Universidad Industrial de Santander, Colombia
Numerical Analysis, 2020-2
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Lab 6. Interpolation

1 Instructions

- Make a **pdf** report including the solution to each point of the practice with name *Lab6_name_lastname.pdf*.
- Send the report and all created files in a rar or zip file with name *Lab6_name_lastname.rar* in the Moodle.
- You are allowed to use internet, notes, and .m files that you have created before.

2 Purposes

- To understand the interpolation and polynomial approximation methods.
- To apply the interpolation and polynomial approximation methods.
- To implement the Lagrange and Newton polynomials in Matlab.

3 Practice

3.1 Understanding

Answer with your own words the following questions:

- (0.2 points) What is interpolation?

- (0.2 points) How to calculate the Taylor polynomial of degree N?

- (0.2 points) How to calculate the Lagrange and Newton interpolation?

- (0.2 points) What applications does the interpolation have?

3.2 Applying

- (0.5 points) Find the quadratic Lagrange polynomial $P_2(x)$ using $y = f(x) = \sqrt{x}$ with the nodes $x_0 = 1$, $x_1 = 1.25$, and $x_2 = 1.5$.

- (0.5 points) Fill in the missing values (?) from the following divided-difference table for $y = f(x)$. Then, find the cubic Newton polynomial $P_3(x)$. *Hint.* The recursive rule for constructing higher-order divided differences is

$$f[x_{k-j}, x_{k-j+1}, \cdots, x_k] = \frac{f[x_{k-j+1}, \cdots, x_k] - f[x_{k-j}, \cdots, x_{k-1}]}{x_k - x_{k-j}}.$$

x_k	$f[x_k]$	$f[\ ,\ ]$	$f[\ ,\ ,\ ]$	$f[\ ,\ ,\ ,\ ]$
$x_0 = 1.0$	3.5	-	-	-
$x_1 = 1.5$	?	?	-	-
$x_2 = 3.5$	103	45.5	11.4	-
$x_3 = 5.0$	491.5	259	61	?

3.3 Implementing

- (0.8 points) Create a Matlab function called *my_LagrangePolynomial_name_lastname()* to find the coefficients of the Lagrange interpolating polynomial C. The arguments of the function must be: a set of points (X,Y). For instance,

```
[ C ] =my_LagrangePolynomial_name_lastname(X,Y);
```

- (0.8 points) Create a Matlab function called *my_NewtonPolynomial_name_lastname()* to find the coefficients of the Newton interpolating polynomial C. The arguments of the function must be: a set of points (X,Y). For instance,

```
[ C ] =my_NewtonPolynomial_name_lastname(X,Y);
```

- (0.4 point) Use the created functions for Lagrange and Newton polynomial to find each interpolating polynomial based on $f(x) = 3\sin^2(\pi x/6)$ with $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $x_3 = 3$, and $x_4 = 4$.

- (0.4 point) In the same plot, compare Lagrange and Newton interpolating polynomials regarding to the real function $f(x) = 3\sin^2(\pi x/6)$. Also, illustrate the given points (x_k, y_k). Discuss what you observe.

3.4 Interpreting

(0.8 points) Determine the degree of the Taylor polynomial $P_N(x)$ expanded about $x_0 = \pi$ that should be used to approximate $\cos(33\pi/32)$ so that the error is less than 10^{-6}. *Hint.* The error term $E_N(x)$ in Taylor polynomials is given by

$$E_N(x) = \frac{f^{(N+1)}(c)}{(N+1)!}(x - x_0)^{(N+1)},$$

for some value $c = c(x)$ that lies between x and x_0.