EECS 391 Written Assignment 4

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November 7, 2017

1. The general version of the product rule is (1):

This can be extended to any number of events (2):

The conditional version of the product rule can be written as (3):

Using (2) from above, this becomes (4):

The probability of the union of Y and **e** can be written as (5):

This leaves us with a factor of , the prior probability of event **e,** in both the numerator and denominator. These are cancelled out and we arrive at the solution (6):

B. The general version of Bayes’ rule is (1):

The conditional version of Bayes’ rule is thus (2):

From part A (6), the right side of (2) can be rewritten as (3):

Also using the result from part A (6), the far right term of (3) becomes (4):

Equating this expression to the far left hand term of (3) results in the desired solution (5):

1. A. **Network C.** The statement indicates the genes of the father, mother, and child are absolutely independent of one another. The only network in Figure 14.20 for which this is true is C because there are no connections between the genes in this network.

B. **Network A and Network B.** The problem states the gene itself is equally likely to be inherited from either of an individual’s parents. It also states there is one gene that influences the handedness of an individual. Network A represents the hypothesis as does network B which show the gene for handedness is dependent on the parents’ genes for handedness.

C. **Network A** is the best description of the hypothesis. The hypothesis claims the handedness of an individual is solely determined by the gene for handedness (with a probability of s) meaning the only connection to handedness should be from the individual’s gene. The gene itself is inherited from either the mother or the father. The actual handedness of the child is conditionally independent of both the genes of the parents and the handedness of the parents. Networks B and C represent that the handedness of the child is directly dependent on the handedness of the parents.

D. Conditional Probability Table ofin terms of s and m where s is the probability the handedness is the same as the gene, and m is the probability of a random mutation flipping the handedness of the gene. The gene of the child does not depend on s because s relates the gene to the handedness and the handedness is an effect of the gene

|  |  |  |
| --- | --- | --- |
| **Gene father** | **Gene mother** | **Gene Child <left, right>** |
| L | L | <1-m, m> |
| L | R | <1/2,1/2> |
| R | L | <1/2, 1/2> |
| R | R | <m, 1-m |

E. We want to find the marginal probability of the child having the left hand gene. To derive this expression, we must hold the variable of interest constant and sum across all possible values for the conditioning variables, in this case, the genes of the mother and the father. This is written as an equation:

We can use the conditional probability table from above and the knowledge to simply the expression.

This simplifies to the expression

F. Given the prior probability of left-handedness is the same for everyone, that would mean q = 0.5. This can be proved by setting and solving the answer to E for q.

However, this contrasts with the actual state of the world because far fewer than 50% of individuals are left handed. According to research, about [5-30% of the world population is left-handed (this can vary by country).](https://www.scientificamerican.com/article/why-are-more-people-right/) This means the hypothesis that genetics is the sole determinant of handiness is incorrect and there must be hidden (latent) variables not accounted for in the network. These could include environmental factors such as societal pressures/traditions. [Studies conducted in twins show](https://www.ncbi.nlm.nih.gov/pubmed/16611467) genetic factors can only explain 25% of the variance in handedness with the rest due to environmental factors. Another explanation is the right hand allele (a version of a gene) may be dominant meaning that if both a left hand and right hand allele are present in a gene for handedness, the right hand gene is expressed in the phenotype (physical manifestation of a genotype). In order to account for these factors, the Bayes Network must be expanded to include all factors which influence handedness. Each of these factors must then have a prior probability and a probability reflecting the factor’s relative influence on handedness in order to specify the full joint distribution.

1. The full Bayes net is pictured below for reference.



1. **(i): FALSE,** this statement claims the probability an individual broke the election law, was indicted, and that the prosecutor is politically motivated are absolutely independent of one another. This is not shown in the network because the probability of indictment is conditionally dependent on both broke election law and a politically motivated prosecutor.

**(ii): TRUE,** this statement claims the probability an individual was jailed is conditionally independent of the probability that the individual was indicted given that the individual was found guilty. This is true as indicated by the network. Indictment indirectly is a cause of jailed, but the outcome of jailed is conditionally independent of indictment once the information that the individual was found guilty is known.

**(iii) TRUE,** this statements claims the probability the prosecutor is politically motivated is conditionally independent of the individual being jailed given that the individual broke the law, was indicted, and was found guilty. The network does not have any direct connections between the politically motivated prosecutor and jailed, and even if they are indirectly related, the probability of a politically motivated prosecutor is conditionally independent of the jailed outcome.

