



Real-Time Geological Inversion for Subsurface Decision-Making

Sergey Alayev saly@norceresearch.no



Picture from my last Geilo Winter School



January,
2017

Outline

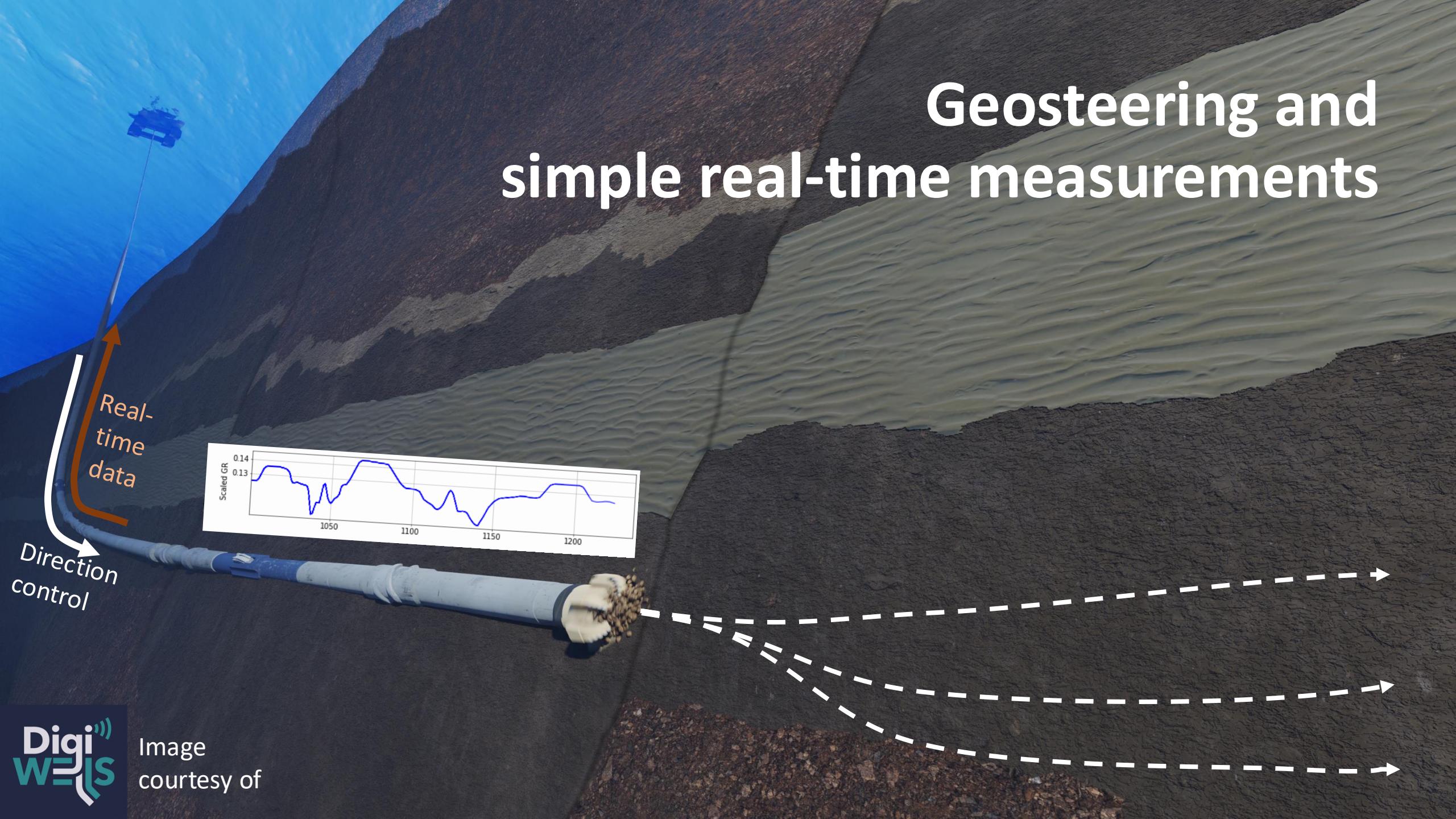
- Lecture 1: Geological inversion for geosteering
 - Overview of geosteering
 - Different types of data and inverse problems
 - Reference problem of sequential inversion
 - Simple inversion
 - Regularization
- Exercise
- Lecture 3: Filtering and multi-modal inversion
 - Particle filter
 - Direct multimodal inversion with machine learning



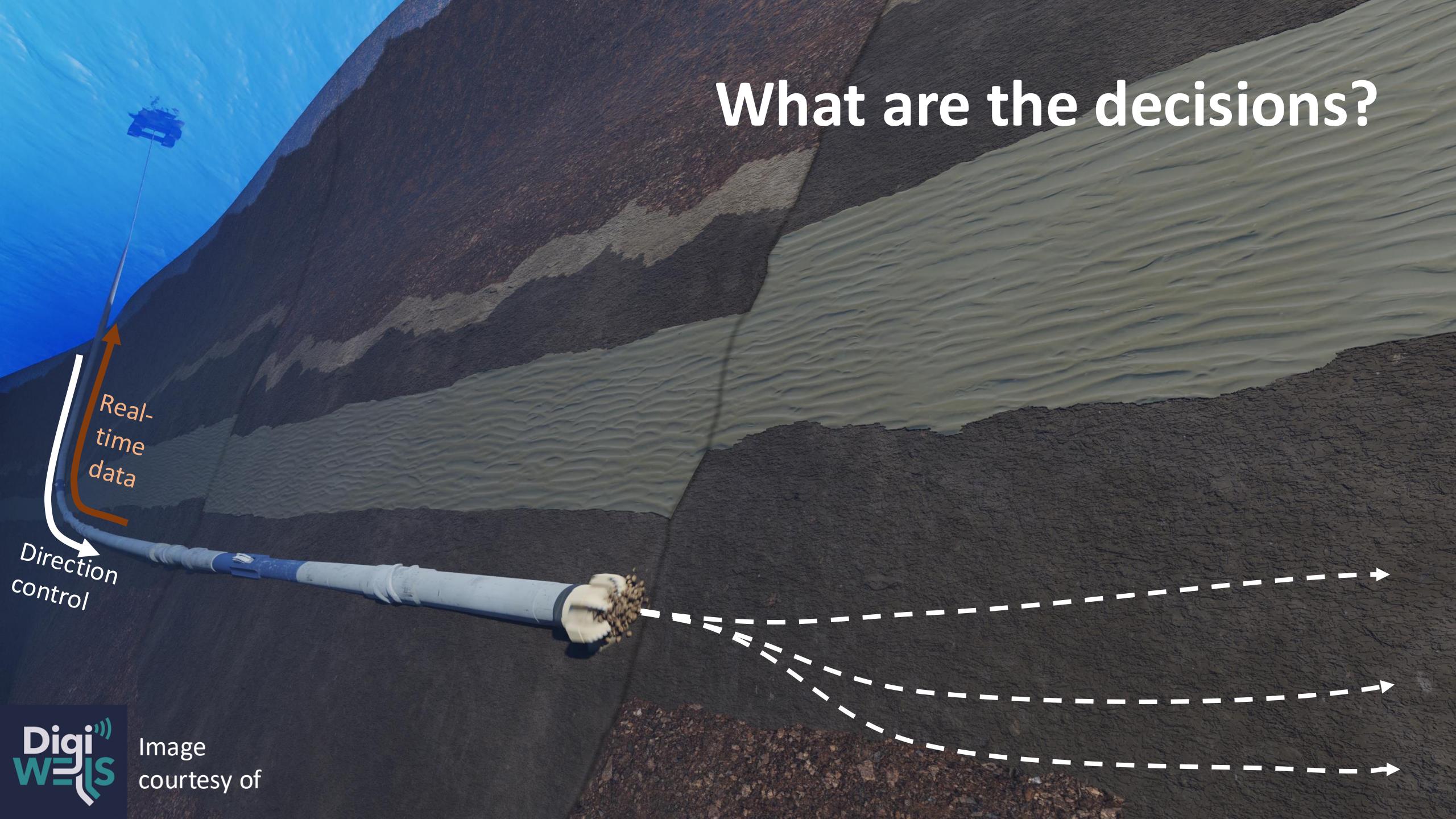
Lecture 1: Overview of geosteering and sequential inversion



Geosteering and simple real-time measurements



What are the decisions?



Bottom hole assembly overview

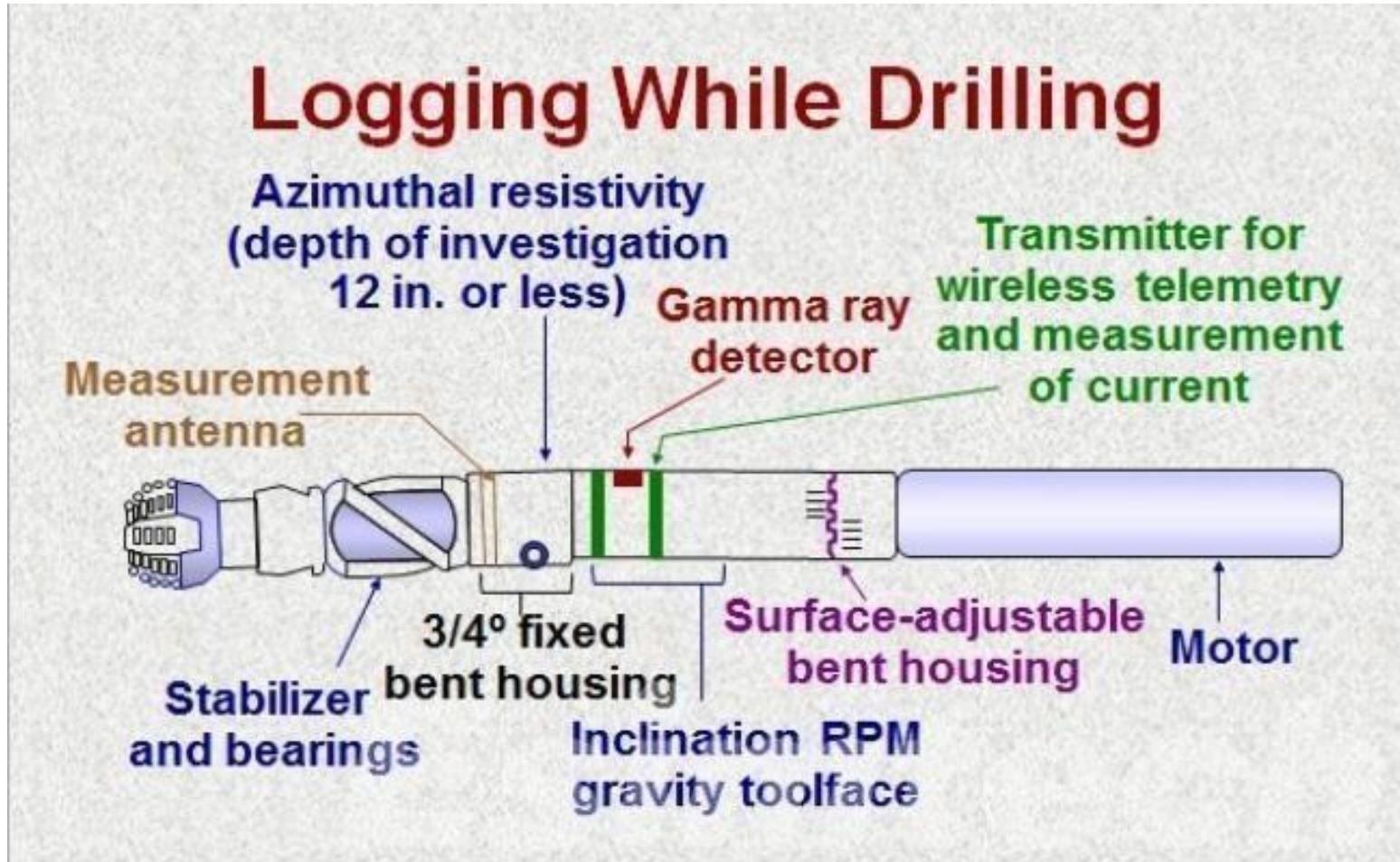
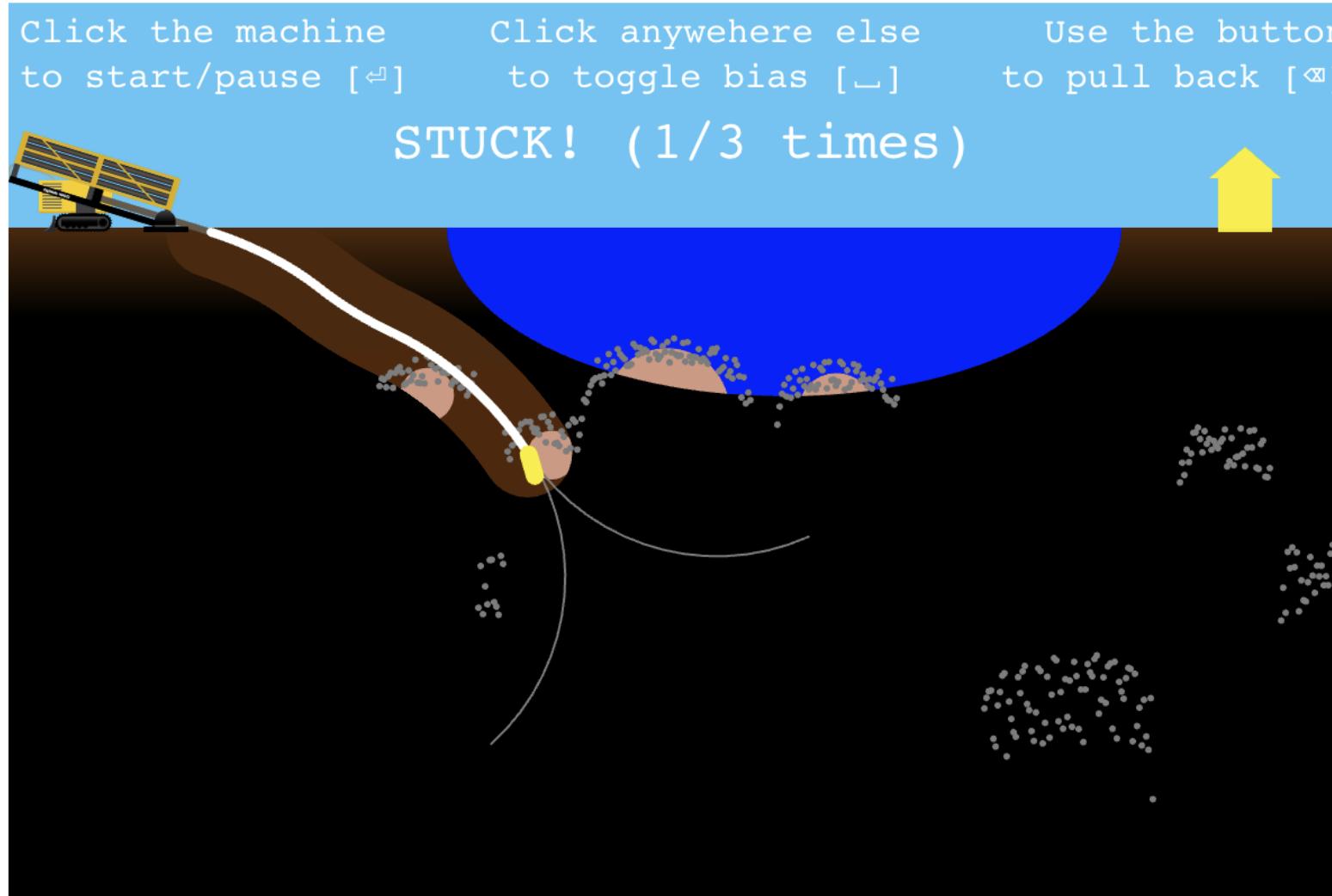


