CS302: Modeling and Simulation
LAB 4 Report

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1. Modeling of Malaria:

The aim of this lab is to model the progress of malaria using the methods developed in the lectures. Malaria involves relationship between humans and mosquitoes. Not all mosquitoes spread malaria. We do not go into the details of this part. The part that is important from a modeling perspective is that out of the total mosquito population only a fraction have the potential to spread malaria. Amongst these we can think of ones which are already infected and the ones which are not infected (susceptible). Amongst humans all have equal chance of being infected with malaria (all are susceptible). A bite from an infected mosquito can lead to malaria in humans. Alternatively, if a susceptible mosquito bites an infected human they can become infected.

We work under the following **assumptions:**

• Since life expectancy of humans is much larger than that of mosquito we assume that the human population is closed with no births, immigration and deaths except from malaria.
• As soon as an infected mosquito bites a susceptible human, it becomes infected.
• As soon as a susceptible mosquito bites an infected human it becomes infected.
• Because of their relatively short life expectancy mosquito's births and deaths are considered.

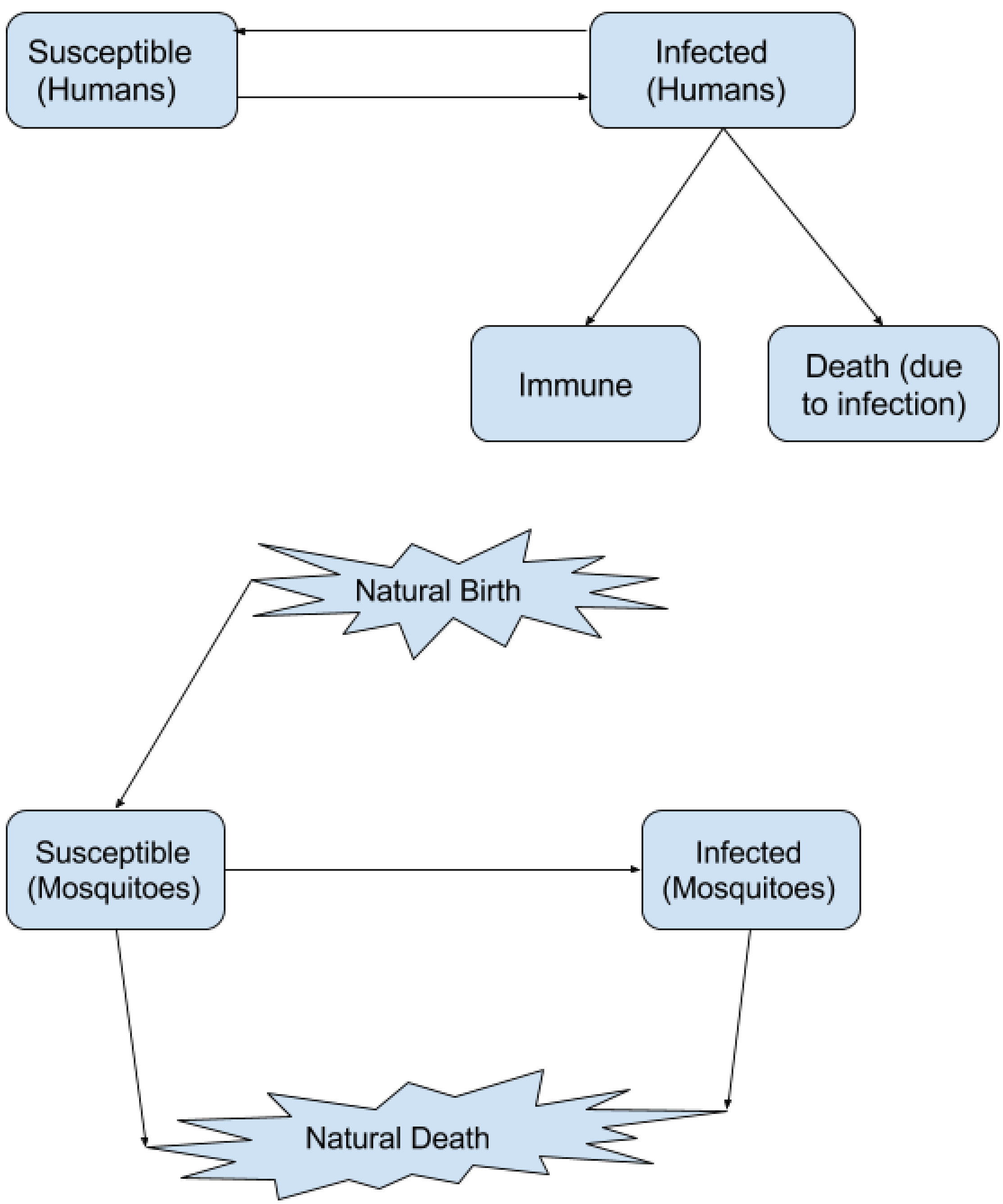
We therefore consider the following processes in malaria spread:

