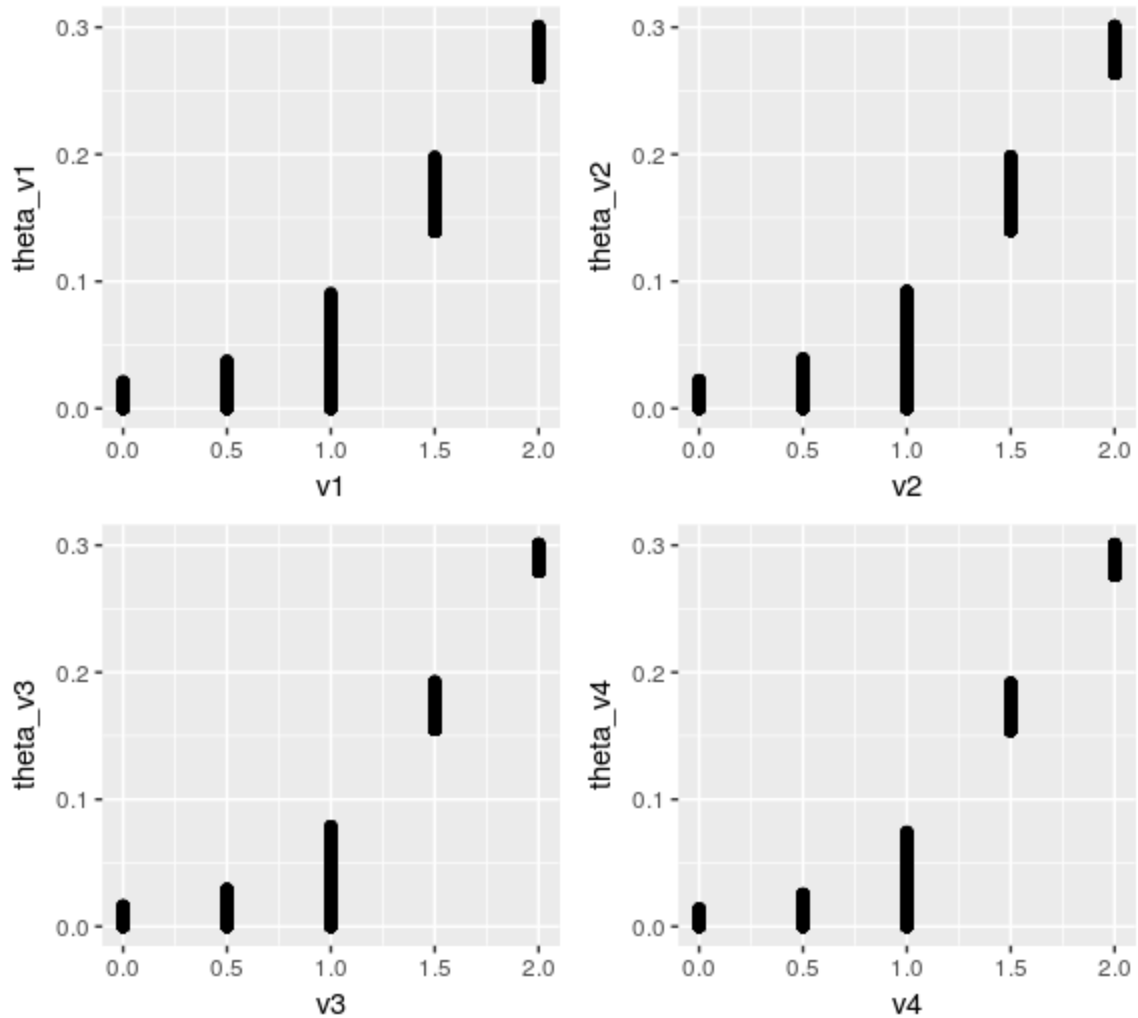
Heatmap

3D Surface

Visualizations

3 plots of surfaces and heatmaps at