World Models

Introduction

Cosmology: science of the universe as a whole

How did the universe evolve to what it is today?

Based on four basic facts:

The universe
- expands,
- is isotropic,
- and is homogeneous.

Isotropy and homogeneity of the universe: "*cosmological principle*".

Perhaps (for us) the most important fact is:

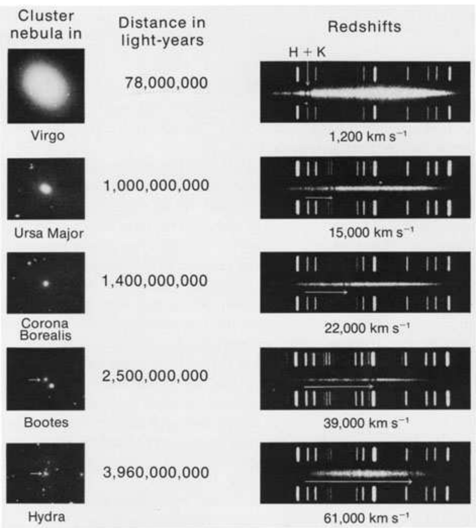
- The universe is habitable for humans.

("*anthropic principle*")

The one question cosmology does not attempt to answer is: How came the universe into being?

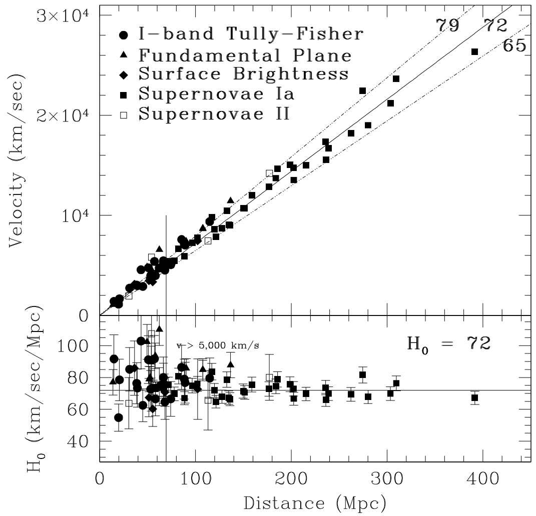
$\Longrightarrow$ Realm of theology!

Redshifts



Hubble: spectral lines in galaxies are more and more redshifted with increasing distance.

Hubble Relation



(Freedman, 2001, Fig.4)

Hubble relation (1929):

The redshift of a galaxy is proportional to its distance: $v = cz = H_0 d$

where H_0: "Hubble constant".

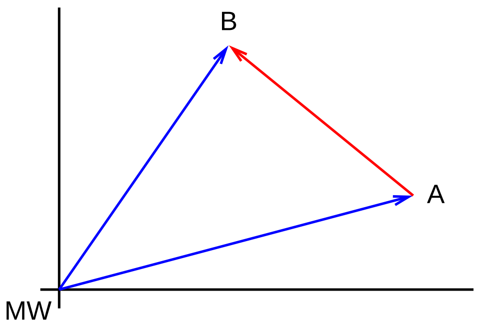
Measurement: determine v from redshift (easy), d with standard candles (difficult)

$\Longrightarrow$ H_0 from linear regression.

Hubble Space Telescope finds

$$H_0 = 72 \pm 8\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$$

Hubble Relation



The expansion law $\boldsymbol{v} = H_0 \boldsymbol{r}$ is unchanged under rotation and translation: isomorphism.

Proof: consider two galaxies at positions $\boldsymbol{r}_A$ and $\boldsymbol{r}_B$:

Hubble: $\boldsymbol{v}_A = H_0\, \boldsymbol{r}_A$ and $\boldsymbol{v}_B = H_0\, \boldsymbol{r}_B$

Galaxy B as seen from galaxy A:

$\boldsymbol{v}_B - \boldsymbol{v}_A = H_0\, \boldsymbol{r}_B - H_0\, \boldsymbol{r}_A = H_0\, (\boldsymbol{r}_B - \boldsymbol{r}_A)$