<u>Humans</u>:
• susceptible humans get infected due to mosquito bite from an infected mosquito.
• Infected humans can recover and become susceptible again.
• Infected humans can die due to malaria.
• Infected humans can become immune.

<u>Mosquitoes</u>:
• Susceptible mosquitoes can become infected when they bite an infected human.
• Mosquitoes can be born. All mosquitoes are born susceptible.
• All mosquitoes can die natural deaths.

Based on these observations:

(a) Draw the compartment model for malaria spread.



(b) Identify all the variables and write differential equations for malaria spread assuming that except interactions takes place all other changes are unconstrained growth and decay.

➔ The **variables** for the malaria spread are:

Sh: Susceptible Humans
Ih: Infected Humans
ImmH: Immuned Humans
Dh: Dead Humans
Sm: Susceptible Mosquitoes
Im: Infected Mosquitoes

➔ The **constants** for the malaria spread are:

k = 0.3 (recovery rate of humans)
l = 0.01 (immunity rate of humans)
w = 0.005 (malaria induced death rate)
b = 0.01 (mosquito birth rate)
d = 0.01 (mosquito death rate)
prob_bite = 0.3 (probability of mosquito biting a human)

➔ The <u>other quantities</u> used are:

prob_infected_human = (Ih)/(Ih+Sh+ImmH)
prob_infected_mosquito = (Im)/(Im+Sm)

➔ The **differential equations** obtained are:

$$\frac{dSh}{dt} = k \cdot Ih - (prob\_bite) \cdot (prob\_infected\_mosquito) \cdot Sh$$

$$\frac{dIh}{dt} = (prob\_bite) \cdot (prob\_infected\_mosquito) \cdot Sh - (k + l + w) \cdot Ih$$