different R values

Visualizations

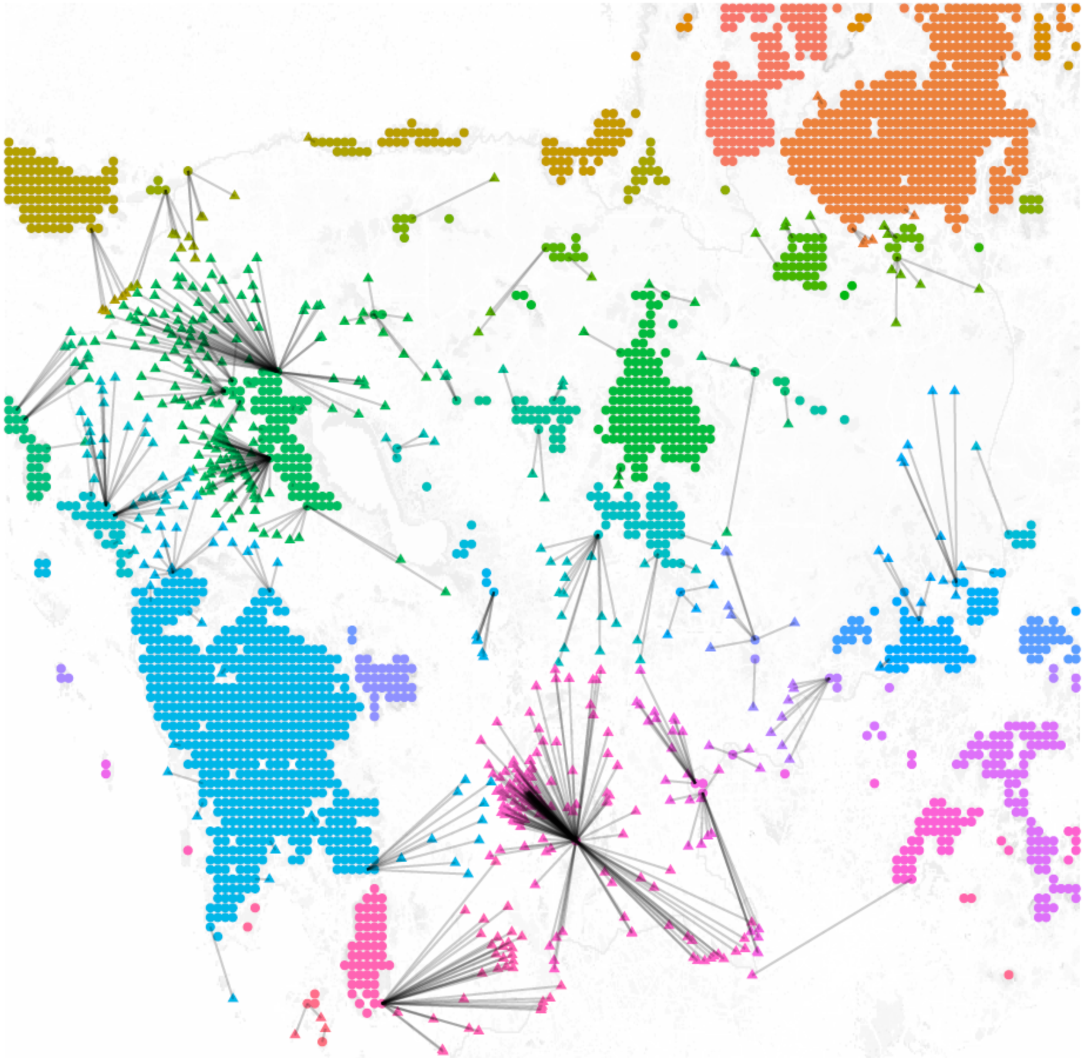


Future Directions

Data, Data, Data