Image from

<https://www.esimtech.com/the-vital-role-of-logging-while-drilling-revolutionizing-oil-exploration.html>

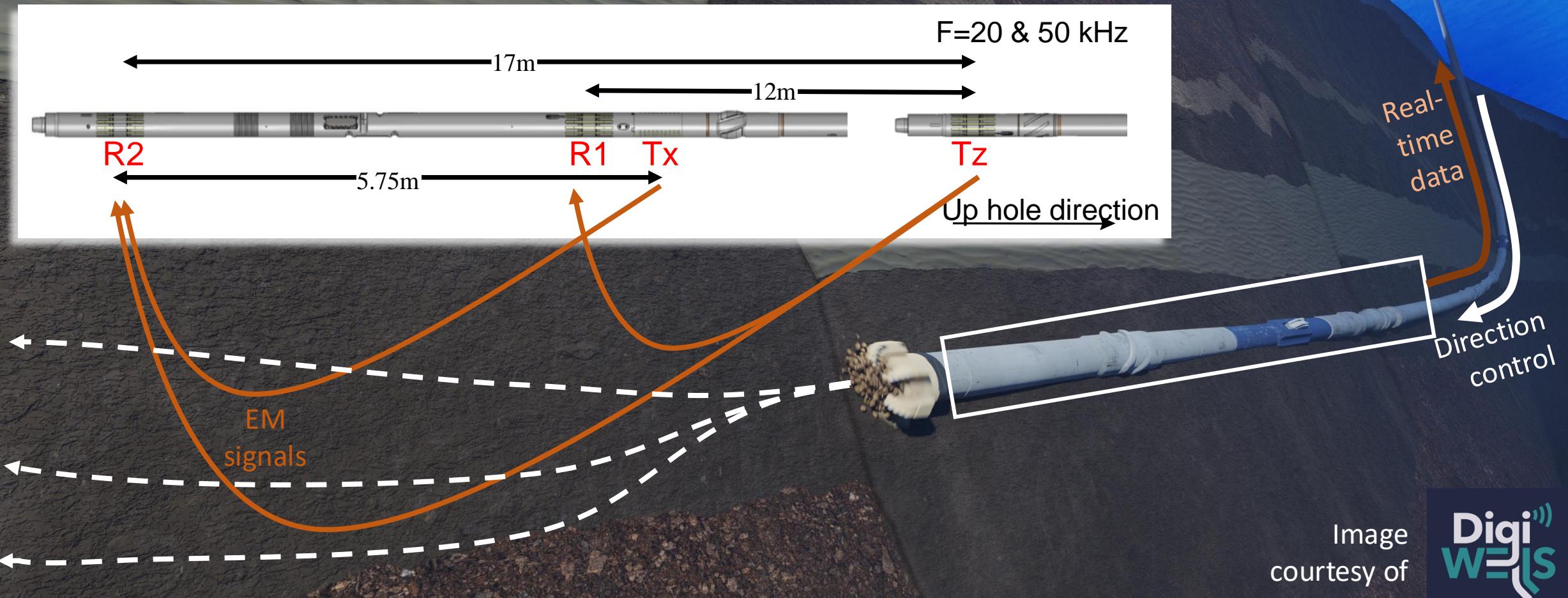
Underbore game shows little-bit how it works



[https://github.com/alin256
/Directional-Boring](https://github.com/alin256/Directional-Boring)



Geosteering and electromagnetic (EM) measurements



Inversion of Triaxial EM Measurements

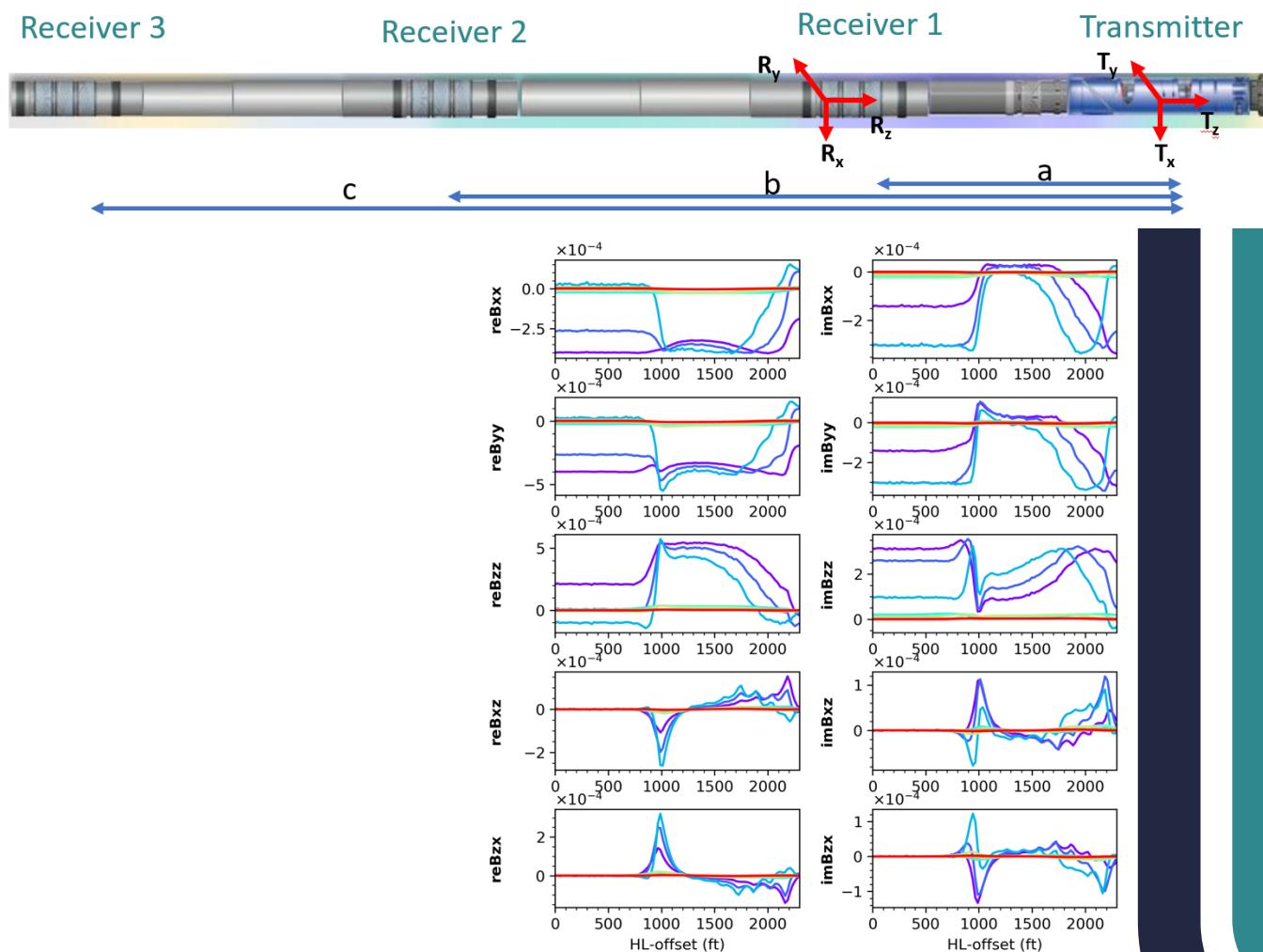
Three tilted antenna coils

9 measured signals

XX, XY, XZ

YX, YY, YZ

ZZ, ZY, ZZ

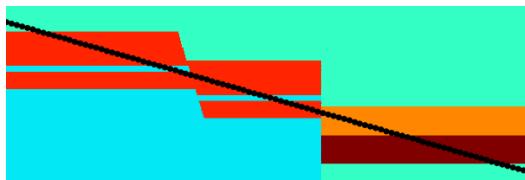
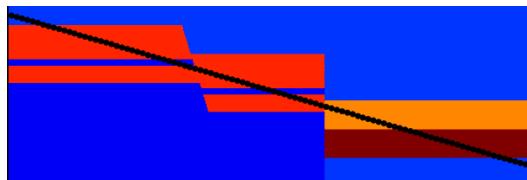


Slides from Nazanin Jahani et al.:

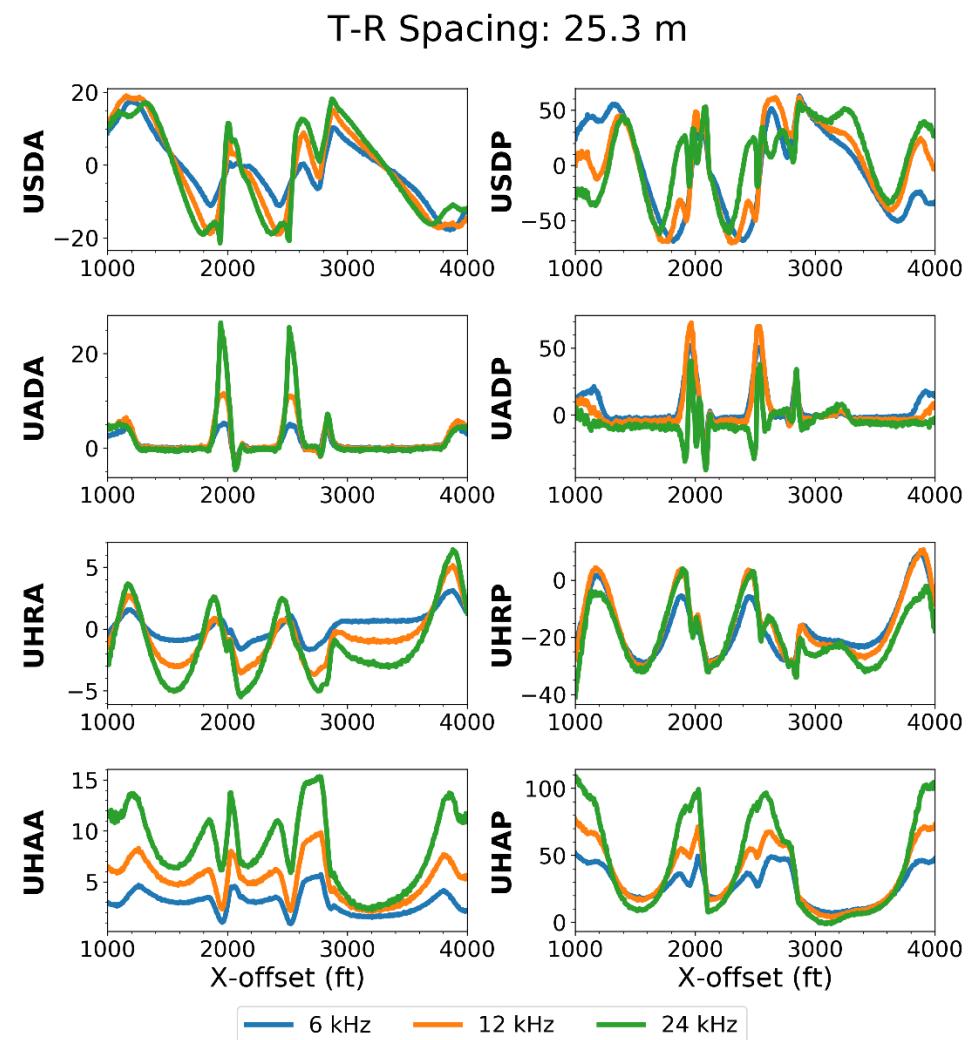
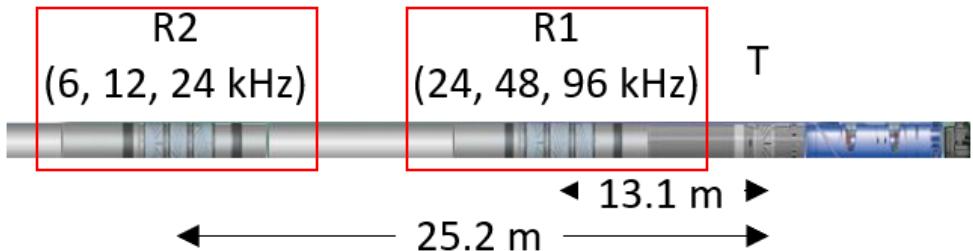
<https://onepetro.org/SPWLAALS/proceedings/SPWLA24/4-SPWLA24/D041S013R002/545536>

| | | |
|------------|------------|------------|
| 24kHz_22ft | 6kHz_55ft | 2kHz_98ft |
| 48kHz_22ft | 12kHz_55ft | 6kHz_98ft |
| 96kHz_22ft | 24kHz_55ft | 12kHz_98ft |