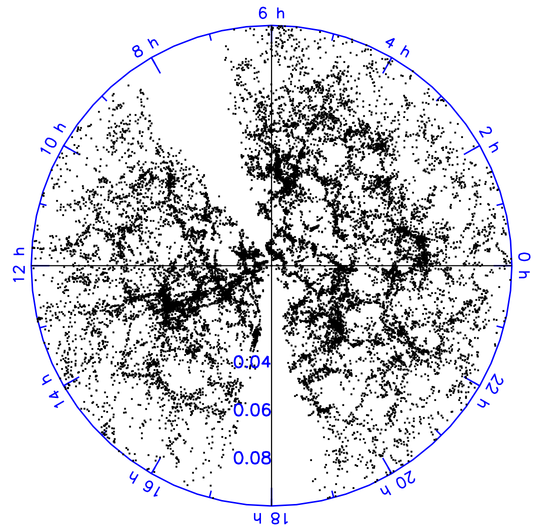
$\Longrightarrow$ observer at A derives the same Hubble law as we in Milky Way.

This is a direct consequence of the homogeneity of the universe.

Despite everything receding from us, we are not at the center of the universe $\Longrightarrow$ Copernican principle still holds.

Copernican principle: We are not at a special place in the universe in time or space.

Homogeneity

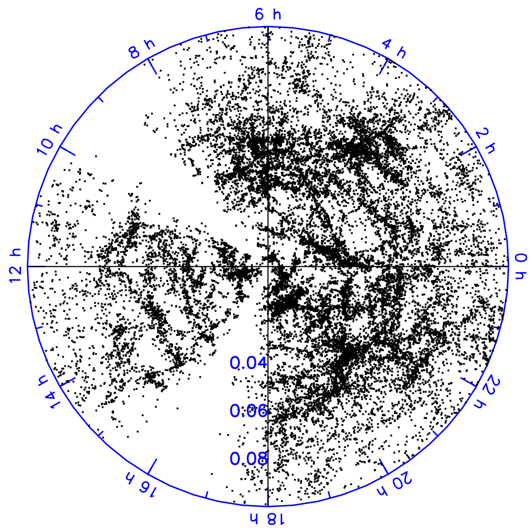


$-30^\circ < \delta < -20^\circ$

Homogeneity: "The universe looks the same, regardless from where it is observed" (on scales $\gg$100 Mpc).

6dF QSO Redshift survey

Homogeneity

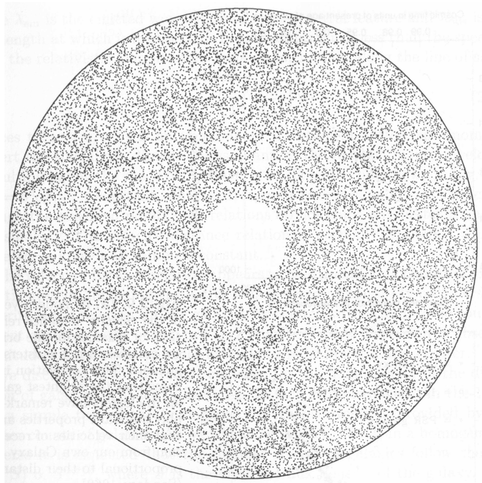


$-60^\circ < \delta < -40^\circ$

Homogeneity: "The universe looks the same, regardless from where it is observed" (on scales $\gg$100 Mpc).

6dF QSO Redshift survey

Isotropy



Peebles (1993): Distribution of 31000 radio sources on northern sky (wavelength $\lambda = 6$ cm)

Isotropy $\Longleftrightarrow$ The universe looks the same in all directions.

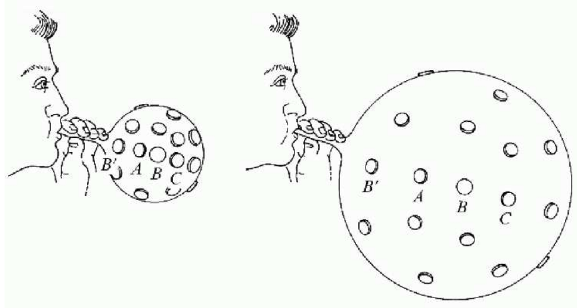
N.B. Homogeneity *does not* imply isotropy, and isotropy around one point does not imply homogeneity!

World Models

A.A. Friedmann (1888–1925)

Friedmann: Mathematical description of the Universe using normal “fixed” coordinates (“comoving coordinates”), plus scale factor R which describes evolution of the Universe.

