EECS 391 Written HW 3

William Koehrsen wjk68

October 12, 2017

1. A. A valid probability distribution must sum to one over all possible values for all variables. The following illustrates that summing down the rows and then summing those results across the columns results in 1. Therefore, all possible outcomes sum to one and this is a valid probability distribution.



B. p(x) is the marginal probability of x. This is calculated by holding x constant and summing over all y values for which X = x. Expressed in a formula this is

Where y is all values of y for which X = x. This calculation is done by summing across the columns of the table.

C. For x and y to be independent, for all values of x and y. That is, the joint probability must be the product of the marginal probabilities for all values in the domain of all variables. In order to confirm or disconfirm this, we need to find all the marginal probabilities and see if the product of the marginal probabilities p(x=i) and p(y=i) is equal to the joint probability p(i, j) for all values of i and j in the domain and range of the distribution. This is shown in the following worksheet:



From these calculations, we can conclude that x and y are independent.

1. A. This statement is **true.** Using the definition of conditional probability:

and

This means that the probability of B and C, must be equal to the probability of A and C, . These statements can further be expressed using the definition of conditional probability.

As the probability of C is assumed not to change between trials, the conditional probability of A given C must equal the conditional probability of B given C. Therefore, .

B. This statement is **false.** The first clause says the conditional probability of A given B and C is equal to the probability of A which means the probability of A is independent of B and C. The second clause states the probability of B given C is equal to the probability of B, which would mean B is independent of C. However, because the probability of A is independent of the probability of B and C, then the probability relationship of B to C does not matter. Say that A is the probability of flipping a fair coin and getting heads (P = 0.5). B and C can be any events because A does not depend on the probability of B and C. Let B represent the probability I will be rained on while going to class tomorrow and C be the probability that the forecast calls for rain tomorrow. If the probability of B on a normal day is 0.1, and the probability of B given C is 0.4, then . That is, B is dependent on C. However, the chance I will be rained on tomorrow has no effect on the coin flip, and .

C. This statement is **true.** The first clause states the probability of A given B is equal to the probability of A meaning A is unconditionally (absolutely) independent of B. Therefore, in the second clause, when A is conditioned on B and C, A will only depend on C. The probability of A given B and C will be equal to the probability of A given only C because A does not depend on B.

1. The capital letters refers to probability vector which is written as <true, false> and must sum to 1 over the domain.



1. This question is asking for the marginal probability of a toothache. To find this, sum up the boxes contained in the toothache section of the chart.
2. The vector probability of a cavity is the vector with all the values possible for the variable cavity in the form <true, false>. The true probability is the sum of the row containing the probability of a cavity under different circumstances and the false probability is 1-true.
3. This question asks for the vector probability of a toothache given that the patient has a cavity. The true probability can be computed by the definition of the conditional. The false probability is 1 – true.
4. This question asks for the conditional probability vector of a cavity given either a toothache or a catch. The true probability can be computed by the definition of the conditional probability. The false probability is 1-true.
5. This is a direct application of Bayes’ Theorem. The probability of having a disease given a positive test depends on the true positive rate of the test, the false positive rate of the test, and the background rate of the disease in the population (the prior probability of having the disease). We want to calculate the probability that someone with a positive test has the disease. Applying Bayes Theorem, we express this mathematically as:

where P(D) is the probability of the disease, P(S) is the probability of no disease, P(P) is the probability of a positive test, and P(N) is the probability of a negative test. The total probability of a positive test is calculated by the total probability rule which states:

The following shows the calculation of the probability that a patient has the disease given a positive test A.





**As can be seen, for test A and for test B. Therefore, a positive result on test B is more indicative of a patient having the virus. Test B produces fewer false positives but also a greater number of false negatives. The choice of which test to use depends on whether one wants to minimize the false positives, which will result in unnecessary treatment, or minimize the false negatives, which will result in a patient with the disease going untreated.**

1. If X and Y are conditionally independent, this can be expressed:

The definition of conditional probability is

In this case, we then want to condition X and Y on Z as well:

We know that X and Y are conditionally independent which results in:

If we cancel out the conditional probabilities of Y given Z, the equation simplifies to:

We can do a similar procedure to prove the conditional probability of Y given Z.

Definition of conditional probability:

Condition on Z:

Use the conditional independence statement:

Simplify the result:

Therefore, the conditional independence statement has proven to be equivalent to the given condition probabilities.

1. A. Without the prior probabilities, in this case the background rates of green and blue taxis in Athens, it is impossible to calculate the probability of the actual color of the taxi. The probability that the taxi was actually blue given that I say it was blue can be expressed:

where P(S) is the probability that I say the taxi *appeared* blue, and P(B) is the probability that the taxi *was* actually blue. Let P(G) be the probability that the taxi *was* green, then the probability that I say the taxi *appeared* blue is expressed:

P(B) and P(G) are the unknown priors that need to be determined in order to solve this problem. These are the prior probabilities that the taxi was actually green or blue which are determined from the distribution of taxis in Athens.

B. Assuming all taxis in Athens are blue or green, this information provides us with the background prior probability rates needed to solve the problem: P(B) = 0.1, P(G) = 0.9. We want to find the probability that the taxi *was* blue given that I say it *appeared* blue. Based on the testing, discrimination between blue and green is 75%. This means that , P(S|G) = 0.25. The probability that I say the taxi appeared blue is calculated:

The relevant probability for the trial is:

**Therefore, we can say that the probability that the taxi *was* blue given that I say it *appeared* blue is 0.25. The probability that the taxi was actually blue depends not only on the reliability of my testimony (my ability to discriminate between green and blue), but also on the distribution of taxi colors in Athens.**