Rutgers University
CS440: Introduction to Artificial Intelligence, Spring 2017
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Homework 3: James Carroll and Joel Carrillo

Bayesian Inference, Temporal State Estimation and Decision Making under Uncertainty

1. Question 1
   (a) i. $P(A, B, C, D, E) =$
   ii. $P(A)P(B)P(C)P(D|A, B)P(E|B, C) =$
   iii. $(0.2)(0.5)(0.8)(0.1)(0.3) = 0.0024$
   (b) i. $P(\neg A)P(\neg B)P(\neg C)P(\neg D)P(\neg E) =$
   ii. $P(\neg A)P(\neg B)P(\neg C)P(\neg D|\neg A, \neg B)P(\neg E|\neg B, \neg C) =$
   iii. $(1 - 0.2)(1 - 0.5)(1 - 0.8)(1 - 0.9)(1 - 0.2) =$
   iv. $(0.8)(0.5)(0.2)(0.1)(0.8) = 0.0064$
   (c) i. $P(\neg A|B, C, D, E) = \frac{P(\neg A,B,C,D,E)}{P(\neg A,B,C,D,E)+P(A,B,C,D,E)}$
   ii. $P(\neg A, B, C, D, E) = P(\neg A)P(B)P(C)P(D|\neg A, B)P(E|B, C) =$
   iii. $(0.8)(0.5)(0.8)(0.6)(0.3) = 0.0576$
   iv. $\frac{0.0576}{0.0576+0.0024} = 0.96$
2. Question 2
   (a) $P(Burglary|JohnCalls = true, MaryCalls = true)$
   Factors:$P(Burglary), P(EQ),$ $P(Alarm|EQ, Burglary)$,$P(JohnCalls|Alarm)$, $P(MaryCalls|Alarm)$
   Elimination Order: EQ, Alarm

   $f_1(Alarm, Burglary) = \sum_{EQ} P(EQ)P(Alarm, EQ, Burglary)$
   $f_2(John, Mary, Alarm, EQ) = \sum_{Alarm} f_1(Alarm, Burglary)P(John|Alarm)P(Mary|Alarm)$
   $P(Burglary|JohnCalls, MaryCalls) = P(Burglary)f_2(John, Mary, Alarm, Burglary)$

   Normalize the above to find $P(Burglary|JohnCalls = true, MaryCalls = true)$
   (b) Approximately 9 operations, including the division for a. Enumerating it, meanwhile, would require 15 operations.
   (c) For variable enumeration, the complexity should be $O(2^n)$. Due to the structure of the network, the complexity for variable elimination should be similarly high.
3. Question 3
   (a) i. By definition: $P(X|MB(X)) = P(X|\{U_1, ..., U_m\}, \{Y_1, ..., Y_n\}, \{Z_1, ..., Z_j\})$

ii. FJPD for parents: $P(Y_i, ..., Y_n) = \prod_{i=1}^{n} P(Y_i|Z_{i1}...)$
iii. End goal: $\alpha P(X|U_1, ..., U_m) \prod_{Y_i} P(Y_i|Z_{i1}...)$

(b) Four states: {{Cloudy=True,Rain=True}, {Cloudy=True,Rain=False}, {Cloudy=False,Rain=True},{Cloudy=False,Rain=False}}

(c) Q =

$$\begin{bmatrix} .5 & 1 & .80 & 1 \\ .05 & 1 & .08 & 1 \\ .4 & 1 & .4 & 1 \\ .5 & 1 & .8 & 1 \end{bmatrix}$$

4. Question 4

(a) $(.7 * 4000) - (.3 * 1400) - 3000 = -620$

(b) $P(Pass) = P(Pass \wedge q^+) \vee P(Pass \wedge q^-) = P(Pass|q^+) + P(Pass|q^-) =$
$(0.8 * 0.7) + (0.35 * 0.3) = 0.655$
$P(\neg Pass|q^+) = .2, P(\neg Pass|q^-) = .65$
$P(q^+|Pass) = P(Pass|q^+) * P(q^+)/P(Pass) = 0.8 * 0.7/0.665 = 0.842$
$P(q^+|\neg Pass) = P(\neg Pass|q^+) * P(q^+)/P(\neg Pass) = 0.2 * 0.7/0.335 = 0.418$
$P(q^-|Pass) = P(Pass|q^-) * P(q^-)/P(Pass) = 0.35 * 0.3/0.665 = 0.158$
$P(q^+|\neg Pass) = P(\neg Pass|q^+) * P(q^+)/P(\neg Pass) = 0.65 * 0.3/0.335 = 0.582$

(c) Utility of $Pass(c1) = (.842 * 4000) - (.158 * 1400) - 3100 = 46.8$
Utility of $\neg Pass(c1) = (.418 * 4000) - (.582 * 1400) - 3100 = -2242.8$
It is therefore better to buy the car if it passes, and better not to buy the car if it fails.

(d) As seen in (a) and (c), there is significant value to taking the car to the mechanic first. It should be done, as we can be assured we are likely making a wiser decision purchasing if the car passes, but we can also safely choose not to buy it if it fails. Not visiting the mechanic makes purchasing the car more unreliable, and spending 100 dollars on a failed car is better than risking a purchase that we are more uncertain of.

5. Question 5a

6. Question 5b

7. Question 5c

(a) 90-cell map starting from (5,4): Actions, Coords, Readings. (ID 1-4)

(b) LLDLDUDDUUDUDLULUUUUDDDLUUUULDLUDLLDULLDUUUDUUULUDLDDLLLDDUU

(c) (4,4),(3,4),(3,4),(2,4),(2,3),(2,4),(2,3),(2,3),(2,4),(2,4),(2,3),(2,4),(2,4),(1,4),(1,5),(0,5),(0,6),(0,7),(0,
,8),(0,8),(0,9),(0,9),(0,8),(0,8),(0,9),(0,8),(0,8),(0,8),(0,7),(0,8),(0,8),(0,8),(0,8),(0,9),(0,9),(0,9),(0,8),
),(0,6),(0,6),(0,6),(0,6),(0,5),(0,5),(0,6),(0,6),(0,6),(0,7),(0,7),(0,8),(0,8),(0,9),(0,9),(0,9),(0,8),(0,9),(0
(0,9),(0,9),(0,8),(0,8),(0,7),(0,7),(0,6),(0,7),(0,6),(0,5),(0,5),(0,6),(0,6),(0,6),(0,7),(0,8),(0,9),(0,8),(0,

(d) TNNNHNHHNNHNNTNNTHHHTHTTHHHHHHHHHHHHHHHHHHHHHHHHHHHHHHTTTTNN

8. Question 5d

9. Question 5e

10. Question 5f
11. Question 5h