World Models

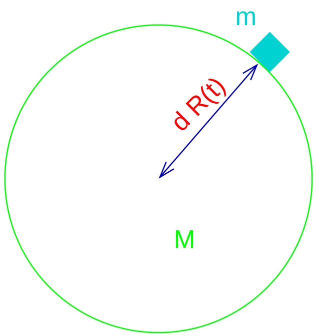


R small R large

Misner, Thorne, Wheeler

Friedmann: Mathematical description of the Universe using normal “fixed” coordinates (“comoving coordinates”), plus scale factor R which describes evolution of the Universe.

Friedmann Equations



Evolution of universe described by Friedmann equations: Dynamics of a mass element on the surface of sphere of density $\rho(t)$ and comoving radius d, i.e., proper radius $d \cdot R(t)$ (McCrea, 1937)

Mass of sphere:

$$M = \frac{4\pi}{3}(d\,R)^3\rho(t) = \frac{4\pi}{3}d^3\rho_0 \quad \text{where} \quad \rho(t) = \frac{\rho_0}{R(t)^3} \tag{19.1}$$

Force on mass element:

$$m\frac{\mathrm{d}^2}{\mathrm{d}t^2}(d\,R(t)) = -\frac{GMm}{(dR(t))^2} = -\frac{4\pi G}{3}\frac{d\rho_0}{R^2(t)}m \tag{19.2}$$

Canceling $m \cdot d$ gives momentum equation:

$$\ddot{R}(t) = -\frac{4\pi G}{3}\frac{\rho_0}{R(t)^2} = -\frac{4\pi G}{3}\rho(t)R(t) \tag{19.3}$$

Multiplying Eq. (19.3) with $\dot{R}$ and integrating yields the energy equation:

$$\frac{1}{2}\dot{R}(t)^2 = +\frac{4\pi G}{3}\frac{\rho_0}{R(t)} + \text{const.} = +\frac{4\pi G}{3}\rho(t)R^2(t) + \text{const.} \tag{19.4}$$

where the constant can only be obtained from GR.

Friedmann Equations

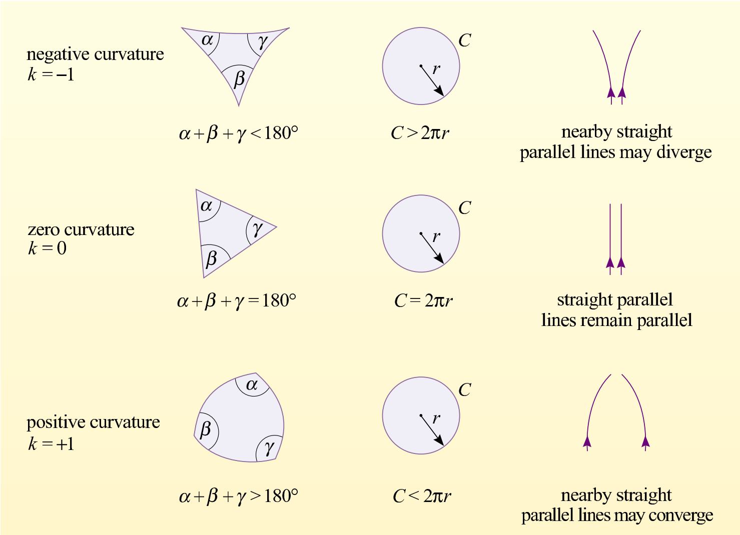
The exact GR derivation of Friedmanns equation gives:

$$\begin{aligned}\ddot{R} &= -\frac{4\pi G}{3}R\left(\rho + \frac{3p}{c^2}\right) + \left[\frac{1}{3}\Lambda R\right] \\ \dot{R}^2 &= +\frac{8\pi G\rho}{3}R^2 - kc^2 + \left[\frac{1}{3}\Lambda c^2 R^2\right]\end{aligned} \tag{19.5}$$

Notes:

1. For $k = 0$: Eq. (19.5) $\longrightarrow$ Eq. (19.4).
2. k determines the curvature of space:
   - $k > 0$: closed universe (finite volume)
   - $k = 0$: flat universe
   - $k < 0$: open universe (infinite volume)
3. The density, ρ, includes the contribution of all different kinds of energy (remember mass-energy equivalence!).
4. cosmological constant Λ introduced by Einstein to ensure stability of the universe. Physics unknown.

