Stars: Structure

The Sun: A typical star (ESA/NASA SOHO)

Stellar Structure

The structure of stars is defined by a set of four coupled differential equations which express the basic conservation and transport quantities always encountered in physics:

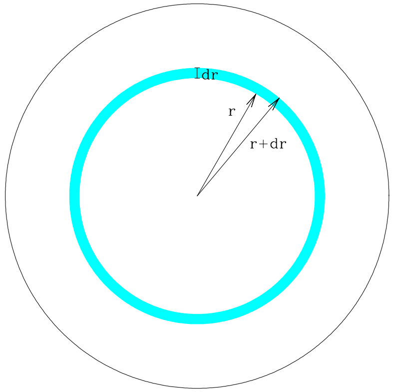
1. Mass conservation
2. Momentum conservation (=hydrostatic equilibrium)
3. Energy conservation
4. Energy transport

and quantities expressing the physical properties of material, mainly:

1. Energy generation
2. Equation of state (=dependence of density of material on physical conditions)

Mass Conservation

Density stratification of a star is defined through mass conservation:



Define M_r as the mass contained within radius r:

$$M_r = \int_0^r 4\pi r^2 \rho(r)\,\mathrm{d}r \tag{9.1}$$

Thus the mass within a spherical shell is

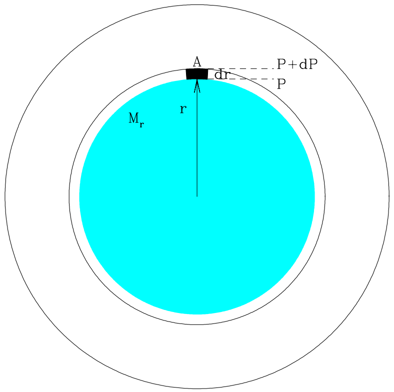
$$\mathrm{d}M_r = 4\pi r^2 \rho\,\mathrm{d}r \tag{9.2}$$

and therefore

$$\frac{\mathrm{d}M_r}{\mathrm{d}r} = 4\pi r^2 \rho \tag{9.3}$$

Hydrostatic Equilibrium

Pressure stratification of a star is defined through hydrostatic equilibrium:



Force on area A by slab of gas of area $\mathrm{d}A$ and density ρ:

$$\mathrm{d}F_g = -\frac{GM_r \mathrm{d}m}{r^2} = -\frac{GM_r\rho}{r^2}\,\mathrm{d}A\,\mathrm{d}r \quad (9.4)$$

Bouyancy:

$$F_P = \mathrm{d}A(P(r+\Delta h) - P(r)) = \mathrm{d}A\Delta P \quad (9.5)$$

Balance of forces:

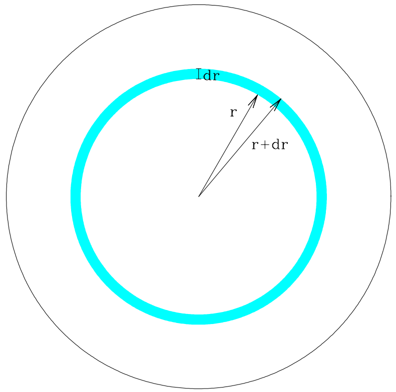
$$-\frac{GM_r\rho}{r^2}\,dA\,dr = dP\,dA \quad (9.6)$$

such that

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\rho(r)\frac{GM(r)}{r^2} \quad (9.7)$$

Energy Conservation

Temperature stratification of a star is defined through energy conservation:



Let ϵ = energy production coefficient, i.e., the energy released per time and unit mass.

Luminosity produced within a spherical shell is

$$\mathrm{d}L_r = \epsilon \mathrm{d}M_r = 4\pi r^2 \rho\epsilon\,\mathrm{d}r \quad (9.8)$$

and therefore

$$\frac{\mathrm{d}L_r}{\mathrm{d}r} = 4\pi r^2\rho\epsilon \quad (9.9)$$

Energy Transport

Energy is transported in stars by

- radiation
- convection
- conduction

In most stars, radiation and convection are important, conduction usually not.

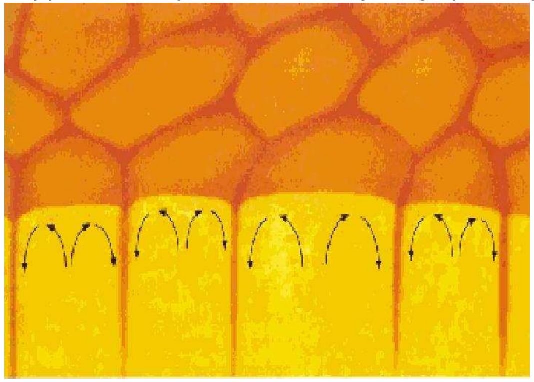
Radiative Transport = diffusive process:

- radiation produced by nuclear fusion (γ-rays)
- mean free path l_{phot} of a photon in center of the Sun: few cm
- photons do random walk to the stellar surface absorbed by ions and reemitted
- number of "scatterings": $N = (R/l_{\text{phot}})^2$: e.g., $R_\odot = 700000\,\text{km}$: $N \approx 10^{20}$
- Diffusion theory:

