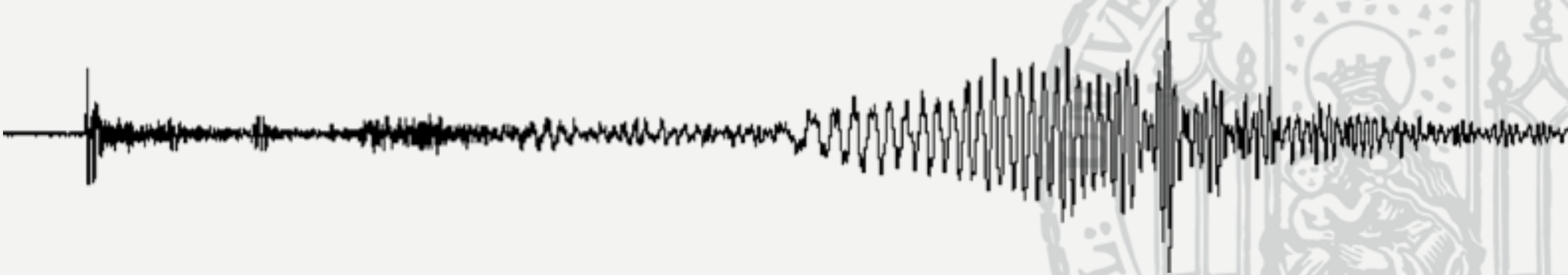
Ceri Nunn

Geophysical Data Analysis

L03 – Observables & Instruments



Observables

What are observables?

Brainstorming …

observables

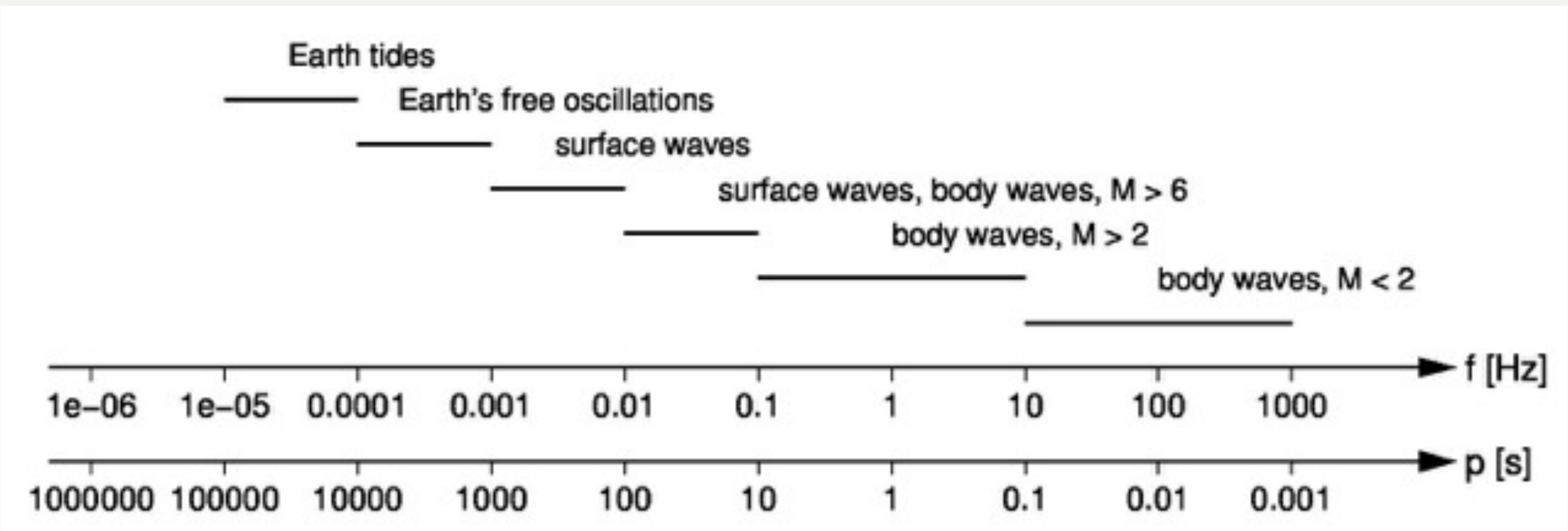
… on the board.

