

Homework 3: James Carroll and Joel Carrillo

Bayesian Inference, Temporal State Estimation and Decision Making under Uncertainty

1. Question 1

- (a) i. $P(A, B, C, D, E) =$
 ii. $P(A)P(B)P(C)P(D|A, B)P(E|B, C) =$
 iii. $(0.2)(0.5)(0.8)(0.1)(0.3) = 0.0024$
- (b) i. $P(\neg A)P(\neg B)P(\neg C)P(\neg D)P(\neg E) =$
 ii. $P(\neg A)P(\neg B)P(\neg C)P(\neg D|\neg A, \neg B)P(\neg E|\neg B, \neg C) =$
 iii. $(1 - 0.2)(1 - 0.5)(1 - 0.8)(1 - 0.9)(1 - 0.2) =$
 iv. $(0.8)(0.5)(0.2)(0.1)(0.8) = 0.0064$
- (c) i. $P(\neg A|B, C, D, E) = \frac{P(\neg A, B, C, D, E)}{P(\neg A, B, C, D, E) + P(A, B, C, D, E)}$
 ii. $P(\neg A, B, C, D, E) = P(\neg A)P(B)P(C)P(D|\neg A, B)P(E|B, C) =$
 iii. $(0.8)(0.5)(0.8)(0.6)(0.3) = 0.0576$
 iv. $\frac{0.0576}{0.0576 + 0.0024} = 0.96$

2. Question 2

- (a) $P(\text{Burglary} | \text{JohnCalls} = \text{true}, \text{MaryCalls} = \text{true})$
 Factors: $P(\text{Burglary}), P(EQ), P(\text{Alarm} | EQ, \text{Burglary}), P(\text{JohnCalls} | \text{Alarm}), P(\text{MaryCalls} | \text{Alarm})$
 Elimination Order: EQ, Alarm

$$f_1(\text{Alarm}, \text{Burglary}) = \sum_{EQ} P(EQ)P(\text{Alarm}, EQ, \text{Burglary})$$

$$f_2(\text{John}, \text{Mary}, \text{Alarm}, EQ) = \sum_{\text{Alarm}} f_1(\text{Alarm}, \text{Burglary})P(\text{John} | \text{Alarm})P(\text{Mary} | \text{Alarm})$$

$$P(\text{Burglary} | \text{JohnCalls}, \text{MaryCalls}) = P(\text{Burglary})f_2(\text{John}, \text{Mary}, \text{Alarm}, \text{Burglary})$$

Normalize the above to find $P(\text{Burglary} | \text{JohnCalls} = \text{true}, \text{MaryCalls} = \text{true})$

- (b)
 (c)

3. Question 3

- (a) i. By definition: $P(X | MB(X)) = P(X | \{U_1, \dots, U_m\}, \{Y_1, \dots, Y_n\}, \{Z_1, \dots, Z_j\})$
 ii. FJPD for parents: $P(Y_i, \dots, Y_n) = \prod_{i=1}^n P(Y_i | Z_{i1} \dots)$
 iii. End goal: $\alpha P(X | U_1, \dots, U_m) \prod_{Y_i} P(Y_i | Z_{i1} \dots)$

- (b) Four states: $\{\{\text{Cloudy}=\text{True}, \text{Rain}=\text{True}\}, \{\text{Cloudy}=\text{True}, \text{Rain}=\text{False}\}, \{\text{Cloudy}=\text{False}, \text{Rain}=\text{True}\}, \{\text{Cloudy}=\text{False}, \text{Rain}=\text{False}\}\}$
- (c) $Q =$