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3}{4ac}\frac{\kappa(r)\rho(r)}{T^3}\frac{L(r)}{4\pi r^2} \quad (9.10)$$

Energy Transport

Convection is much more difficult to deal with, no simple self-consistent hydrodynamical treatment, approximate (so called mixing-length) theory



$$\frac{\mathrm{d}T}{\mathrm{d}r} = \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{\mathrm{d}P}{\mathrm{d}r} \quad (9.11)$$

Structure equations

Stellar structure governed by four coupled differential equations:

Mass structure
(mass conservation)

$$\frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2 \rho(r)$$

Pressure structure
(hydrostatic equilibrium)

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\rho(r)\frac{GM(r)}{r^2}$$

Temperature structure
(e.g. radiative transfer)

$$\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{3}{4ac}\frac{\kappa\rho(r)}{T^3}\frac{L(r)}{4\pi r^2}$$

Energy conservation

$$\frac{\mathrm{d}L}{\mathrm{d}r} = 4\pi r^2 \rho(r)\epsilon(r)$$

plus "equation of state" ($P = P(T,\rho)$), Opacities $\kappa(T,\rho,Z)$ = interaction of radiation with gas, energy generation ($\epsilon = \epsilon(T,\rho,Z)$),...

Stellar model: numerical solution of stellar structure equations.

Energy generation: Overview

Main sequence: Nuclear fusion of Hydrogen into Helium:

$$4\mathrm{p} \longrightarrow {}^4_2\mathrm{He} + E$$

How much energy is gained?

Particle physics: express mass as "rest energy equivalent" via $E = mc^2$

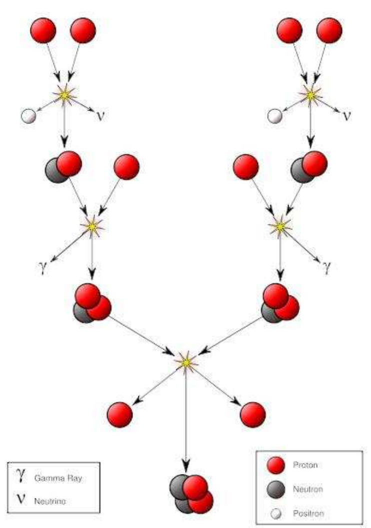
(and call it "mass"...); usually use energy units of MeV, 1 MeV $= 1.602 \times 10^{-13}$ J

	mass of 4 protons (4×938 MeV):	3752 MeV
$-$	mass of ${}^4_2$He:	3727 MeV
	mass defect Δmc^2:	25 MeV

In the fusion of hydrogen to helium, 0.7% of the available rest mass energy is converted to energy.

Two main burning cycles: proton-proton chain and the CNO cycle.

Energy generation: Proton-Proton chain



For moderate central temperatures, He is produced using the proton-proton chain.

First, two protons create a deuteron:

$$^1\mathrm{H} + {}^1\mathrm{H} \longrightarrow {}^2\mathrm{H} + \mathrm{e}^+ + \nu_\mathrm{e} \tag{9.12}$$

This process is slow (happens once for a nucleon per 10^{10} years)

Then an additional proton is attached:

$$^2\mathrm{H} + {}^1\mathrm{H} \longrightarrow {}^3\mathrm{He} + \gamma \tag{9.13}$$

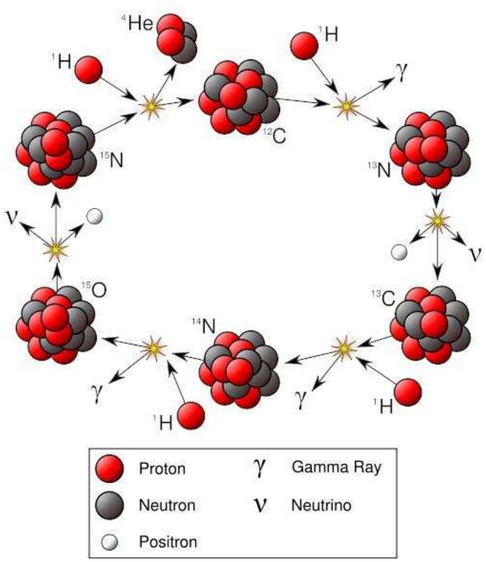
and two helium nuclei can form an α-particle:

$$^3\mathrm{He} + {}^3\mathrm{He} \longrightarrow {}^4\mathrm{He} + 2\,{}^1\mathrm{H} \tag{9.14}$$

This is the so called pp I-cycle, minor variations of the theme exist (pp II, pp III cycles), but pp I dominates.

pp chain dominates for $T \lesssim 2 \times 10^7$ K, $\epsilon_{\mathrm{pp}} \propto T^5$;
Sun: 98.4%.

Energy generation: CNO cycle



The CNO cycle (Bethe-Weizsäcker-cycle) requires the presence of C, N, and O isotopes as catalysts.

CNO cycle has slightly smaller energy release than pp-cycle because of higher neutrino losses.

