



# Neural Sequence Models: From RNNs to Attention

COM6513/4513 Natural Language Processing

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Readings:

[Chapter 13](#) from Jurafsky and Martin

# **Attendance Monitoring/Questions?**

**TG-YD-HQ**

# Recap

Why is the chain rule essential for language modeling, and what practical assumption do n-gram models make to make it computationally feasible?

# Recap

If a perfect language model (perplexity = 1) existed, would that mean it understands language?

# Lecture Plan

- Neural Language Models Beyond Fixed Context
- Recurrent Neural Networks
- LSTM and GRU
- Structural Limits of Recurrence
- Attention

# The Language Modeling Pipeline

Language Modeling System =

1.  Training Data (Corpora)
  2.  Input Representation (Tokenization)
  3.  Model Architecture
  4.  Optimization (Training Objective)
- 
5.  Evaluation
  6.  Deployment / Generation

$$P(w_1, \dots, w_T) = \prod_t P(w_t | w_{<t})$$

# Issues of N-gram LM?



1

Go to [wooclap.com](https://wooclap.com)

2

Enter the event code in the top banner



Enable answers by SMS

Event code  
**XAOPRJ**

# Issues of N-gram LM?

- Data sparsity
- Exponential growth of parameters
- No generalization across similar words
- Fixed length context

# From Counts to Continuous Representations

## N-gram Models

- Computing probabilities by counting
- Discrete one-hot word representations
- Separate parameter for each context
- No sharing across similar words
- Exponential growth:  $O(V^n)$

$$P(w_1, \dots, w_T) = \prod_t P(w_t | w_{<t})$$

$$P(w_t | w_{t-n+1:t-1}) = \frac{C(w_{t-n+1:t})}{C(w_{t-n+1:t-1})}$$

# From Counts to Continuous Representations

## Neural Models

$$P(w_1, \dots, w_T) = \prod_t P(w_t \mid w_{<t})$$

- Replace counting with a learned function
- Words represented as dense vectors (embeddings)
- Parameters shared across contexts
- Generalization via distributed representations

$$P(w_t \mid \text{context}) = \text{softmax}(f_\theta(\text{context}))$$

# From Counts to Continuous Representations

## Neural Models

$$P(w_1, \dots, w_T) = \prod_t P(w_t \mid w_{<t})$$

- Replace counting with a learned function
- Words represented as dense vectors
- Parameters shared across contexts
- Generalization

$$P(w_t \mid \text{context}) = \text{softmax}(f_\theta(\text{context}))$$

Moving from memorizing observed patterns to learning continuous representations that generalize

# From Counts to Continuous Representations

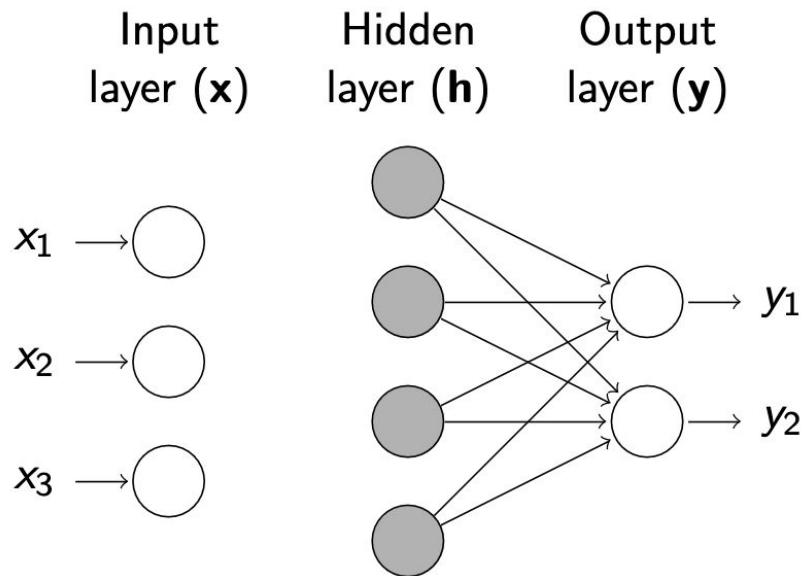
## Neural Models

$$P(w_1, \dots, w_T) = \prod_t P(w_t \mid w_{<t})$$

- Replace counting with a learned function
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- Parameters shared across contexts
- Generalization

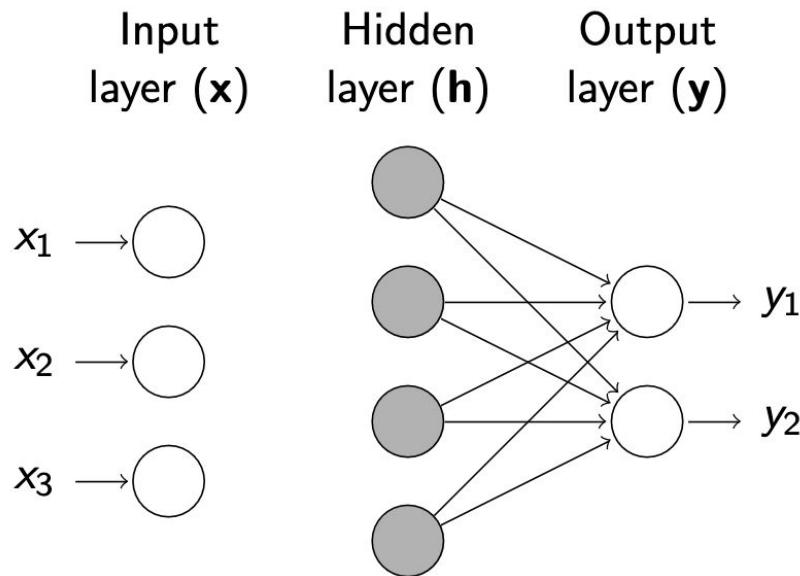
$$P(w_t \mid \text{context}) = \text{softmax}(f_\theta(\text{context}))$$

# FF Neural Networks (recap)



$$\begin{aligned}\mathbf{h} &= g(\mathbf{x}^T \mathbf{W}_h) \\ \mathbf{y} &= \text{softmax}(\mathbf{h}^T \mathbf{W}_o) \\ \mathbf{W}_o &\in \mathcal{R}^{h \times y}\end{aligned}$$

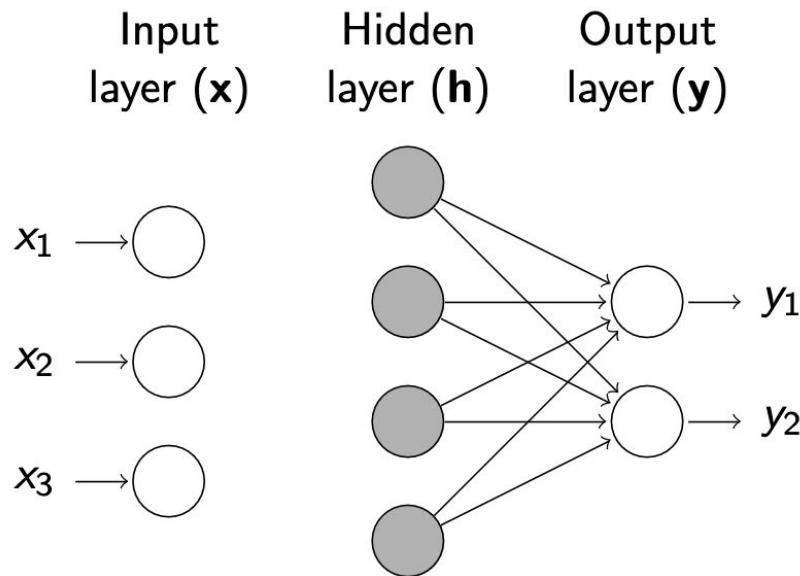
# FF Neural Networks (recap)



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Output dimension?

# FF Neural Networks (recap)



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Why softmax?

# Why Softmax

Softmax ensures:

- ✓ Non-negative probabilities
- ✓ Sum to 1
- ✓ Differentiable (trainable with gradient descent)

$$P(w_t = i \mid \text{context}) = \frac{e^{z_i}}{\sum_j e^{z_j}}$$

# Why Softmax

**Enables Maximum Likelihood Estimation**

Training objective:

$$\mathcal{L} = -\frac{1}{T} \sum_{t=1}^T \log P(w_t^{\text{true}} \mid w_{<t})$$

# Training: Stochastic Gradient Descent (SGD)

**Input:**  $D_{train} = \{(x_1, y_1) \dots (x_M, y_M)\}$ ,  $D_{val} = \{(x_1, y_1) \dots (x_D, y_D)\}$ ,  
*learning rate  $\eta$ , epochs  $e$ , tolerance  $t$*

*initialize  $\mathbf{w}$  with zeros*

**for each epoch  $e$  do**

*randomise order in  $D_{train}$*

**for each  $(x_i, y_i)$  in  $D_{train}$  do**

*update  $\mathbf{w} = \mathbf{w} - \eta \nabla_{\mathbf{w}} L(\mathbf{w}; x_i; y_i)$*

*monitor training and validation loss*

**if** previous validation loss – current validation loss; smaller than  $t$

**break**

**return  $\mathbf{w}$**

# Backpropagation (recap)

**Forward Pass:** Calculates activations and outputs using inputs and current weights

$$a^{[l]} = \sigma(W^{[l]}a^{[l-1]} + b^{[l]})$$

**Backward Pass (Error Propagation):** Computes gradient of loss w.r.t. weights using the chain rule

$$\delta^{[L]} = \frac{\partial L}{\partial a^{[L]}} \cdot \sigma'(z^{[L]}) \quad \delta^{[l]} = (W^{[l+1]T} \delta^{[l+1]}) \cdot \sigma'(z^{[l]})$$

**Weight Update:** Adjusts weights in the direction that reduces loss.

$$W^{[l]} = W^{[l]} - \alpha \frac{\partial L}{\partial W^{[l]}}$$

# What FF Neural LMs Improve Over N-grams?

- Data sparsity
- Exponential growth of parameters
- No generalization across similar words
- Fixed length context

# What FF Neural LMs Improve Over N-grams?

- ✓ Data sparsity
- ✓ Exponential growth of parameters
- ✓ No generalization across similar words
- ✗ Fixed length context

# Fixed Context

FF LM assumes:

- Fixed window size
- Context length chosen in advance
- No memory beyond window

$$P(w_t \mid w_{t-n+1:t-1})$$

# Fixed Context

Increasing window size:

- Larger input dimension
- More parameters
- More computation

$$P(w_t \mid w_{t-n+1:t-1})$$

But still finite

Doesn't remove Markov assumption

# What Do We Actually Want?

True Language Modeling Goal

$$P(w_t \mid w_{<t})$$

Conditioning on all previous words

# What Do We Actually Want?

True Language Modeling Goal

$$P(w_t \mid w_{<t})$$

Conditioning on all previous words

- Storing the entire sequence explicitly
- Increasing parameters with sequence length

# What Do We Actually Want?

True Language Modeling Goal

$$P(w_t \mid w_{<t})$$

Conditioning on all previous words

- Maintaining a running summary of everything seen so far

# What Do We Actually Want?

True Language Modeling Goal

$$P(w_t \mid w_{<t})$$

Conditioning on all previous words

- Maintaining a running summary of everything seen so far

$$h_t = f(h_{t-1}, x_t)$$

# Recurrent Neural Networks

## Core Idea

Maintain a dynamic hidden state that summarizes the past:

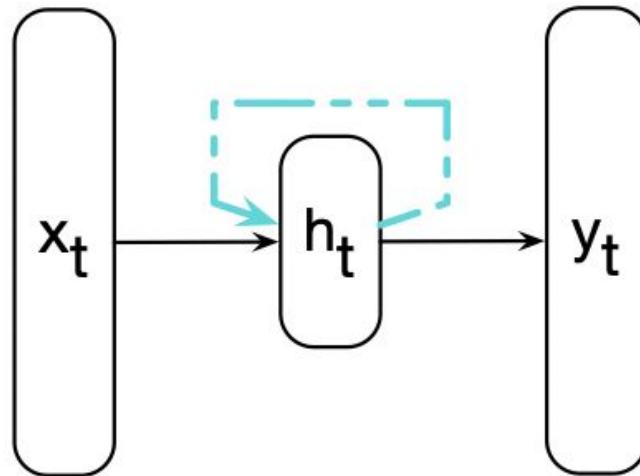
- $h_t$ : running summary of all previous words
- Same function used at every time step
- Parameters shared across time

