Cartograms Work Backwards

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Summary

Cartograms work backwards. At least, diffusion-based cartograms do, if you are willing to reverse time in the physical analogy that generates them. Why is this useful? New types of visualisation may be created - by considering patterns defined in 'cartogram space and mapped back into ordinary cartographic projections, or by identifying grid warps that map one population density on to another — for example night-time to day-time populations in a region, or shifts in residential population over long periods of time. Here we explain how backward cartograms may be created, and gives examples of their use.

KEYWORDS: Cartogram, Visualisation, Geocomputation, Population density, Map projection.

1 Introduction

Cartograms have been used as a tool for visualising population-based geographical patterns for a number of decades. Essentially a cartogram is a kind of map projection in which the area allocated to any given region is proportional to the population in that region. A relatively recent and now frequently used kind of cartogram is the Gastner and Newman (2004) diffusion-based cartogram. This cartogram is defined by physical analogy, in particular the diffusion of a fluid with heterogeneous density. For such a fluid, diffusion is governed by the equation

$$\nabla^2 \rho(\mathbf{x}, t) - D \frac{\partial \rho(\mathbf{x}, t)}{\partial t} = 0 \tag{1}$$

where $\rho(\mathbf{x}, t)$ is the density of the fluid at location $\mathbf{X}$ and time t. D is a coefficient of diffusion. This is derived from the Fick's first and second laws of diffusion, which respectively state that fluids flow along the direction of maximum density reduction, and that mass is conserved in fluid motion (Fick, 1995). D is the constant of proportionality between the rate of change of density, and the rate of flow. An appropriate choice of units allow this to assume the value 1, without loss of generality. A general solution to this equation if the density function ρ is known at time zero, and D is 1, is

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$$\rho(\mathbf{x},t) = \frac{1}{4\pi t}\int_{\mathbb{R}^2} \rho(\mathbf{x},0)\exp\left(-\frac{\|\mathbf{x}-\mathbf{y}\|^2}{4t}\right)\, d\mathbf{y} \tag{2}$$

Although these equations may appear daunting to those not familiar with multivariate calculus, the last result can be summed up in a more general way:

"If we have raster image representing density of some fluid in 2D at time zero, the density at some time t after this can be obtained by applying a 'Gaussian blur' filter to the initial image. The bandwidth of the filter is a proxy for time, so that the density after a greater time is more 'blurred' than at an earlier time."

It is possible to approximate fluid diffusion and Fick's laws using Photoshop or ImageMagick! This is illustrated in Figure 1, showing changes in density as intensity plots, with darker coloring implying higher density. Here the intensity of the initial image is derived from population density by local authority districts in the East Midlands calculated using the 2011 UK Census (Office for National Statistics, 2016). Major high density areas are Leicester, Nottingham, Derby, Northampton and Lincoln. Areas in the rectangle, but outside of the East Midlands region are set to the aggregate mean density level.

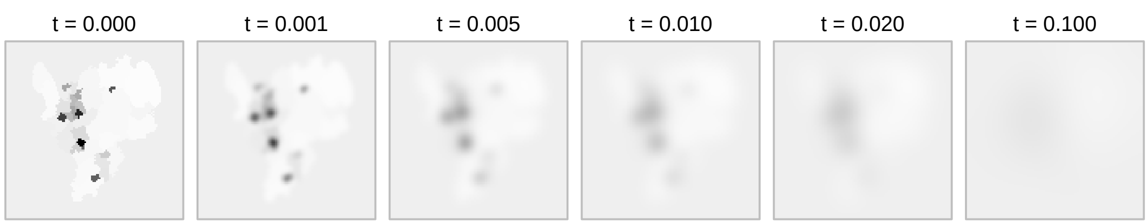

Figure 1: Diffusion process at six points in time

As time passes, the density evens out, so that when t is large the density becomes more or less uniform. At this point of very little spatial variation (i.e. when $\nabla\rho(\mathbf{x},t) \approx \mathbf{0}$) then very little diffusion occurs and the fluid becomes static.

This demonstrates how fluid diffusion is simulated, but how can this be exploited to create cartograms? The answer comes by noting that the fluid diffuses from an initial hetergeneous state towards a final homogeneous state. If some particles were distributed in the fluid according to its initial density at $t = 0$, and then were carried with the fluid as it flows they would be uniformly distributed at the final state. Essentially, the mapping of particle positions from initial to final states is a cartogram transform.

Similarly, if particles are distributed as a regular lattice initially, their final positions if they follow the fluid movement will provide the guide points for a cartogram 'warp grid'. This is illustrated in Figure 2 for the same initial density and time points used in Figure 1.