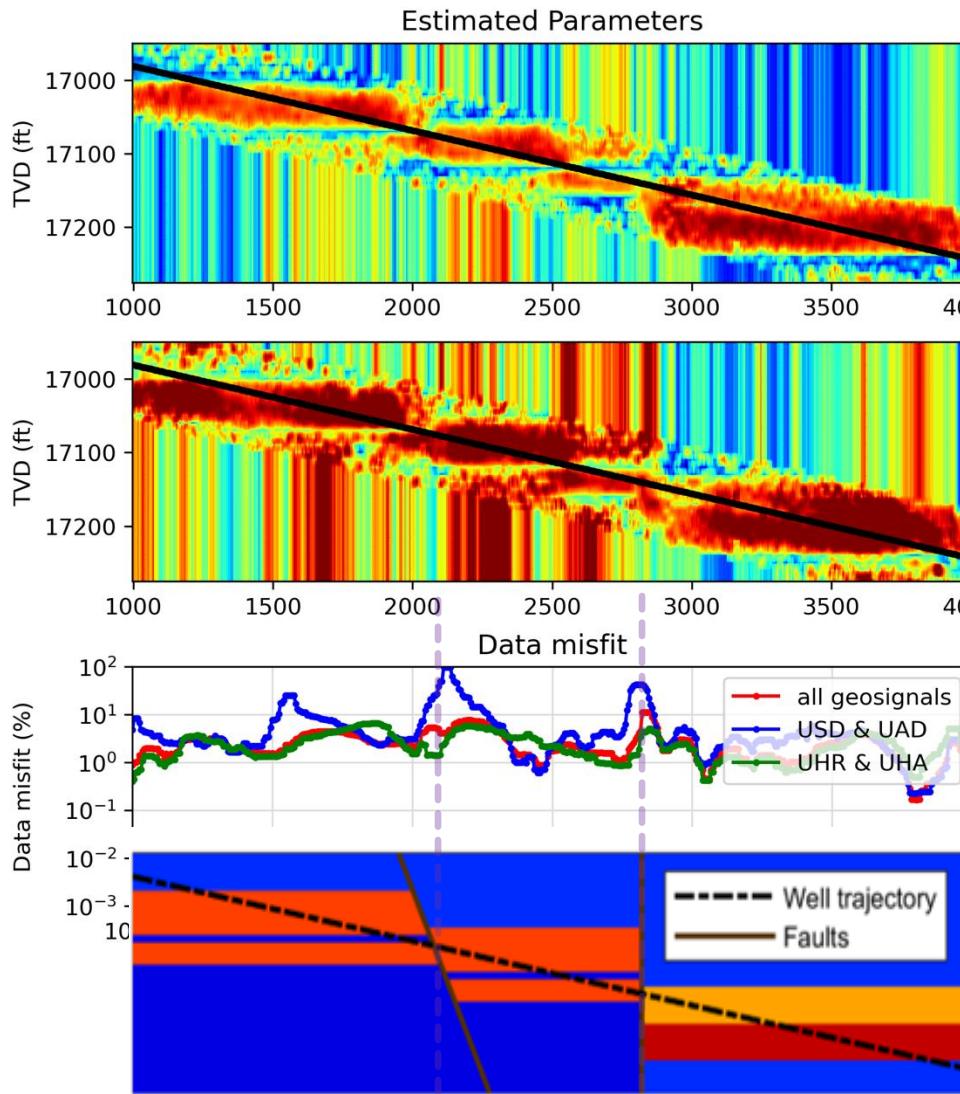
Anisotropic Two-Fault Model: Synthetic Data



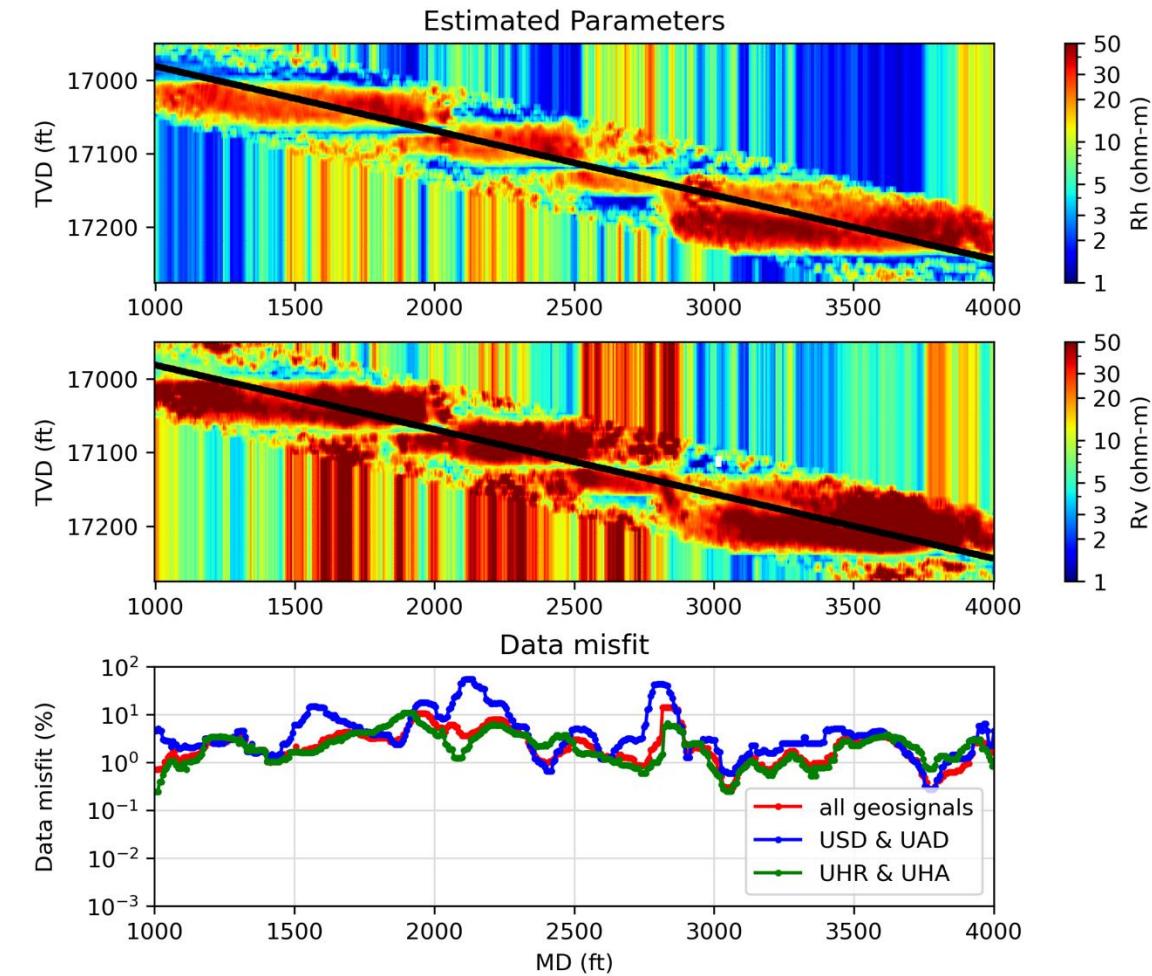
- Synthetic 3D geo-signal data from **LogSimulator3D_v2.3**
- + 2% Relative Noise



First Pass



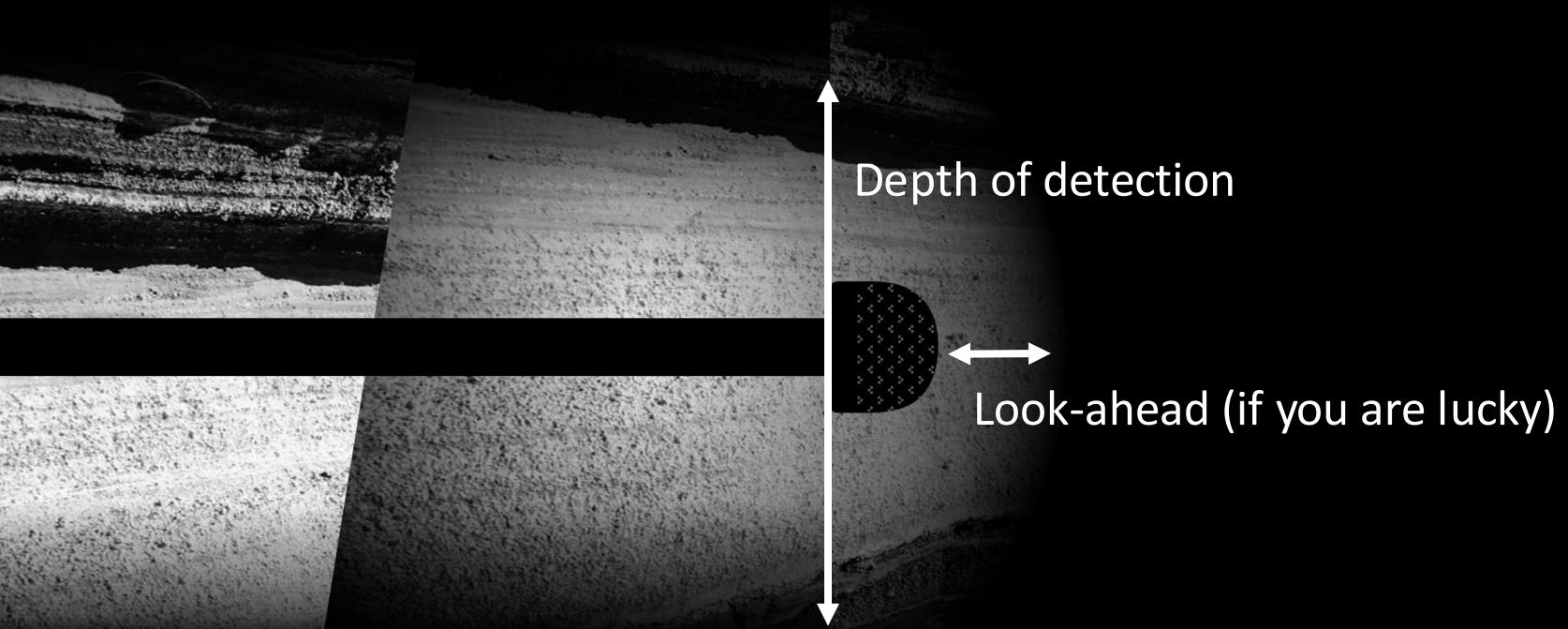
Second Pass



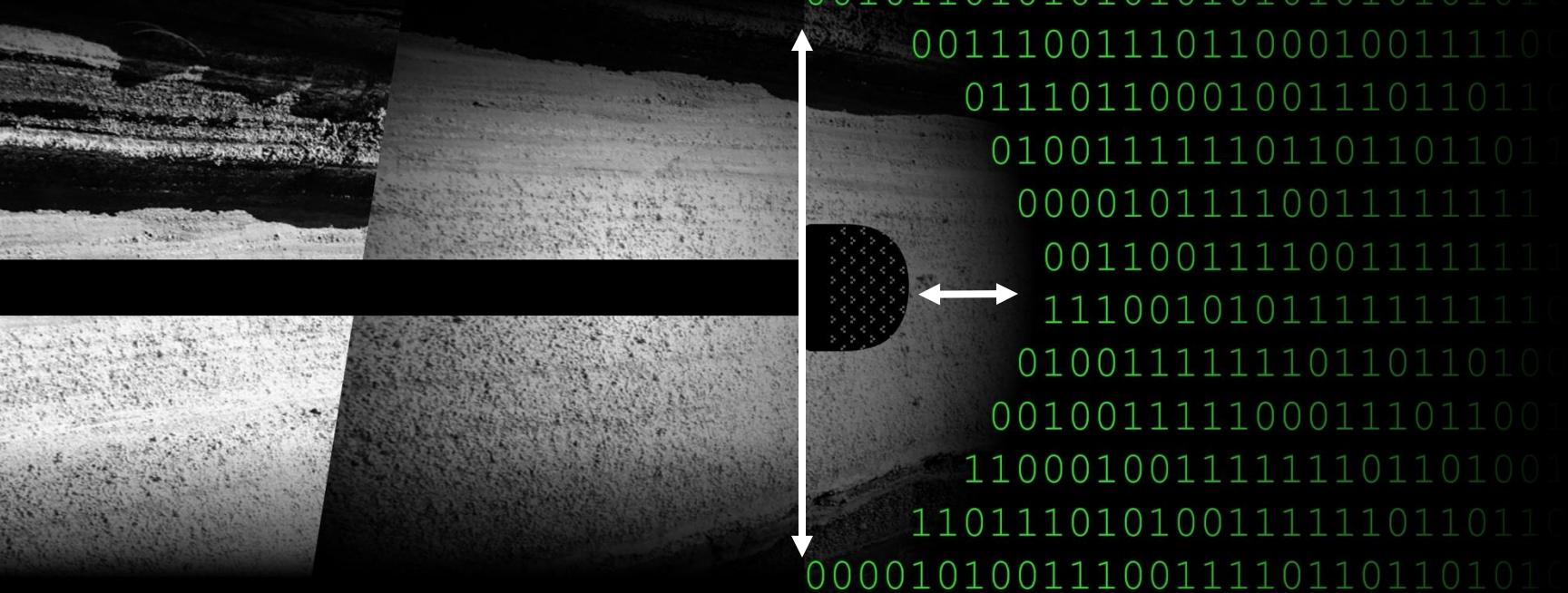
How do you use this inversion?