$$h_t = f(h_{t-1}, x_t)$$

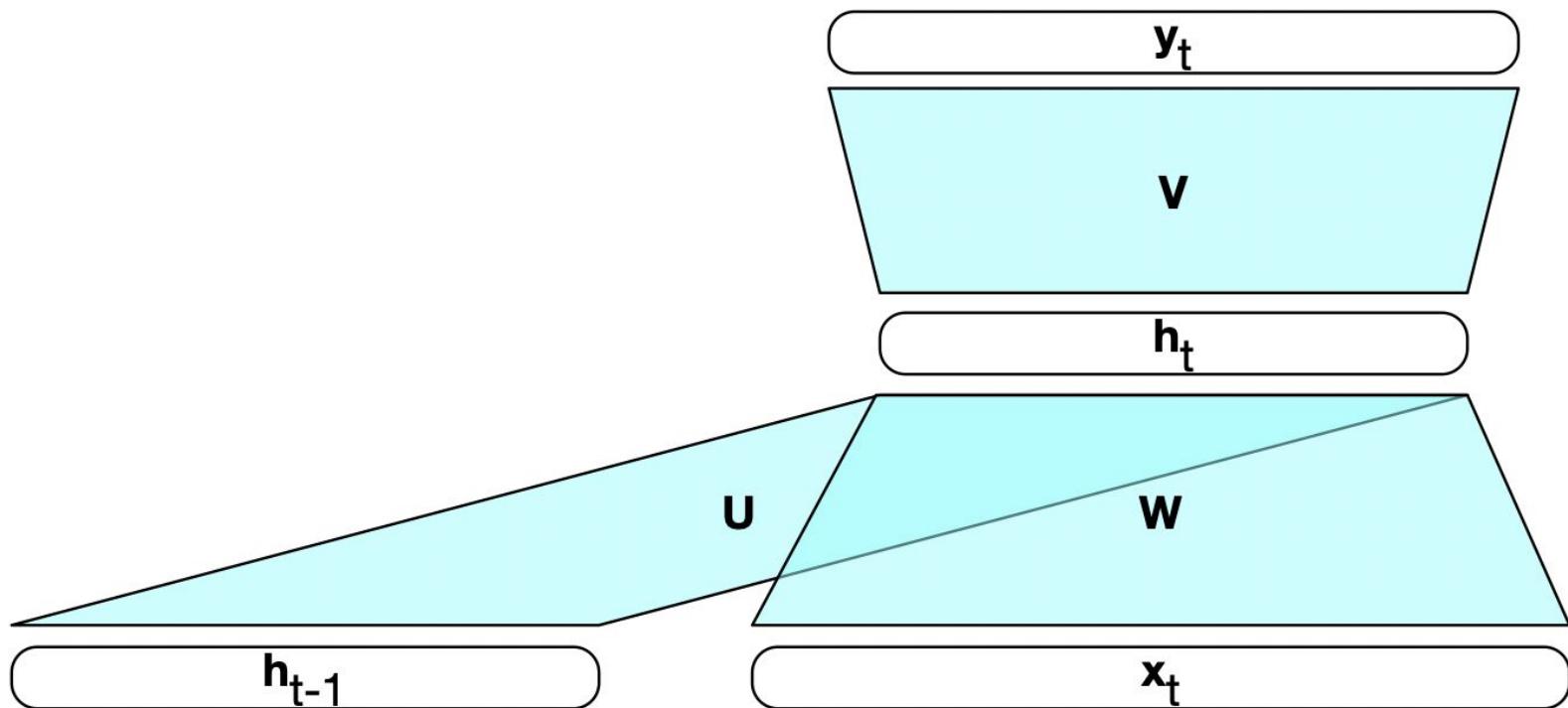
# Elman Networks [Elman, 1990]

Simple recurrent networks

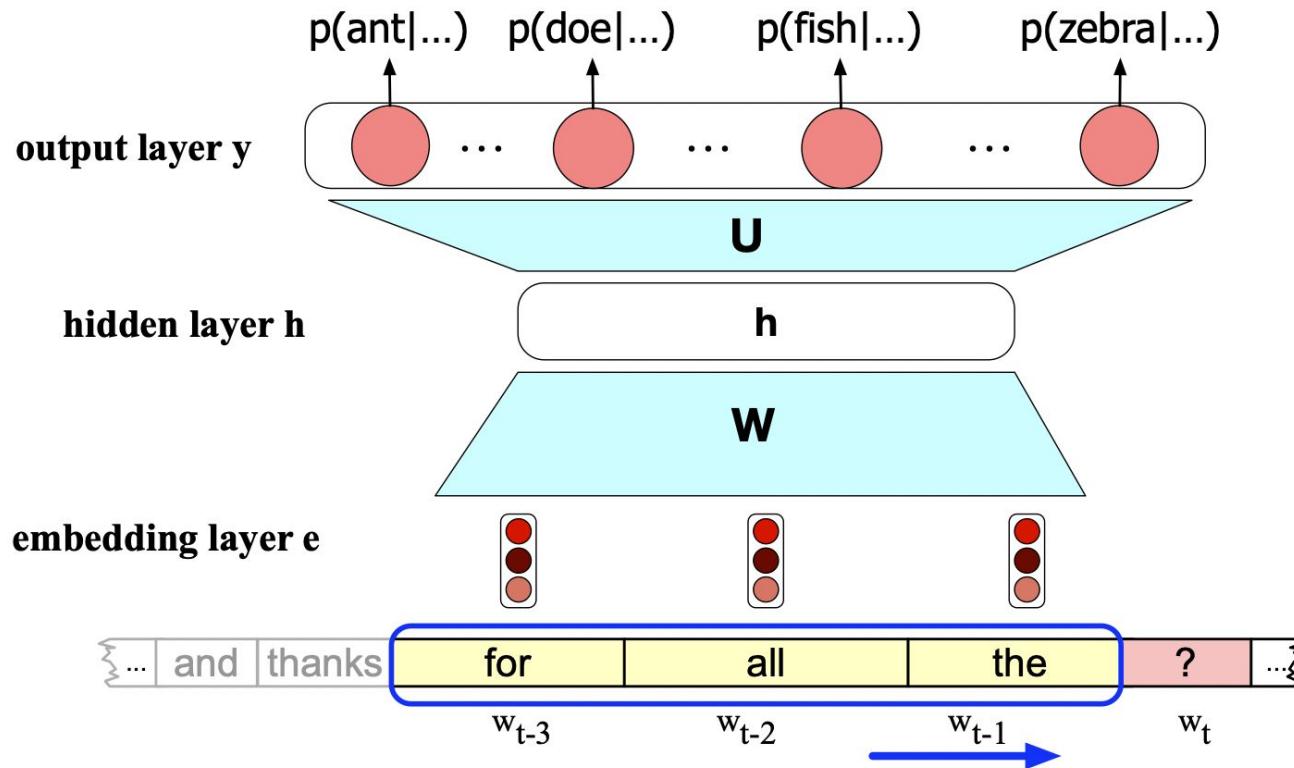
- $X_t$ : input vector  $x$  at time  $t$
- $H_t$ : a form of memory with no fixed-length limit



# Elman Networks [Elman, 1990]



# Comparison to FFNN



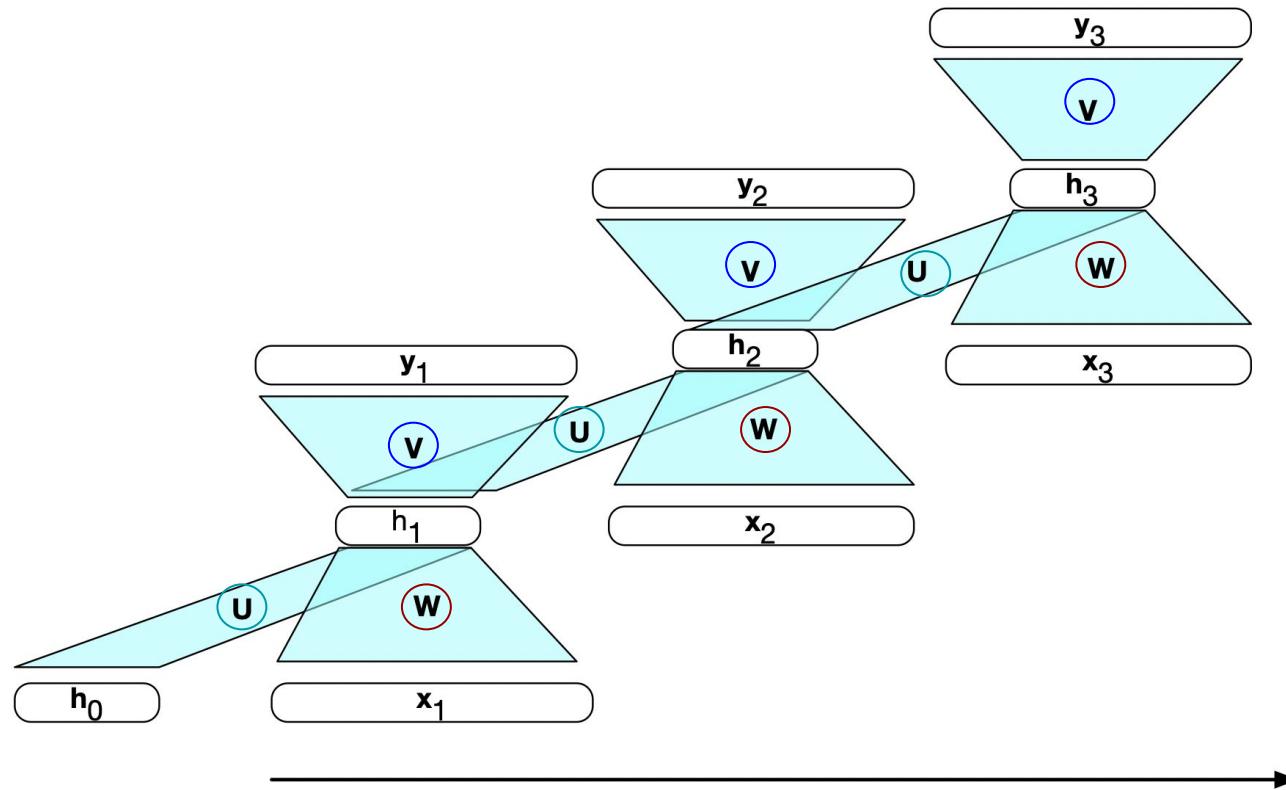
# Forward inference

**function** FORWARDRNN( $\mathbf{x}$ , *network*) **returns** output sequence  $\mathbf{y}$

```
 $\mathbf{h}^0 \leftarrow 0$ 
for  $i \leftarrow 1$  to LENGTH( $\mathbf{x}$ ) do
     $\mathbf{h}_i \leftarrow g(\mathbf{U}\mathbf{h}_{i-1} + \mathbf{W}\mathbf{x}_i)$ 
     $\mathbf{y}_i \leftarrow f(\mathbf{V}\mathbf{h}_i)$ 
return  $\mathbf{y}$ 
```