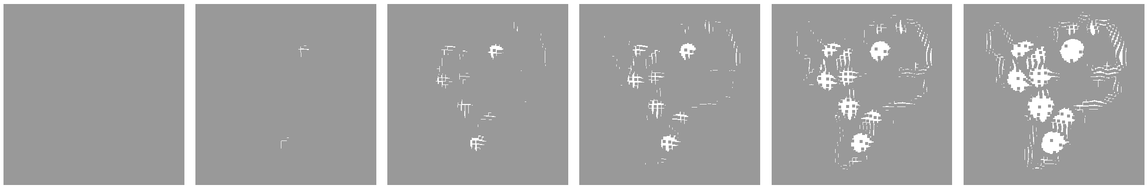

Figure 2: Diffusion Transit of regular lattice at six points in time

Carrying out simulations of fluid diffusion as demonstrated here therefore leads to a set of interpolation points that can be used to form the basis of a cartogram transform - and this is essentially how cartograms such as those in WorldMapper (`https://worldmapper.org`) are created. The initial intensities are usually related to human density (or some other density variable, e.g. density for some other kind of animal, or some subset of the population) and simulations such as these are run to create the transformation.

2 Time Going Backwards

The information in the previous section shows how *existing* diffusion-based cartograms work, but offers nothing new - except perhaps that adopting this temporal snapshot approach allows the making animations demonstrating the process, which may provide a helpful visual explanation. However, an interesting observation may be made. If the flow of time were reversed - or equivalently fluids were made to flow in the opposite direction along the gradients of steepest change in density - one could start with the homogeneous fluid and flow towards the initial heterogeneous state. Thus, the (now in the realms of science fiction) diffusion equation becomes Equation 3 (assuming D=1).

$$\nabla^2 \rho(\mathbf{x}, t) + \frac{\partial \rho(\mathbf{x}, t)}{\partial t} = 0 \tag{3}$$

Implimenting this is very simple. The original approach used to create Figure 2 can be set out as below:

1. Note initial locations of particles. Here this is a rectangular lattice of points.
2. Set time $t = 0$.
3. For each particle find the *slope* of the density (maginitude and direction). Multiply by a constant to obtain the particle's velocity, $\mathbf{v}$.
4. For a small time interval δt compute the change in position of the particle $\mathbf{v}\delta t$.

5. Update t to $t + \delta t$. Apply a Gaussian blur to the density based on the new value of t.

6. Repeat from step 3 until t is sufficiently large, and velocities are neglible.

To create a reverse cartogram, set the initial value of t in step 2 to a large value, and in other steps, replace δt with $-\delta t$. The result of this (with an initial rectangular lattice distrubuted in the diffused fluid) is shown in Figure 3. Here the transformation 'pinches in' the high density places such as Nottingham, as the uniform density distrubution 'inverse-diffuses' to a distribution of higher density in the urban centres.

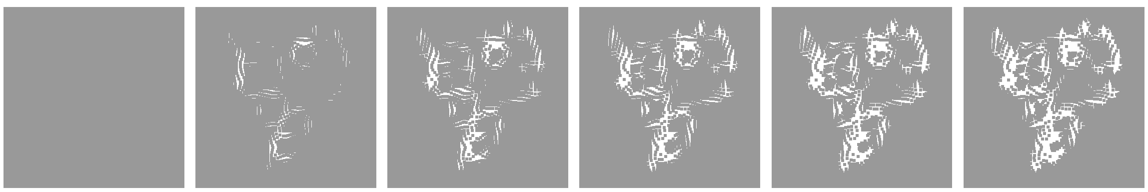

Figure 3: Reverse diffusion transit of regular lattice at six points in time

3 Applications

Attention has thus far been focussed on the *how* aspects of inverse cartograms. In this section, the *why* aspects will be considered, by outlining some applications.

3.1 Isodems

Here, an *isodem* is defined in a similar way to an *isochrone* - essentially a set of isodems are a set of contours centred on a location, with the areas between each contour pair containing the same population. One way to create an isodem diagram is to draw this as a series of conentric circles in cartogram space, and then use an inverse cartogram transform to return to standard geographical space. This is illustrated in Figure 4.

The isodems are also shown against an OpenStreetMap backdrop - Figure 5. They are centred around Leicester, and demonstrate trends in population density. Note that towards the south east of the city, where Welford Road Cemetary also Nelson Mandela park are located, the contours are further apart as there are fewer residents in these places.