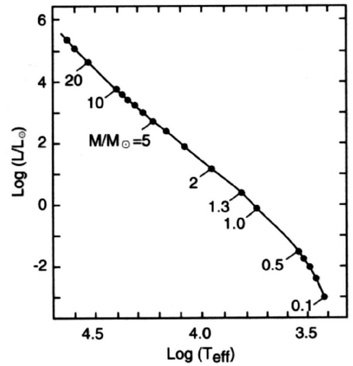
Reaction $^{14}\mathrm{N} + \mathrm{p} \longrightarrow {}^{15}\mathrm{O} + \gamma$ is th slowest reaction (one million years).

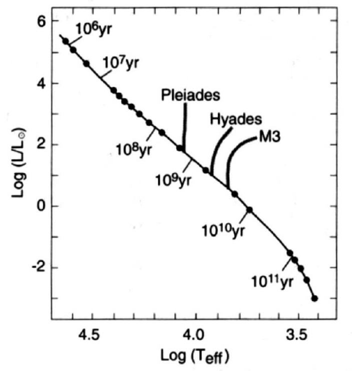
CNO cycle dominates above 2×10^7 K, $\epsilon_{\mathrm{CNO}} \propto T^{17}$;
Sun: 1.6%.

Wikipedia

Zero Age Main Sequence

Main Sequence in the Hertzsprung-Russell-Diagram:





Masses along the HRD

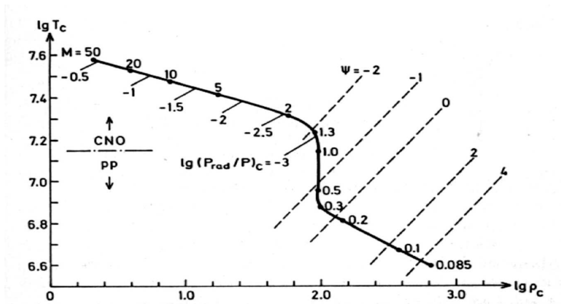
life times along the HRD

mass limits: Min.: 0.08$M_\odot$ (no H-burning for lower masses)

Max.: $\approx$ 100 $M_\odot$ (radiation pressure too high, $p_{\rm rad} \sim T_{\rm eff}^4$)

Zero Age Main Sequence

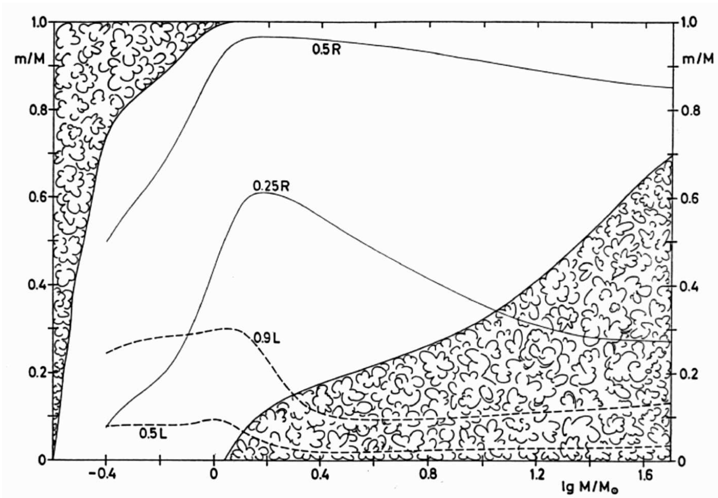
Properties of Main Sequence stars:



Central temperatures & densities

- strong change at $\approx 1M_\odot$
- low masses ($< 0.5M_\odot$): low temperature, very high density
- high masses ($> 1.3M_\odot$): high temperature, low density
- low mass stars burn H in the pp-chains
- high mass stars burn H in the CNO-cycle

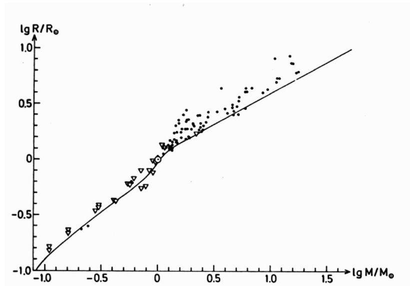
Zero Age Main Sequence



Kippenhahn-diagram: Internal Structure of Main Sequence stars

Zero Age Main Sequence

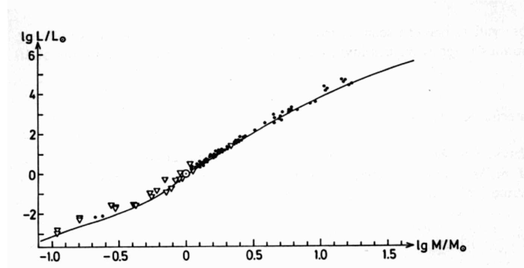
Empirical test of main sequence models:



Mass–Radius relation

$R \sim M^a$

lower main sequence: $a \approx$ 0.8
upper main sequence: $a \approx$ 0.6



Mass–Luminosity relation

$L \sim M^b$

$b =$ 2.3 for $M <$ 0.43 $M_\odot$
$b =$ 4.0 for $M >$ 0.43 $M_\odot$