Modeling Voter Behaviour and Electoral Dynamics using Cellular Automata.

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The goal of the project is to study models which are able to capture voting behaviour and observe the long term emergent behaviour of the entire voting system. This is a fairly complicated problem to tackle. Thus, we begin with models based on simple rules and binary states and slowly develop them using more complicated rules and introducing variety in the states of the agents. Since the problem has elements which dynamically evolve with time we have chosen to use cellular automaton as the simulation technique.

INTRODUCTION

We have implemented a number of models. We start with basic, simplistic models and then build up on them adding more sophisticated elements leading to richer models giving better insights. Paper on 'review of Galam models' [1] gives a number of useful models to begin with and also suggests some extensions.

BACKGROUND, IMPORTANCE AND RELEVANCE

The dynamics of voting is a very interesting problem and has direct real life applications. The voting problem falls under opinion dynamics, which is a part of a larger field of sociophysics. The models which we have discussed in this report are general and are applicable to rumor spreading, national issue opinion, etc. Based on the decision making characteristics of individuals, we explore the global behaviour of the entire population.

BASIC MODELS

Basic 1: Voter model with local interaction -Linear voter model

It is one of the simplest models for modeling voter dynamics.

Rule:

- For every time step, an agent is selected at random.
- The agent further selects a random neighbour in his Moore Neighbourhood and takes his opinion.

Because of the rule, the agents seem very gullible in nature. They simply take the opinion of one of their neighbours. It can be shown that the model is actually an absorbing Markov chain. The absorbing states are the grid being either completely black or white. Thus, the model given enough time must go to some absorbing state.

Implementation:

- We use a square grid of size 10*10 with periodic boundary conditions for a certain number of time steps (around 1000).
- We initialize the grid with either black or white color randomly and allow it to evolve. We repeat this a large number of times to ensure we understand the average behaviour of the system.

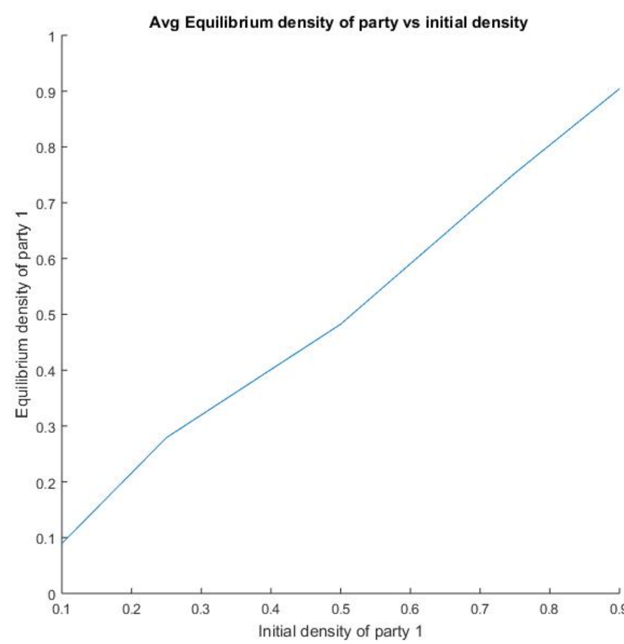

FIG. 1. Linear relation between expected density and initial density

Results:

- Here we have allowed the opinions to evolve for 1000 iterations. Interestingly, the model shows a linear relation between the average density after 1000 iterations and the initial density.

- The reason why it is so difficult to vanquish the weaker opinion is because of the rule chosen. Here, even if 1 white cell is present in a neighbourhood dominated by black cells, it still has the potential to create a new white cell. Contrast this with majority models where once a local majority is achieved the weaker opinion can never win locally which leads to a much faster converge to a single opinion on a global scale.

- The voters in this model are highly turbulent and do not believe in taking group decisions. They may choose to take an opinion which is not the most popular one locally. The majority opinion is only established in an average sense, it is not a part of a rule.

- No stable equilibrium state is reached even after a long time. Clusters are observed which remain stationary. However, the agents at the boundaries of the clusters keep oscillating between 0 and 1 state.

