

CS302: Modeling and Simulation

Lab-6 Report

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Cellular Automata Simulation

In this lab we will use cellular automata approach to study heat conduction in rod. The relevant chapter (Diffusion: Overcoming differences) from the book is provided as reference. You are supposed to understand the discussion in the chapter and attempt at least two of project questions at the end of the chapter.

Reference for codes: <https://ics.wofford-ecs.org/files/toolbox/MATLAB.zip>

The codes shared at the book's web page have been used as reference.

We have attempted project 1 and project 7 from the *Projects* section of the text.

1.

a. Determine how long it takes, t , for the bar modeled in the module to reach equilibrium, where from time t to time $t + 1$ the values in each cell vary by no more than plus or minus some small value, such as ± 0.001 .

Ans.

The model of diffusion given in the module is as follows:

- The base of the model of diffusion given in the module is Newton's law of heating and cooling, which states that the rate of change of the temperature with respect to time of an object is proportional to the difference between the temperature of the object and the temperature of its surroundings.
- Similarly, the the change in a cell's temperature Δ_{site} , from time t to time Δt is a diffusion rate parameter (r) times the sum of each difference in the temperature of a neighbor ($neighbor_i$) and the cell's temperature ($site$), as follows:

$$\Delta_{site} = r \sum_{i=1}^8 (neighbor_i - site), \text{ where } 0 < r < 1/8 = 0.125$$

→ Hence,

$$site + \Delta_{site} = (1 - 8r)site + r \sum_{i=1}^8 neighbor_i, \text{ where } 0 < r < 0.125$$

For the given problem,

→ We have to find the time t required to reach equilibrium, where from time t to time $t + 1$ the values in each cell vary by no more than plus or minus some small value.

→ Here, we have chosen the small value(threshold) to be 0.001.

→ After each instant of time, for each cell, we find the difference between the cell's temperature at that time, and that at the previous instant.

→ If for some instant, this difference for each cell is less than a pre-decided threshold, then the state is said to be in equilibrium, and that time instant is noted as the time to reach equilibrium.

→ Parameters taken:

Diffusion rate $r = 0.05$

Grid size: 10 x 30

HOT = $50^{\circ}C$

COLD = $0^{\circ}C$

AMBIENT = $25^{\circ}C$

→ Key for display:

Black - HOT

White - COLD

Gray - AMBIENT

→ Initial state:

Grid at $t=0$ (initial state)



→ Result:

The time to reach equilibrium is **1551**.

→The intermediate grids before reaching equilibrium:

Grid at t = 300



Grid at t = 600



Grid at t = 1000



→The grid at equilibrium:

Grid at t=1551 (equilibrium state)



→The grid after equilibrium:

Grid at t = 2000 (after equilibrium)



Observations:

- The spreading of heat can be seen in the intermediate grids, as it evolves from the initial state.
- Once the equilibrium state is reached, very less change is observed in the values, and so the grid remains more or less constant.

b. Repeat Part a, applying heat and cold for 10 time steps and then removing such heating and cooling.

Ans.

→ The steps and the parameters taken remain same as in the previous part of the question.

→ Here, we do not apply hot and cold after 10 time steps.

→ Key for display:

Black - HOT

White - COLD

Gray - AMBIENT

→ Initial state:

Grid at t=0 (initial state)



→ Result:

The time to reach equilibrium is **436**.

→The intermediate grids before reaching equilibrium:

Grid at $t = 10$



Grid at $t = 100$



Grid at $t = 250$



→The grid at equilibrium:

Grid at t = 436 (equilibrium state)



→The grid after equilibrium:

Grid at t = 600



Observations:

- The spreading of heat can be seen in the intermediate grids, as it evolves from the initial state.
- Once the equilibrium state is reached, very less change is observed in the values, and so the grid remains more or less constant.

As compared to previous part:

- The equilibrium is reached at a much earlier time instant.
- The reason is that the constant application of heat and cold at designated locations on the grid is not there.
- So, the diffusion affects those regions as well, and the overall grid comes to the AMBIENT state in lesser time.

2. Model and visualize a situation in which diffusion tends to occur more in one direction than another, say more from the east than from the west. Thus, design a filter that favors directional diffusion. Such a configuration should be used in modeling diffusion on the surface of flowing water. Give your assumptions and discuss the results.

Ans. __




Unbiased Filter :

0.0625	0.125	0.0625
0.125	0.25	0.125
0.0625	0.125	0.0625

Biased filter from West to East :

1.5×0.0625	0.125	0.5×0.0625
1.5×0.125	0.25	0.5×0.125
1.5×0.0625	0.125	0.5×0.0625

KEY :

	Hot
	Intermediate
	Cool

Assumptions:

The most important assumption we make based on our intuition in order to simulate the heat flow in a rod is reflecting boundary conditions. This ensures that the transfer is not affected by the surroundings.

Observations :

Following are the figures with a bias of West to East for different time values :

West to East biased : $t = 0$



West to East biased : $t = 100$



West to East biased : $t = 200$



West to East biased : $t = 300$



West to East biased : $t = 400$



So, as expected, the source will affect the cells to its east the most and as the time increases, this effect sprawls over the neighbouring cells.

This is the reason why we observe the effect of Cold source being defeated by that of the hotter source on the west most column, as its effect on its west is handicapped.

So, there being 2 hot sources, we expected the figure to be of higher temperature, which is what we see at the equilibrium (too many red cells)!!

Absorbing boundary conditions (effect of surroundings taken into Consideration) :

West to East biased : $t = 400$
Surrounding constant temp = Intermediate



Observation :

One can observe the West to East bias here although even at $t=400$, the effect of temperature spread is moderated because of the intermediate boundary conditions. !!!