EECS 391 Written HW 3

William Koehrsen wjk68

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1. A . A valid probability distribution must sum to one over all possible outcomes. The following illustrates that summing down the rows and then summing those results across the columns results in 1. Therefore, this is a valid probability distribution.



B. p(x) is the marginal probability of x. This is calculated by holding x constant and summing over all y values for which X = x. Expressed in a formula this is

Where Rx is the domain of all probabilities for which X = x. This calculation is done by summing across the columns in this case.

C. For x and y to be independent, for all values of x any y. That is, the joint probability is the product of the marginal probabilities. In order to confirm or disconfirm this, we need to find all the marginal probabilities and see if the product of the marginal probabilities p(x=i) and p(y=i) is equal to the joint probability p(i, j) for all values of i and j in the domain and range of the distribution. This is shown in the following worksheet:



1. A. This statement is **true.** Using the definition of conditional probability:

and

This means that the probability of B and C, must be equal to the probability of A and C, . These statements can further be expressed using the definition of conditional probability.

As the probability of C does not change between trials, the conditional probability of A given C must be equal to the conditional probability of B given C. Therefore, .

B. This statement is **false.** The first clause says the conditional probability of A given B and C is equal to the probability of A by itself which means the probability of A is independent of B and C. The second clause states the probability of B given C is equal to the probability of B by itself which means B is independent of C. However, because the probability of A is independent of the probability of B and C, then the probability relation of B to C does not matter. Say that A is the probability of flipping a fair coin and getting heads (P = 0.5). B and C can be any events because A does not depend on the probability of B and C. Let B represent the probability I will be rained on while going to class tomorrow and C be the probability that the forecast calls for rain tomorrow. If the probability of B on a normal day is 0.1, and the probability of B given C is 0.4, then . That is, B is not independent of C. However, the chance I will be rained on tomorrow has no effect on the coin flip, and in this example.

C. This statement is **false.** The first clause states the probability of A given B is equal to the probability of A, so A is unconditionally independent of B. The second clause states the probability of A given B and C is equal to the probability of A given C alone. This would be true if A and B were conditionally independent on C, but it is not true in the case of absolute independence. For example, suppose A is the probability of a tail when flipping a fair coin and B is the probability that it will rain tomorrow. A is absolutely independent of B because the knowledge of the weather does not affect the result of the coin flip. If C is then the probability of a green traffic light, then C would also be independent of A. In this case we can express the probabilities:

In order for the second clause to be true, there would need to be an additional factor of P(c) multiplying P(a|c). This example shows the difference between conditional and absolute independence.

We can use this result to show

1. A probability vector is written as <true, false> and must sum to 1.



1. The probability of a toothache is the marginal probability of a toothache. Sum up the boxes contained in the toothache section of the chart.
2. The vector probability of a cavity is the vector in the form <true, false>. The true value is the sum of the row containing the probability of a cavity under different circumstances.
3. This is the vector probability of a toothache given that there is a cavity. The true value can be computed by the definition of the conditional.
4. This is the conditional probability of a cavity given either a toothache or a catch. The true value can be computed by the definition of the conditional probability.
5. This is a direct application of Bayes’ Theorem. The probability of having a disease given a positive test depends not only on the false positive rate of the test, but also the background rate of the disease in the population. We want to calculate the probability that someone with a positive test has the disease. Applying Bayes Theorem, we express this mathematically as:

where P(D) is the probability of the disease, P(S) is the probability of no disease, P(P) is the probability of a positive test, and P(N) is the probability of a negative test. The total probability of a positive test is calculated by the total probability rule which states:

The following images show the probability that a patient has the disease given a positive test for tests A and B.





**As can be seen, for test A and for test B. Therefore, a positive result on test B is more indicative of a patient having the virus.**

1. If X and Y are conditionally independent, this can be expressed:

The definition of conditional probability is

In this case, we then want to condition on Z:

We know that X and Y are conditionally independent which results in:

If we cancel out the conditional probabilities of Y, the equation simplifies:

We can do a similar procedure to prove the conditional probability of Y given Z.

Which simplifies to the result:

Therefore, all of the statements have been proven to be equivalent.

1. A. Without the prior probabilities, in this case the background rates of green and blue taxis in Athens, it is impossible to calculate the probability of the actual color of the taxi. The probability that the taxi was actually blue given that I say it was blue can be expressed:

where P(S) is the probability that I say it was blue, P(B) is the probability of a blue taxi in Athens, and let P(G) be the probability of a green taxi in Athens.

P(B) and P(G) are the unknown priors that need to be determined in order to solve this problem.

B. Assuming all taxis in Athens are blue or green, this information provides us with the background rates needed to solve the problem: P(B) = 0.1, P(G) = 0.9. We want to find the probability that the taxi was blue given that I say it was blue. Based on the testing, discrimination between blue and green is 75%. This means that the , P(S|G) = 0.25. The probability that I say the taxi is blue is calculated:

The needed probability is:

**Therefore, we can say that the probability that the taxi was blue given that I say it was blue is 0.25.**