Basic 2: Nonlinear voter models

The linear and nonlinear voter models are popular voter models. In this subsection, we have presented a brief overview of nonlinear model.

Rule [2]:

- For every time step, an agent is selected at random.

- The Von Neumann neighborhood of the agent is considered. The sum of the opinions of the five agents is considered (The opinion is either 0 or 1). A probability mass function is used to decide whether the agent remains with party/opinion 1. Changing the mass function gives us different models for modelling voter dynamics. Note that in all the models, the agents are not strong-minded, i.e. a person having a particular opinion doesnt hesitate to change it, if the neighborhood forces him to.

We have considered two sub-models [3] :

- Majority rule model - The agent always goes with the majority in the group/locality. Intuitively, we expect clusters being formed of both the parties. These reflect the party strongholds. This reflects crowd mentality of being influenced by the majority rather than sticking to your own opinion.

- Threshold model - In this model, the agents' thinking is binary. If the neighborhood has absolute majority, they remain in the same state. Otherwise, the agent flips a coin to decide the party. Illiterate voters, who have no preference for any party because of lack of knowledge can be modelled using this model.

Sum of opinions in neighborhood	0	1	2	3	4	5
p.m.f for Majority model	0	0	0	1	1	1
p.m.f for Linear voter model	0	0.2	0.4	0.6	0.8	1
p.m.f for Threshold model	0	0.5	0.5	0.5	0.5	1

FIG. 2. Probability(of converting to party 1) functions for various models

Implementation:

- We use a square grid with periodic boundary conditions for a certain number of time steps.

- We initialize the grid of size 50*50 randomly with two colors - black and white, which represent the two opinions and allow it to evolve according to the rules.

Results:

1. Majority rule model -

- If the initial densities are equal - Equilibrium is reached after around 15000 time steps. Clusters can be observed in the grid as expected. The count of 0s and 1s remains close to 1250.

- If the initial densities are unequal(Eg. 30%-70%) - Equilibrium is reached after around 15000 time steps. The one with initial majority quickly captures more territory. The density of the minority decreases from 30% to 8%.

2. Threshold model -

- If the initial densities are equal - Formation of clusters is very rare because of the probability mass function. The agents keep changing their states often, with no stable equilibrium reached.

- If the initial densities are unequal(Eg. 30-70) - An interesting phenomena is observed in this case. The densities become equal after about 10,000 time steps. In the real world scenario, if a party has majority, but the voters are inclined to vote randomly if an absolute consensus is not reached, then the densities will eventually become equal. Thus the party with initial majority loses out if people behave irrationally.

The results are summarized in the graph in Fig. 3. As the initial density increases, the majority model follows a logistic curve, the linear model follows a straight line with slope 1 and the threshold remains constant at value 0.5.

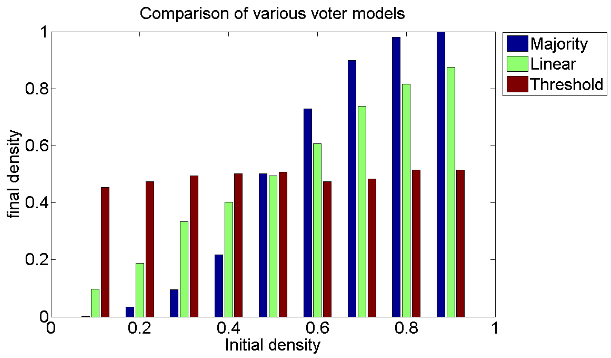

FIG. 3. Comparison of long term behaviour of majority model, linear model and threshold model.

Basic 3: Random discussion groups [4]

In this model, spatial locality (local neighbourhood) is not considered. Instead, agents form randomly sized discussion groups and interact with one another. The entire group selects a particular party according to the majority in the group. In the real world, this model can be used to model voter behaviour due to interactions between voters in offices, markets, clubs, public transport facilities, etc. A **discussion cycle** is defined as one cycle of discussions in which each agent in the grid participates in a discussion and may change his/her opinion.

