TD 1 - Linear Algebra and Gaussian vector

Exercice 1

Let X and Y be two random variables with normal distribution $\mathcal{N}(0, 1)$,

1. What is the value of $\mathrm{Cov}(X, Y)$? What if they are independent?

2. Plot the density of X and a histogram corresponding to a sample $X_1, \cdots, X_n$ for different values of n on the same graph. Comment.

3. Using R, construct two samples of size n from the distribution $\mathcal{N}(0, 1)$, and calculate the empirical covariance for different values of n. What do you observe? We assume that X and Y are independent.

   (a) Plot the pairs of points (X, Y) on the same graph.

   (b) What is the distribution of $\begin{pmatrix} X \\ Y \end{pmatrix}$?

   (c) Let $Z = 2X + Y$. What is the value of $\mathrm{Cov}(X, Z)$?

   (d) What is the distribution of $\begin{pmatrix} X \\ Z \end{pmatrix}$?

Exercice 2

Consider

$$M = \begin{pmatrix} 1 & 3 & 5 \\ 1 & 3 & 5 \end{pmatrix}.$$

1. Is it possible to calculate

$$M * \begin{pmatrix} 1 \\ 3 \\ 7 \end{pmatrix}?$$

2. Is it possible to calculate

$$M * \begin{pmatrix} 1 \\ 3 \end{pmatrix}?$$

3. Is the matrix M invertible?

4. Consider the matrix

$$A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

Let b the vector

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Find the solution of

$$Ax = b.$$