- ???

Reality (simplified)



Our project: DISTINGUISH



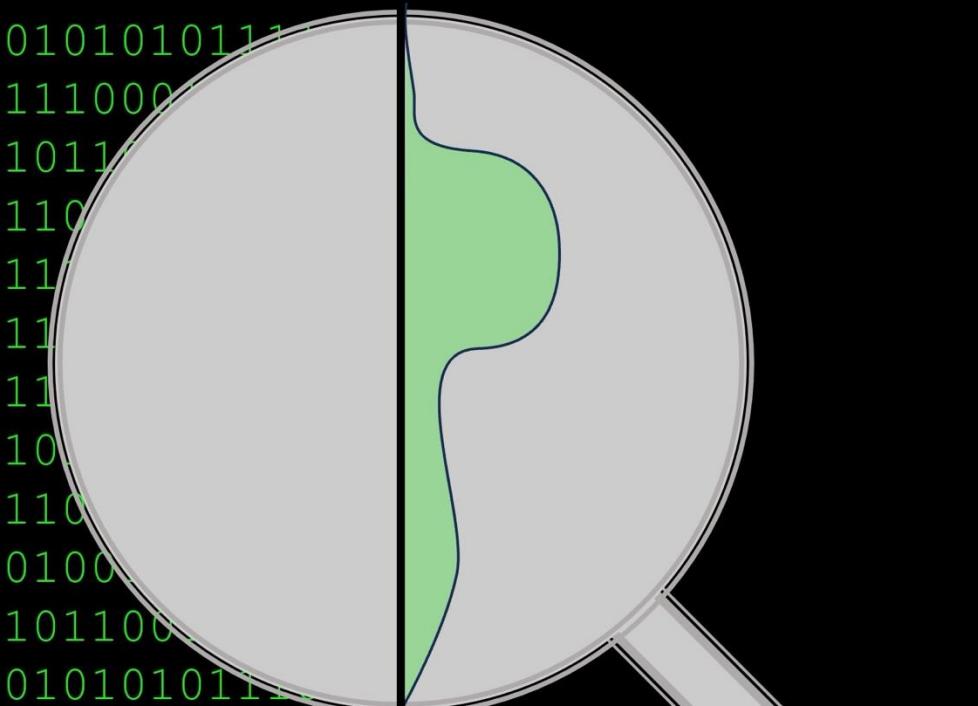
Our project: DISTINGUISH converts data to prediction



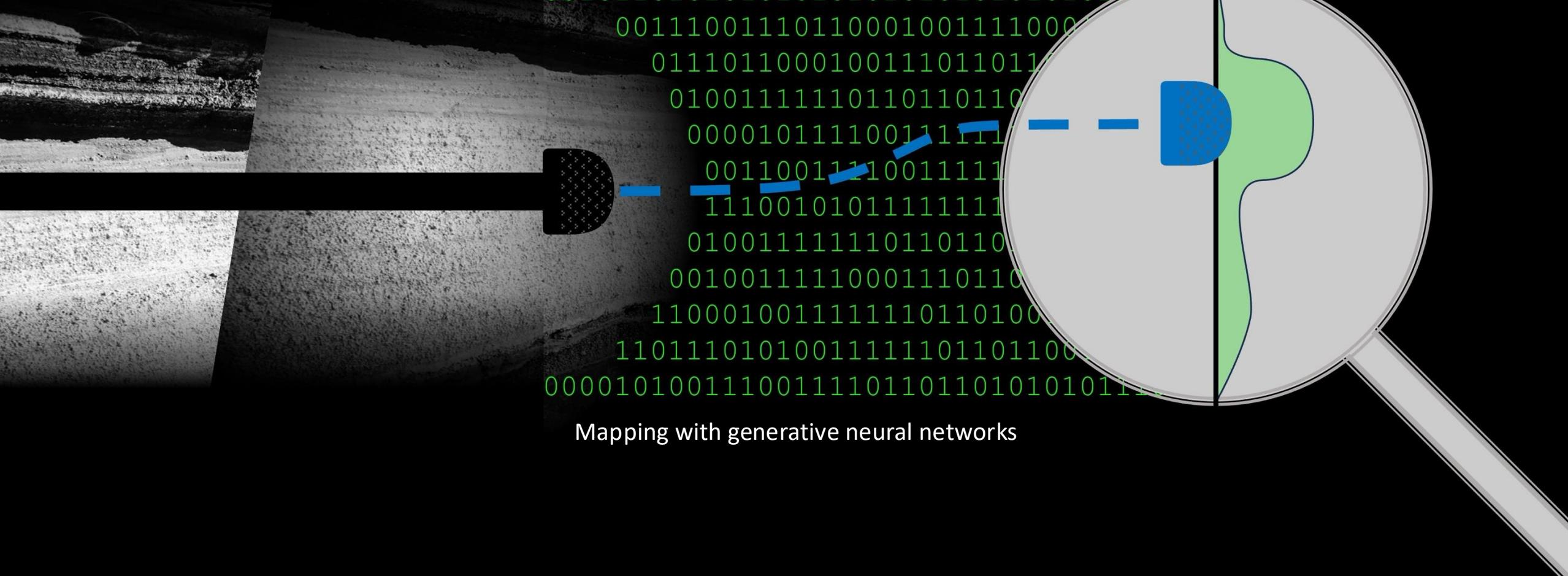
We want Probabilistic decision-support



001011010101010101010101010101010111
001110011101100010011110001
0111011000100111011011
0100111110110110110
000010111100111111
00110011110011111
11100101011111111
01001111110110110
00100111110001110110
1100010011111110110100
11011101010011111101101100
00001010011100111101101101010111



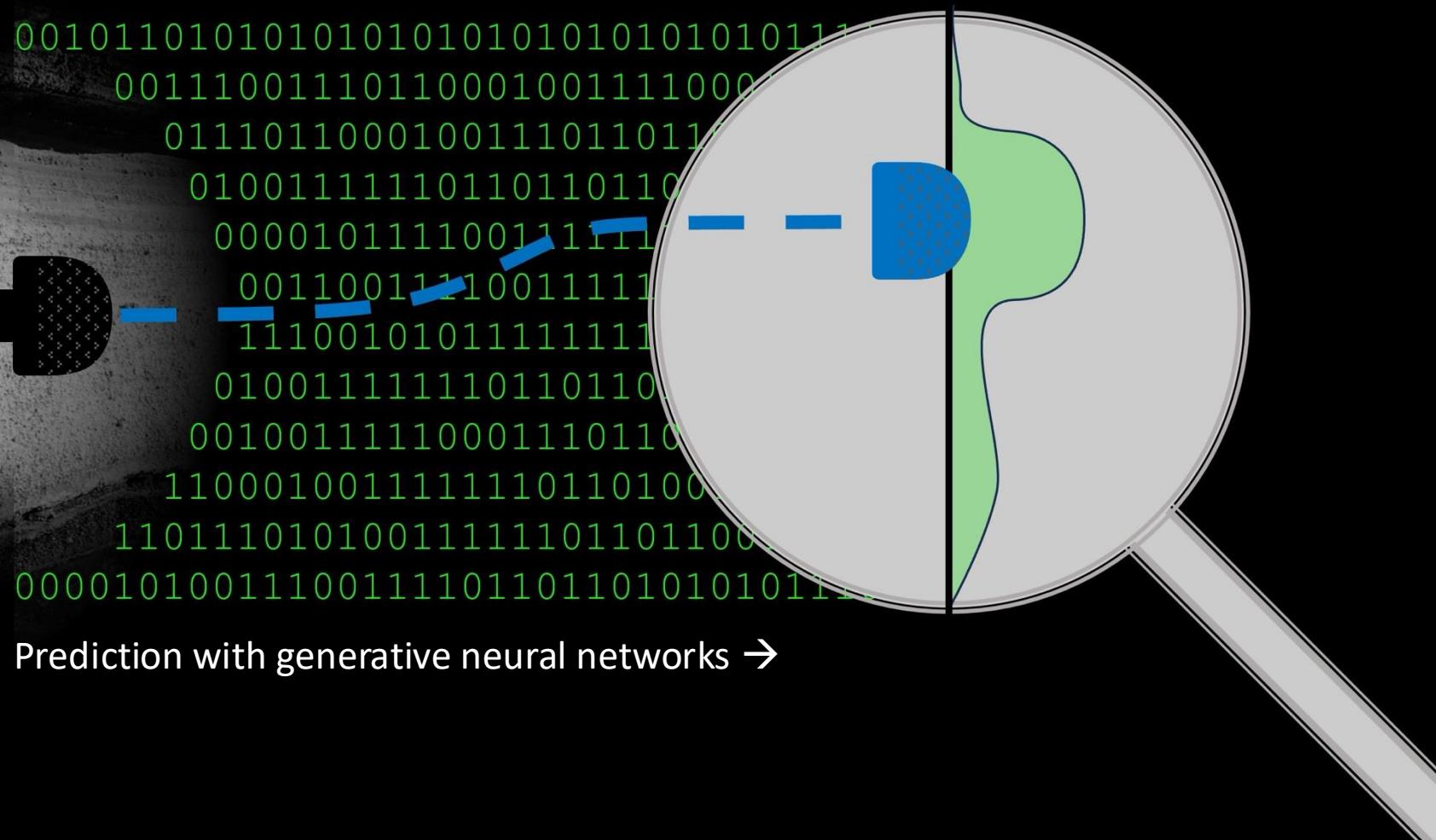
We want Probabilistic decision-support



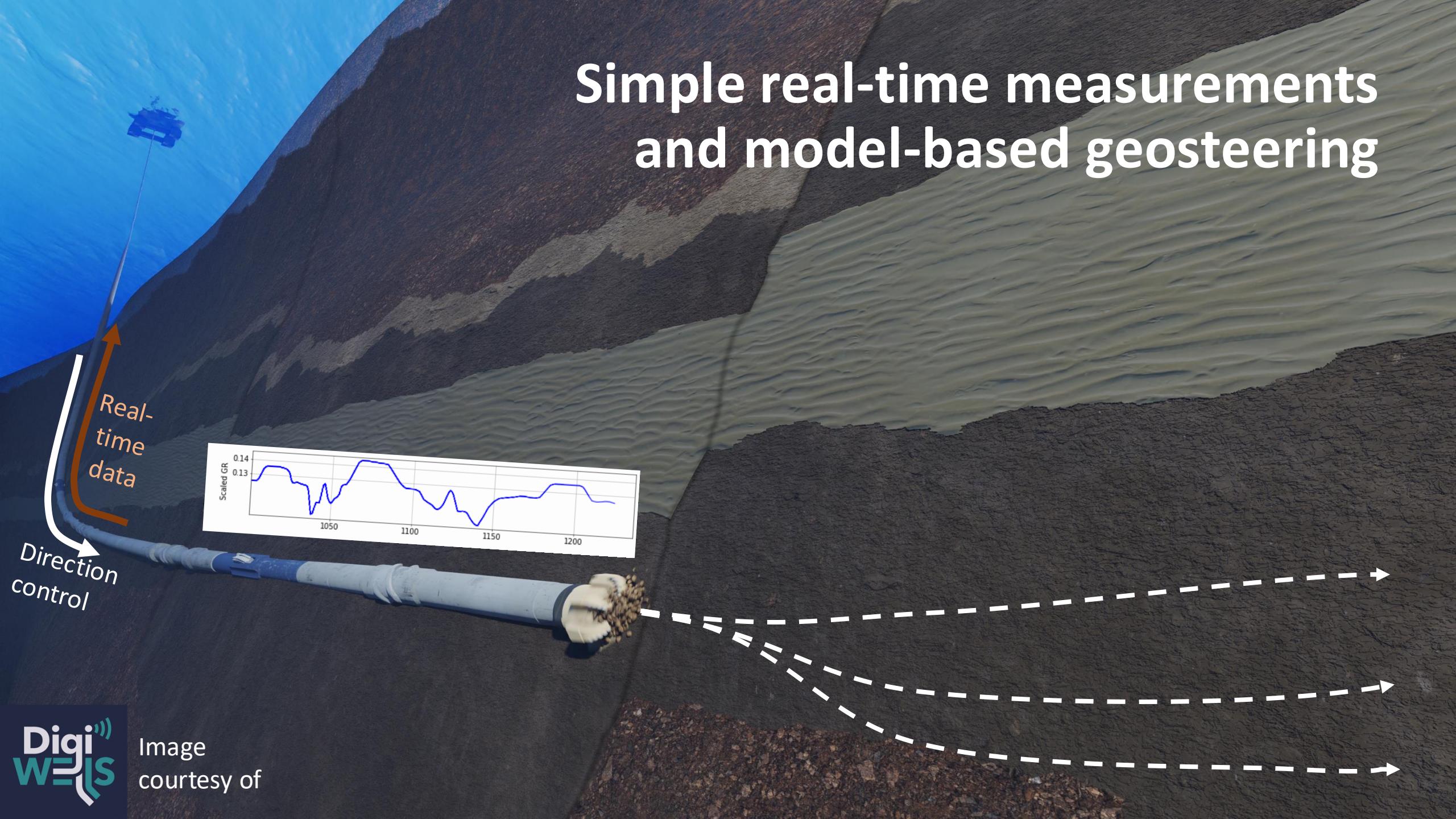
DISTINGUISH: Probabilistic decision-support