Rule:

- In each discussion cycle, the entire population is divided into randomly sized discussion groups. The size of the discussion group is decided by a probability mass function.
- The entire group adopts a single opinion decided by majority. In case of a tie, opinion 0 is adopted. This represents some sort of bias for opinion 0.

Implementation:

- We initialize the grid of size 20*20 randomly with the two opinions and allow it to evolve according to the rules mentioned above.

Results: Refer to the figure 5. The following are the important results: We have run the simulation for this model for two different probability mass functions of the discussion group size. One function favors even sized groups and the other function favors odd sized groups. The probability functions used are given in the table below :

Size of Group	1	2	3	4	5	6
Probability for odd dominance	0	0.3	0.6	0.05	0.05	0
Probability for even dominance	0	0.3	0.3	0.3	0.1	0

FIG. 4. Probability function for group size

The results for two functions given below explain the observations from Fig. 5. Note that the density represents the proportion of opinions in favor of party 1.

- The final density values for initial densities less than 0.5 is zero. This results from the fact that the initial density as well as the bias favors party 0, and hence the final density moves to 0 after 10-15 discussion cycles. This holds for both even as well as odd dominant discussion groups.
- For the simulation favoring odd sized groups, the possibility of a tie is less. Hence, the final density is determined majorly by the initial density of the population. This can be seen from Fig. 5. If we consider only odd sized groups, then the maximum initial density with which party 0 can win will be close to 0.5.
- For the simulation favoring even sized groups, the possibility of a tie is relatively more. Hence, the final density is determined by the initial density of the population as well as the bias in case of a tie. This can be seen from Fig. 5. For an initial density of upto 0.7, the party 0 is able to win the majority(90%) in the long run. An initial density greater than 0.8 guarantees that party 1 will win in the long run.

EXTENSIONS:

1. Effect of Radicals [6]

This model builds on the majority rule basic model. In this model the effect of radicals is considered where the radicals never change their opinion. Effect of radicals of only one opinion is considered. Radicals of both opinions

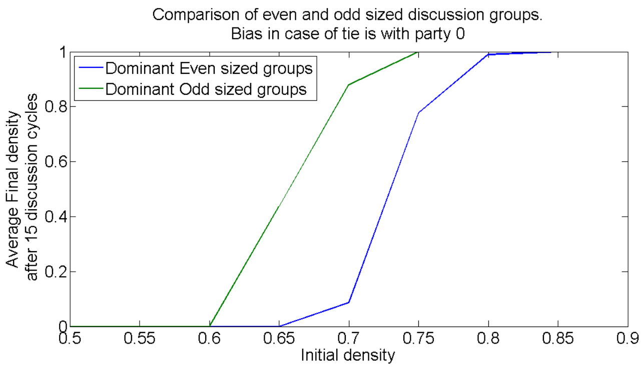

FIG. 5. Comparison of even size dominated and odd size dominated discussion groups

just dilute the effect seen by the radicals of only one opinion.

Rules:

- A person is randomly chosen in each iteration.
- If the person is a radical, then there is no effect of majority rule.
- If the person is normal, then majority rule is applied in Von Neumann neighbourhood.

Implementation:

- We use a square grid of size 50*50 with some initial distribution and it is varied to study the effect of radicals.

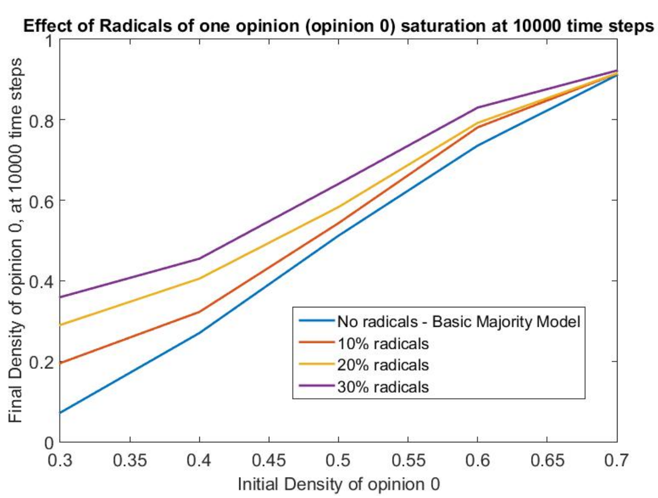

FIG. 6. Variation in final density when initial density and radicals of single opinion are varied.