$\mathbf{U}$ ,  $\mathbf{V}$  and  $\mathbf{W}$  are shared across time

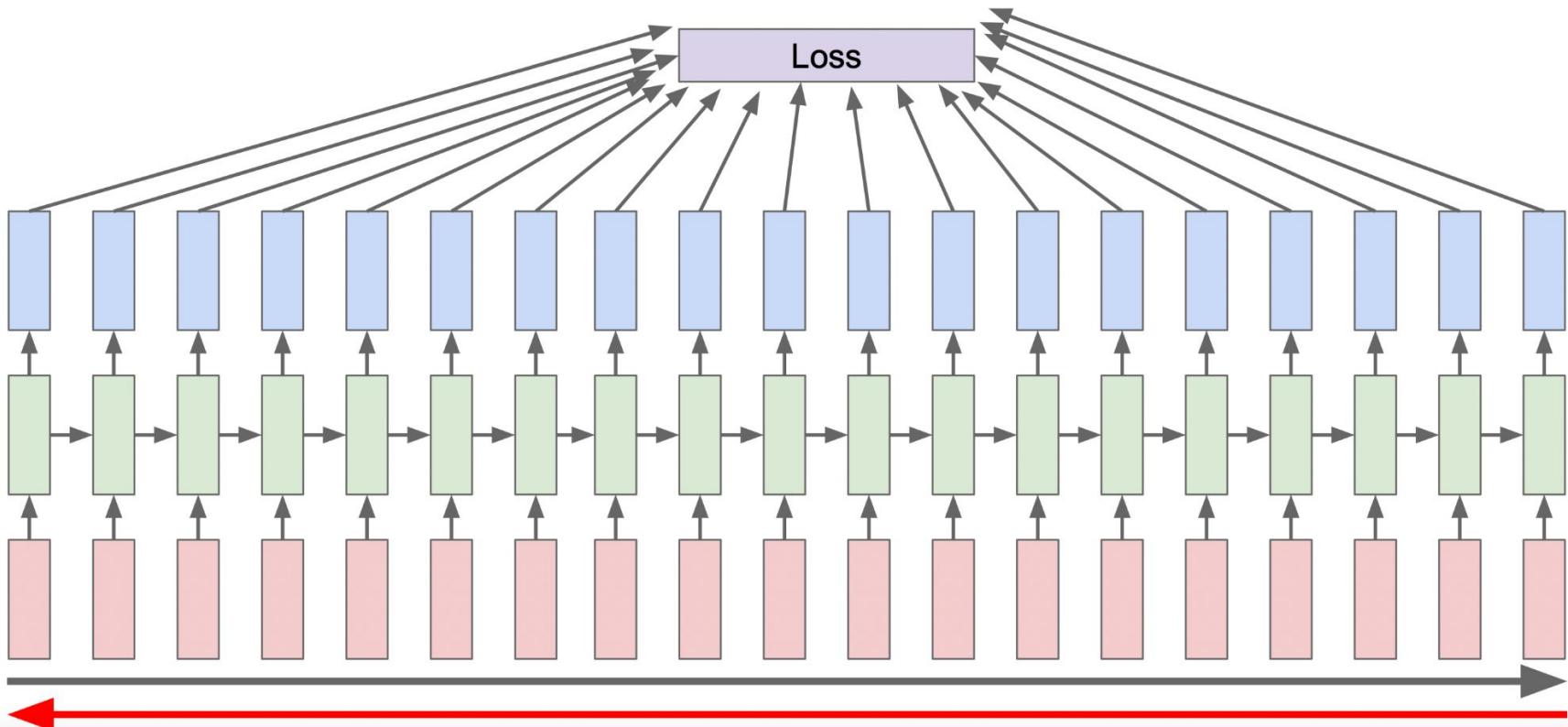
# Forward Inference



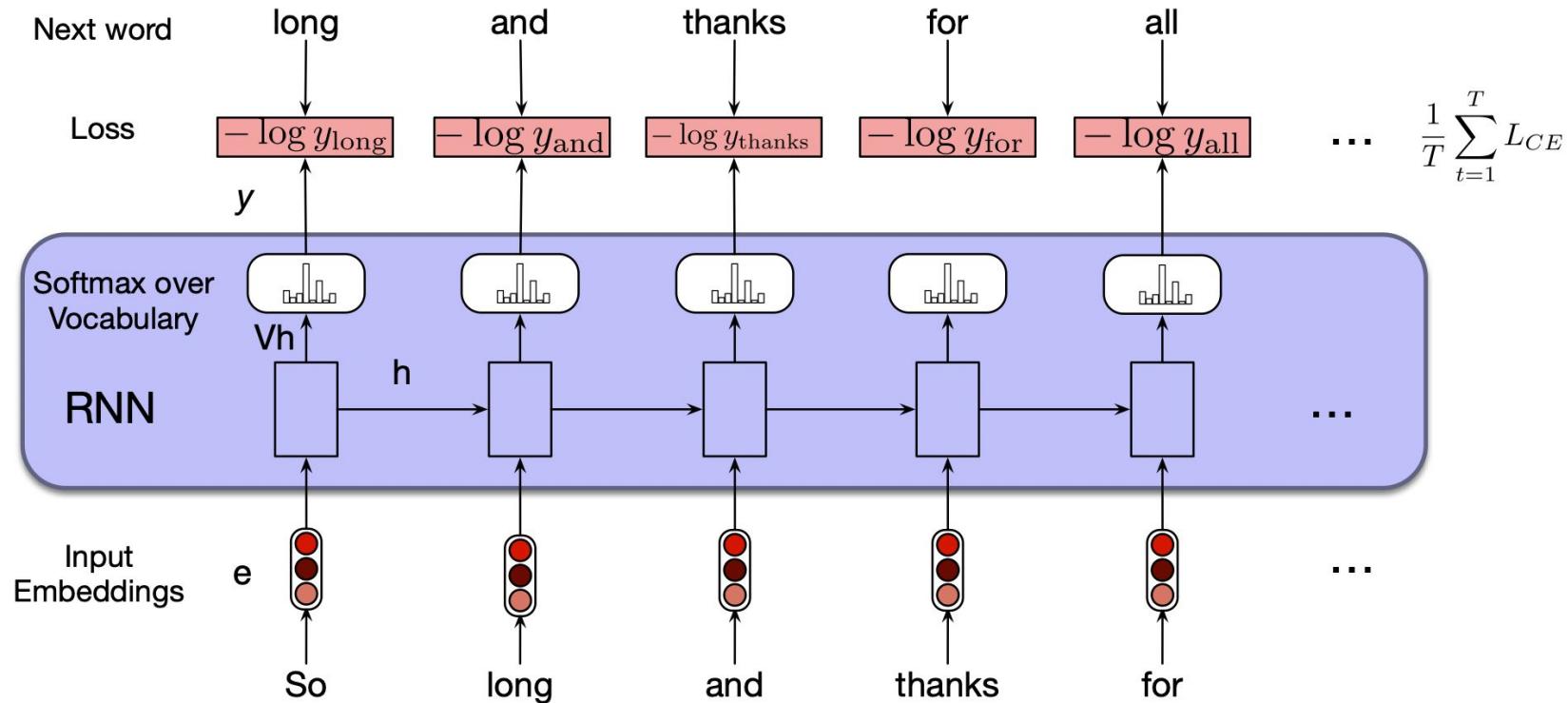
# Backpropagation through time (BPTT)

- Unfold the computational graph, and use backpropagation
- Can apply any general-purpose gradient-based techniques

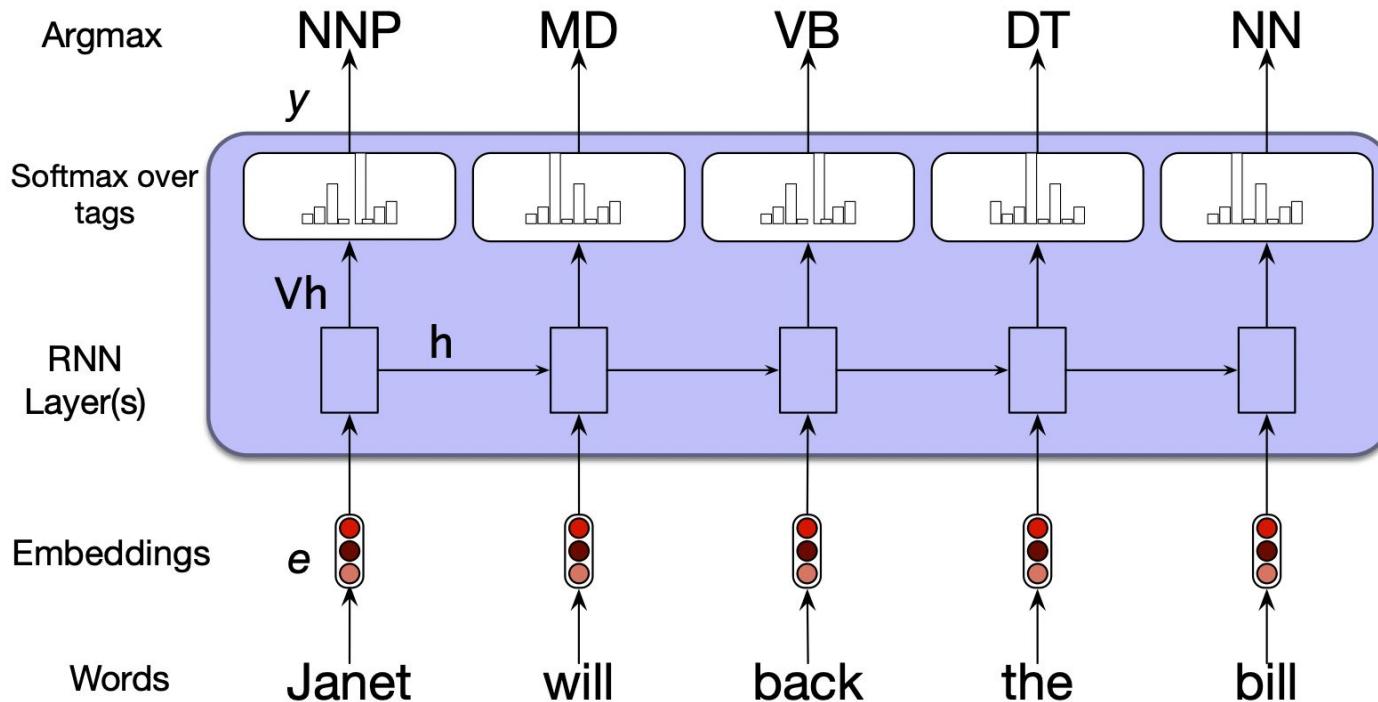
# Backpropagation through time (BPTT)



# RNN for LM

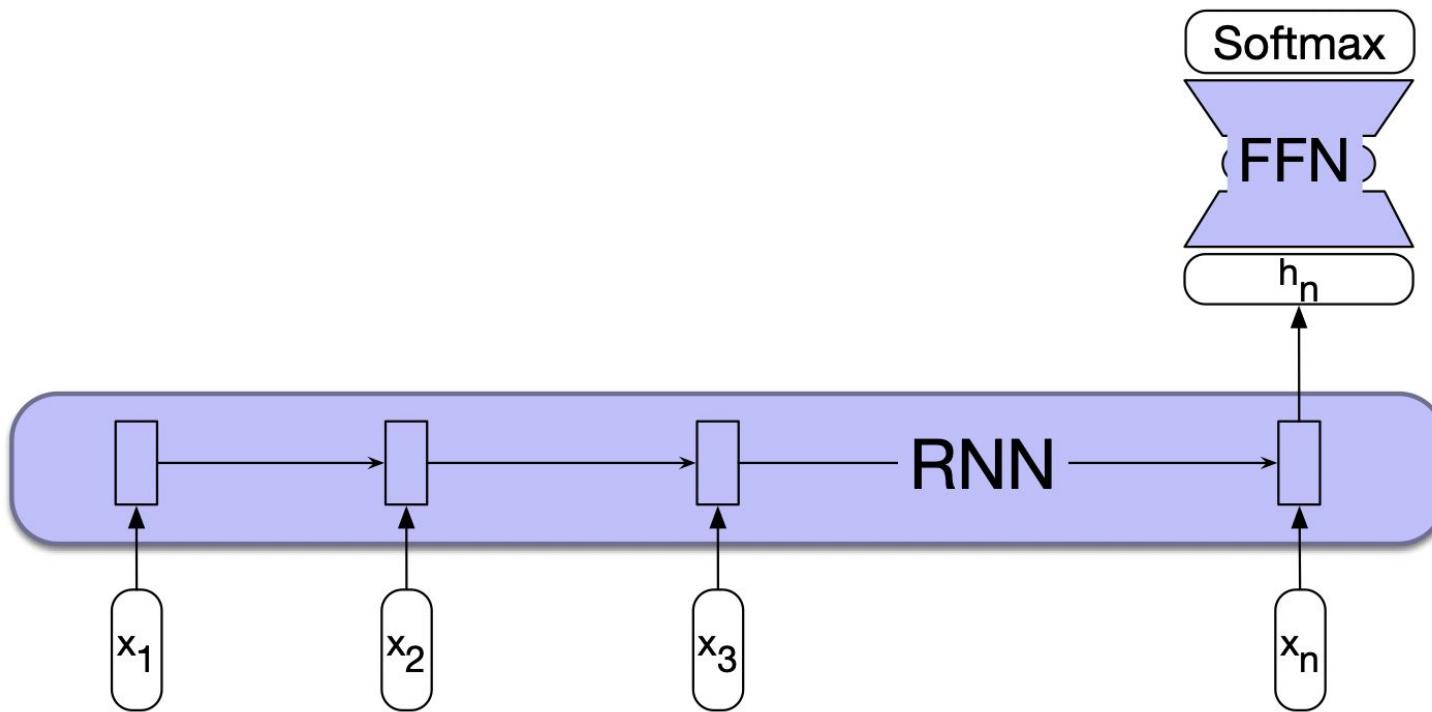


# RNN for Sequence Labeling



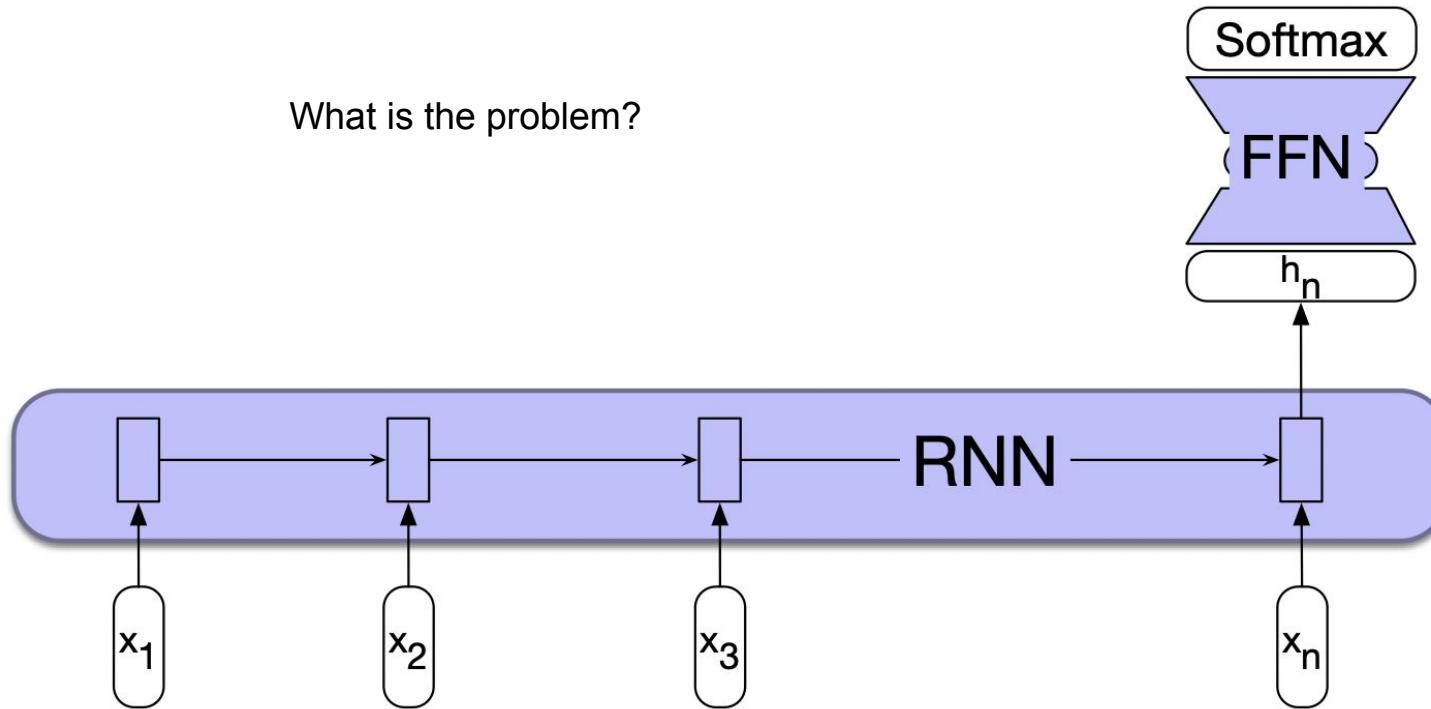
# RNN for Classification?

# RNN for Classification



# RNN for Classification

What is the problem?



