Geostatistics

Homework No.4b

Giorgos Raptakis
374030

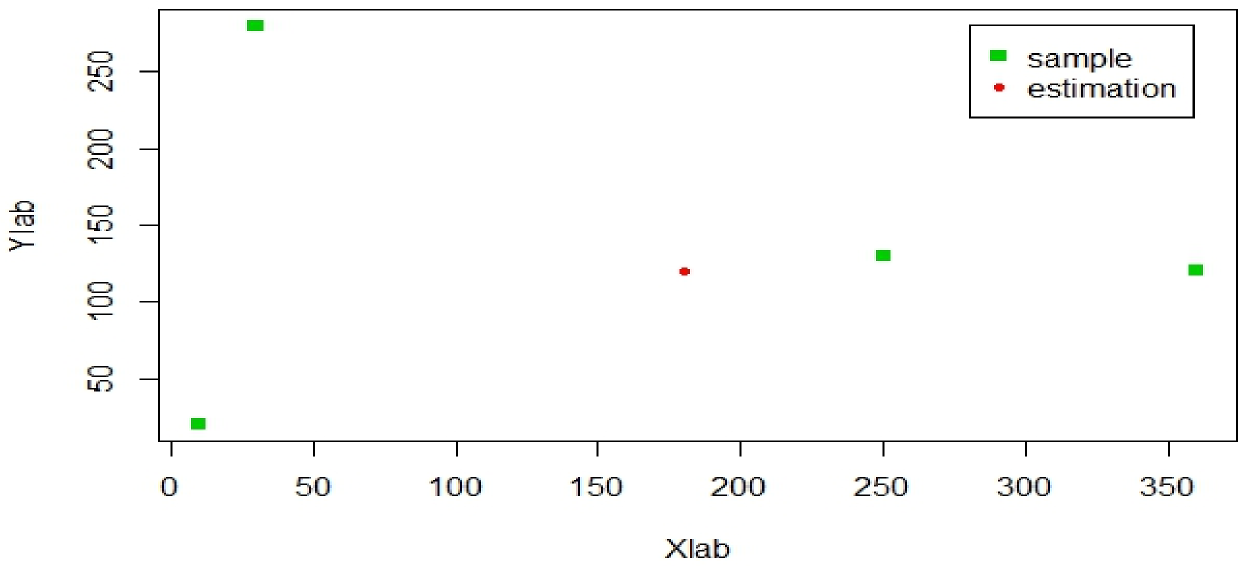
Contents

Task a

We make a plot to show the sampling locations and the estimation location of our points.

Index	Easting	Northing	Measurement
1	10	20	40
2	30	280	130
3	250	130	90
4	360	120	160

Estimation location: x0 = (180,120)



Task b

We write the function of covariance for the lag h and created the plot of the associated covariance model:

$$Cov(h) = 2000 \cdot e^{-\frac{h}{250}}$$

$$\gamma(h) = Cov(0) - Cov(h)$$

```
Ch <- function(h) {
c<-2000*exp(-(h)/250)
}
```

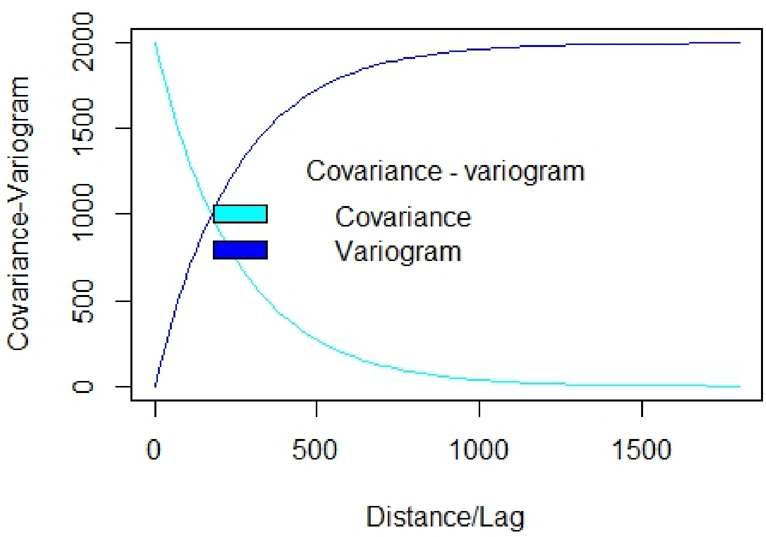
Create a plot of the associated covariance model.

```
Plot (Ch, xlim=c(0,1800), type='l', col = 45, xlab = "Distance/Lag",  ylab = "Covariance-Variogram")

variogram = function(h){Ch(0)-Ch(h)}

plot(variogram, xlim=c(0,1800), type='l', col = 'blue',add = TRUE)

legend("center", inset=.05,    bty="n", cex=1,   title="Covariance - variogram",
     c("Covariance", "Variogram"),fill=c(45,'blue'))
```