Results:

- In FIG.6, it is observed that as the number of radicals increase, the saturation point density achieved is more. Hence, purple graph is always above the others.
- When 30% people of the total population are radicals, the final density of the opinion is always greater than the initial density. This means that irrespective of initial density, due to presence of radicals, the opinion of radicals is able to expand.
- For 20% radicals, for initial density 0.4, the final density is almost 0.4 indicating that the equilibrium (initial density = final density) has shifted from 0.5 in the basic majority model to 0.4 due to impact of radicals. This is because, probability of choosing a person of opinion 0 and it not being a radical is less than probability of choosing a person of opinion 1. Therefore, people of opinion 1 decrease over time.
- For initial density > 0.5 in 20% radicals case, the final density is always better than the basic model because radicals never change their opinion and hence improve the spread of opinion.
- For 10% radicals the behaviour is much closer to the basic case because impact of radicals is less significant.
- For initial density > 0.6, the difference between the 4 cases of different radical percentages decreases because the behaviour is largely guided by the majority rule as the opinion 0 is in majority

2. Tri-Party Model

We generalize Galam's model of opinion spreading by introducing three competing species [7]. This is an extension to the Random discussion groups model in which spacial locality is not considered and instead agents form discussion groups of size 3 and interact with one another. The entire group selects a particular party according to the majority in the group.

Rule:

- In each discussion cycle, the entire population is divided into discussion groups of size 3.
- The entire group adopts a single opinion decided by majority of the 3 people.
- In case of a tie, opinion A is adopted with probability α, B is adopted with probability β and C is adopted with probability $1 - \alpha - \beta$.

Implementation:

- We initialize a 30*30 grid randomly according to some initial densities and allow it to evolve.
- Density of population supporting A, B and C is plotted as a function of number of discussion cycles.