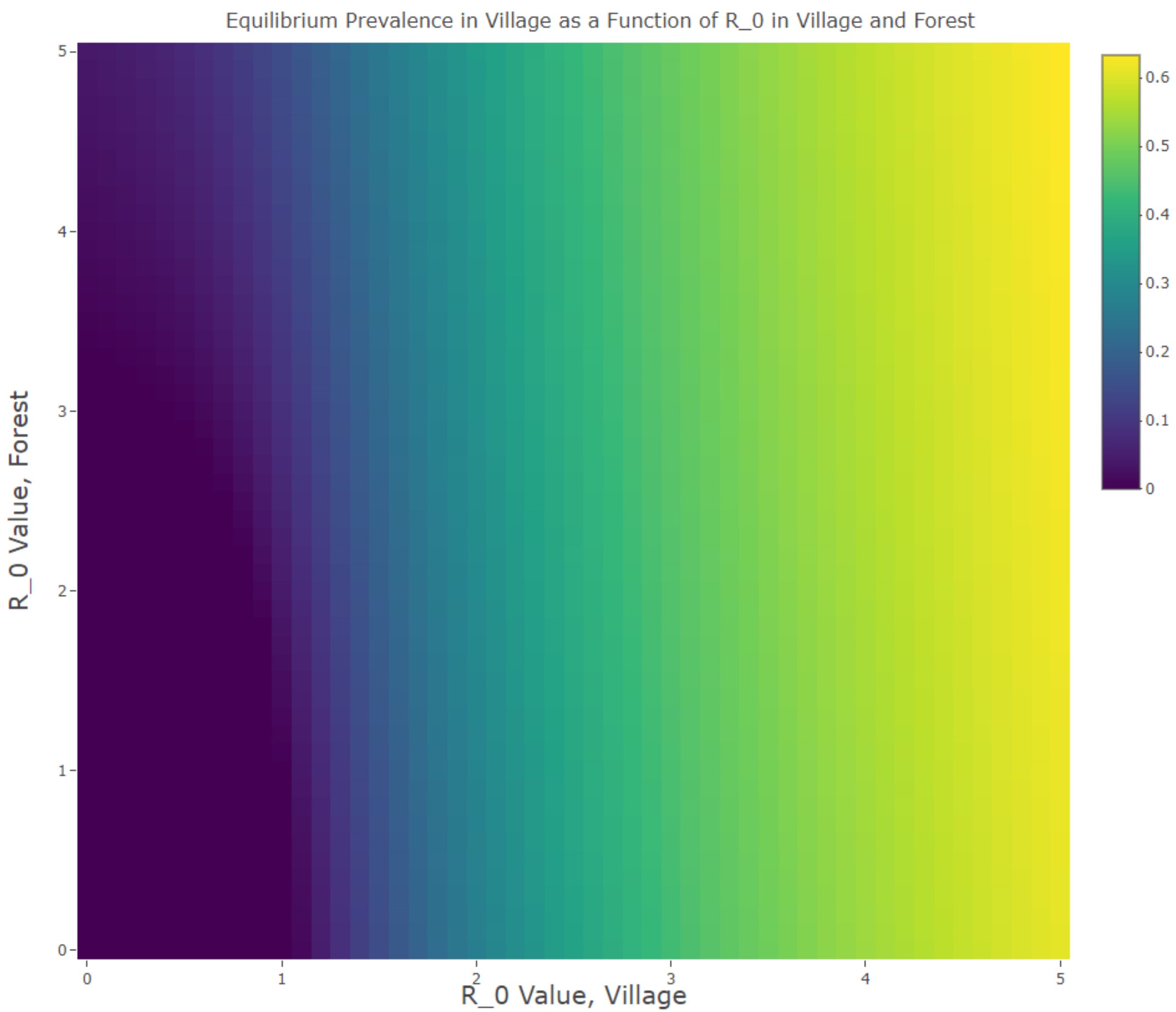
$$\Psi = \begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix}$$