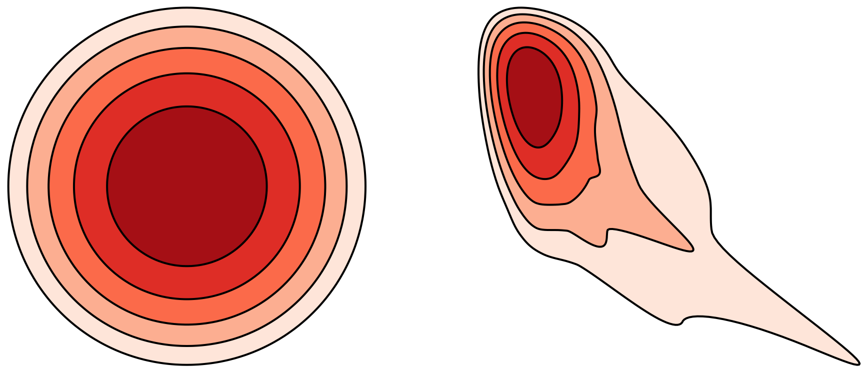

Figure 4: Isodems in cartogram space (left) and normal map space (right)

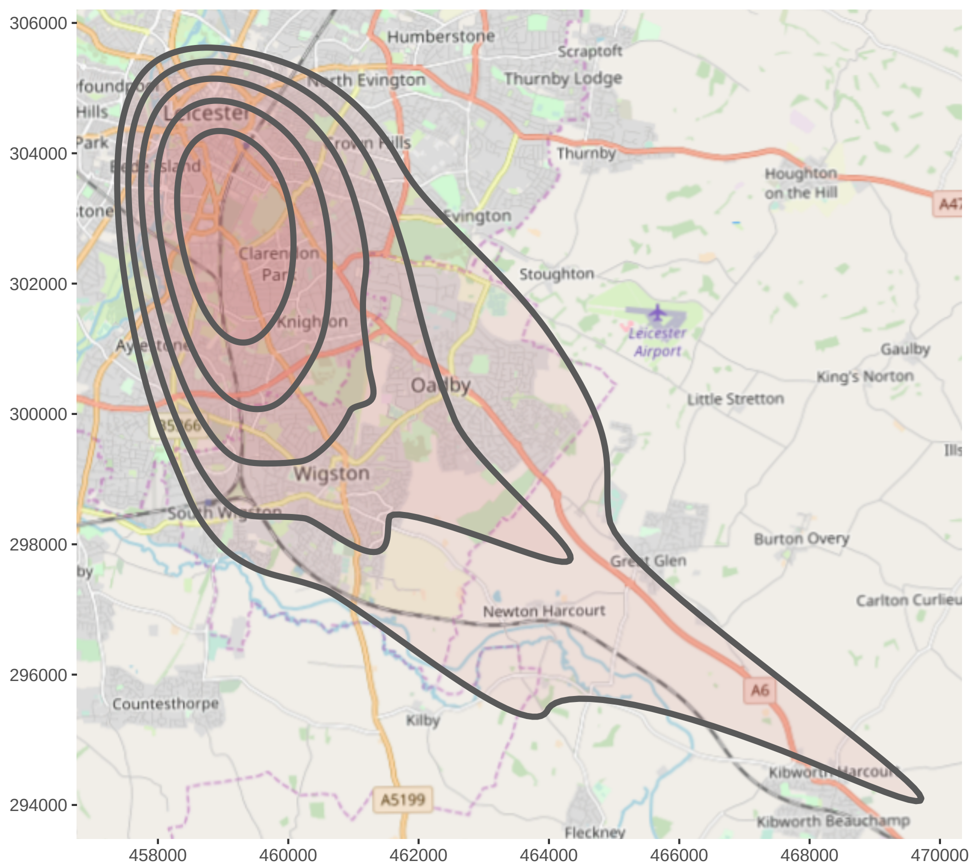

Figure 5: Isodems shown against OpenStreetMap backdrop

3.2 Generalised Cartograms

A standard cartogram provides a transformation C_1 from a standard map with population density function $P_1(\mathbf{x})$ to a transformed map with a uniform population density. We could denote this as in Equation 4, with U denoting a U uniform density function.

$$P_1 \xrightarrow{C_1} U \tag{4}$$

The inverse cartogram transformation C_1^{-1} can be thought of as the inverse transform - as in Equation 5

$$U \xrightarrow{C_1^{-1}} P_1 \tag{5}$$

We could similarly define a cartogram transform C_2 with inverse C_2^{-1} working in the same way, for a population density P_2. With this done, it can be seen that applying Equation 5 for C_2^{-1} and Equation **??** for C_1 we have the compound transformation in Equation 6.

$$P_1 \xrightarrow{C_1} U \xrightarrow{C_2^{-1}} P_2 \tag{6}$$

That is, composing the transformations C_1 and C_2^{-1} (which will be denoted as $C_{1\to 2}$) warps the initial map with population density P_1 into a new map with population density P_2. Such a transformation essentially represents a movement of population. By warping an initial map, the entities (for example people) associated with an initial location to a new one. For example, if P_1 was night-time population, and P_2 the day time population, then the warping would represent the movement of people from residential location to workplace location. Overlaying a warped regular lattice on a background map provides a novel way of visualising population flow - illustrated in Figure 6. Here the generalised transform is based on the mapping from the night time population of the East Midlands (from the 2011 Census) to an approximation of the day time population of the East Midlands, based on a combination of night time population counts and travel to work data from the 2011 Census (Office for National Statistics, 2011).