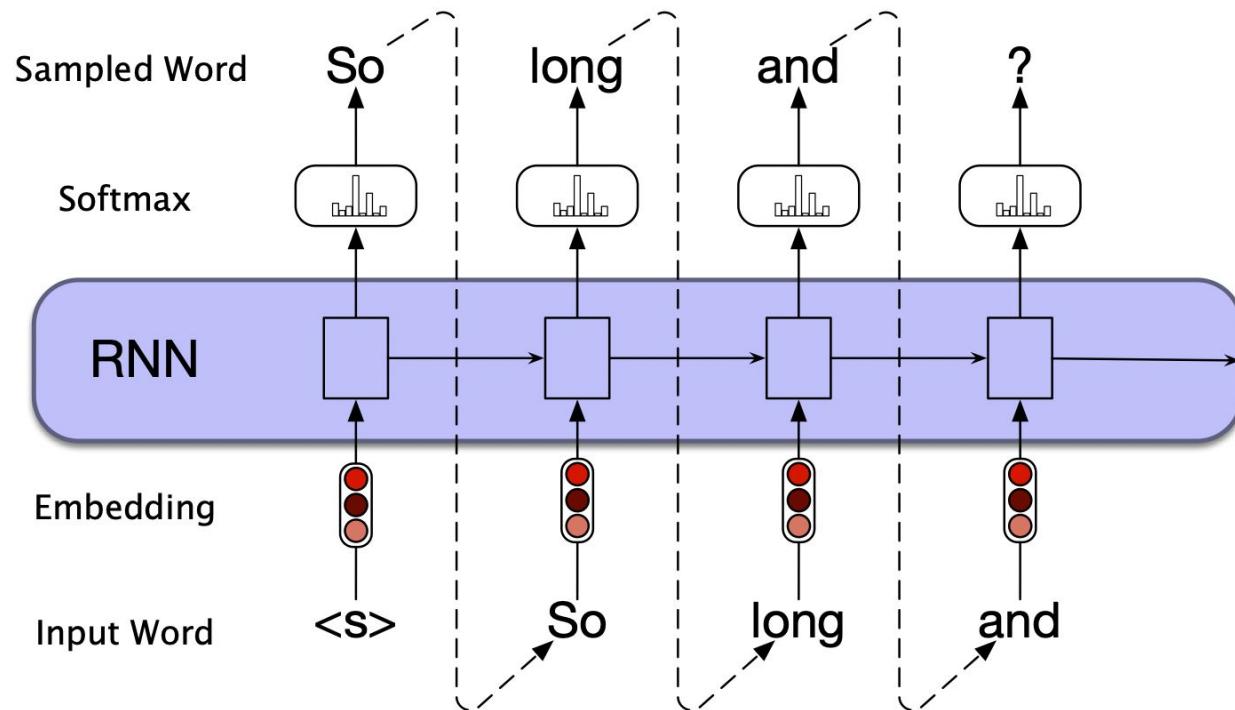
# RNN for Text Generation

- “Autoregressive” text generation
  - What is the input?

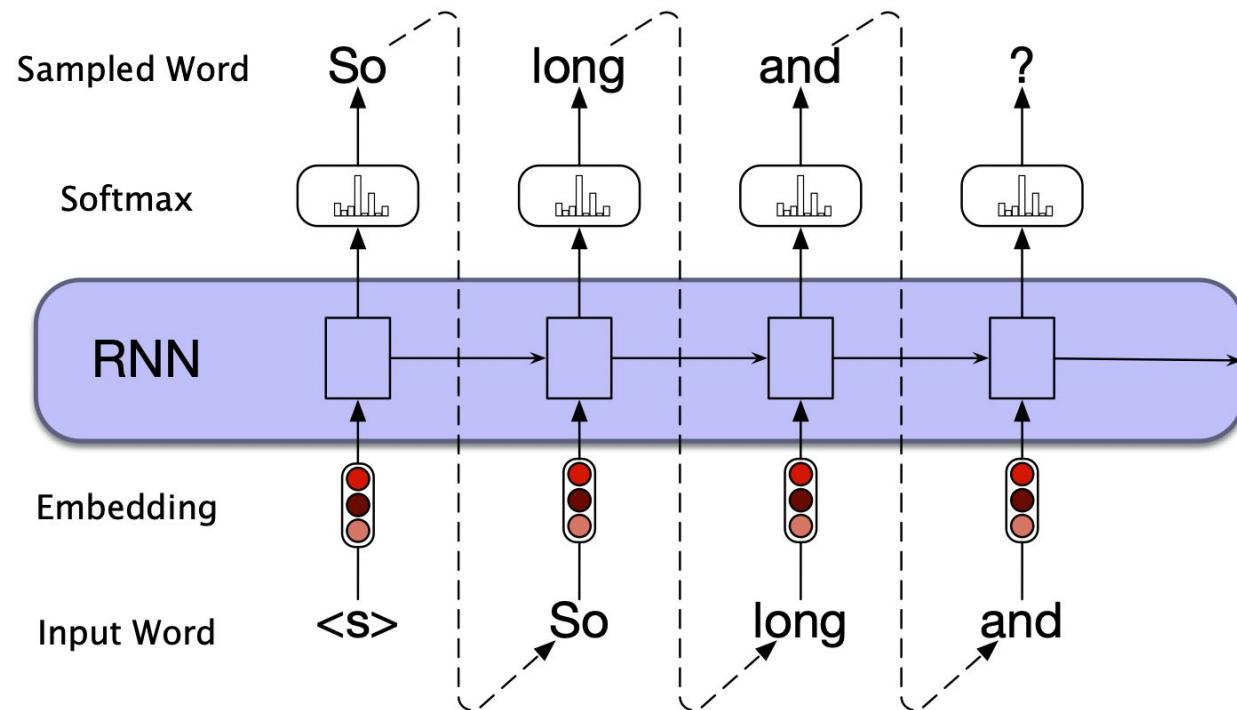
# RNN for Text Generation

- “Autoregressive” text generation
  - Begin the sequence with a special token, e.g., <s>
  - Sample a word in the output from the softmax distribution given the start symbol
  - Use the word embedding of the generated word as the input for generating the next word
  - Continue until a max length or generating an end symbol (</s>)

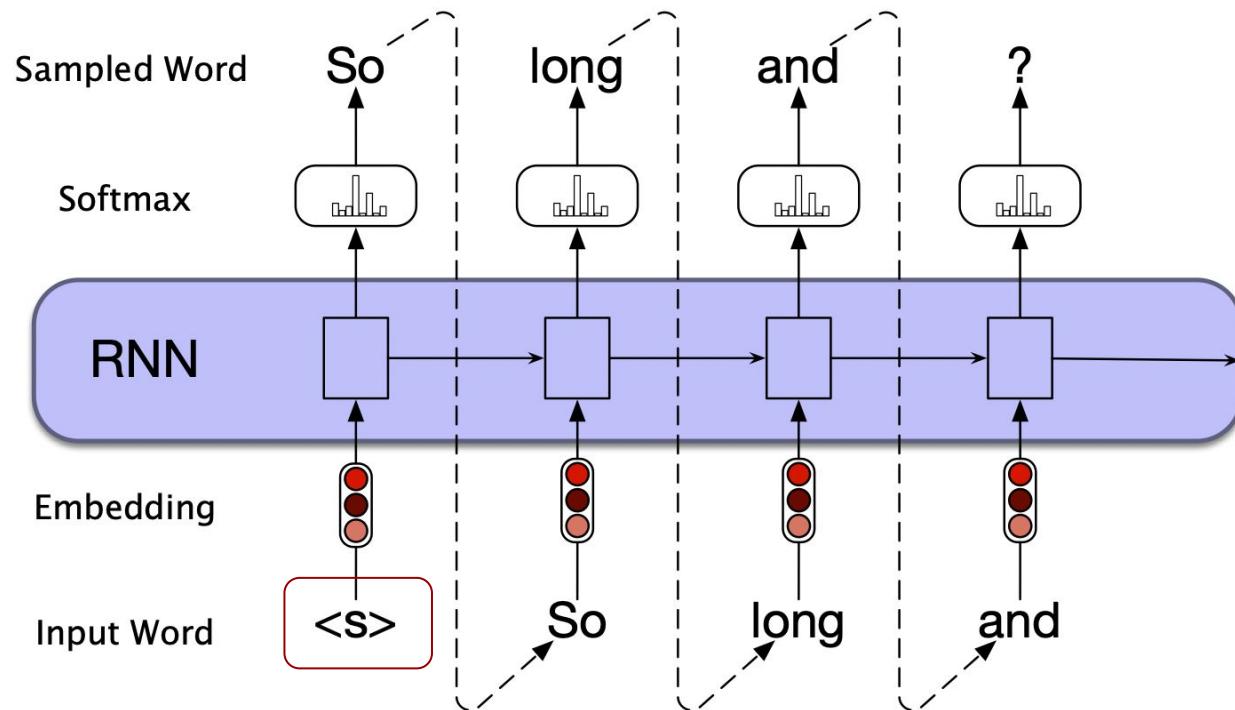
# RNN for Text Generation



# How can we do better?



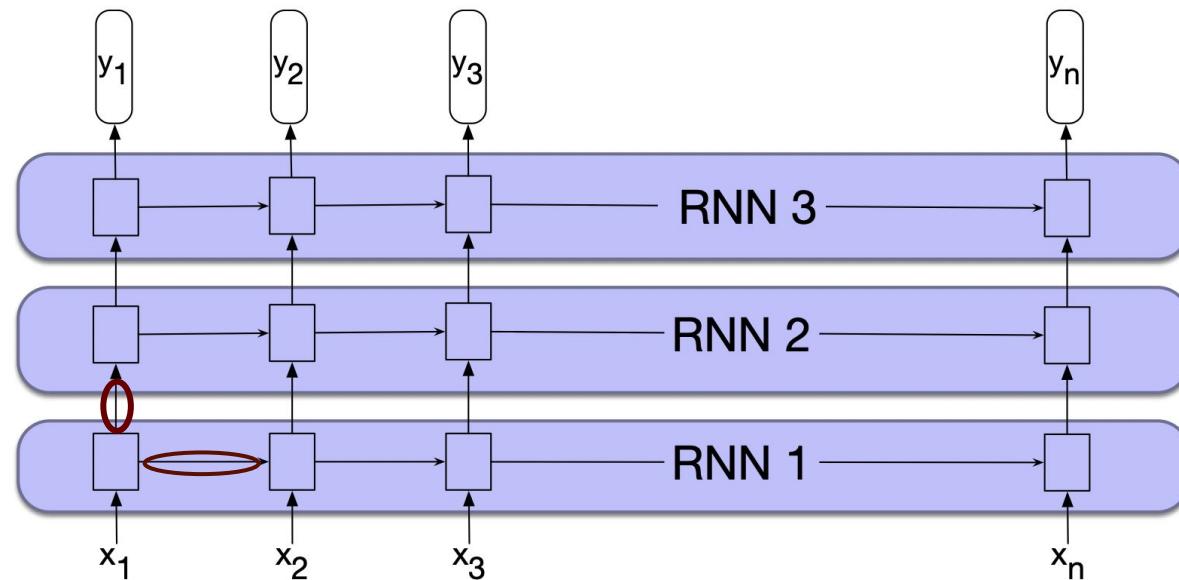
# How can we do better?



# RNN Variations

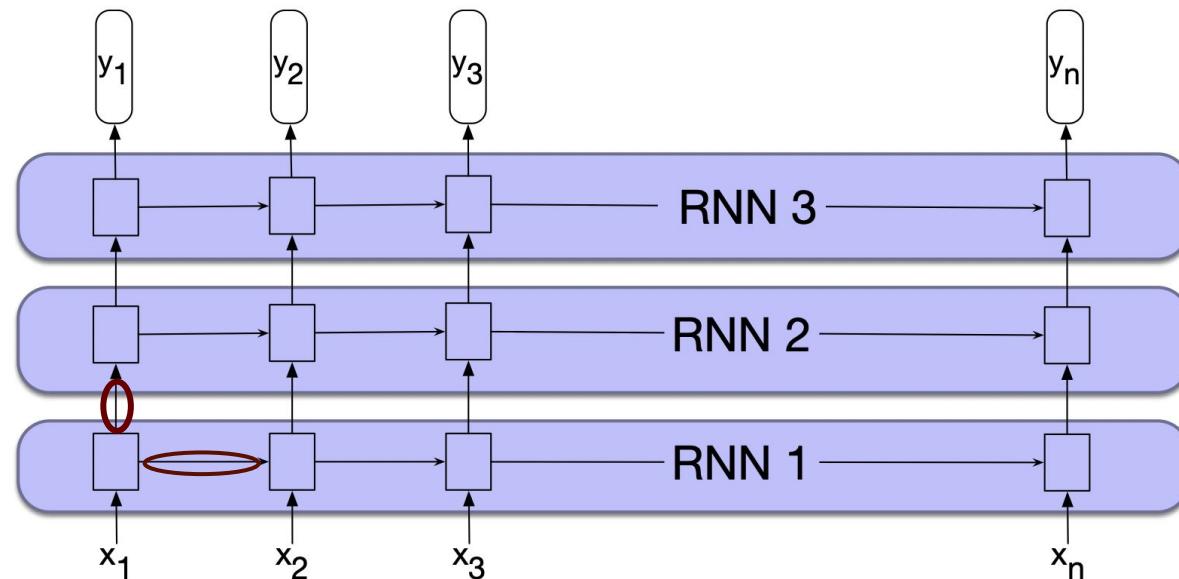
- Stacked RNN
- Bidirectional RNN

# Stacked RNN



# Stacked RNN

Induces representations at differing levels of abstraction

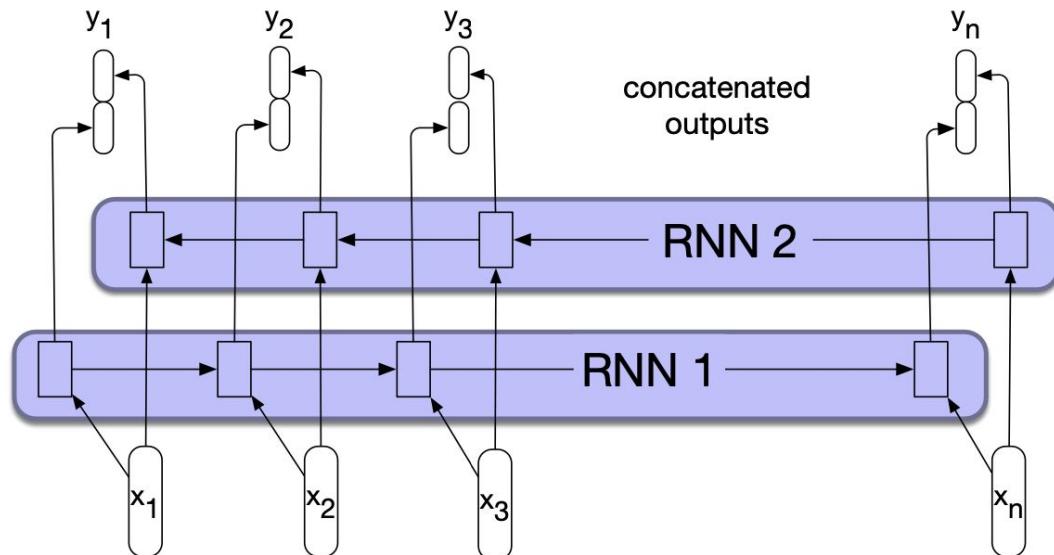


# Bidirectional RNNs

$$\mathbf{h}_t^f = \text{RNN}_{\text{forward}}(\mathbf{x}_1, \dots, \mathbf{x}_t)$$

