Uncertainty in Geo-science: A Workshop on Hazard Analysis

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OUTLINE

Problem Statement

Study of physical phenomena involved with uncertainty in input parameters:

- **Intrusive**: change the original governing equation.
  - Polynomial Chaos Stochastic Galerkin.
  - Perturbation methods
  - ...
- **Non-intrusive**: *do not* change the original governing equation.
  - Monte Carlo & LHS
  - Important sampling Methods
  - Non-intrusive spectral projection (NISP)
  - ...

Latin Hypercube Sampling (LHS)

Generating a sample set of uncertain values from equally probable intervals of the probability density function.

1. Select the pdf
2. Select the number of samples
3. Divide the pdf function into equal probability intervals.
4. Generate random samples on each interval.

Advantage:

- Much less number of samples require for convergence compared to MC
- No need to change the original solver (like any other non-intrusive method)

Polynomial Chaos Quadrature (PCQ)

The basic idea comes from projection theory that each function can be written as an expansion of a series of orthogonal function:

$$h(\eta) = \sum_i a_i \Psi_i(\eta) \tag{1}$$

So any uncertain parameter in the model can be expressed as above. The inner product of $h(\eta)$ and jth basis fucntion is

$$< h(\eta), \Psi_j(\eta) >=< \sum_i a_i \Psi_i(\eta), \Psi_j(\eta) > \tag{2}$$

Due to linear property of the inner product and orthogonality of basis:

$$< h(\eta), \Psi_j(\eta) >= a_i < \Psi_i(\eta), \Psi_j(\eta) > \tag{3}$$

$$a_i = \frac{< \Psi_i(\eta), \Psi_j(\eta) >}{< h(\eta), \Psi_j(\eta) >} \tag{4}$$

Polynomial Chaos Quadrature (PCQ)

The intergration is then approximated using the concept of the quadrature points.
By definition, inner product with respect to certain distribution is:

$$\begin{aligned} < h(\eta), \Psi_j(\eta) > &= \int_{-\infty}^{\infty} h(\eta)\Psi_j(\eta)p(\eta)d\eta \\ &= \sum_i h(\eta_i)\Psi_j(\eta_i)w_i \end{aligned} \tag{5}$$

$$< \Psi_j(\eta), \Psi_j(\eta) >= \sum_k \Psi_j(\eta_k)\Psi_j(\eta_k)w_k \tag{6}$$

Instructions for PCQ script

The basic procedure of running the PCQ script:

- Generate sample points and quadrature weight
- Run simulation with sampled parameters
- Parse output file and extract desired properties
- Prepare for coefficient computing
- compute coefficient by PCQ (or by solving system of equations)
- Plot plume height (or mass loading) as a function of random parameters
- Plot histogram of plume height (or mass loading) for specific distribution of input parameters

Instructions for PCQ script

- **Generate sample points and quadrature weight**:
  - The Gaussian Quadrature points generator should be called accordingly based on distribution that you selected: Hermite for Gaussian, Legendre for Uniform, Leguerre for Gamma, and Jacobi for Beta (modify smplingx=...)
  - Transfer and scale sample points (do not need modification)
- **Run simulation with sampled parameters**:
  Leave it as it is
- **Parse output file and extract desired properties**:
  Leave it as it is
- **Prepare for coefficient computing**:
  Leave it as it is
- **compute coefficient by PCQ (or by solving system of equations)**:
  Leave it as it is

Instructions for PCQ script

- **Plot plume height (or mass loading) as a function of random parameters**:
  Maybe need to modify the xlable and ylabel, title...
- **Plot histogram of plume height (or mass loading) for specific distribution of input parameters**:
  You can try different number of bins ect. depends on you.

Thank You ...