$$\frac{dImmH}{dt} = l \cdot Ih$$

$$\frac{dDh}{dt} = w \cdot Ih$$

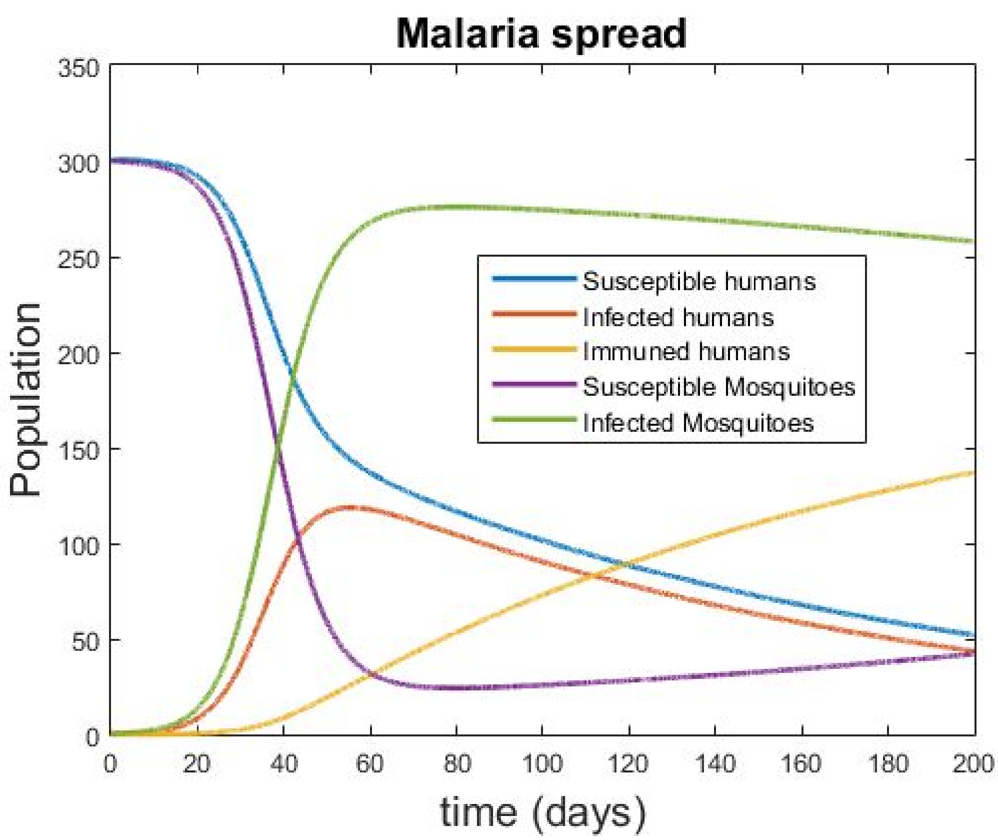
$$\frac{dSm}{dt} = b \cdot (Sm + Im) - (prob\_bite) \cdot (prob\_infected\_human) \cdot Sm - d \cdot Sm$$

$$\frac{dIm}{dt} = (prob\_bite) \cdot (prob\_infected\_human) \cdot Sm - d \cdot Im$$

(c) Using the following information in your model implement on the computer and comment on the observations (Make a single figure with all the susceptibles and infected)**:**

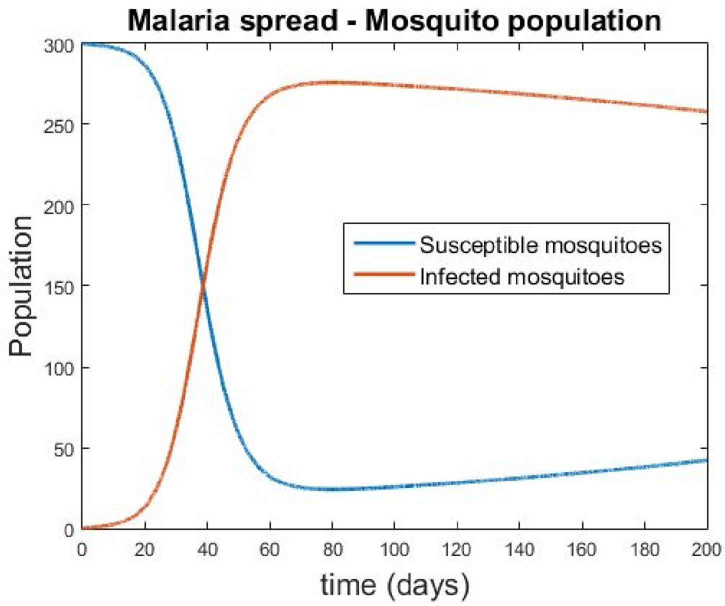
i. (Initial values (t=0)):
Susceptible humans = 300, Infected human = 1, immune humans = 0, susceptible mosquitoes = 300, infected mosquito= 0.
ii. Constants:
recovery rate of humans = 0.3, immunity rate = 0.01, malaria induced death rate = 0.005, mosquito birth rate = 0.01, probability of mosquito biting a susceptible human = probability of mosquito biting an infected human = 0.3.