$$\mathbf{h}_t^b = \text{RNN}_{\text{backward}}(\mathbf{x}_t, \dots, \mathbf{x}_n)$$

$$\mathbf{h}_t = [\mathbf{h}_t^f ; \mathbf{h}_t^b]$$



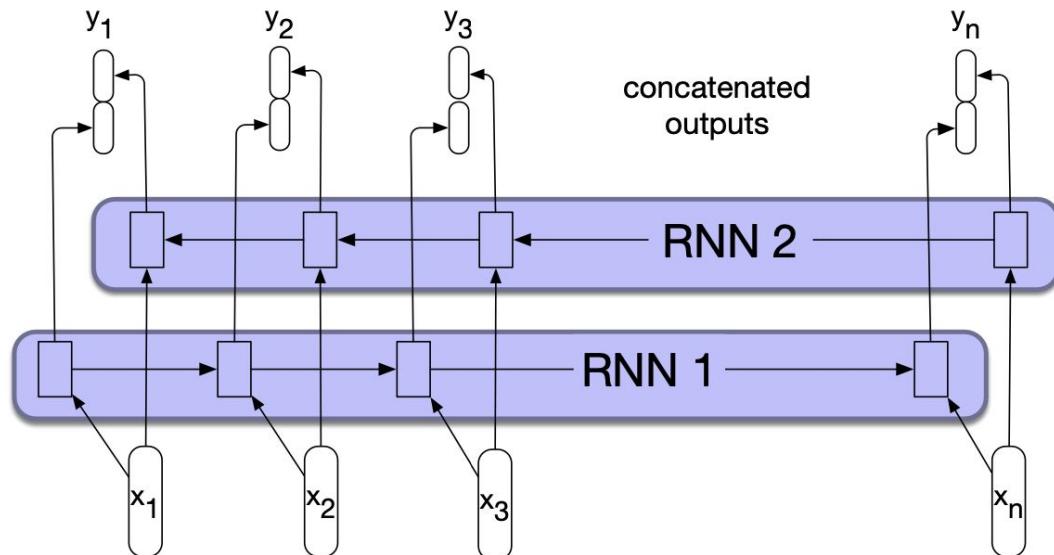
# Bidirectional RNNs

Difference with Stacked RNN?

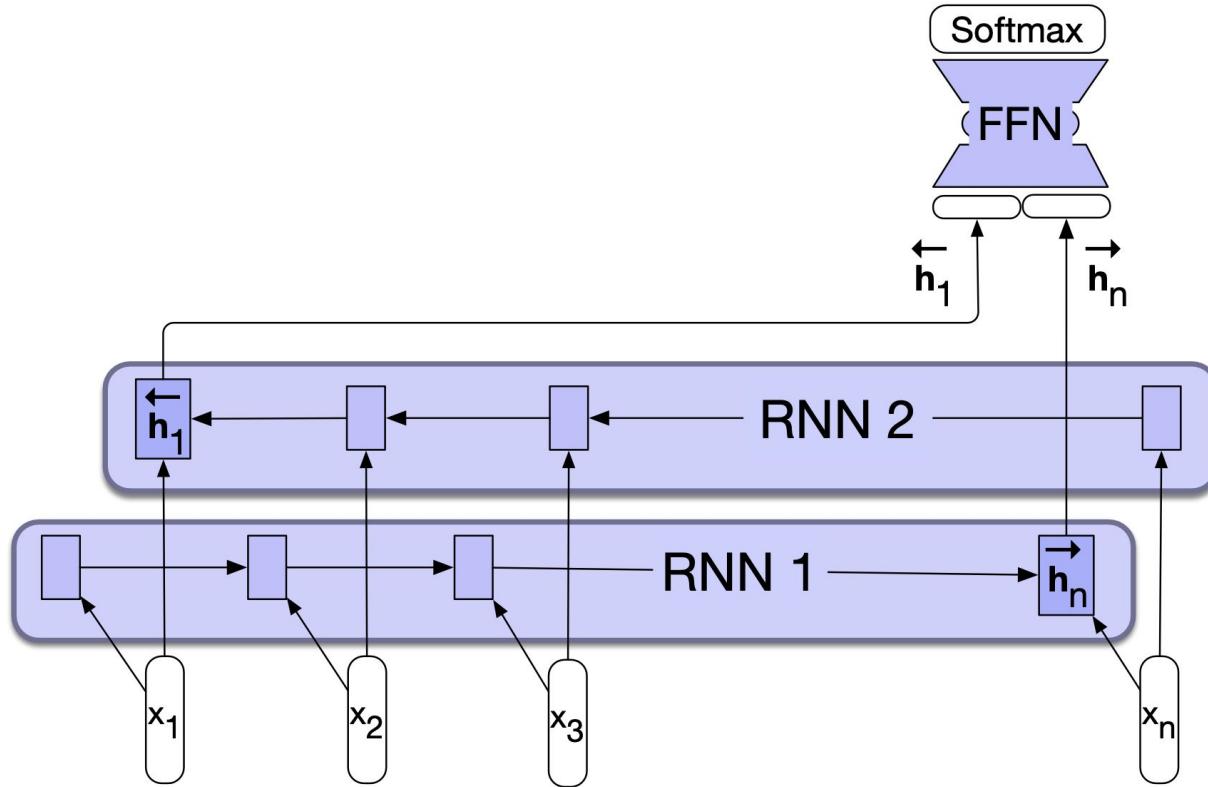
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$$\mathbf{h}_t = [\mathbf{h}_t^f ; \mathbf{h}_t^b]$$



# Bidirectional RNNs



# Problem?

# Problem

- Exploding or vanishing gradients for long sequences
  - Gradients can shrink exponentially when weights are small
  - Gradients grow exponentially when weights are big
  - Typical feed-forward neural nets often do not have many hidden layers
- Memory is Uncontrolled
  - Hidden state must store everything
  - No mechanism to decide what to keep or discard
- Single Vector Bottleneck
  - All past information compressed into a single vector

# What Should a Better Recurrent Model Do?

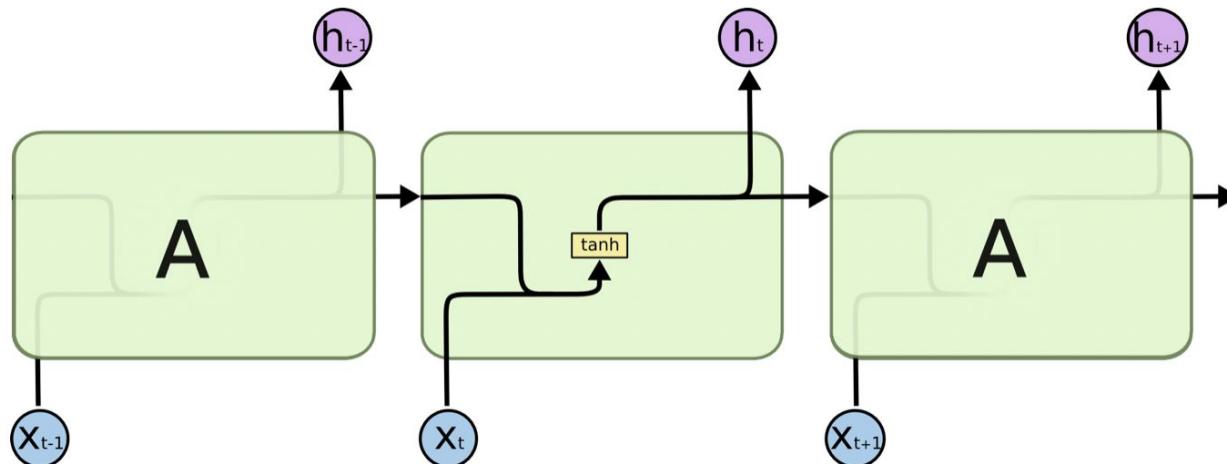
- ✓ Preserve important information over long distances
- ✓ Forget irrelevant information
- ✓ Allow gradients to flow across many time steps

# LSTM (Hochreiter and Schmidhuber, 1997)

- Controlled Memory via Cell State
  - Introduces a dedicated memory vector
- Context management
  - Removing information no longer needed
  - Adding information likely to be needed later
- Gates to control the flow of information