Data, Data, Data

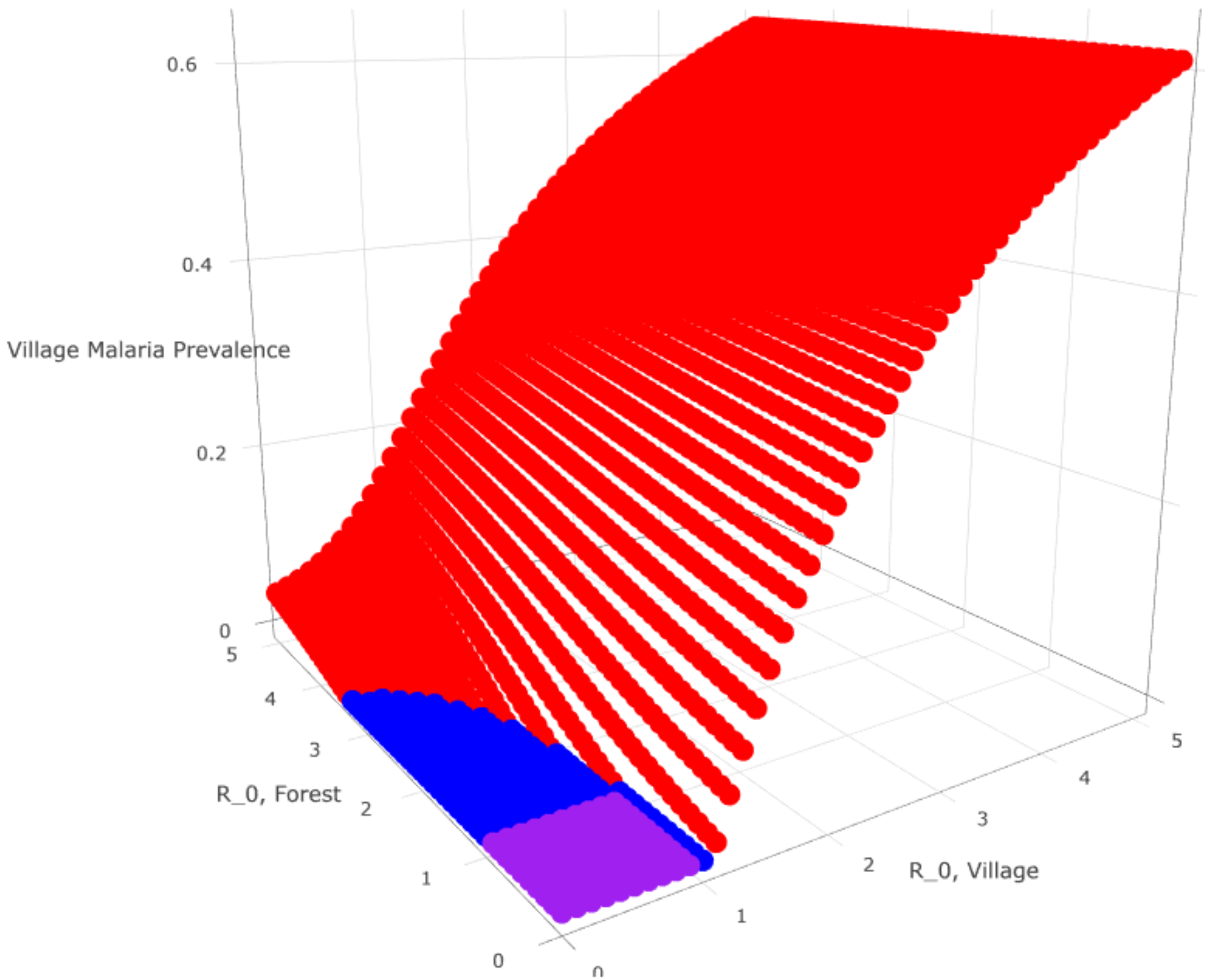


Model Interface

Questions?


Equilibrium Prevalence in Village as a Function of R_0 in Village and Forest
R_0 Value, Forest
R_0 Value, Village
0
1
2
3
4
5
0.6
0.5
0.4
0.3
0.2
0.1
0

Malaria Prevalence in the Village as a Function of R_0



Simple Example: One Village, One Forest

$$0 = \left(\boldsymbol{\Psi}\left(\mathbf{R}^* \circ \frac{\boldsymbol{\Theta}^*}{cS\boldsymbol{\Theta}^* + 1}\right)\right) \circ (\mathbf{H} - \mathbf{X}) - \mathbf{X}$$

$$\frac{\boldsymbol{\Theta}^*}{cS\boldsymbol{\Theta}^* + 1} = \frac{\begin{bmatrix} \Theta^*_{V_{\text{only}}} \\ \Theta^*_{V \leftrightarrow F} \end{bmatrix}}{\begin{bmatrix} cS\Theta^*_{V_{\text{only}}} + 1 \\ cS\Theta^*_{V \leftrightarrow F} + 1 \end{bmatrix}} = \begin{bmatrix} \dfrac{\Theta^*_{V_{\text{only}}}}{cS\Theta^*_{V_{\text{only}}} + 1} \\ \dfrac{\Theta^*_{V \leftrightarrow F}}{cS\Theta^*_{V \leftrightarrow F} + 1} \end{bmatrix}$$

Simple Example: One Village, One Forest

$$\left(\boldsymbol{\Psi}\left(\mathbf{R}^* \circ \frac{\boldsymbol{\Theta}^*}{cS\boldsymbol{\Theta}^* + 1}\right)\right) = \boldsymbol{\Psi}\begin{bmatrix} R_V^* \\ R_F^* \end{bmatrix} \circ \begin{bmatrix} \dfrac{\Theta_{V_{\text{only}}}^*}{cS\Theta_{V_{\text{only}}}^* + 1} \\ \dfrac{\Theta_{V\leftrightarrow F}^*}{cS\Theta_{V\leftrightarrow F}^* + 1} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 1-p & p \end{bmatrix} \begin{bmatrix} R_V^* \cdot \dfrac{\Theta_V^*}{cS\Theta_V^* + 1} \\ R_F^* \cdot \dfrac{\Theta_F^*}{cS\Theta_F^* + 1} \end{bmatrix}$$

$$= \begin{bmatrix} R_V^* \cdot \dfrac{\Theta_V^*}{cS\Theta_V^* + 1} \\ (1-p) \cdot R_V^* \cdot \dfrac{\Theta_V^*}{cS\Theta_V^* + 1} + p \cdot R_F^* \cdot \dfrac{\Theta_F^*}{cS\Theta_F^* + 1} \end{bmatrix}$$

Vectors of Variables

For n locations,

$$\mathbf{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} \quad \mathbf{V} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}$$

For m unique human populations,

$$\mathbf{H} = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_m \end{bmatrix} \quad \mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_m \end{bmatrix}$$