Task c

We calculate the distances between our sampling points and then we calculate the covariance matrix.

```
Dist = matrix(NA,nrow=4 ,ncol=4)

for( i in 1: 4 ) {

 for( j in 1:4 ) {

  Dist[i,j] = ( (data[i,1]-data[j,1])^2.0 + (data[i,2]-data[j,2])^2.0 )^0.5 } }

C = matrix(1,nrow=5,ncol=5)

C[1:4,1:4] = Ch(Dist)

C[5,5] = 0
```

The result of the covariance matrix C is:

	V1	**V2**	**V3**	**V4**	**V5**
1	2000.0000	704.7408	695.6677	466.3239	1
2	704.7408	2000.0000	689.3991	461.2466	1
3	695.6677	689.3991	2000.0000	1285.7378	1
4	466.3239	461.2466	1285.7378	2000.0000	1
5	1.0000	1.0000	1.0000	1.0000	0

We should mention that the Covariance matrix is built showing the dispersion of variables around the mean. The more the dispersion is around mean then the more they vary.

Task d

The examination of the matrix after the command: (print(round(C,2))):

1	2000.00	704.74	695.67	466.32	1
2	704.74	2000.00	689.40	461.25	1
3	695.67	689.40	2000.00	1285.74	1
4	466.32	461.25	1285.74	2000.00	1
5	1.00	1.00	1.00	1.00	0

- The values on the diagonal are Variances. They are Cov(0) which are equal of sill.
- The meaning and import of the values in row and column 5 have values equal of one, which imply that the summation of the estimated weights are one

Task e

After we load the library MASS we use the command ginv() to invert the C matrix (Ci=ginv(C)).

```
require(MASS)

Ci=ginv(C) ##inverse covariance matrix

I=Ci%*%C
```

The identity matrix {Ci%*%C} have in diagonal values equal of one and the other features equal of zero.

The values of Ci :

1	5.259954e-04	-2.473128e-04	-0.0001842889	-9.439368e-05	0.2945225
2	-2.473128e-04	5.234835e-04	-0.0001819869	-9.418383e-05	0.2962280
3	-1.842889e-04	-1.819869e-04	0.0009418133	-5.755375e-04	0.1100292
4	-9.439368e-05	-9.418383e-05	-0.0005755375	7.641150e-04	0.2992203
5	2.945225e-01	2.962280e-01	0.1100291783	2.992203e-01	-1013.8863482

The values of matrix Ci shows the estimated weight of the corresponding given point.

Task f

We computed the distanced between the unknown point and the sampled points and use the Cov() function to calculate the D vector

```
DI<- c(NA[1:4])

for( i in 1:4 ) {

  DI[i] = ( (data[i,1]-180)^2.0 + (data[i,2]-120)^2.0 )^0.5}

D = Ch(DI)

D[5] = 1
```

D= 908.6674 831.8349 1507.2766 973.5045   1.0000

Task g

For the interpretation of the covariance matrix D we find out that the values with the biggest values shows strongest relation between the estimation and sampling points. That means that the points with lower distant have bigger value in covariance matrix and the points with bigger distance have lower value.

Task h

We calculate the weight vector w = Ci%*%D :

	V1
1	0.19708695
2	0.14096178
3	0.65047363
4	0.01147764
5	-42.71382033

The weight matrix shows the weight values of each sampling point for the calculation of the measured value of the estimation point. The aggregation of the first four features are equal of one, in order to have the best linear unbiased estimation.

Task i

For the calculation of the Kriging estimate at position (180E, 120N) we use the equation: z_0=W*Z, and the result is equal of 86.58756. Which is a reasonable value if we consider that our point affected more from the point 3 with z-value equal of 90.

Task j

For the calculation of the kriging standard error at (180E, 120N) we use the given equation.

$$\sigma_{ok}^2 = \sigma_0^2 - (\mathrm{W}^t \cdot \mathrm{D}[1:4] - \mu)$$

σ_0^2 : sill effect (Cov(h=0))

μ : the lagrange paramater, w[5]

σ_{ok}^2 is 541.1841