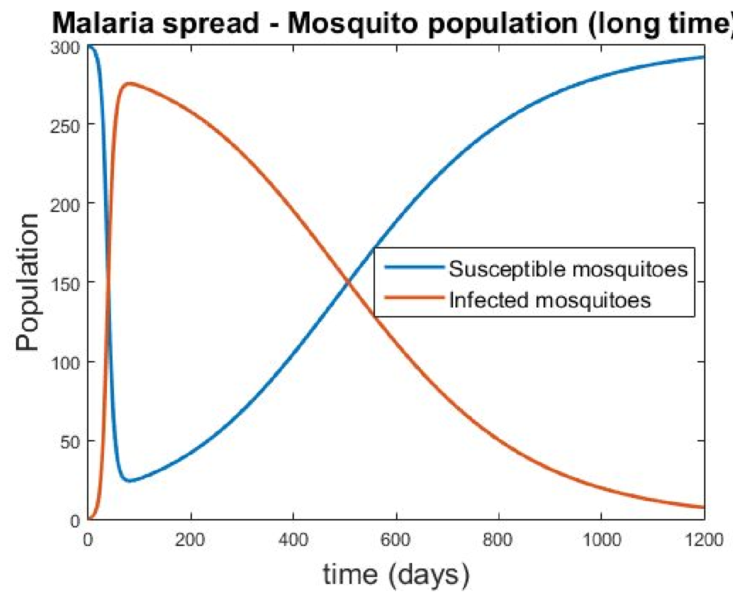
➔ **For the above mentioned initial conditions, we obtain the following graph:**



<u>Observations</u>:

For mosquitoes:

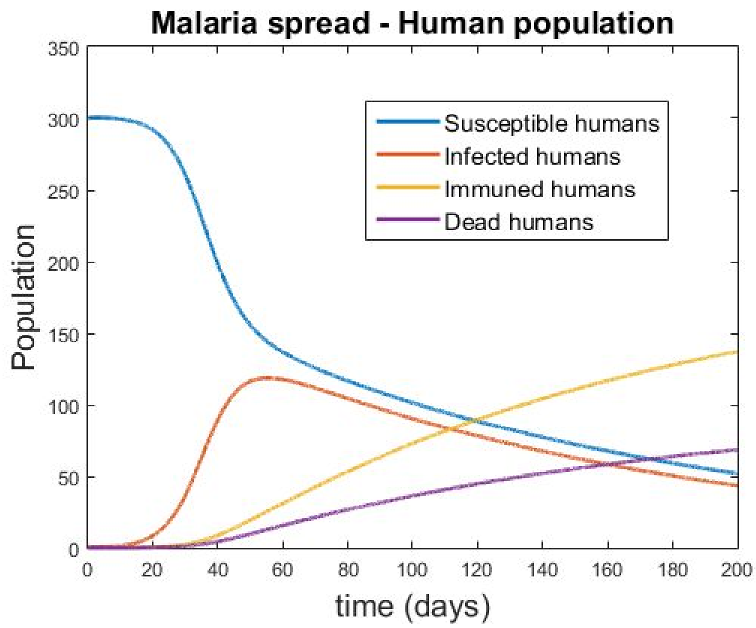


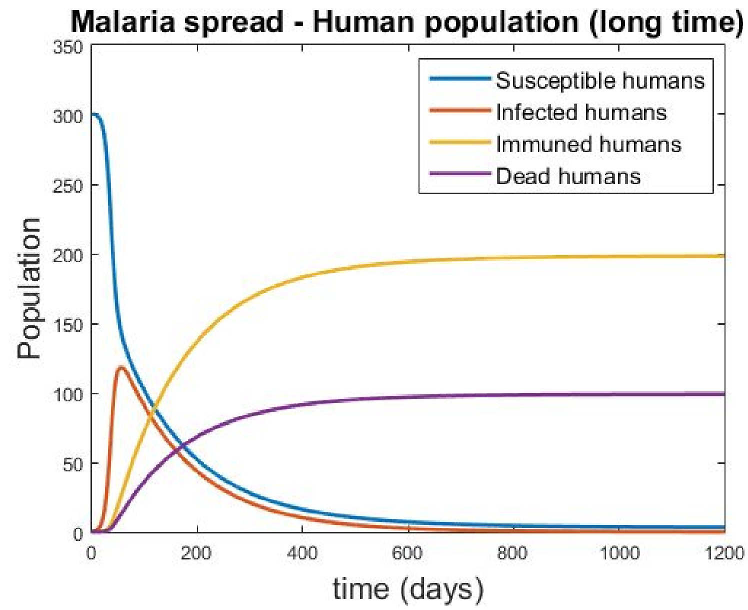


- ➔ **Till about 80 days, susceptible mosquitoes decrease with time.**
- ➔ **Correspondingly, infected mosquitoes increase.**
  - The number of infected humans increase with time (for most part of this period).
  - As the number of infected humans increase, the probability of a susceptible mosquito coming in contact with an infected human increases, and so the number of infected mosquitoes increase.
  - The birth rate and death rate of the mosquitoes has been taken as a constant.
  - So, the total population of mosquitoes remains constant (300).