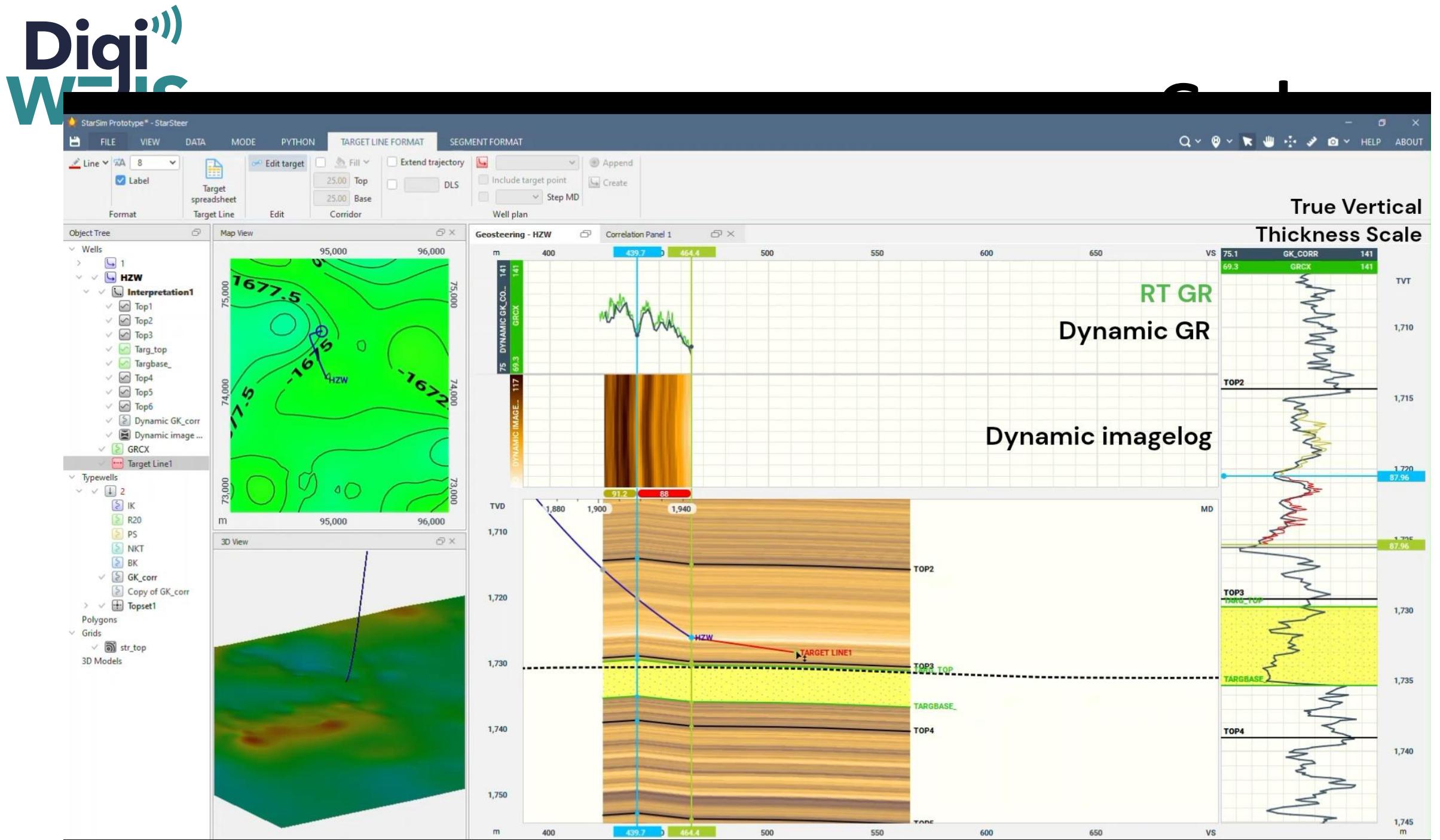


Well with look-around information

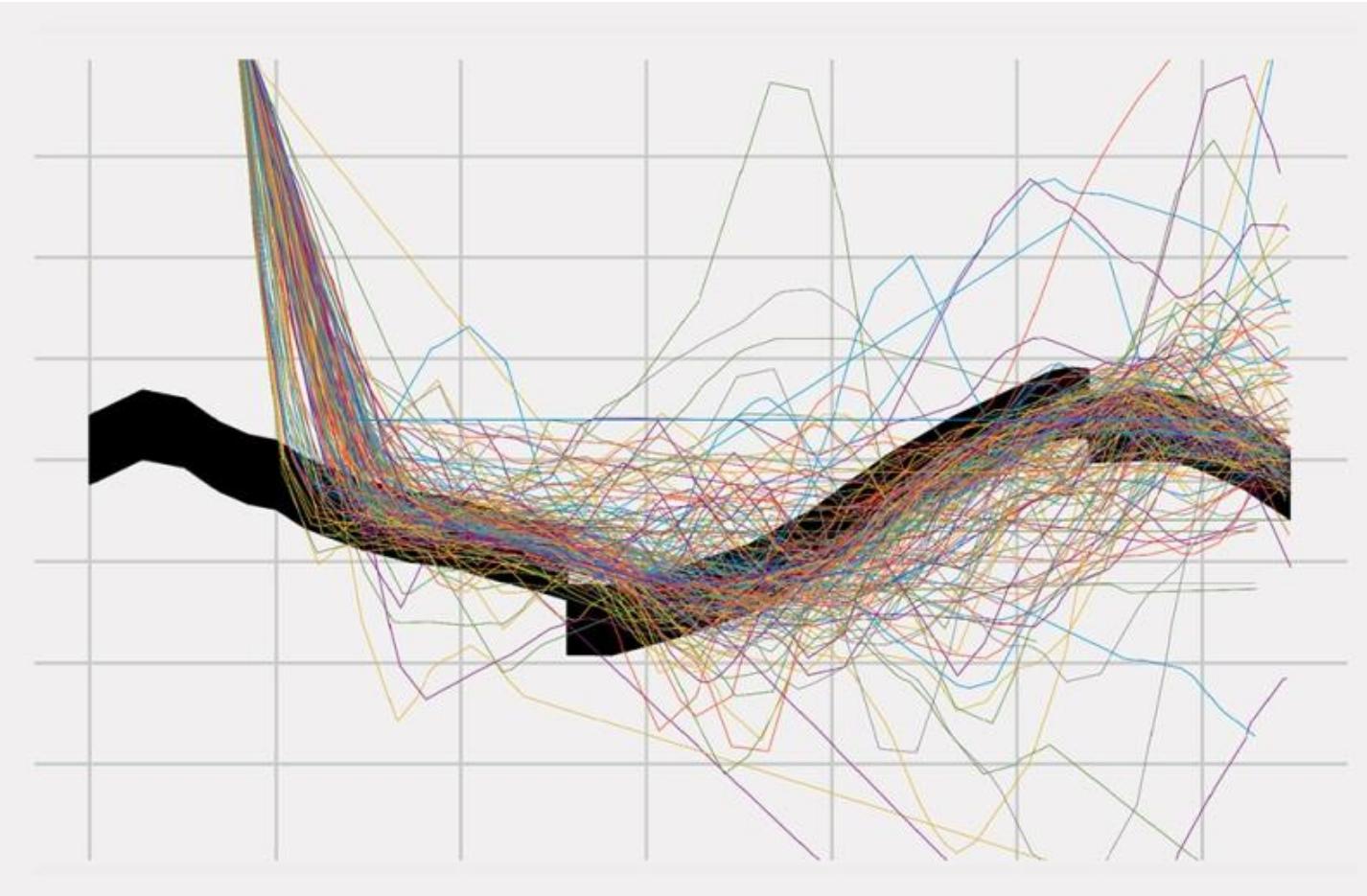


Simple real-time measurements and model-based geosteering

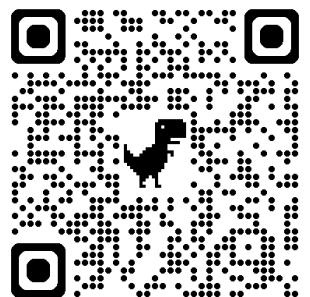




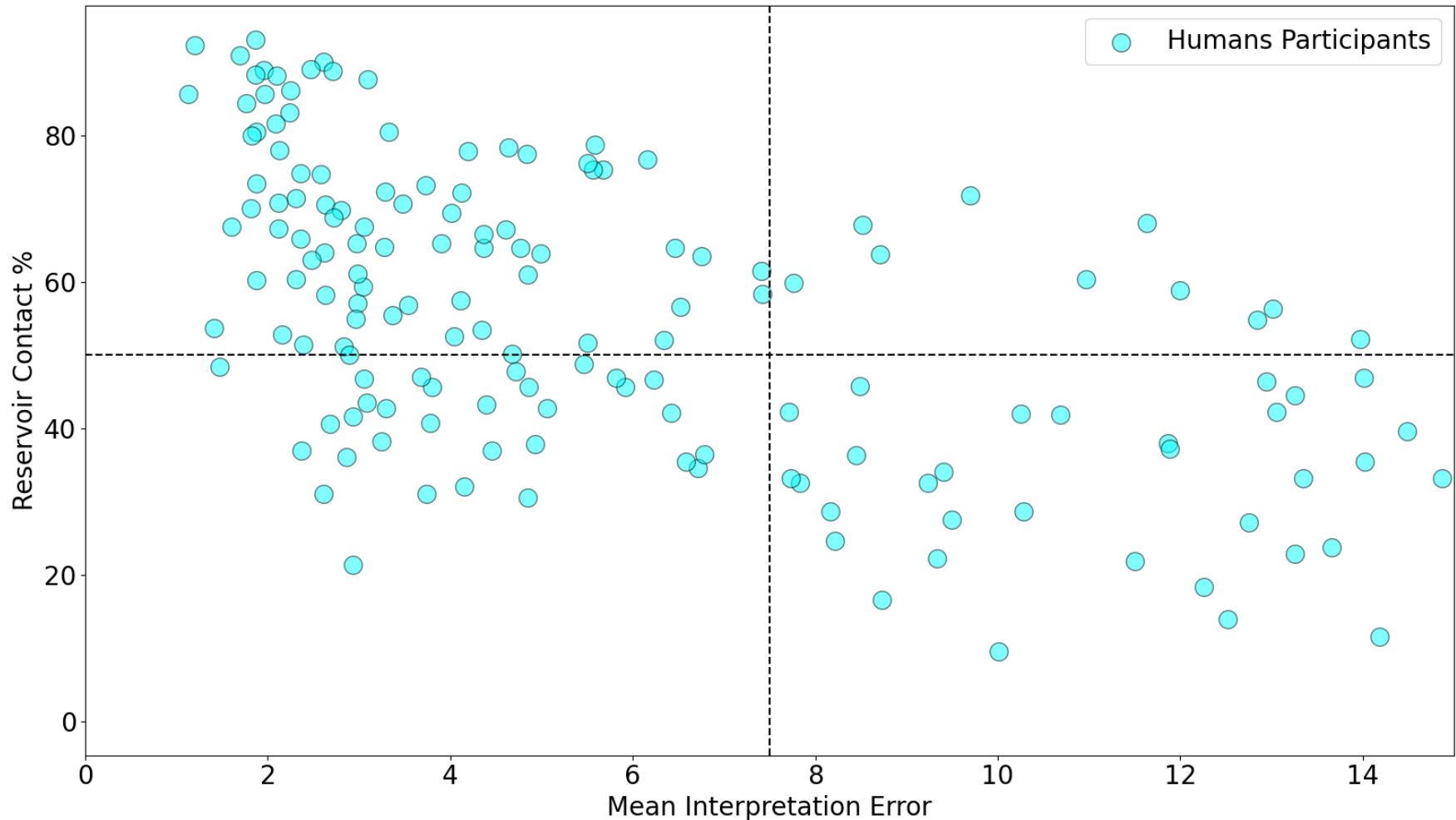
Geosteering decisions are challenging



[What Can We Learn After 10,000 Geosteering Decisions? | SPE/AAPG/SEG
Unconventional Resources Technology Conference | OnePetro](#)



The decisions depend on interpretation



Geological Interpretation

- the science (and art) of inferring the geology at some depth from data

Inverse problem

- (With cleverly defined regularizations and constraints)

Inverse problem

Geology with ideal thin layer cake



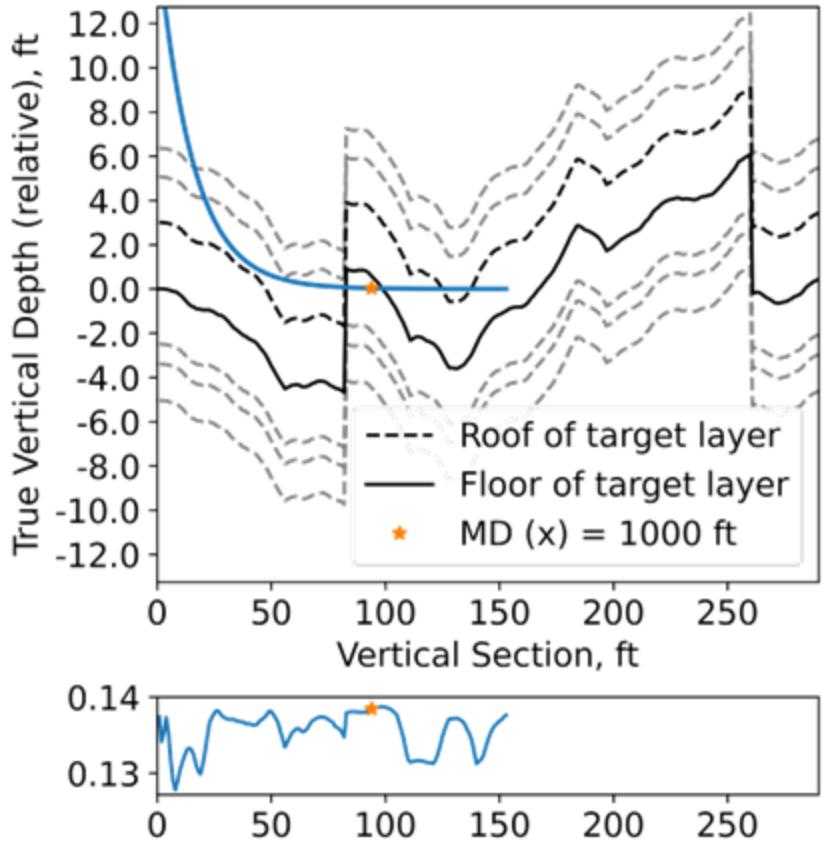
Layer cake with complex boundary



Inverse problem formulation

The most important slide

Find unknown geological strata



Assumptions:

- Layers have fixed thickness
- We can correlate to an offset well log

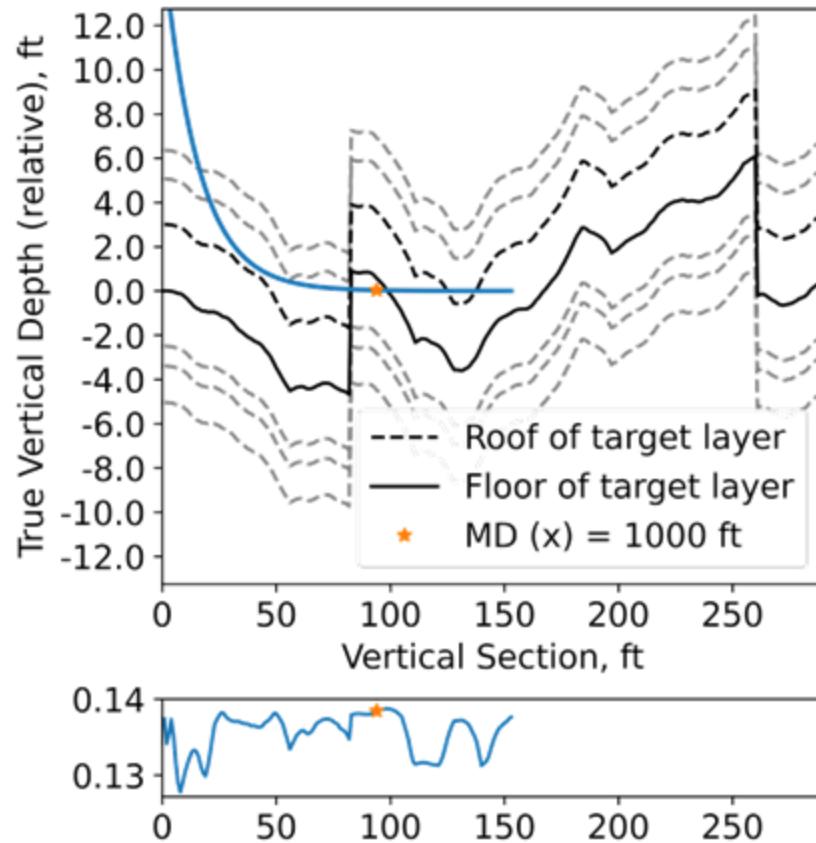
Stratigraphy can contain

- Varying dips (inclinations)
- Vertical faults

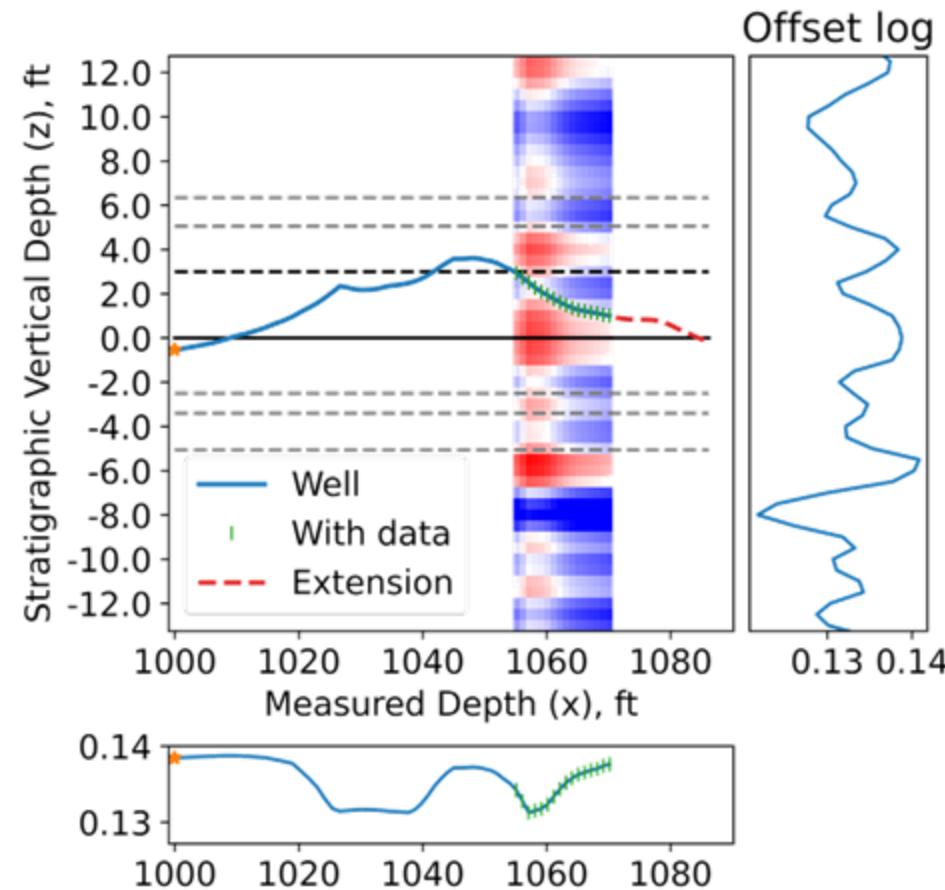
Inverse problem formulation

The most important slide

Find unknown geological strata

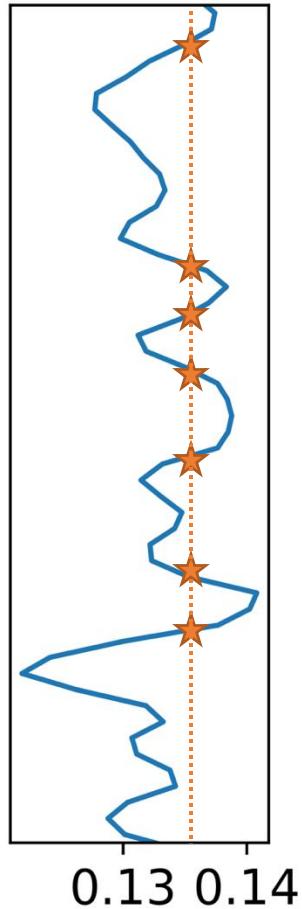


Find the well trajectory's **Boundary function**



Interpretations
of a single data point

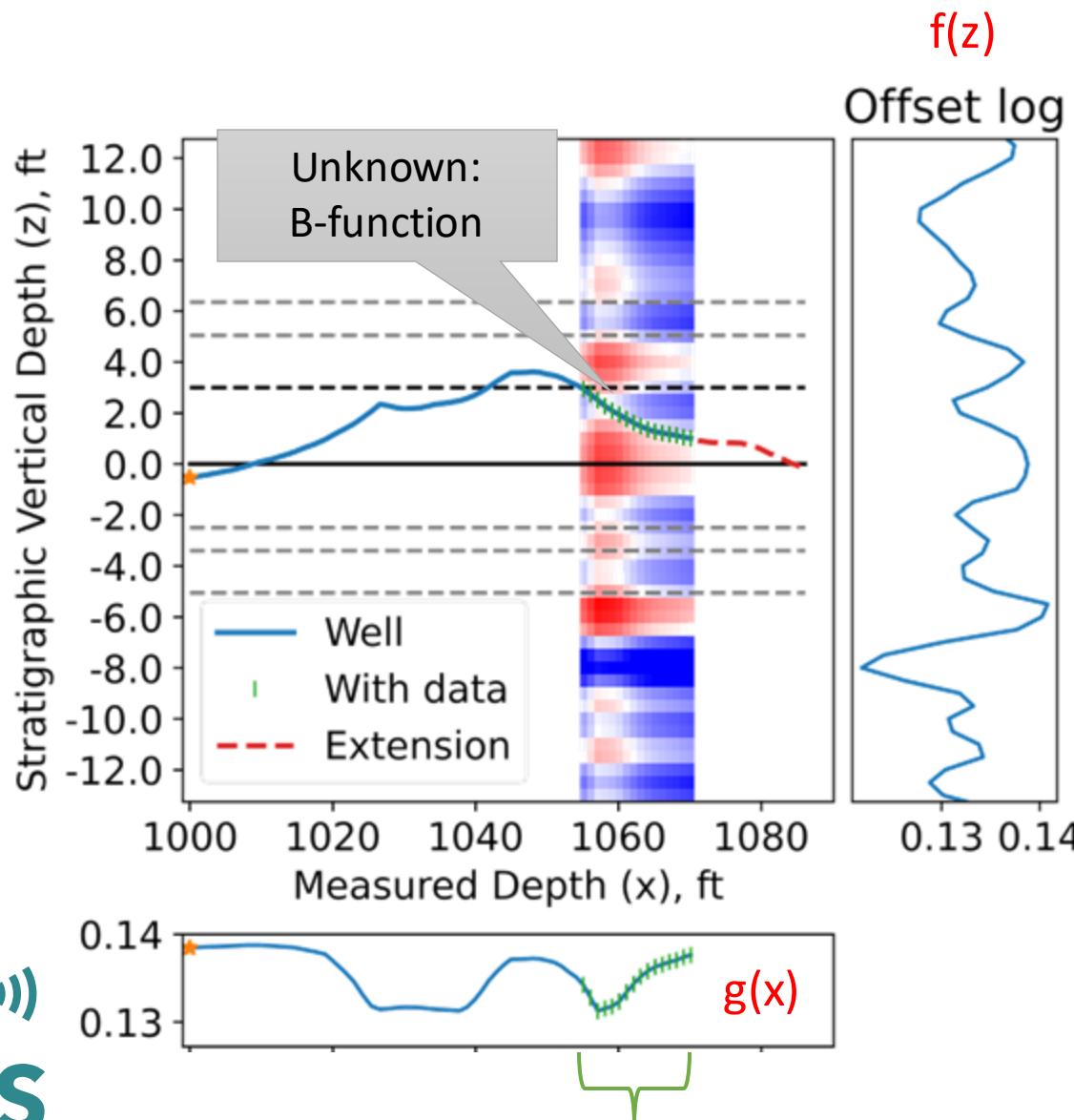
Offset log



Difficulty with low-data geosteering

- Log data has multiple depth interpretations
- Picking one interpretation consistent with geology is difficult

Another way of visualization



Find the well trajectory's **Boundary function** representing relative geology

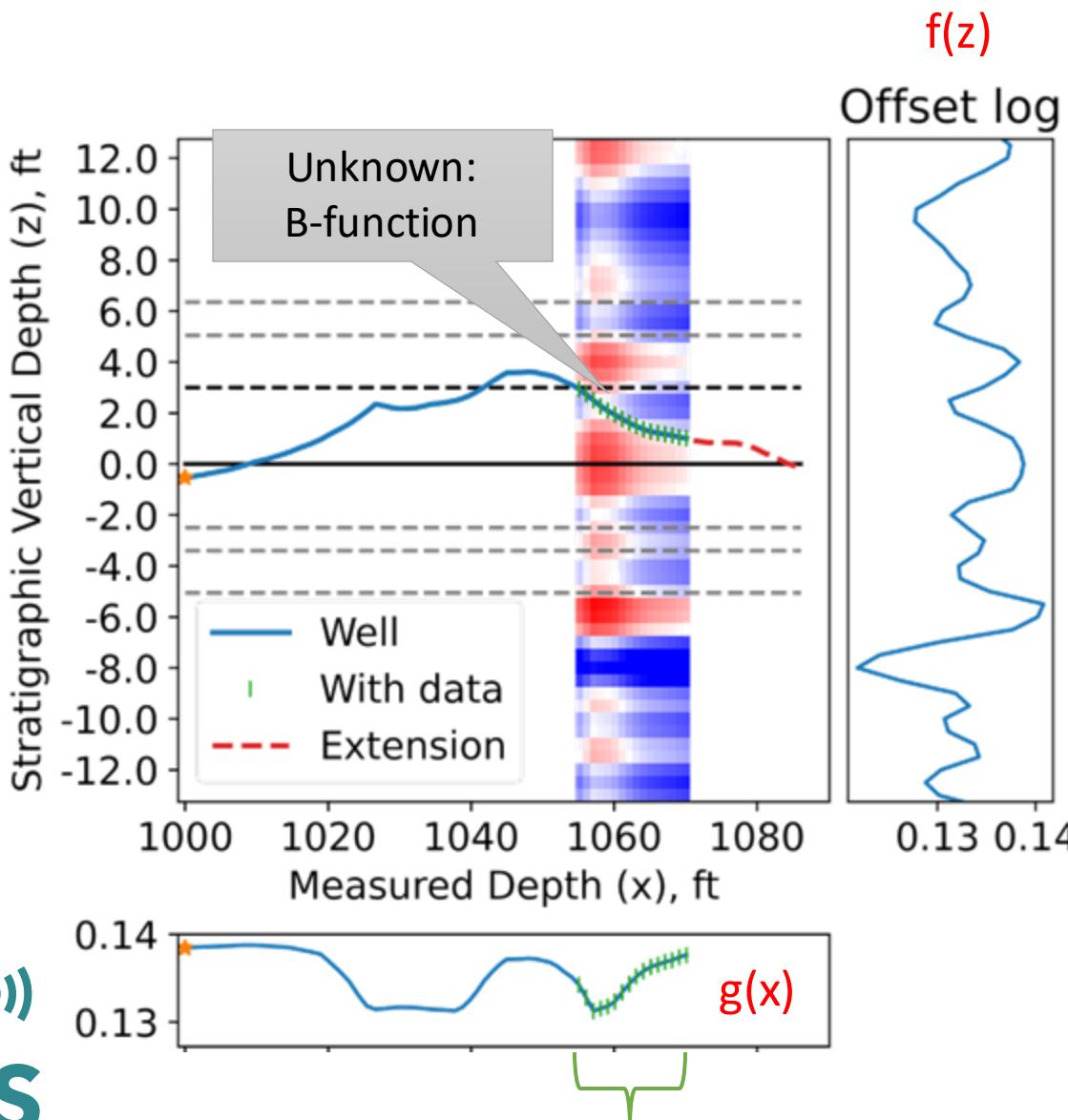
$$\begin{aligned} &\text{Find } b(x) \\ &f(b(x)) = g(x) \end{aligned}$$