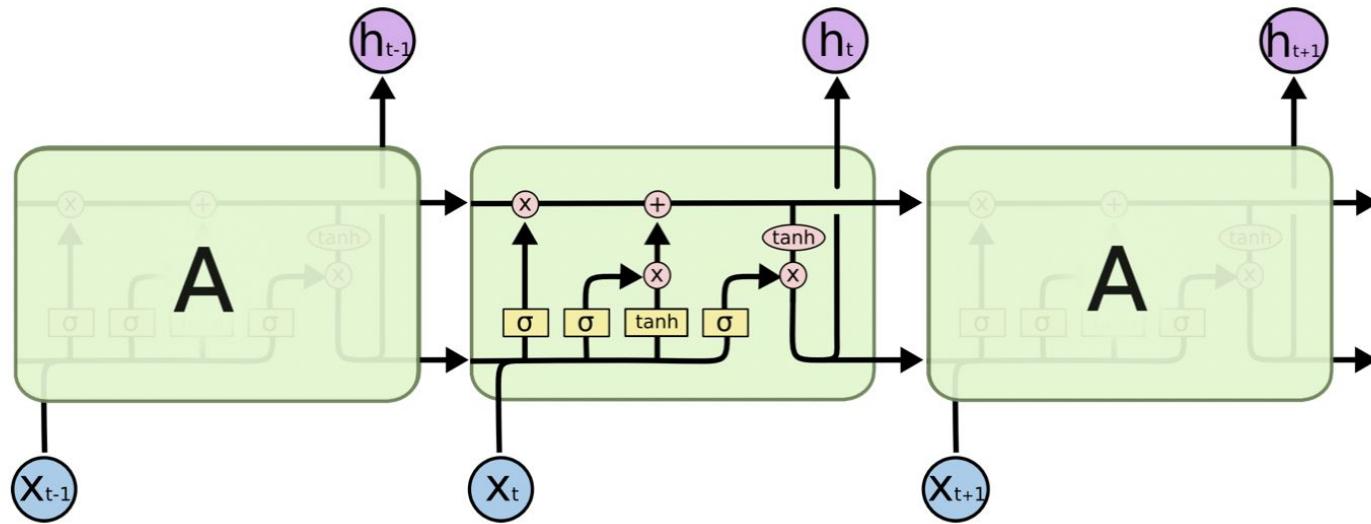
# RNN

The repeating module in a standard RNN contains a single layer



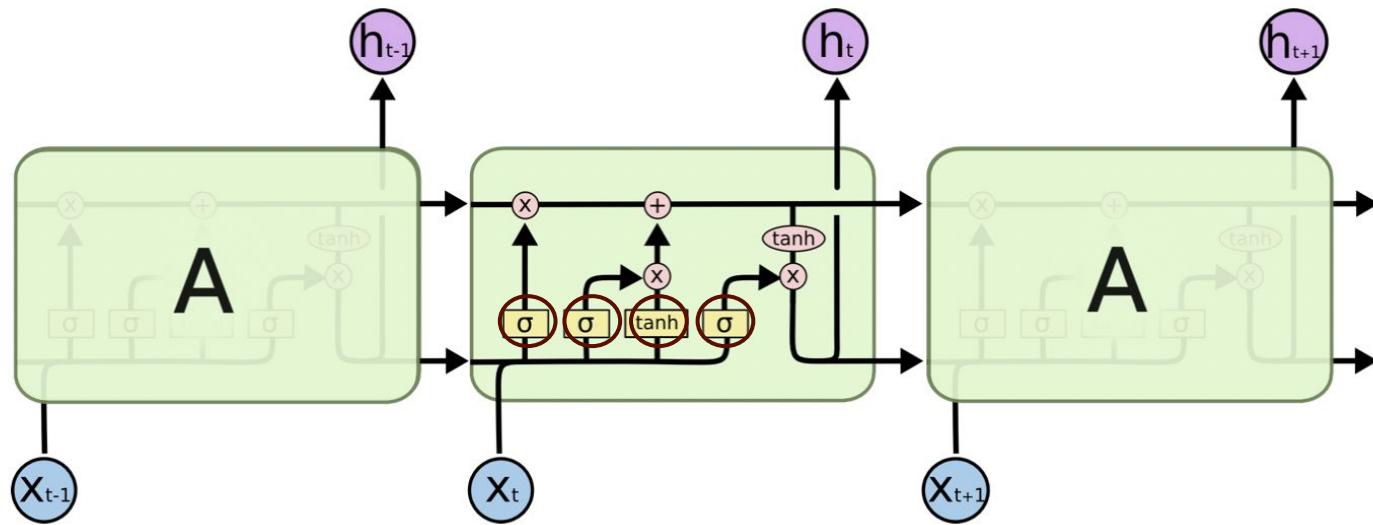
# LSTM

The repeating module in an LSTM contains four interacting layers



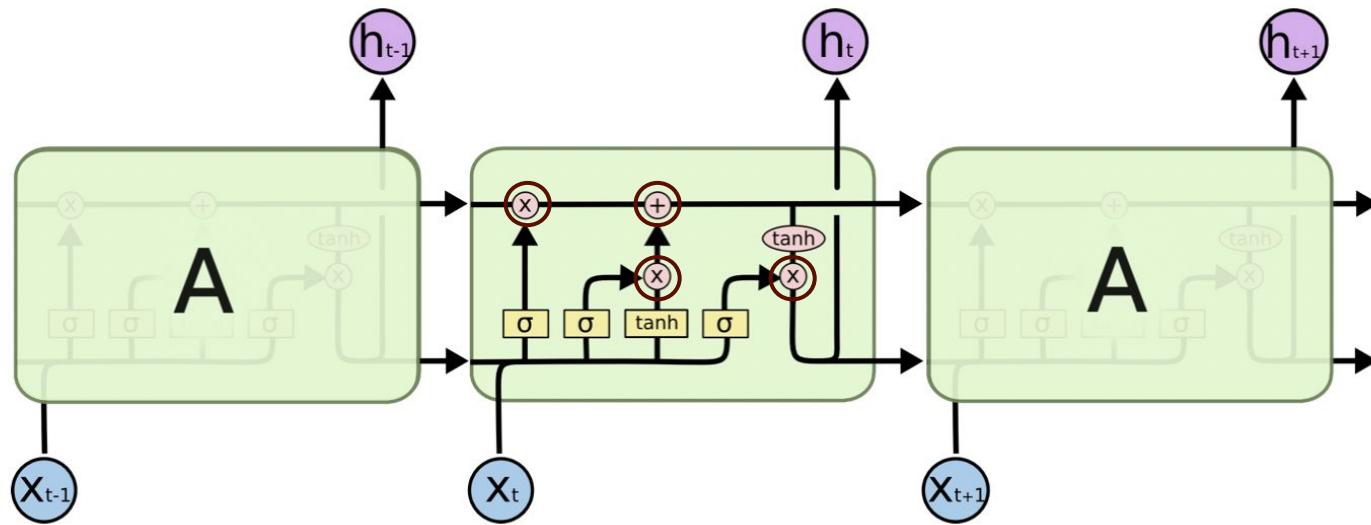
# LSTM

Neural network layers

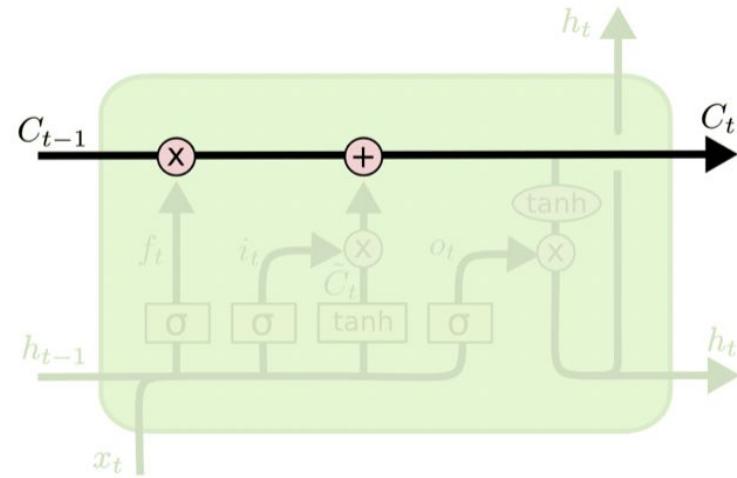


# LSTM

Pointwise operations

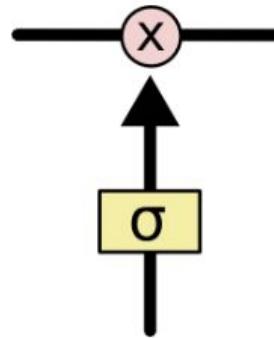


# Cell state



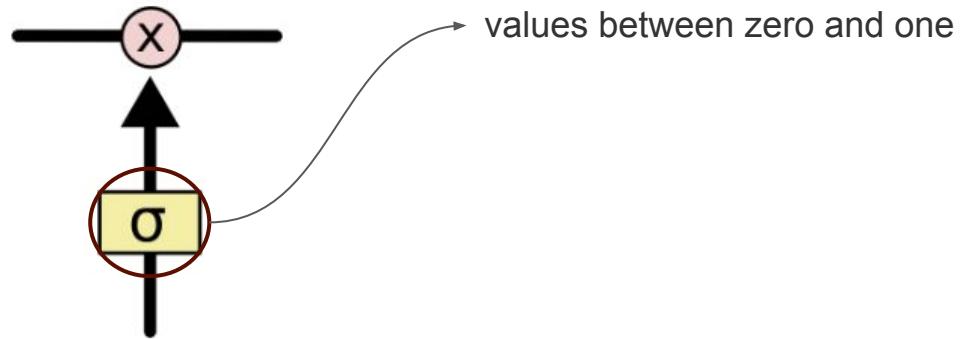
# Gates

- Optionally let information through
- Sigmoid neural net layer + a pointwise multiplication operation



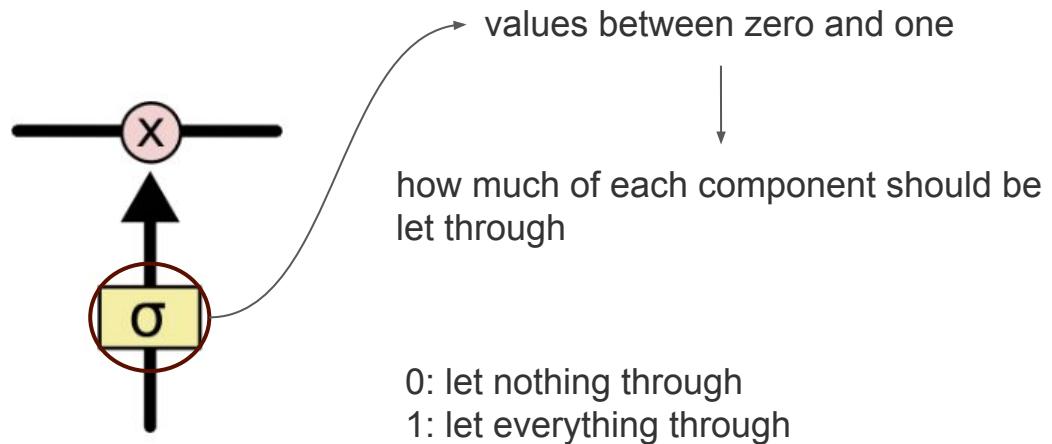
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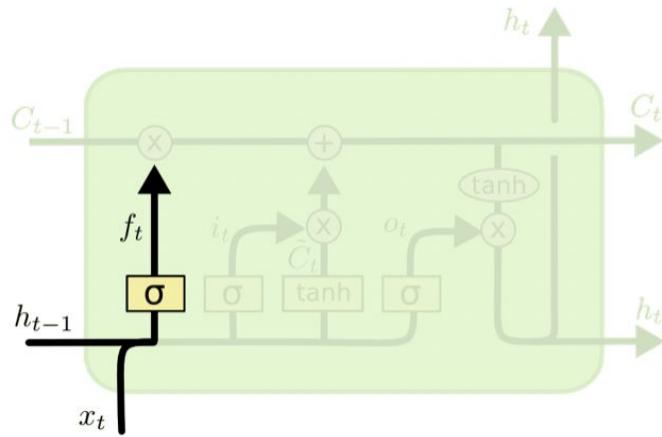
# Gates

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# Forget Gate Layer

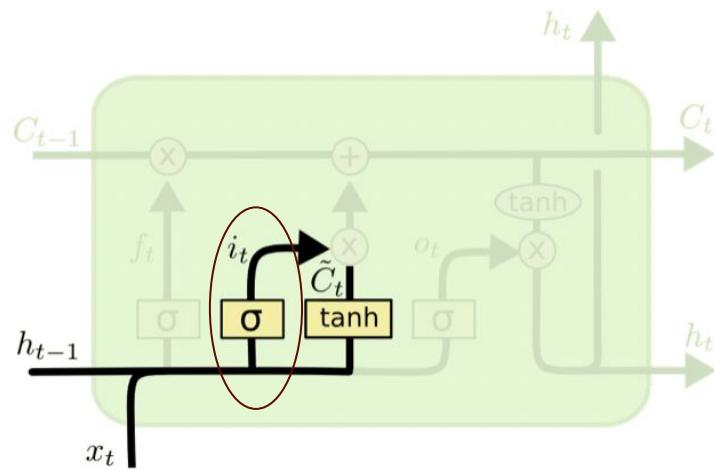
What information to throw away from the cell state



$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$$

# Input Gate Layer

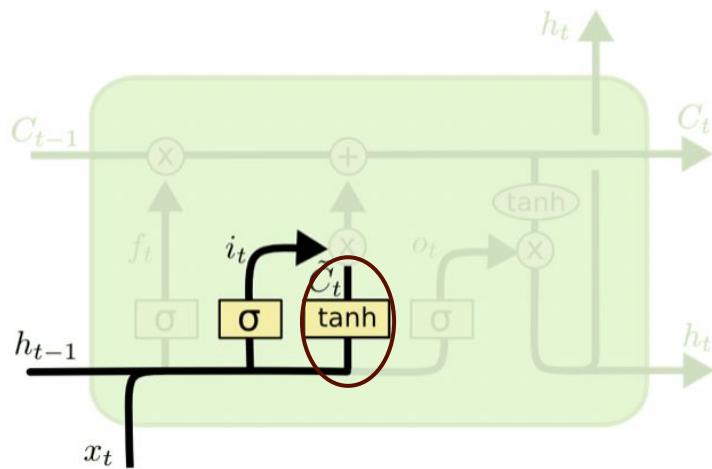
which values to update



$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i)$$
$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$

# Input Gate Layer

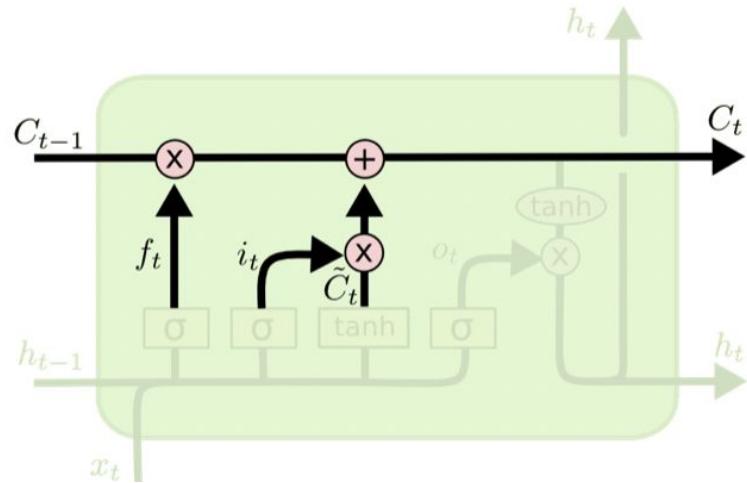
vector of new candidate values that could be added to the state  
(candidate memory update)



$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$

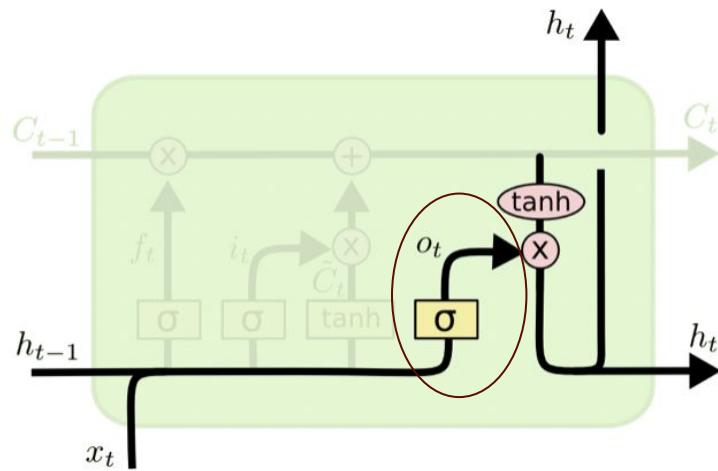
# Updating Old Cell State



$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

# Output

what parts of the cell to output

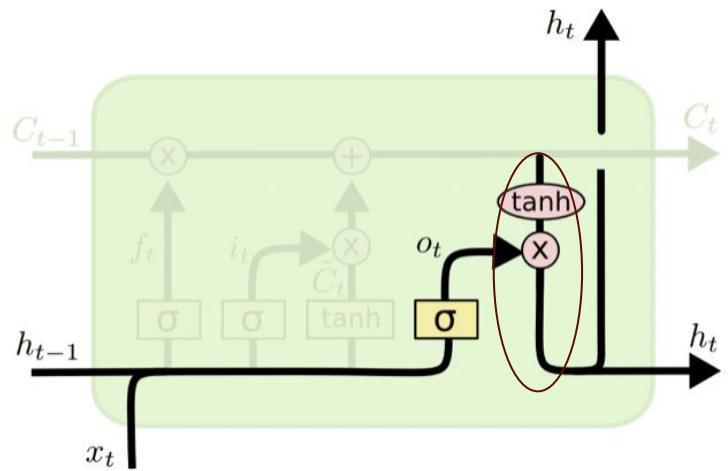


$$o_t = \sigma(W_o [h_{t-1}, x_t] + b_o)$$

$$h_t = o_t * \tanh(C_t)$$

# Output

tanh (cell state): pushing values to [-1, 1]  
multiply cell state by the output gate