- ➔ **After this period**, the number of infected mosquitoes start to decrease, while the number of susceptible mosquitoes start increasing.
  - During this period, the number of infected humans are decreasing with time.
  - So, lesser infected humans are there to come in contact with a susceptible mosquito and leading to mosquito being infected.
  - There is birth of mosquitoes, but the rate of getting infected has decreased, and so the number of susceptible mosquitoes increase.
  - With the number of infected mosquitoes tending to zero, the vast majority of the mosquitoes are susceptible.

For humans:





➔ **The number of susceptible humans decrease with time.**
- Initially, when the number of infected mosquitoes are less, the number of susceptible humans decrease slowly.
- Then, as the number of infected mosquitoes become large, the susceptible humans start decreasing faster with time.
- After the number of infected humans reach a peak value, the number of susceptible humans continue to decrease, but not as rapidly.

➔ **The number of infected humans increase with time, reach a peak, then decrease.**
- Initially, when the number of infected mosquitoes are less, the number of infected humans increase slowly.
- Then, as the number of infected mosquitoes become large, the infected humans start increasing faster with time.
- As the number of infected humans increase, the number of infecteds becoming immune, becoming dead, or becoming susceptible again also increase.
- And so, the number of infected humans decrease after reaching a peak value.

➔ **The number of immune humans, and dead humans increase with time.**
- The humans become immune or dead only if they are infected.
- Initially, when less humans are infected, the immune and dead population is less.
- With time, they increase with the increase in the number of infected humans.
- The rate of immunity is higher than the death rate due to infection, and so the number of immune humans increase faster than the dead humans.

After long time:

- ➔ **Eventually, the number of infected humans approach zero.**
  - Most of the humans are either dead or immune.
  - So, the susceptible humans also tend to zero, leading to no increase in the number of infected humans.

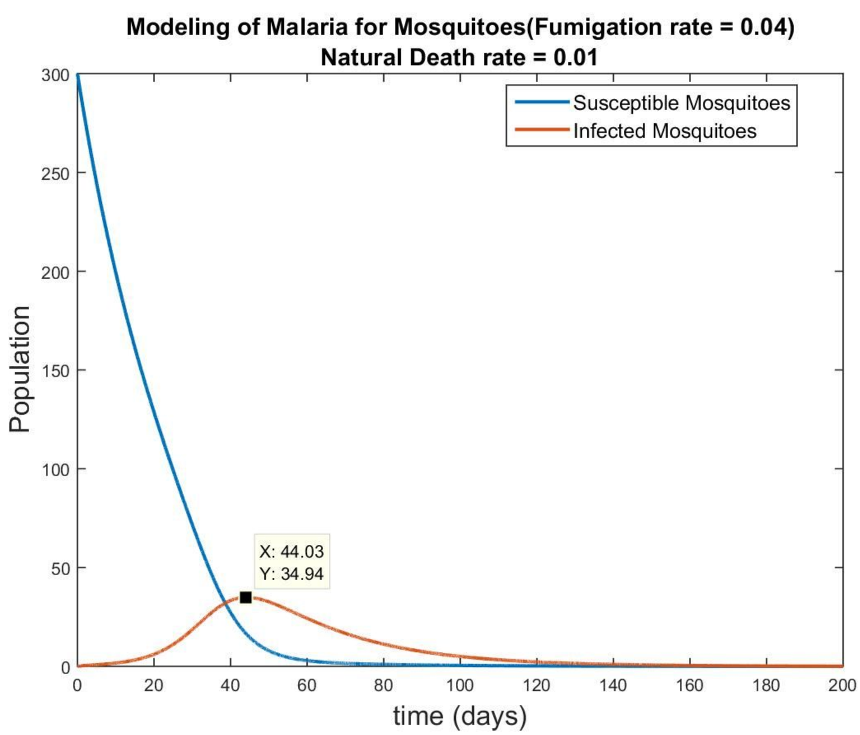
(d) If we take into account spraying or fumigation which quantity changes in your equations. How does this change reflect in the figure that you made in part (c).