$b(x)$ – boundary function
 $f(z)$ – offset (reference) measurements
 $g(x)$ – actual measurements

Inputs
Offset well log
Current well log

→ Heatmap image

Another way of visualization



Find the well trajectory **Boundary function** representing relative geology

Find $b(x)$

$$h_v(f(b(x))) = h_l(g(x))$$

$b(x)$ – boundary function

$f(z)$ – offset (reference) measurements

$g(x)$ – actual measurements

In general, there might be
a measurement function

h_v, h_l – direction-dependent
measurement functions

Inputs

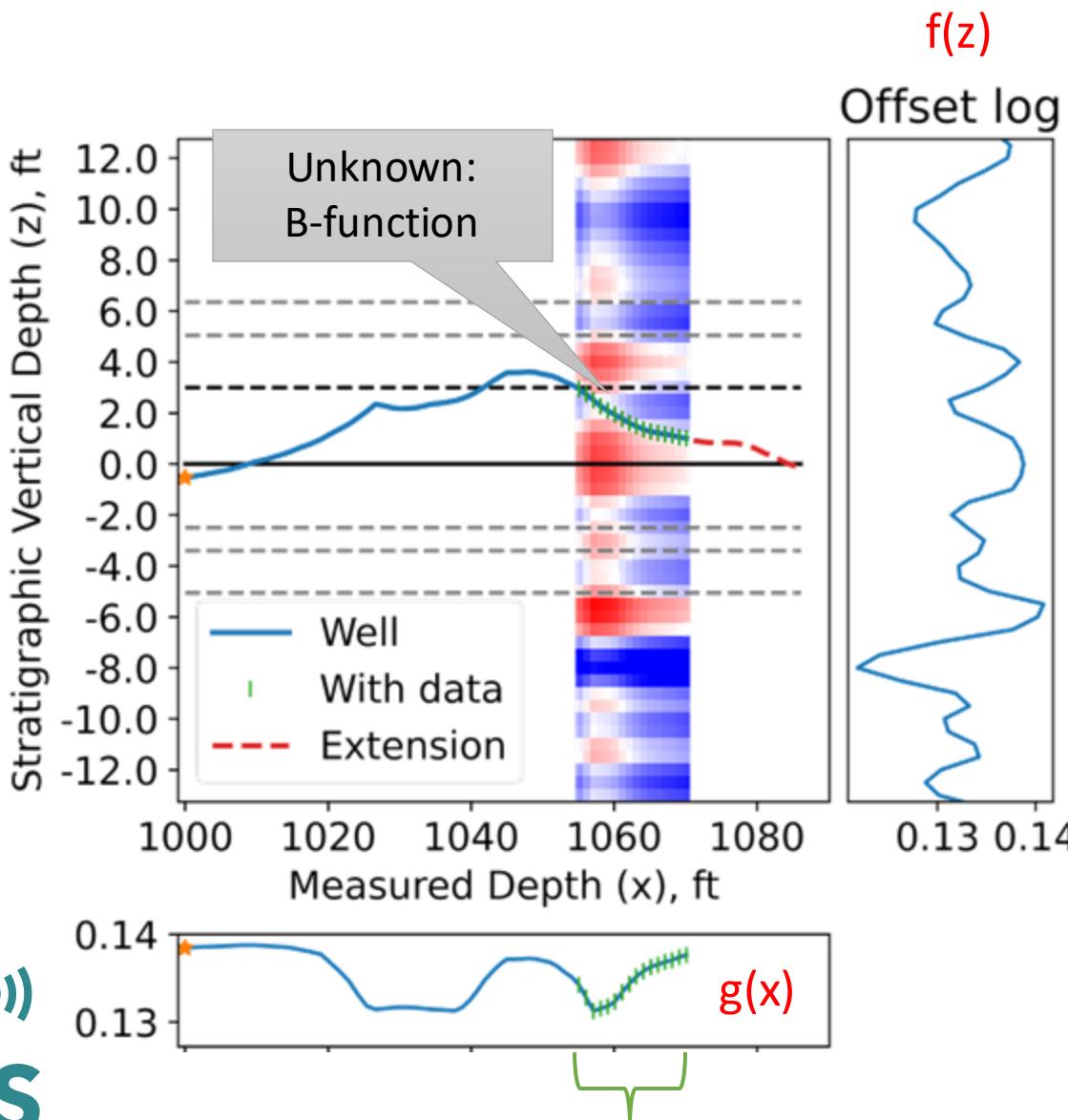
Offset well log

Current well log



Heatmap
image

Another way of visualization



Find the well trajectory's
Boundary function
representing relative geology

Heatmap image is useful to understand the data:

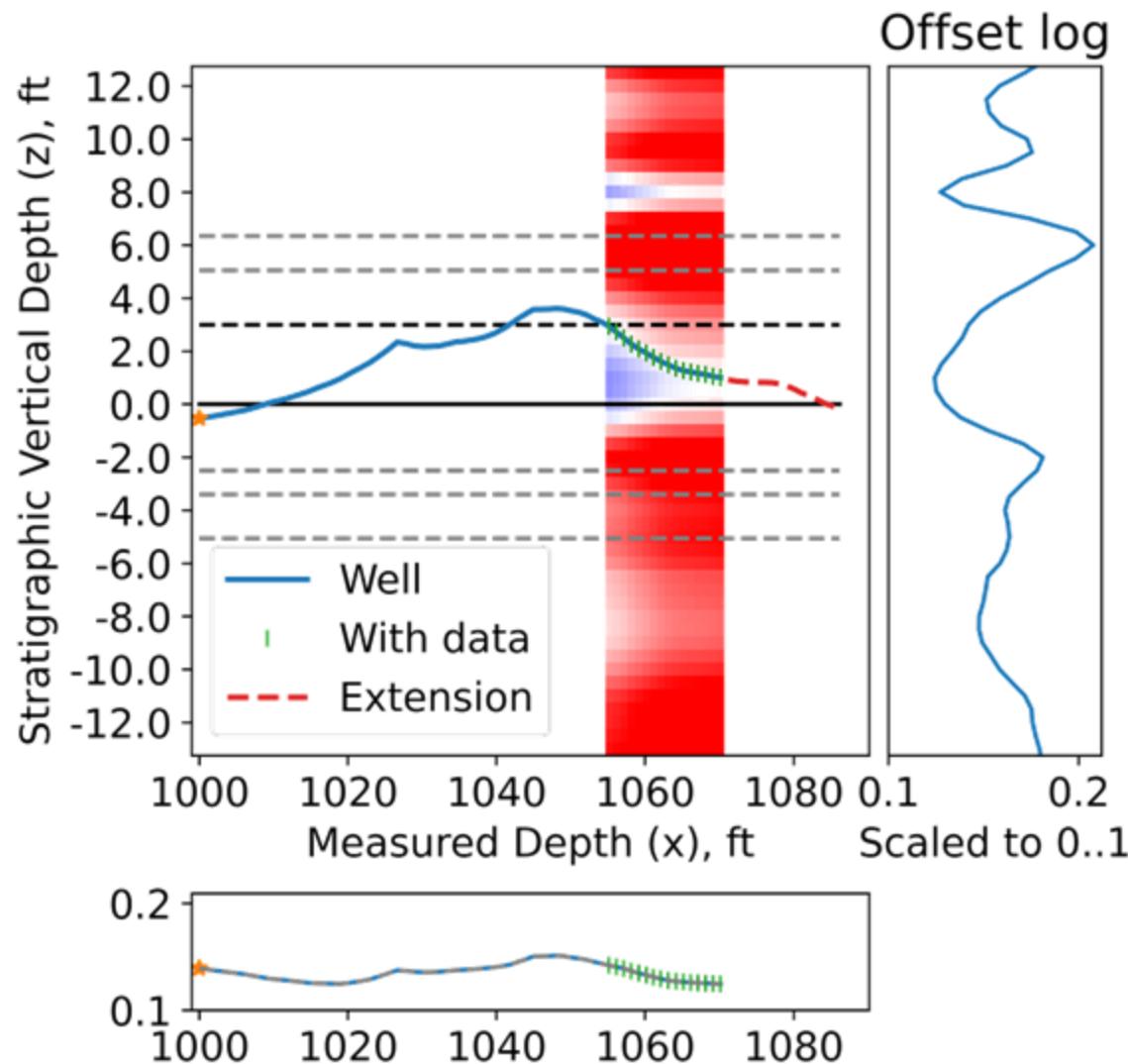
$$H(x, z) = f(z) - g(x)$$

Find $b(x)$
 $H(x, b(x))=0$

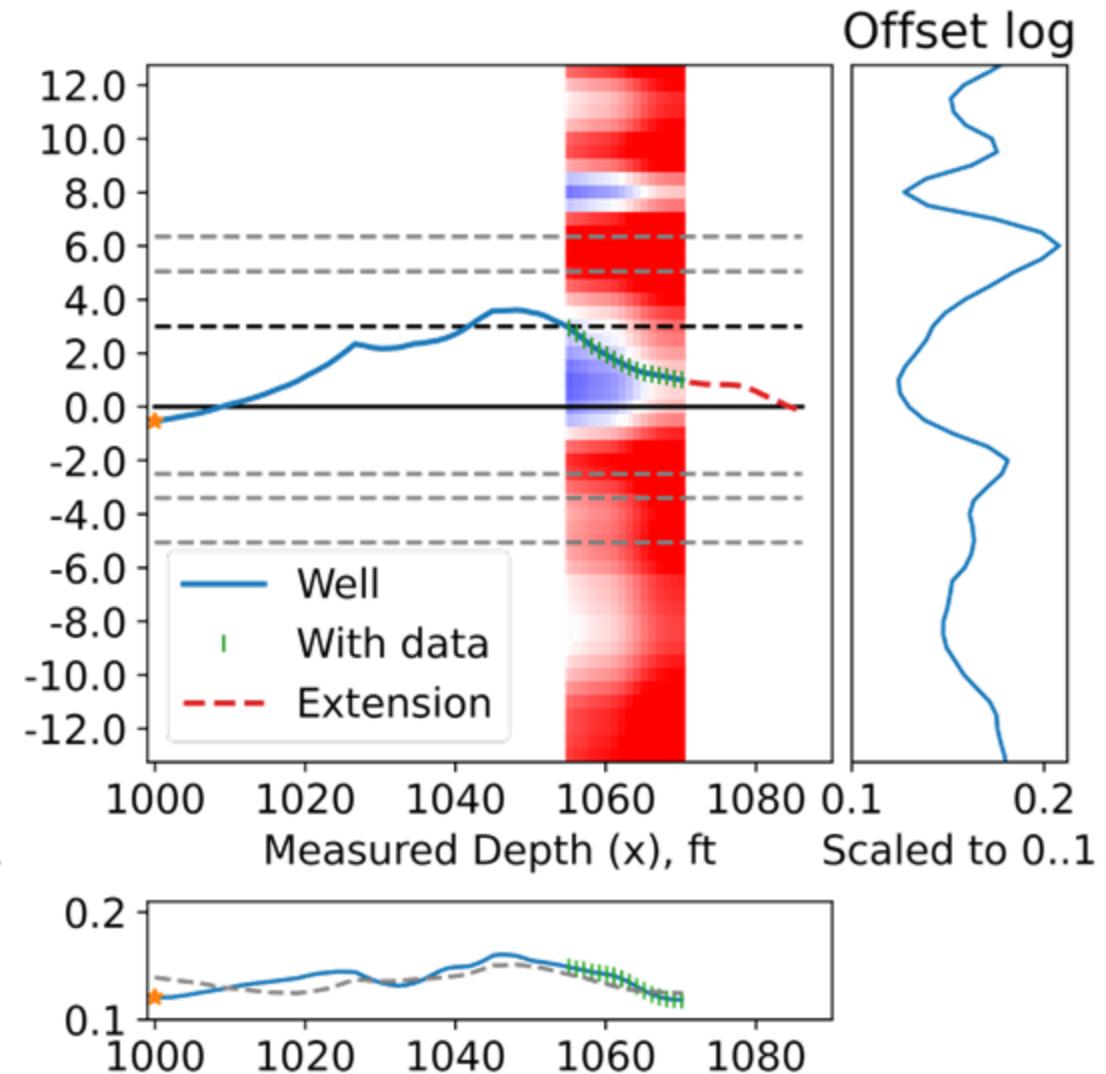
Inputs
Offset well log
Current well log

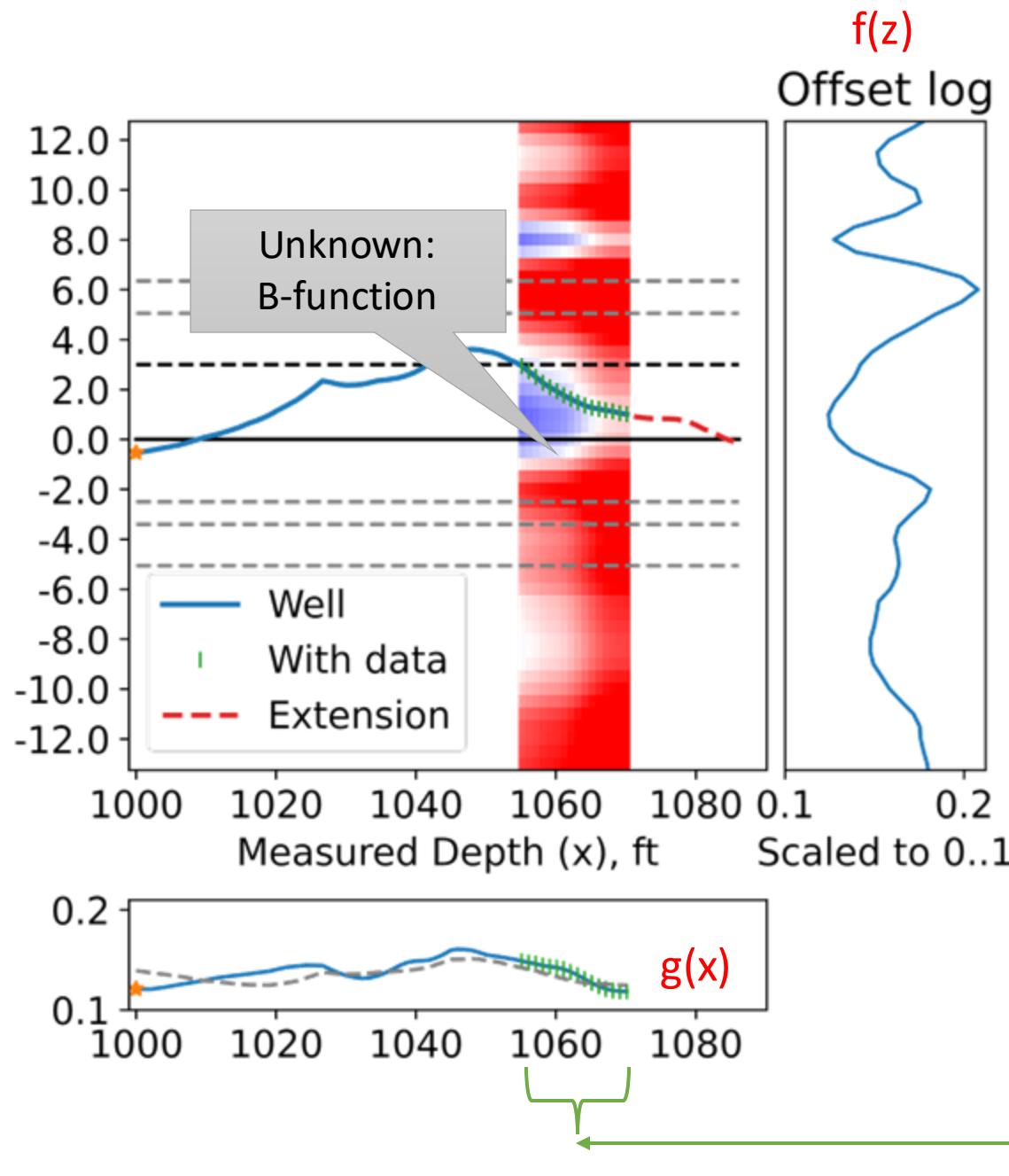
→ Heatmap
image

No noise



With noise





Boundary function to boundary

- $DZ_{geo}(x) = Z(x) - b(x)$
 - DZ – relative geological depth
 - $b(x)$ – boundary function in the inverse problem
 - $Z(x)$ – lateral well's depth
- For exercise purposes
 - $Z(x) = 0$
 - The well is horizontal

Other domains with fitting to reference data

- Autonomous vehicle positioning
- ???

The toy inverse problem

$$b^*(x) = \underset{b(x)}{\operatorname{argmin}}(\|f(b(x)) - g(x)\|_2^2)$$

Linear approximation:

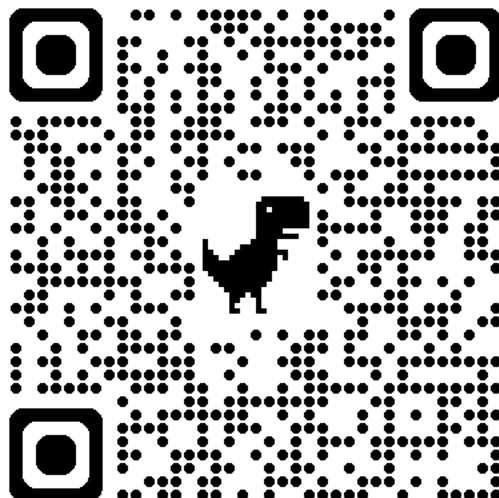
- $b(x) = a^*x + c$

Sequential inverse problem

- Find a 1D boundary function representing 1D geology
- We know a solution
 - $b_{i-1}(x)$ from x_{i-1} to x_i
- Find B function $b_i(x)$
 - $f(b_i(x)) = g(x)$
- For x from x_i to x_{i+1}
 - $b_i(x_i) \approx b_{i-1}(x_i)$
 - b is reasonably continuous unless geological faults
- b is “reasonable” geologically
 - $t(x)$ – coarse scale geological trend

What is reasonable geologically?

- As a mathematician I would like to know that too...
- Data competition for generating 1D geology on Kaggle:
 - Small monetary prizes
 - Possibility to publish your work in a benchmark journal article
 - Submission deadline in June 2025



<https://www.kaggle.com/competitions/geology-forecast-challenge/overview>

What we solve

$$b_i^*(x) = \operatorname{argmin}_{b_i(x)} (\|f(b_i(x)) - g_i(x)\|_2^2 f(b_i(x)))$$

With slow geological variation, start with linear or piecewise linear

- $b_i(x) = a_i x + c_i \quad x \text{ in } x_i \dots x_{i+1}$

Additional data for regularization given multi-modality:

- Your previous solution was $b_{i-1}^*(x)$, $x \text{ in } x_{i-1} \dots x_i$
- Coarse scale trend is $t(x)$

Better linear form

- $b_i(x) = a_i x + c_i$

Parametrized with data points:

- $b_i(x) = (z_{i+1} - z_i)/(x_{i+1} - x_i) * (x - x_i) + z_i$

Data-related parameters z_i
simplify formulating regularization and constraints

Alternative function

Piecewise linear within each interval:

- $b_i(x) = a_{i_1}x + c_{i_1}$ x in $x_{i-1} \dots x_*$
- $b_i(x) = a_{i_2}x + c_{i_2}$ x in $x_* \dots x_i$

B-spline

- $b_i(x) = p_n(x)$
- $p_n(x)$ is polynomial of degree n

Inverse solvers in scipy

curvefit

- Inputs
 - $f(b_i(x))$
 - $g_i(x)$
- Solves
 - Find p , $f(b_i(x; p)) \approx g_i(x)$
 - $p = \operatorname{argmin} \|f(b_i(x; p)) - g_i(x)\|_2^2$
- Parameterers
 - initial guess
 - Parameter constraints

See also underlying methods

- `least_squares`
- `leastsq`

These methods use gradient decent to solve non-linear least-squares problem

*The geological inversion is definitely not convex

Tikhonov Regularization

Fit to data:

$$\bullet p = \operatorname{argmin} \|f(b_i(x; p)) - g_i(x)\|_2^2$$

Fit to beliefs (prior):

$$\bullet p = \operatorname{argmin} \|b_i(x; p) - \tilde{b}(x)\|_2^2$$

Regularized problem:

$$\bullet p = \operatorname{argmin} \left(\|f(b_i(x; p)) - g_i(x)\|_2^2 + \kappa \|b_i(x; p) - \tilde{b}(x)\|_2^2 \right)$$

In the context of probabilistic estimation

$$p = \operatorname{argmin}_p \left(\|f(b_i(x; p)) - g_i(x)\|_2^2 + \kappa \|b_i(x; p) - \tilde{b}(x)\|_2^2 \right)$$

- Tikhonov solution is **Maximum A Posterior (MAP)** of probabilistic solution, if
- $\kappa = \sigma^2 / \gamma^2$
 - σ^2 - variance of measurement errors w
 - γ^2 - variance (uncertainty) in beliefs errors w

NEW: Reformulate using `least_squares`

- Input is the function that returns vector of residuals
 - $\|f(b_i(x; p)) - g_i(x)\|_2^2 + \kappa \|b_i(x; p) - \tilde{b}(x)\|_2^2$
- Equivalent to:
 - $f_k(x) = f(b_i(x_k; p)) - g_i(x_k), \quad k < n = \text{len}(x)$
 - $f_k(x) = \sqrt{\kappa} (b_i(x_{k-n}; p) - \tilde{b}(x_{k-n})), \quad k \geq n = \text{len}(x)$
- Starting guess vector
 - $param_0$ (x_0 in docs)
- Check auxiliary variables
- Check minimize function
- `minimize F(param) = 0.5*sum(rho(f_k(param)**2), k = 0, ..., m - 1)`
subject to `lb <= param <= ub`

Reformulate using `minimize`

- Input is the minimization functional
 - $\|f(b_i(x; p)) - g_i(x)\|_2^2 + \kappa \|b_i(x; p) - \tilde{b}(x)\|_2^2$
- Starting guess vector
 - p_0
- Check auxiliary variables
- Check minimize function

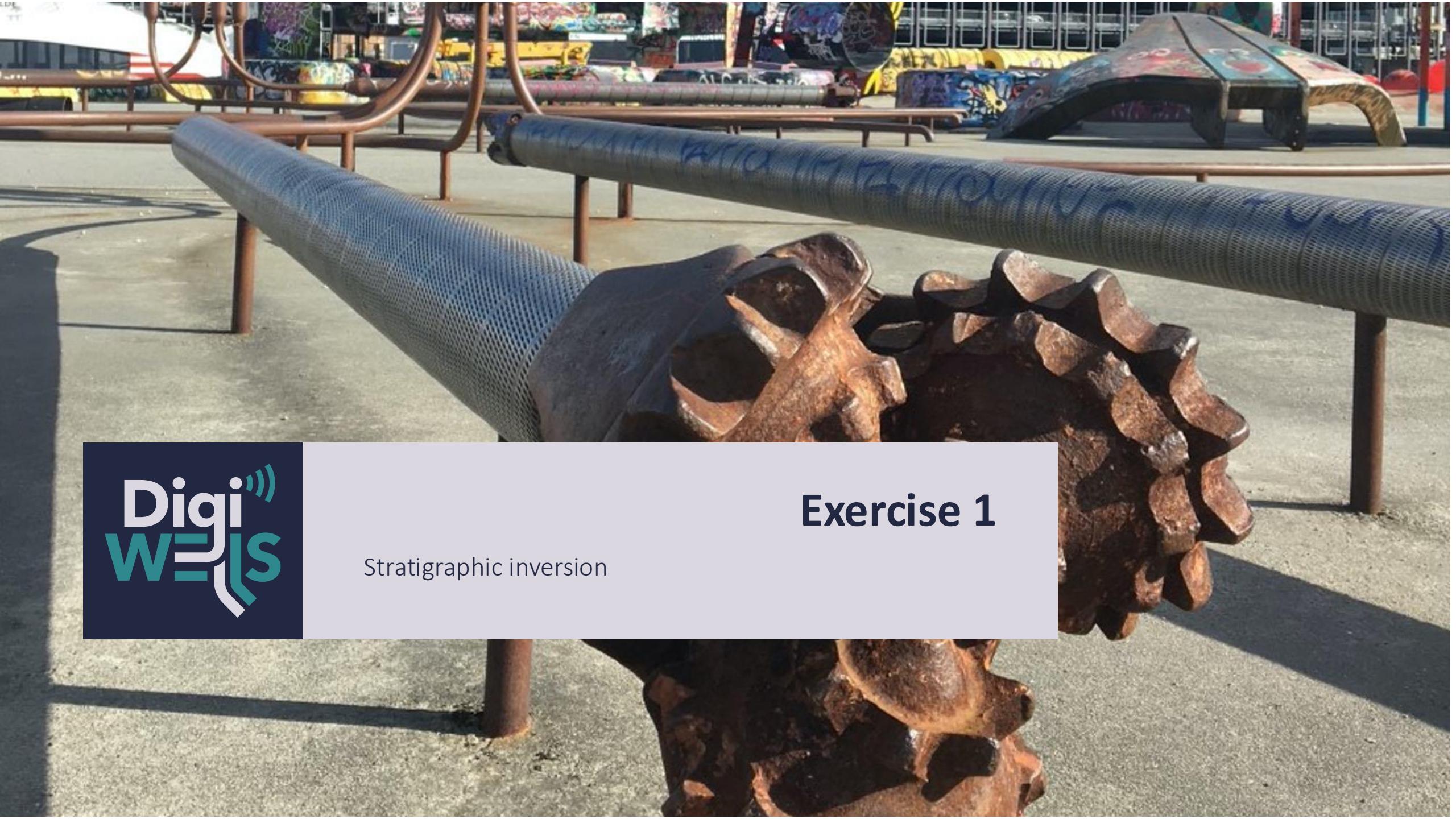
Learnings

- Overview of geosteering as a sequential inverse problem
- Formulation of stratigraphic interpretation
- Basic preparation for exercise



Stratigraphic inversion

Exercise 1



Expected Learnings

- Hands-on experience with an optimization libraries for inverse problems: `scipy`
- Representing simple inverse problem as curve fitting
- Ability to formulate an inverse problem with regularizations and constraints

Expected Prize

not distributed to

Candies ~~taken from~~ kids on Halloween

Github link

- https://github.com/geosteering-no/inversion_school_geosteering



Scipy links

- https://docs.scipy.org/doc/scipy-1.15.0/reference/generated/scipy.optimize.curve_fit.html
- <https://docs.scipy.org/doc/scipy-1.15.0/reference/generated/scipy.optimize.leastsq.html#scipy.optimize.leastsq>
- https://docs.scipy.org/doc/scipy-1.15.0/reference/generated/scipy.optimize.least_squares.html#scipy.optimize.least_squares
- <https://docs.scipy.org/doc/scipy-1.15.0/reference/generated/scipy.optimize.minimize.html>