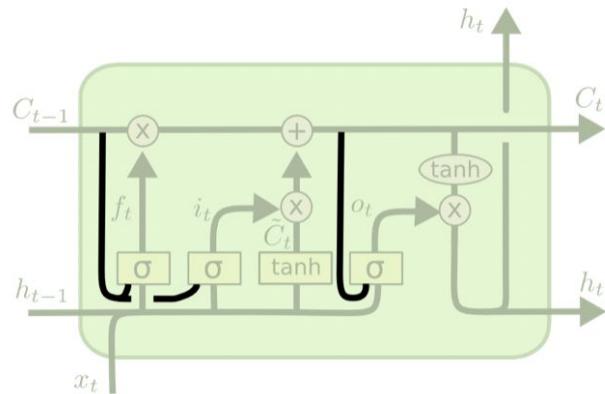
$$o_t = \sigma (W_o [ h_{t-1}, x_t ] + b_o)$$

$$h_t = o_t * \tanh (C_t)$$

# LSTM Variants

Gers & Schmidhuber (2000)

letting the gate layers look at the cell state



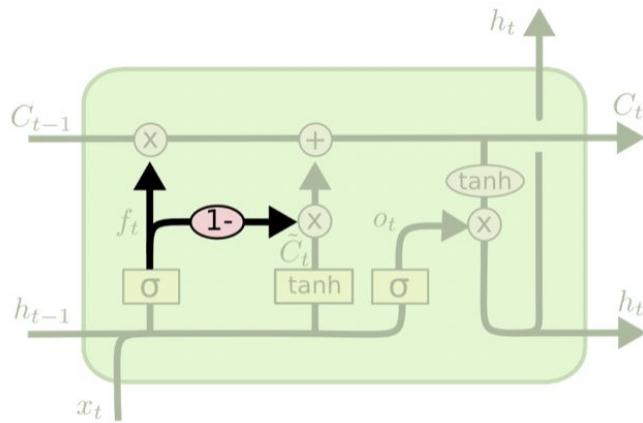
$$f_t = \sigma(W_f \cdot [C_{t-1}, h_{t-1}, x_t] + b_f)$$

$$i_t = \sigma(W_i \cdot [C_{t-1}, h_{t-1}, x_t] + b_i)$$

$$o_t = \sigma(W_o \cdot [C_t, h_{t-1}, x_t] + b_o)$$

# LSTM Variants

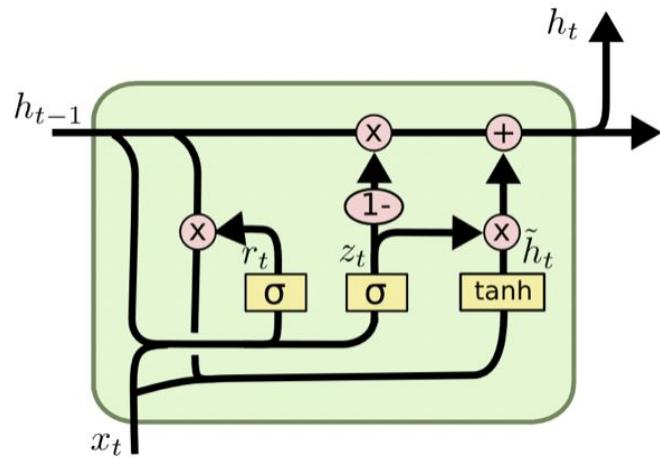
coupled forget and input gates



$$C_t = f_t * C_{t-1} + (1 - f_t) * \tilde{C}_t$$

# LSTM Variants

Gated Recurrent Unit (GRU) [Cho, et al., 2014]



$$z_t = \sigma (W_z \cdot [h_{t-1}, x_t])$$

$$r_t = \sigma (W_r \cdot [h_{t-1}, x_t])$$

$$\tilde{h}_t = \tanh (W \cdot [r_t * h_{t-1}, x_t])$$

$$h_t = (1 - z_t) * h_{t-1} + z_t * \tilde{h}_t$$

# Why Gated RNNs Still Matter?

Even though Transformers dominate:

Gated RNNs introduced three foundational ideas:

- ✓ Additive state updates (stabilize gradients)
- ✓ Gated information flow (learn what to keep/forget)
- ✓ Learned dynamic memory

# From LSTM to Residual Connections

LSTM cell update

$$C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t$$

- Preserve previous state and add controlled update

Transformer Residual Connection

$$x_{l+1} = x_l + \text{Attention}(x_l)$$

- Preserve previous representation
- Add learned modification

# From LSTM to Residual Connections

- Stabilize gradients
- Enable deep networks
- Prevent information collapse

$$C_t = f_t \odot C_{t-1} + i_t \odot \tilde{C}_t$$

$$x_{l+1} = x_l + \text{Attention}(x_l)$$

# From Gates to Gated MLP Blocks

In LSTM, gates decide:

- What to keep
- What to update

Some modern LLMs replace the FFN with a Gated FFN (SwiGLU activation)

- Llama, Qwen, Mistral, Gemma

$$\text{FFN}(x) = W_2 \text{ReLU}(W_1 x)$$

$$(W_1 x) \odot \text{Activation}(W_2 x)$$

# From Memory Control to Mixture-of-Experts

LSTM Gates Route Information

- control memory flow

$$\text{Output} = \sum_k g_k(x) \cdot \text{Expert}_k(x)$$

Mixture-of-Experts (MoE)

- $g_k$  are learned routing weights
- control compute flow

# Recurrence Revisited: State-Space Models

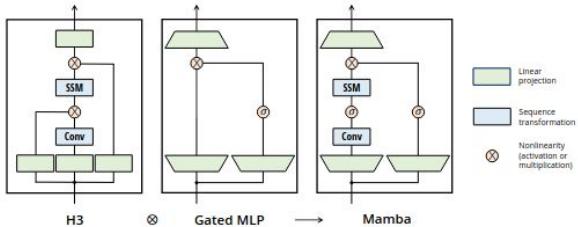


Figure 2: (Architecture.) Our simplified block design combines the H3 block, which is the basis of most SSM architectures, with the ubiquitous MLP block of modern neural networks. Instead of interleaving these two blocks, we simply repeat the Mamba block homogeneously. Compared to the H3 block, Mamba replaces the first multiplicative gate with an activation function. Compared to the MLP block, Mamba adds an SSM to the main branch. For  $\sigma$  we use the SiLU / Swish activation (Hendrycks & Gimpel, 2016; Ramachandran et al., 2017).

## Linear-Time Sequence Modeling with Selective State Spaces

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## EFFICIENTLY MODELING LONG SEQUENCES WITH STRUCTURED STATE SPACES

Albert Gu & Karan Goel & Christopher Ré  
Department of Computer Science, Stanford University  
{albertgu,krng}@stanford.edu, chrismre@cs.stanford.edu

Transformers dominate  
But research is rediscovering structured  
recurrence (for efficiency)

# The Structural Limitation of Recurrence

# The Structural Limitation of Recurrence

All information must pass through a single evolving state

- Information from token 1 must pass through many steps
- Compression bottleneck
- Sequential dependency

# The Structural Limitation of Recurrence

If we increase the hidden dimension of an RNN, do we eliminate the compression bottleneck?

Why or why not?

# The Structural Limitation of Recurrence

All information must pass through a single evolving state

- Information from token 1 must pass through many steps
- Compression bottleneck
- Sequential dependency

Do we really need to compress the past into one vector?

# Attention

Instead of:

State evolution

Let each token:

- Look at all others
- Select relevant information
- Compute a weighted combination

$$h_i = \sum_j \alpha_{ij} x_j$$

# Attention

Consider a sequence of length n

How many transformations must information from the first token undergo before it can influence the last token in:

An RNN?

A self-attention layer?

# Attention

## First Successful Use of Attention

Published as a conference paper at ICLR 2015

### NEURAL MACHINE TRANSLATION BY JOINTLY LEARNING TO ALIGN AND TRANSLATE

Dzmitry Bahdanau  
Jacobs University Bremen, Germany

KyungHyun Cho    Yoshua Bengio\*  
Université de Montréal

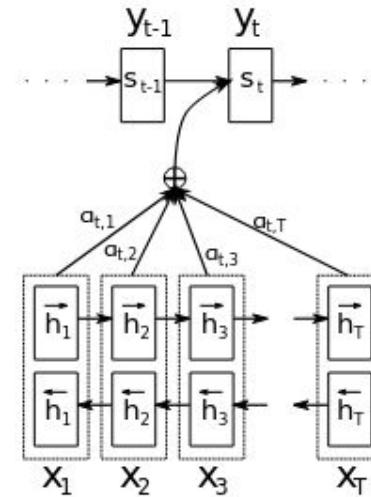
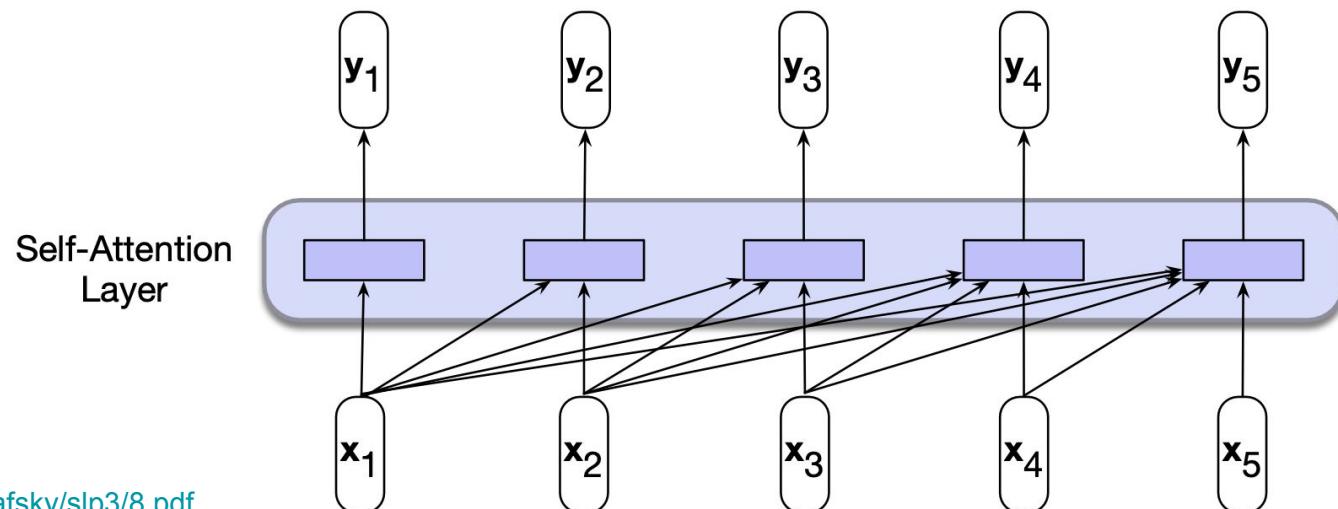


Figure 1: The graphical illustration of the proposed model trying to generate the  $t$ -th target word  $y_t$  given a source sentence  $(x_1, x_2, \dots, x_T)$ .