Friedmann Equations



Jones & Lambourne: An Introduction to Galaxies and Cosmology

Hubble's Law

The variation of $R(t)$ implies Hubble's Law:

Small scales $\Longrightarrow$ Euclidean geometry

Proper distance between two observers with comoving distance d:

$$D(t) = d \cdot R(t) \tag{19.6}$$

Expansion $\Longrightarrow$ D changes:

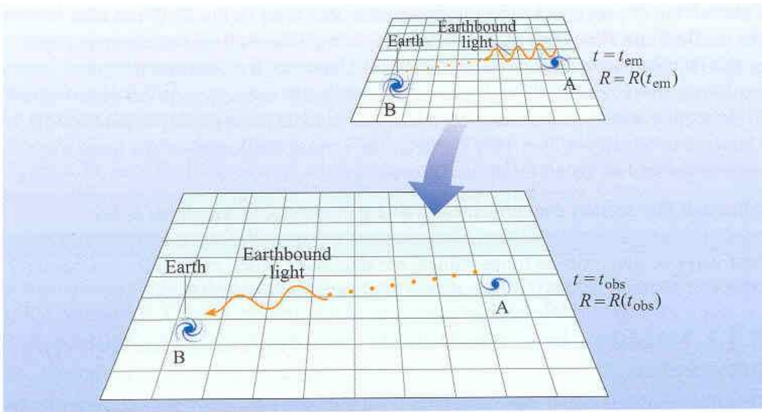
$$\frac{\Delta D}{\Delta t} = \frac{R(t+\Delta t)d - R(t)d}{\Delta t} \quad \text{and for } \lim_{\Delta t\to 0} \quad v = \frac{\mathrm{d}D}{\mathrm{d}t} = \dot{R}\, d = \frac{\dot{R}}{R} D =: H\, D \tag{19.7}$$

$\Longrightarrow$ Identify local Hubble "constant" as

$$H = H(t) = \frac{\dot{R}(t)}{R(t)} \tag{19.8}$$

$\Longrightarrow$ Hubble "constant" is time-dependent! $\Longrightarrow$ "Hubble parameter"

Hubble's Law



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Expansion of space leads to the redshift:

$$z = \frac{R(t_0)}{R(t_{\text{em}})} - 1 \quad \Longrightarrow \quad \frac{R(t_{\text{em}})}{R(t_0)} = \frac{1}{z+1} \tag{19.9}$$

Critical Density

Looking at the energy equation for $\Lambda = 0$,

$$\dot{R}^2 = +\frac{8\pi G\rho}{3} R^2 - kc^2 \tag{19.10}$$

we find that the evolution of the Hubble parameter is:

$$\left(\frac{\dot{R}}{R}\right)^2 = H(t)^2 = \frac{8\pi G\rho(t)}{3} - \frac{kc^2}{R^2} \tag{19.11}$$

and therefore

$$k \cdot \frac{c^2}{R(t)^2 H(t)^2} = \frac{8\pi G}{3H(t)^2}\rho(t) - 1 = \frac{\rho(t)}{\rho_{\text{crit}}} - 1 = \Omega - 1 \tag{19.12}$$

where Ω is called the critical density:

$$\Omega = \frac{\rho}{\rho_{\text{crit}}} \quad \text{where} \quad \rho_{\text{crit}} = \frac{3H^2}{8\pi G} \tag{19.13}$$

currently: $\rho_{\text{crit}} \sim 1.67 \times 10^{-24}\,\text{g}\,\text{cm}^{-3}$ (3...10 H-Atoms m^{-3}).