1. In order to reason with a Bayes Net, the probability postulate must be broken down into smaller chunks that can be evaluated from the net. The known prior probabilities are . The remaining probabilities must therefore be expressed as conditional probabilities which can be found in the tables. The probability postulate can be expressed in common terms as the probability that the individual was jailed given that he broke the election law, was indicted, was found guilty, and the prosecutor was not politically motivated.

The probability of guilty verdict is not a prior and must be expressed as a conditional.

Likewise, the probability of an indictment is not known a priori. It must also be expressed as a conditional.

The probability statement is now in terms of known conditionals and priors. To solve, the numbers can be substituted:

Given the information, we can conclude the probability an individual will be jailed given that he broke the law, was indicted, was found guilty, and the prosecutor is not politically motivated is 29.16%. This probability of a jailed outcome may seem low, but as Professor Lewicki discussed in class, the more specific the situation, the less likely that exact situation will occur. There are 25 possible combinations of variables, and each one in isolation is unlikely which contrasts with our intuition because we only ever see one version of reality.

1. The probability an individual goes to jail given he broke the law, has been indicted, and faces a politically motivated prosecutor can be expressed in a postulate as:

As in part B, this can be simplified until we know all the conditionals and prior probabilities in the expression:

Substituting in the given number yields:

In understandable terms, the probability that an individual goes to jail given he broke the election law, was indicted, and faces a politically motivated prosecutor is 6.56%.

1. If indicted is False, then found guilty is independent of its other parents, broke the law and politically motivated prosecutor. This is because the probability that an individual is found guilty is 0 if the individual was not indicted.
2. A presidential pardon is when an individual is found guilty, but has his/her sentence shortened by the president. In the United States, a presidential pardon can only be used for an individual [found guilty of a federal crime.](https://www.pbs.org/newshour/politics/presidents-pardon-power-works) Assuming the presidential pardon is independent of any other variables, it will only be connected to the jailed node. That is, the presidential pardon will only affect the probability of the individual being jailed because the pardon can be definition only occur after an individual has been found guilty. If the presidential pardon is True, the probability of jail is 0.0 and the probability of jail without the presidential pardon is can be any valid probability, in this case it is assigned 0.8. The presidential pardon is represented by the following addition to the Bayes Net:



P P(J|P)

P = T 0.0

P = F 0.8

P

1. A. The Bayes Net for this situation is represented below:

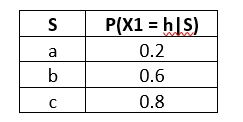
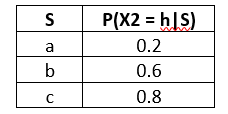
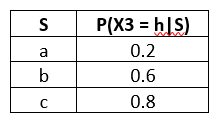
|  |
| --- |
| **P(S = a): 1/3** |
| **P(S = b): 1/3** |
| **P(S = c): 1/3** |

S

X1

X2

X3



The S node represents the selection of the code. Each coin is equally likely to be selected and there are three coins yielding the a priori probabilities of 1/3 for a, b, and c. The conditional probability tables relate the probability of the variables X1, X2, and X3 coming up heads given the selection of a particular coin. These rates represent the biased probability of the coin given in the problem statement.

B. The outcome of each coin is independent of the others, and the assignment of the coins does not matter for the problem formulation (in other words, X1 = h, X2 = h, and X3 = t is the same as X1 = t, X2 = h, X3 = h). The three expressions we need to solve for to determine the most likely coin are:

These equations provide a chance to put Bayes’ Rule into practice. I will demonstrate with just the first equation, but the second two follow the same process:

The probability in the denominator can be expanded using the total probability rule:

The denominator is the same for all three cases as is the prior probability in the numerator. The denominator can be determined by substituting in the known values:

The results of the coin flips are conditionally independent of one another which means the first term in the numerator can be expressed in terms of the known conditionals and priors:

This expression can be evaluated for all coins as is shown above in the denominator calculation:

The denominator is the same for all coins, so there is no need to figure out the actual probabilities to declare that Coin 2 is the most likely to have come out of the bag given the results 2 heads and 1 tail. However, for curiosity (and to check my answer), I will go ahead and calculate the percentages by normalizing with the denominator and multiplying by the prior:

**Given the result of 2 heads and 1 tail, Coin 2 is the most likely to have been selected at 47.4%.**

1. A. The outcome of a given match depends only on the difference in quality between the teams. Therefore, in order to express the probabilities for the outcome of a match, the most intuitive way to do so is to express the probability of an outcome based on the differences in quality. The following shows the conditional probability table for the outcome of a match conditioned on the quality of the two teams in the match. There are seven possibilities for

Conditional Probabilities

|  |  |  |
| --- | --- | --- |
|  | **Probability (1 win|)** | **Probability (Tie|)** |
| -3 | 0.05 | 0.05 |
| -2 | 0.1 | 0.15 |
| -1 | 0.2 | 0.2 |
| 0 | 0.3 | 0.4 |
| 1 | 0.6 | 0.2 |
| 2 | 0.75 | 0.15 |
| 3 | 0.9 | 0.05 |

The other probabilities to define are the posteriors, which in this case are the quality of the teams.