The 'pinching' on the grid in this map illustrates a transformation taking the night-time population to their places of work. As an alternative view to a typical travel-to-work area (TTWA) map (Coombes et al., 1986) this shows a more continuous flux, rather than a set of hard boundaries between zones. Since TTWAs are designed for reporting official statistics they are intentionally a set of crisp boundaries, although the zones have an incomplete level of containment. This approach gives a complimentary visualisation of spatial trends in commuting, providing a less 'crisp' perspective and exchanging the facility of statistical reporting for a more direct view of the non-binary nature of containment.

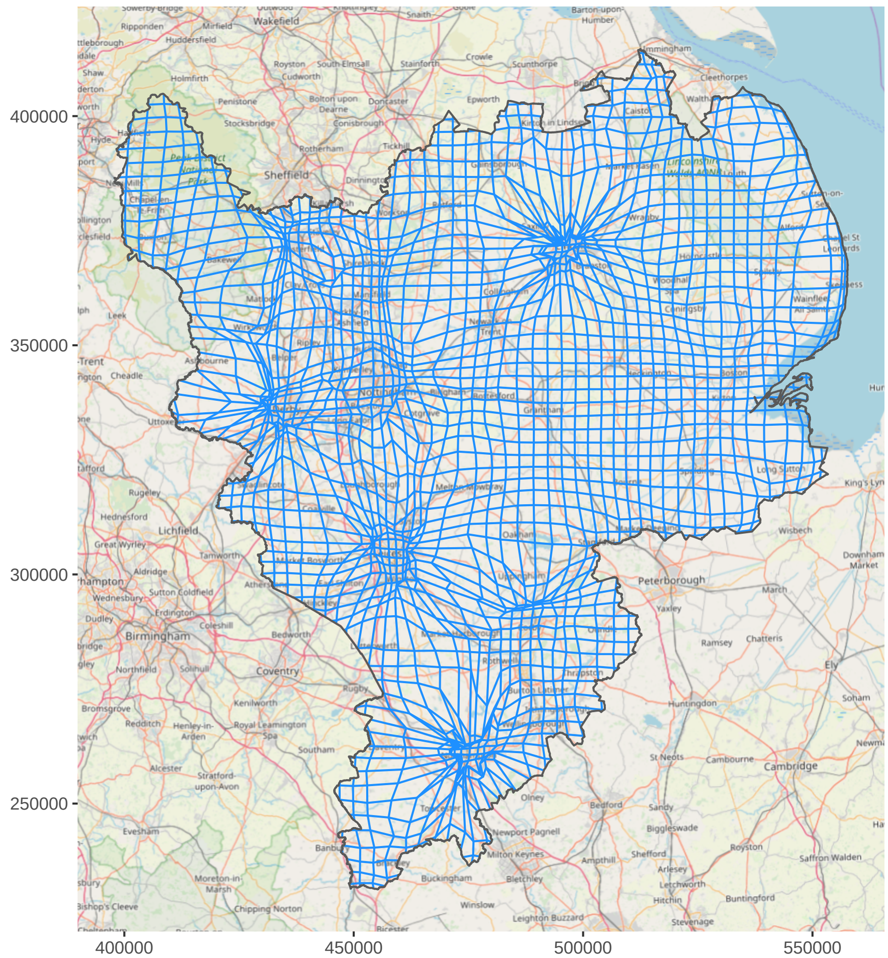

Figure 6: Generalised cartogram mapping night-time to day-time population

4 Conclusion

The approaches here extend the ideas of diffusion cartograms by

- Introducing the 'reverse cartogram'.
- Introducing the 'generalised cartogram.

These provide visual tools that could be used in a similar way to standard cartograms, but are perhaps more informative when showing warped objects on a more convential map projection, and used to show trends in population distribution, or redistribution via travel to work (as shown above), or migration, or population change over time. The temporal nature of the diffusion analogy used to generate the cartograms also lends itself to animation based displays, which will be demonstrated in the proposed talk.

5 Acknowledgements

The contribution of Science Foundation Ireland (Investigators Programme Grant 15/IA/3090 - Building City Dashboards) is gratefully acknowledged.

6 Biography

Chris Brunsdon is Director and Martin Charlton is Deputy Director of the National Centre for Geocomputation at Maynooth University, Ireland.

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