- ➔ Introducing fumigation essentially accentuates the death rate of the mosquitoes. Hence, the quantity which changes is the mosquito death rate.

<u>Aims of this question</u> :

- ➔ The main aim of this question is to analyse, how the human and mosquito population change after introducing fumigation for each of the compartments.
- ➔ Also, to determine what shall be the death rate due to fumigation to make an efficient impact over the infected mosquito population.

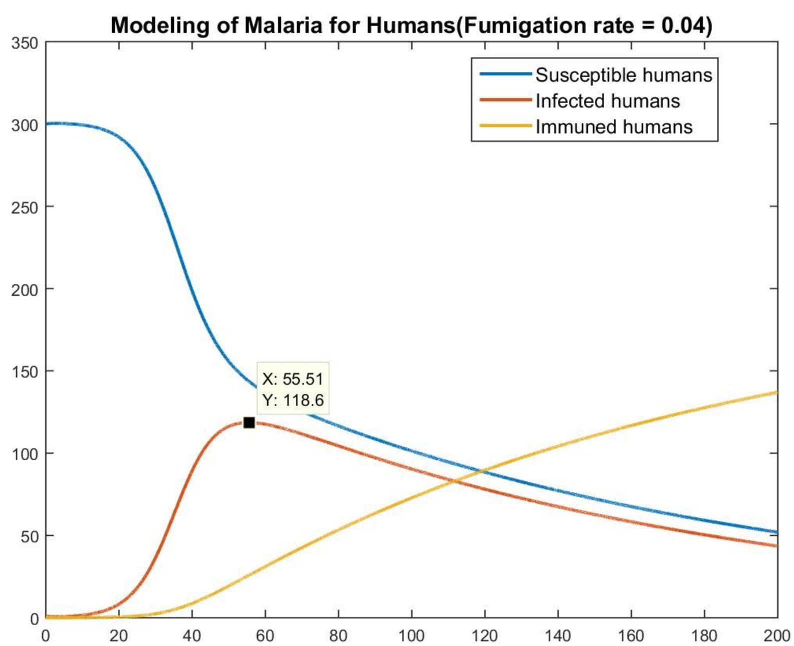
Statement : So, we chose the death rate due to fumigation(0.04) > natural death rate(0.01).
Reason : Obviously, if we introduce fumigation, we want the mosquitoes to die at a much faster rate than the current rate.



Observation :

1. **As compared to the part (c) curve, the susceptible population monotonously decreases.**
   ➔ **Explanation :**
      - Now, since the overall death rate is natural death_rate + fumigation death_rate = 0.01 + 0.04 = 0.05.
      - With a high death rate we expect the mosquito population to decline rapidly, due to the added death term due to fumigation.

2. **The infected population decreases early.**
   ➔ **Explanation :**
      - Nothing new, just as in previous observation, a particular <u>increased</u> percent of mosquito population is eliminated at every instant.
      - So, this limits the increase in infected mosquito population, and as soon as that decreased peak is reached, the peak is reached, the population starts decreasing.