The following prior probabilities can apply to all teams:

Prior Probabilities

|  |  |
| --- | --- |
| **Quality** | **Probability (Q = q)** |
| 0 | 0.25 |
| 1 | 0.25 |
| 2 | 0.25 |
| 3 | 0.25 |

These prior probabilities can be used to calculate the probability of any difference in quality between two teams.

|  |  |
| --- | --- |
|  | **Probability ()** |
| -3 | 1/16 = 0.0625 |
| -2 | 2/16 = 0.125 |
| -1 | 3/16 = 0.1875 |
| 0 | 4/16 = 0.25 |
| 1 | 3/16 = 0.1875 |
| 2 | 2/16 = 0.125 |
| 3 | 1/16 = 0.0625 |

The outcome of a match is completely dependent on the difference in quality of the two teams in the match. Therefore, the conditionals relating the outcome of a match to the difference in quality between the two teams and the prior probabilities of the quality of the teams is enough to specify the complete joint distribution.

B. The Bayes’ Net can be drawn as follows:

QC

QB

QA

A vs B

A vs C

B vs C

The Bayes Net combined with the conditional probabilities and the priors can be used to calculate any possible probability statement. The Bayes Net with all of the conditional and prior probabilities specified represents the full joint probability distribution.

C. I will solve for the conditional probability of the outcome of the third match using inference by enumeration. This starts with the general expression for inference:

where α is a normalization constant to makes the probabilities sum to 1, X is the query event, e is the list of evidence variables, and y is the possible values of the unobserves variables.

From the information given in the problem (A beats B and draws with C), we need to calculate the posterior probability of the outcome of the third game. This requires assessing the probability of the difference in quality based on the evidence.

This requires assessing the probability of the quality of both B and C given the evidence. The results of the games are independent from one another, and the quality of a team not involved in the match is independent of the outcome of the match.

This can then be expressed in terms of Bayes’ Rule:

The same expression can be written for any quality value of B or C:

In the case of a tie game, that information does not given us any indication of the quality of the teams involved. In other words, we can gain no information from a tie, so the conditional probability for the quality of C is identical to the prior probability for the quality of C.

Going back to the equation for the probability of , this can be expressed by the general inference procedure:

Here, the normalization constant will be:

The chance of a given quality of a team is always 0.25, and the probability on the bottom can be worked out in general for any two teams 1 and 2 given the conditional and prior probabilities:

With the result that

We can now compute the probability of any quality of B

The results are the following two tables for the quality of B and C

|  |  |  |
| --- | --- | --- |
| **Quality** |  |  |
| 0 | 0.408 | 0.25 |
| 1 | 0.296 | 0.25 |
| 2 | 0.192 | 0.25 |
| 3 | 0.104 | 0.25 |

To make sure we are on the right track, we can sum the columns for the quality of B and C and confirm that both of them add up to 1 as expected. The next step is to calculate the probabilities of the difference between teams B and C.

|  |  |
| --- | --- |
|  | **Probability ()** |
| -3 | (0.104) \* 0.25= **0.026** |
| -2 | (0.104 + 0.192) \* 0.25 = **0.074** |
| -1 | (0.104 + 0.192 + 0.296) \* 0.25 = **0.148** |
| 0 | (0.408 + 0.296 + 0.192+ 0.104) \* 0.25 = **0.25** |
| 1 | (0.408 + 0.296 + 0.192) \* 0.25 = **0.224** |
| 2 | (0.296 + 0.408) \* 0.25 = **0.176** |
| 3 | (0.408) \* 0.25 = **0.102** |

Again, these probabilities must sum to 1 which can be confirmed. The final step is now to categorize the probabilities of each of the possible outcomes of the match knowing the probabilities for the difference quality:

Therefore, the final answer is

**Probability C beats B = 0.4715 = 47.15%**

**Probability C ties B = 0.2183 = 21.83%**

**Probability B beats C = 0.3102 = 31.02%**

**The results include the somewhat surprising conclusion that the probability that B beats C is greater than the probability of a tie. However, this should be easier to realize once the background rates are taken into account. The background rate of a tie at 21.875% is much lower than that of a win for either team at 39.0625%.**