# Self-Attention?

# Self-attention

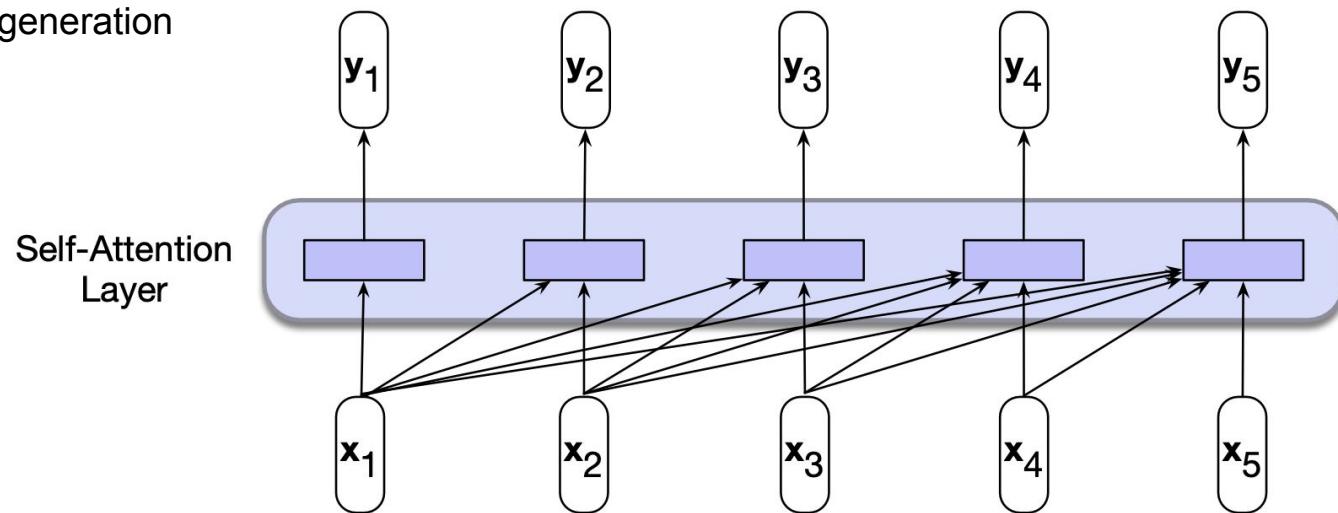
- Mapping input sequences to output sequences of the same length
- Has access to all of the inputs up to and including  $x_i$  for processing  $x_i$
- Computations for each item is independent of all the other computations



# Self-attention

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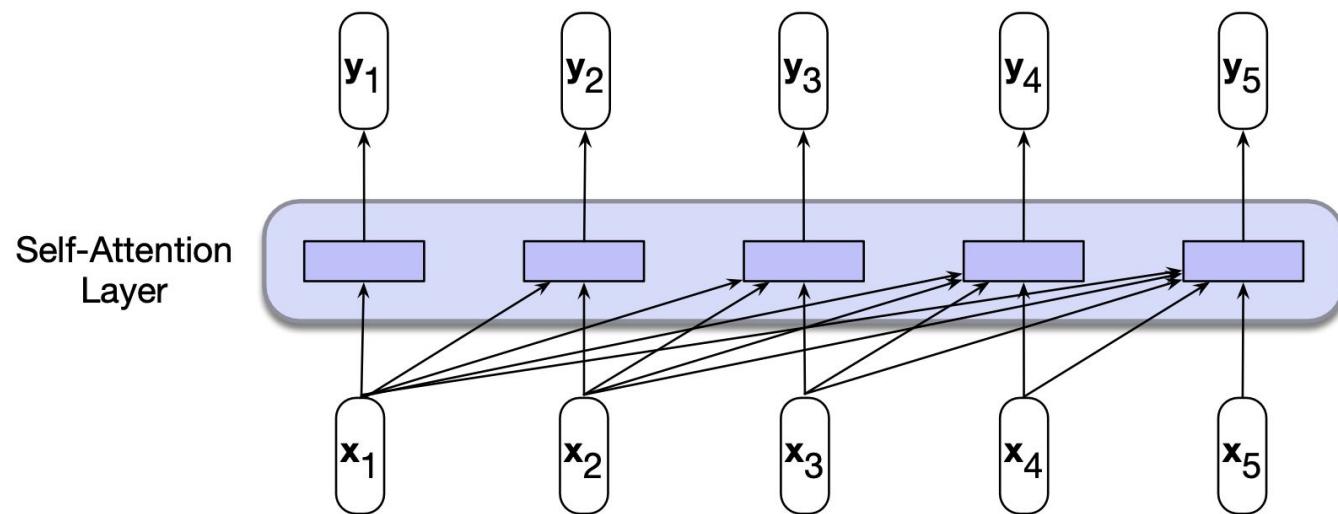
LMs and autoregressive generation



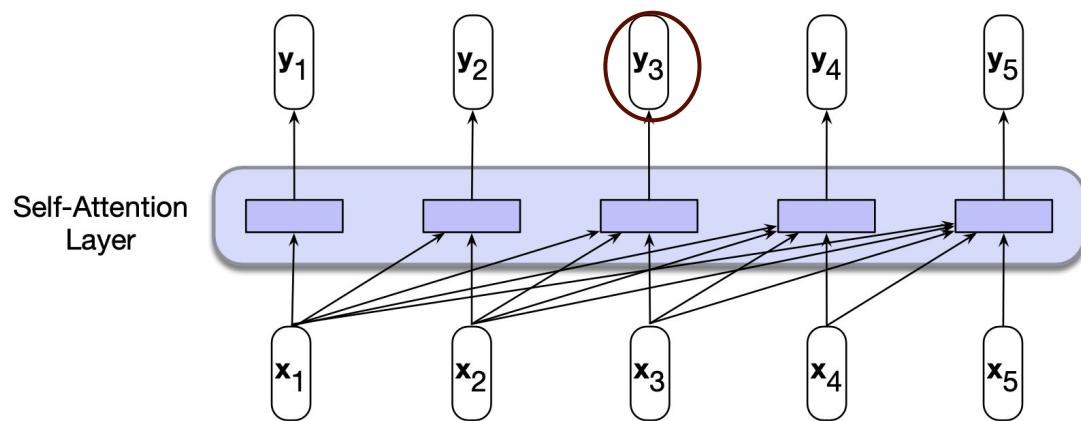
# Self-attention

- Mapping input sequences to output sequences of the same length
- Has access to all of the inputs up to and including  $x_i$  for processing  $x_i$
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Parallel computations

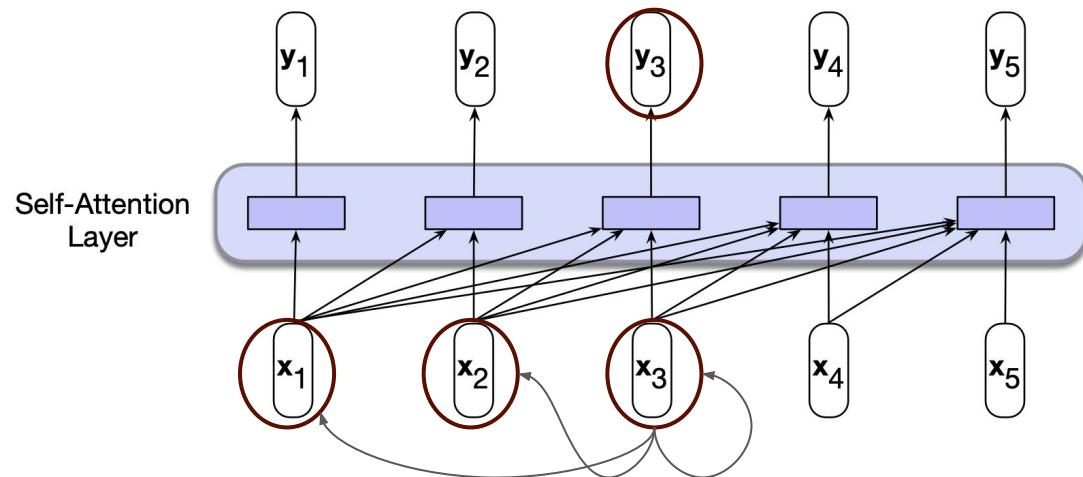


# Self-attention



# Self-attention

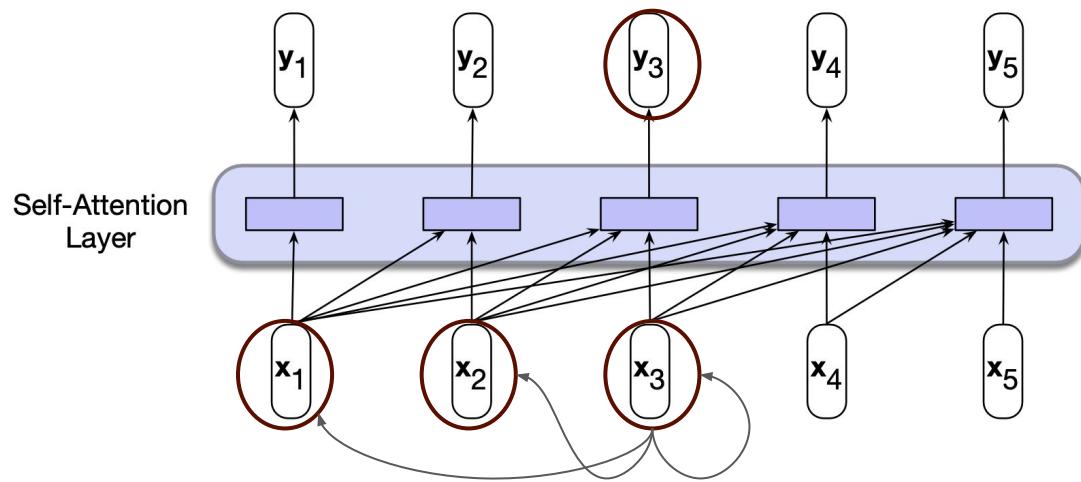
comparing an item of interest to a collection of other items to reveal their relevance in the given context



# Self-attention

comparing an item of interest to a collection of other items to reveal their relevance in the given context

$$\text{score}(\mathbf{x}_i, \mathbf{x}_j) = \mathbf{x}_i \cdot \mathbf{x}_j$$



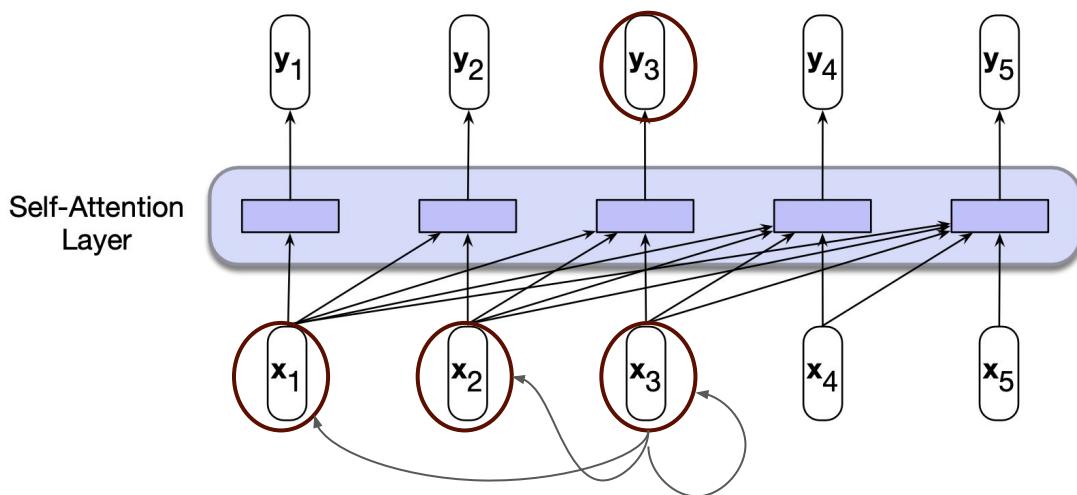
# Self-attention

comparing an item of interest to a collection of other items to reveal their relevance in the given context

$$\text{score}(\mathbf{x}_i, \mathbf{x}_j) = \mathbf{x}_i \cdot \mathbf{x}_j$$

normalization to get probability distribution

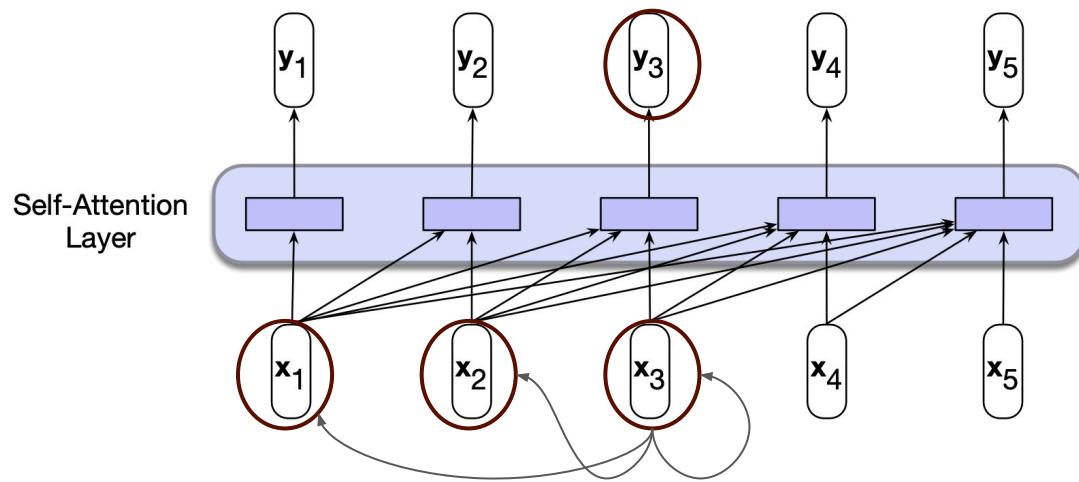
$$\begin{aligned}\alpha_{ij} &= \text{softmax}(\text{score}(\mathbf{x}_i, \mathbf{x}_j)) \quad \forall j \leq i \\ &= \frac{\exp(\text{score}(\mathbf{x}_i, \mathbf{x}_j))}{\sum_{k=1}^i \exp(\text{score}(\mathbf{x}_i, \mathbf{x}_k))} \quad \forall j \leq i\end{aligned}$$



# Self-attention

weighted sum using this distribution

$$\mathbf{y}_i = \sum_{j \leq i} \alpha_{ij} \mathbf{x}_j$$



# Question

Are RNNs and Self-Attention Aware of Position?

Consider a sentence:

“The cat chased the mouse.”

Would the model know if we permute the words?

“Mouse the chased cat the.”

# Wrap-Up

<b>Model</b>	<b>Core Idea</b>	<b>Limitation</b>
N-gram	Fixed window counts	
FFNN LM	Distributed representations	
RNN	Dynamic state	
LSTM/GRU	Gated memory	
Attention	Dynamic context access	

# Wrap-Up

<b>Model</b>	<b>Core Idea</b>	<b>Limitation</b>
N-gram	Fixed window counts	No generalization
FFNN LM	Distributed representations	Fixed context size
RNN	Dynamic state	Hard to train, compression
LSTM/GRU	Gated memory	Still sequential, compression
Attention	Dynamic context access	Quadratic cost

# Questions?