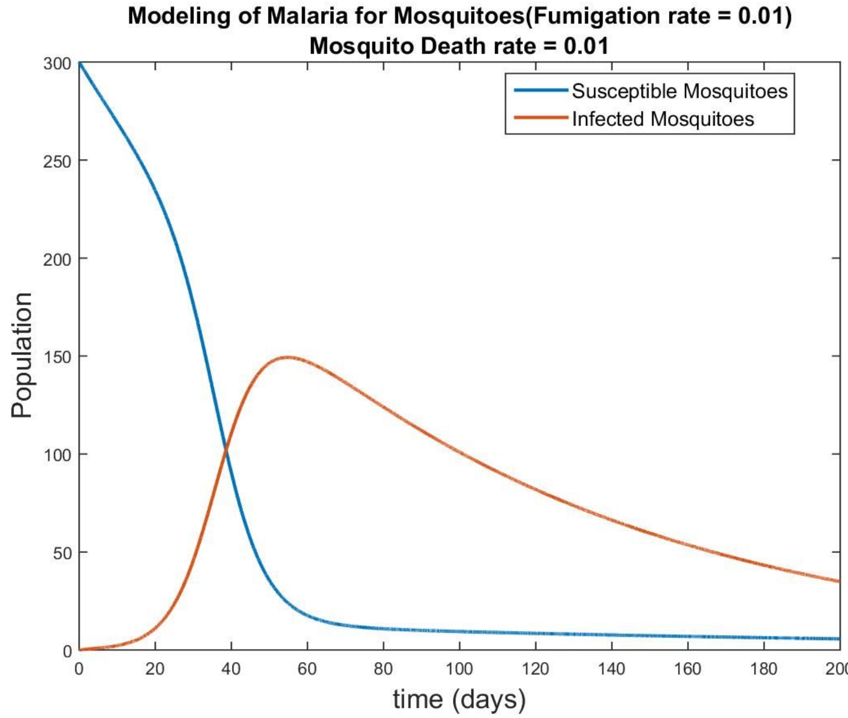


Observation : The graph is same as in part (c) for the Human populations

➔ **Explanation :** The curve is same as despite introducing the fumigation, the model is such that there's no effect on the human population. The reason being, the mosquito-related term in the rates concerned with population of humans is a ratio.

And this ratio remains same, as the same fraction is subtracted from the numerator as well as denominator.

Note(Important) : The actual constant which should change as we decrease the mosquito population is the probability of biting, but we keep that as constant for no good reason as guided by the model.



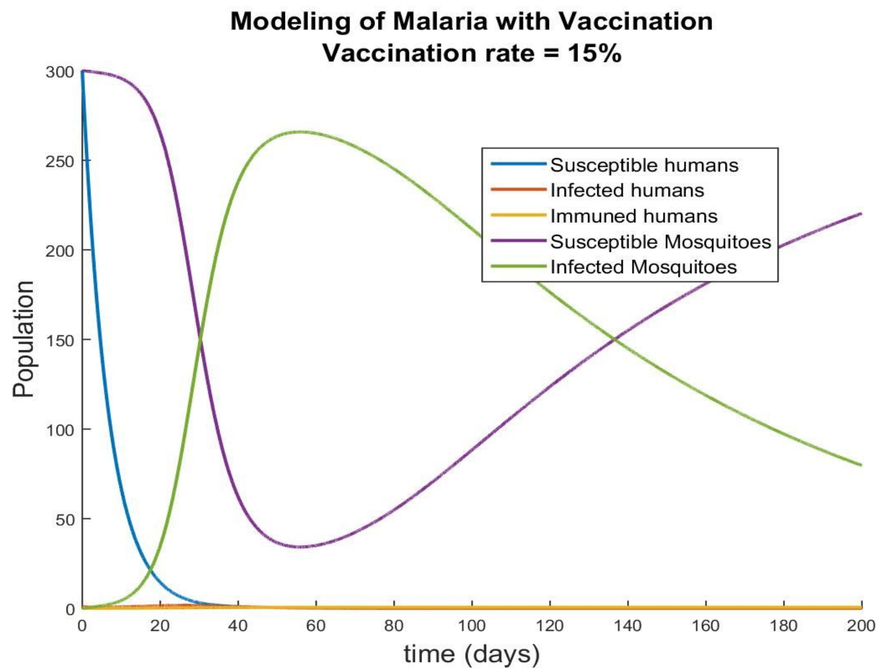
Observation :