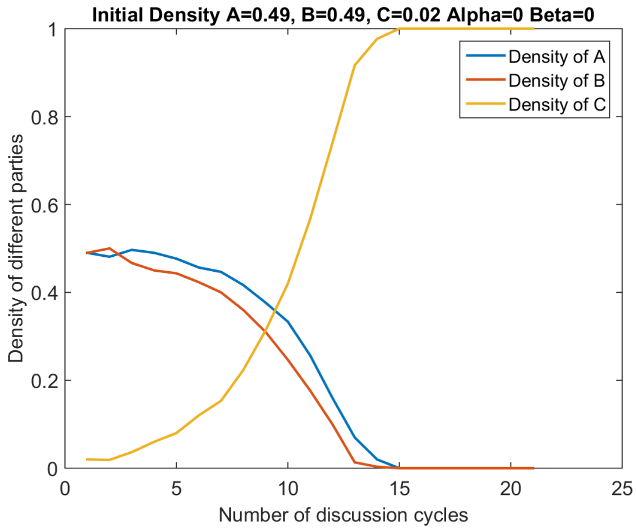

FIG. 7. Tri Party Simulation with initial density less than 50%

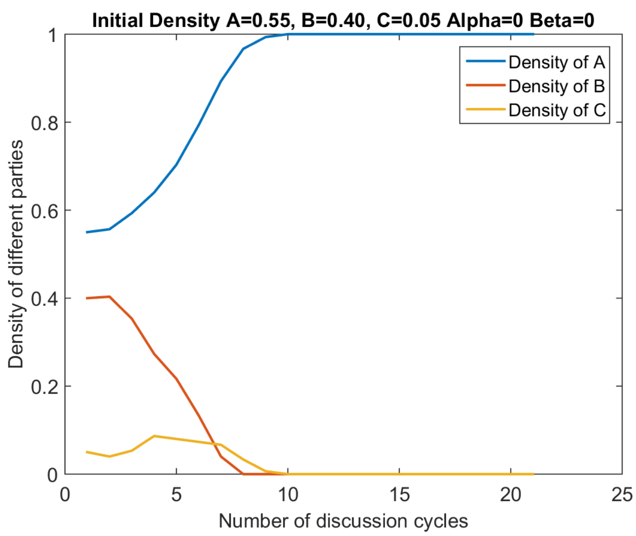

FIG. 8. Tri Party Simulation with initial density greater than 50%

Results:

- This model always results in polarization i.e. one party always wins but the time it takes to polarize depends on the initial condition. If the initial density of an opinion is very high then the polarization time is very less.
- When the initial density of A is 0.55 and B is 0.40 and the values of α and β are such that in case of a tie opinion C is given to the discussion group. A emerges as a winner. This is seen in FIG.8.
- When the initial density of A is lowered to 0.49 and B is given 0.49, we surprisingly observe that C emerges as the winner even though the initial density of C is very less. This can be explained mathematically using expected value of number of conversions to C after a discussion. This can be observed in FIG.7.
- On experimenting we observe that when one of the opinions has initial density greater than 0.5, it eventually comes out as the winner irrespective of what happens on a tie. If all the opinions have initial density less than 0.5 then result of tie becomes an important factor in deciding who eventually succeeds.

3. The effect of a biased mass media on a population where consensus is difficult to achieve.

Here, we examine the effect of a biased external agent(media) on a society which takes its decisions either by consensus based internal discussions or by taking the media's opinion. We have loosely based the model on the one given in [5]. Note, the success of an internal discussion is pretty low because it is consensus based which is very difficult to achieve.

Rule:

- For each time step, pick a random cell. Appoint cells in its Von Neumann neighbourhood as its advisers.
- If they are able to reach a consensus, the cell takes their opinion.
- If this fails, the person takes the opinion of the media with some probability p.

Expectation:

- The consensus is a pretty strict requirement. For 4 people to agree, we have a $\frac{1}{8}$ probability, which means that 7 out of 8 times, the person will consider the medias opinion.
- In such a situation, if the media is persuasive (high p), then we find that the grid is quickly influenced by the medias opinion. If the media is not that influential, then the model is stuck in a status-quo indecisive situation.

Implementation:

- We perform the experiment for 2 situations, one where media is persuasive and the other where it is not.

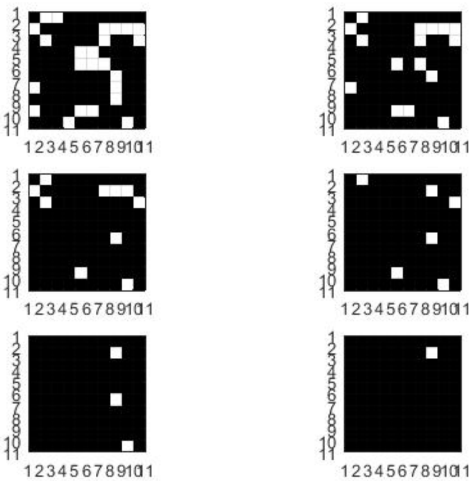

FIG. 9. Media's bias deciding opinion

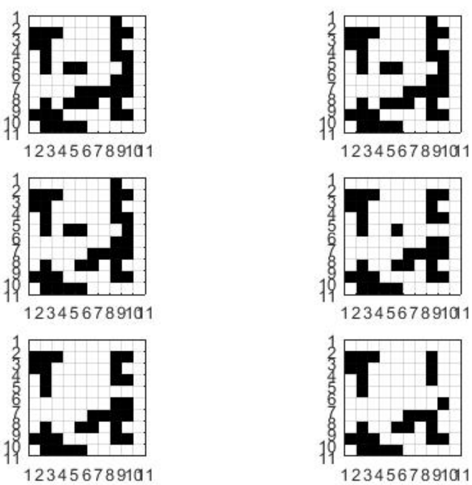

FIG. 10. Media not influential in consensus model

Note: In Fig 9 and 10, start from top left and move rightwards and then down.

