Universidad Industrial de Santander, Colombia
Numerical Analysis, 2020-2
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Lab 7. Curve Fitting

1 Instructions

- Make a **pdf** report including the solution to each point of the practice with name *Lab7_name_lastname.pdf*.
- Send the report and all created files in a rar or zip file with name *Lab7_name_lastname.rar* in the Moodle.
- You are allowed to use internet, notes, and .m files that you have created before.

2 Purposes

- To understand the least-squares method.
- To implement the least-squares method in Matlab.
- To interpret problems which can be solved by the least-squares method.

3 Practice

3.1 Understanding

Answer with your own words the following questions:

- (0.2 points) What is curve fitting?

- (0.2 points) How to calculate the least-squares line?

- (0.2 points) What is data linearization?

- (0.2 points) What applications do the least-squares have?

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3.2 Implementing

- (1.0 point) Create a Matlab function called *my_lsline_name_lastname()* to construct the least-squares line $y = Ax + B$ that fits the N data points (x_k, y_k) with $k \in \{1, \cdots N\}$. For instance,

```
[ A,B ] =my_lsline_name_lastname(X,Y);
```

- (0.5 point) Use the created function to find the least-squares line $y = f(x) = Ax + B$ for the data and calculate $E_2(f)$.

x_k	y_k	$f(x_k)$
-8	6.8	7.32
-2	5.0	3.81
0	2.2	2.64
4	0.5	0.30
6	-1.3	-0.87

- (0.5 points) Plot the least-squares line and the given points from before item. Discuss what you observe.

3.3 Interpreting

Kepler's laws of planetary motion describe the motions of the planets in the solar system. Specifically, the third Kepler's law establishes the square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit. This law captures the relationship between the distance of planets from the Sun, and their orbital periods. Then,

$$T^2 = CR^3, \tag{1}$$

where T is the orbital period, R is the semi-major axis of its orbit, and C is the Kepler's constant. Here, C is given by

$$C = \frac{4\pi^2}{\mathcal{G}M}, \tag{2}$$

where $M = 1.989 \times 10^{30}$ [Kg] is the mass of the Sun and $\mathcal{G} = 6.674 \times 10^{-11}$ [Nm^2/Kg^2] is the universal gravitational constant.

- (0.8 points) Use the created function to find the least-squares line $y = Ax + B$ for the planetary data, as shown in Table 1, considering

$$\log_{10}(T^2) = \log_{10}(R^3) + \log_{10}(C), \tag{3}$$

  where $y = \log_{10}(T^2)$, $x = \log_{10}(R^3)$, $B = \log_{10}(C)$ and $A = 1$.

- (0.8 points) Construct a sketch similar to illustrate in Fig. 1 from the least-squares line computed previously.

- (0.6 points) Find an approximation of the universal gravitational constant $\mathcal{G}$ through the least-squares line computed previously. Compare the estimated gravitational constant regarding the theoretical value using relative error. *Hint.* Use $C = 10^B$ and formula (2). *Be careful with the units.*

Planet	**Period [days]**	**Semi-major axis** $\times 10^6$**[km]**
Mercury	87.97	57.91
Venus	224.70	108.70
Earth	365.26	149.60
Mars	686.98	227.92
Jupiter	4332.59	778.57
Saturn	10759.22	1433.53
Uranus	30685.40	2872.46
Neptune	60189.00	4495.06

Table 1: Planetary data: orbital parameters.

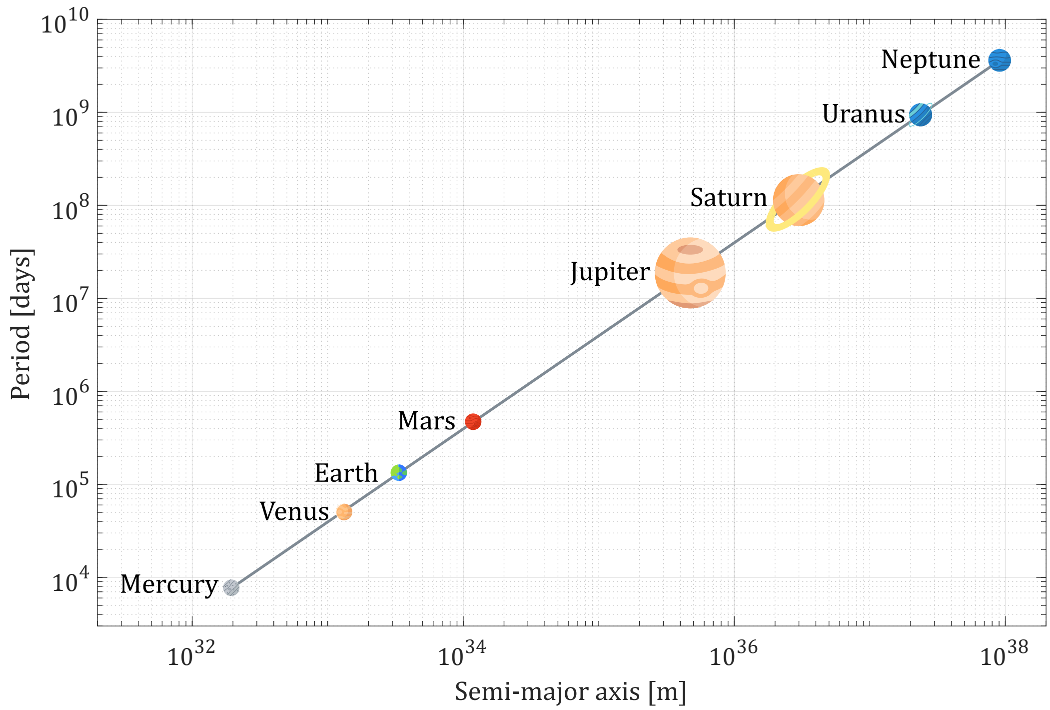

Figure 1: Relationship between the orbital periods of planets and the distance of planets from the Sun in a log-log plot.