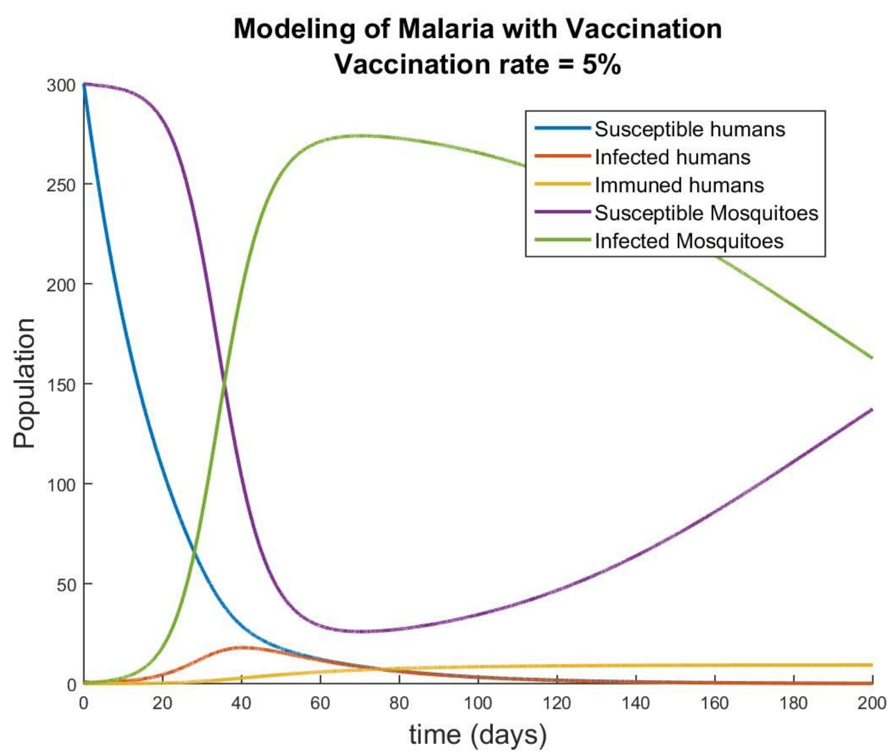
1. **The Infected mosquitoes don't die down within the studied time duration:**
2. **The graph behaves like the one in part (c) .**

Explanation : The reason for both the observations is due to less overall death rate for mosquitoes. Since the rate is less, the decline has to be slow for both the compartments unlike in the previous case of 4% death rate due to fumigation.

(e) If instead of spraying we went for vaccination what would you change and again how does it impact malaria spread.

__Assumption__ **: Once a person has been vaccinated, he/she has 0 probability of catching the disease again.**





Observations :

⇒ Common to both the curves shown :

1. **The population of humans in all the compartments is less as compared to (c).**

Explanation : Since the vaccination model assumes that the already vaccinated won't ever catch the disease, basically a new compartment of "Vaccinated" is introduced.
This compartment feeds from the susceptible population, leading to decline in the susceptible population, which is the ultimate source of the rest of the compartments.
Hence the decline.

2. **The infected mosquitoes initially increases despite a sharp decline in infected humans as compared to (c) .**

Explanation : The rate of infected mosquitoes has a positive human-dependent term. This term (a ratio : probability of infected person) has the susceptible humans term in the denominator. Due to vaccination, in initial stages, the population of Susceptible humans are affected the most. Hence the ratio shoots up initially, leading to large number of infected mosquitoes.

3. **The susceptible mosquitoes increase and Infected mosquitoes decrease in later stages.**

Explanation : In later stages, the term corresponding to probability_of_infected_person decreases as now the numerator also dies down due to substantial decrease in the susceptible humans. Hence the susceptible mosquitoes are negatively related with this term, so they increase whereas the infected mosquitoes(positively related) decrease.

⇒ Differences between the curves with different vaccination rates :

1. **Higher vaccination rate observes less human population in all compartments**

Explanation : The higher vaccination rate takes out more from the susceptible human population which is the prime contributor(directly or indirectly) to every other human compartment. Hence the decreased human population.

2. **Higher vaccination rate observes rapid change in mosquito population.**

Explanation : Higher vaccination rate implies that the probability_of_infected_person term varies rapidly. First Increases due to decrease in susceptible humans and then decreases due to consequent transfer of large amount of humans in the "vaccinated" compartments.

Hence we see, for 15% vaccination rate, that the susceptibles and infected mosquitoes intersect within the time period of study, whereas they are still about to do so for 5% vaccination rate.