Ω describes the curvature of the universe:

$\Omega > 1 \Longrightarrow k > 0$: closed | $\Omega = 1 \Longrightarrow k = 0$: flat | $\Omega < 1 \Longrightarrow k < 0$: open

Critical Density

World Model: Evolution of R as a function of time

Solution of Friedmann equations depends on boundary conditions:

1. Value of H as measured today (H is time dependent!)
2. Density Parameter of universe

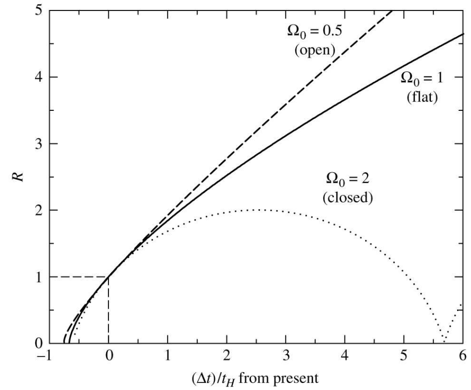
Note: total Ω is sum of:

1. Ω_m: Matter, i.e., everything that leads to gravitative effects
   Ω_m in baryonic matter is $\lesssim$3%, but note there might be "nonbaryonic dark matter" as well!
2. $\Omega_\Lambda = \Lambda c^2/3H^2$: contribution by cosmological constant Λ
   (Λ is often called "dark energy" for PR reasons)

Hubble time: Assume an empty universe ($\Omega = 0$): linear expansion

$\Longrightarrow$ age of the Universe: $t_H = v/d = 1/H_0$ is called Hubble time

Critical Density

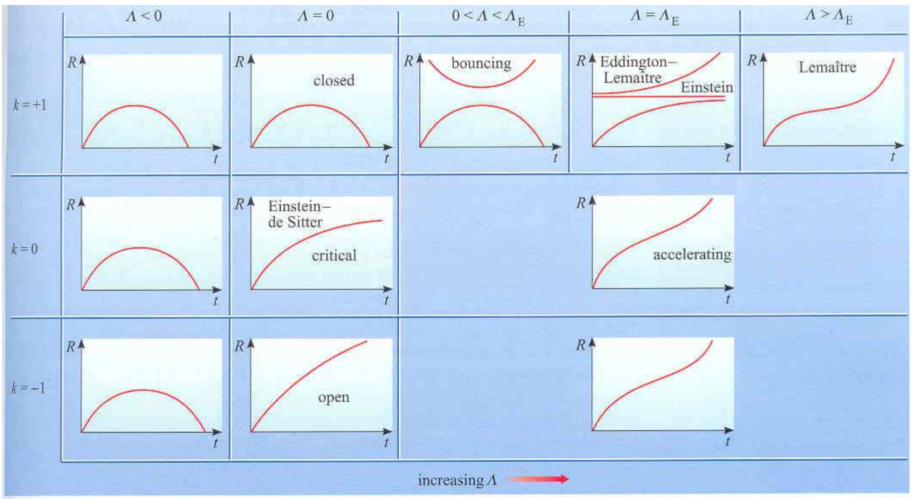


Carroll & Ostlie, Fig. 29.5

$\Omega > 1 \Longrightarrow$ bounces back | $\Omega = 1 \Longrightarrow$ expands forever, stops at $t = \infty$

| $\Omega < 1 \Longrightarrow$ expands forever

Critical Density



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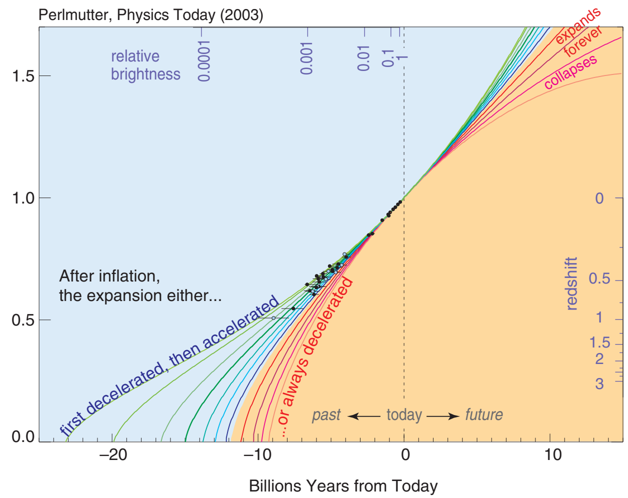
Many different kinds of world models are possible, depending on Ω und Λ.

Age of the Universe



Note: Extrapolation backwards gives age of universe as *roughly* $1/H_0$!

for $H_0 = 72\,\mathrm{km\,s^{-1}\,Mpc^{-1}} = 2.3 \times 10^{-18}\,\mathrm{s^{-1}}$, giving an age of 13.6 Gyr.