Results:

1. Strong Media Influence: Ref. Fig 9
    - Here we have set the probability of a person accepting medias opinion as high.
    - Hence, the media is much more effective than the internal consensus based system and hence quickly turns public opinion towards its favoured candidate.
    - This shows us the peril of consensus based discussion and a biased media's dangerous influence
2. Weak Media Influence: Ref. Fig 10
    - But building a consensus is as difficult as before. Thus, the situation is indecisive, and the state of the automaton is stagnant. Nothing much happens in the system.
    - In fact it is this indecision which allows a dominant media to quickly sway public opinion.

4. The effect of a biased mass media on a population which takes decisions based on majority

Here the groups take decisions based on majority rather than complete consensus.

Observations:

- Note that it is much easier to achieve majority in a Von Neumann Neighbourhood (4 cells around the chosen one).
- Thus, media's effect is gentler as compared to the above section. Thus, this models dynamics will be more influenced by the internal majority based discussions.
- We know that in the basic majority model the initial density is very important. The one which has a larger initial density eventually comes out on top. Thus, if the biased media supports the one with a larger initial density then the result is a no-brainer. It is as if a strong opponent is given an added advantage against a weaker opponent.
- The following graph shows effect when media supports weaker/minority opinion at the start.

Results:

- If the media supports a candidate then the extra push ensures that that candidate is able to achieve a majority.
- Hence when p = 1, the blue graph shows that the equilibrium point shifts so that the favoured opinion can afford a smaller density.
- Here, for initial density above 0.5, the favoured opinion wins hands down. For lower values of p, the equilibrium point is rightwards closer to 0.5. The critical point beyond which victory is certain also shifts rightwards.
- Thus, the media basically biased the results in favour of a candidate and majority model shows much richer dynamics than the boring consensus based situation.

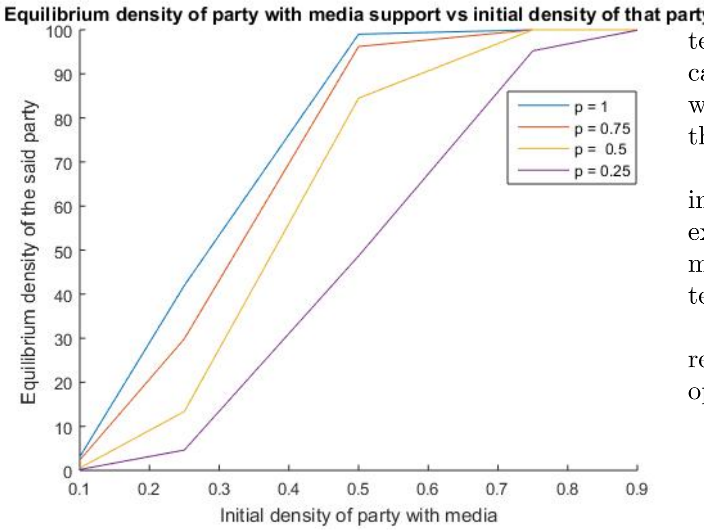

FIG. 11. Equilibrium density of media opinion vs initial density for different values of media persuasion

CONCLUSION

The aim was to experiment with different voter models which would be able to simulate the voter dynamics and allow us to get insights about the real world process.

We implemented all the models from scratch. For most of the models, we referred to academic papers. One model - extension of majority model to include effect of media was devised by us. This was needed because the model in the paper was not providing useful insights.

Most of the models fell into two types: Obvious and Interesting. The basic models essentially fell in the former category. Here, the models behaved according to what we expected. This was because the rules were simple or there was not much which could happen with the model.

However, there were some models in which we saw interesting behaviour which was not very obvious. For example, Tri-party system and effect of mass media on majority model. Also, note that we had developed extensions each capturing different factors.

On the whole, the models capture and emulate many real world situations which play a role in shaping voter opinion.

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