Observables - some examples

Acceleration/Velocity/Displacement
Strain
Rotations
Geomagnetic field
Gravitimetric field
Temperature (heat flow)
Radioactivity (dating)
Petrology/Sedimentology
Palaeontology (especially for dating)
Chemistry
-bulk composition
-trace element ratios
-thin section

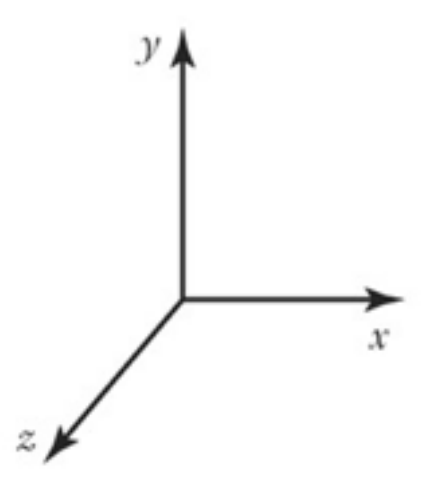
Period ranges

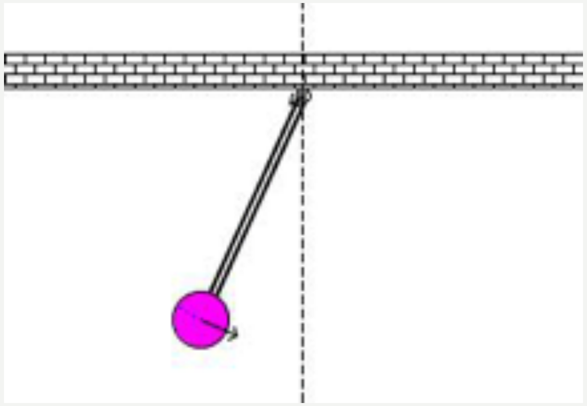
Laboratory signals	0.000001 – 0.001s
Seismic exploration	0.001 – 0.1s
Sound	0.001 – 0.1s
Earthquakes	0.01 – 100s
Coseismic deformation	1 – 1000s
Eigenmodes of the Earth (Normal Modes)	1000s
Postseismic deformation	> 10000s



Translation

- *deformation in direction of 3 orthogonal axis*
- *usually denoted by vector **u** with appropriate connection to strain tensor*
- *can be measured as displacement **u**, velocity **v**, or acceleration **a***

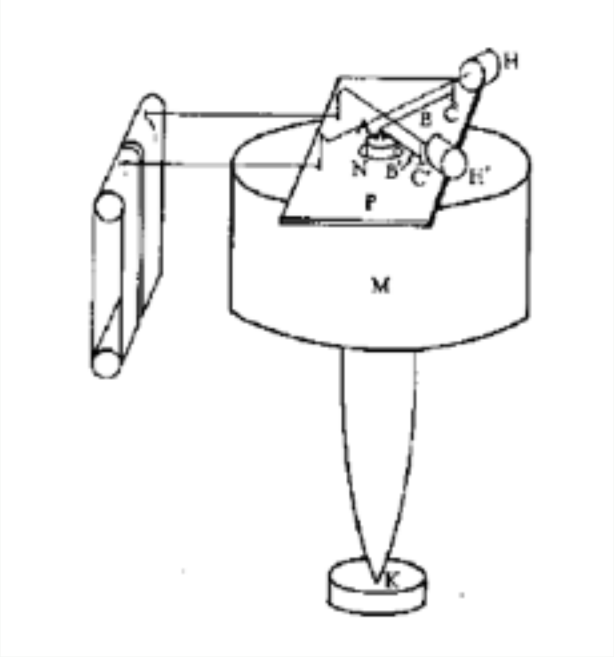




Translation – displacement

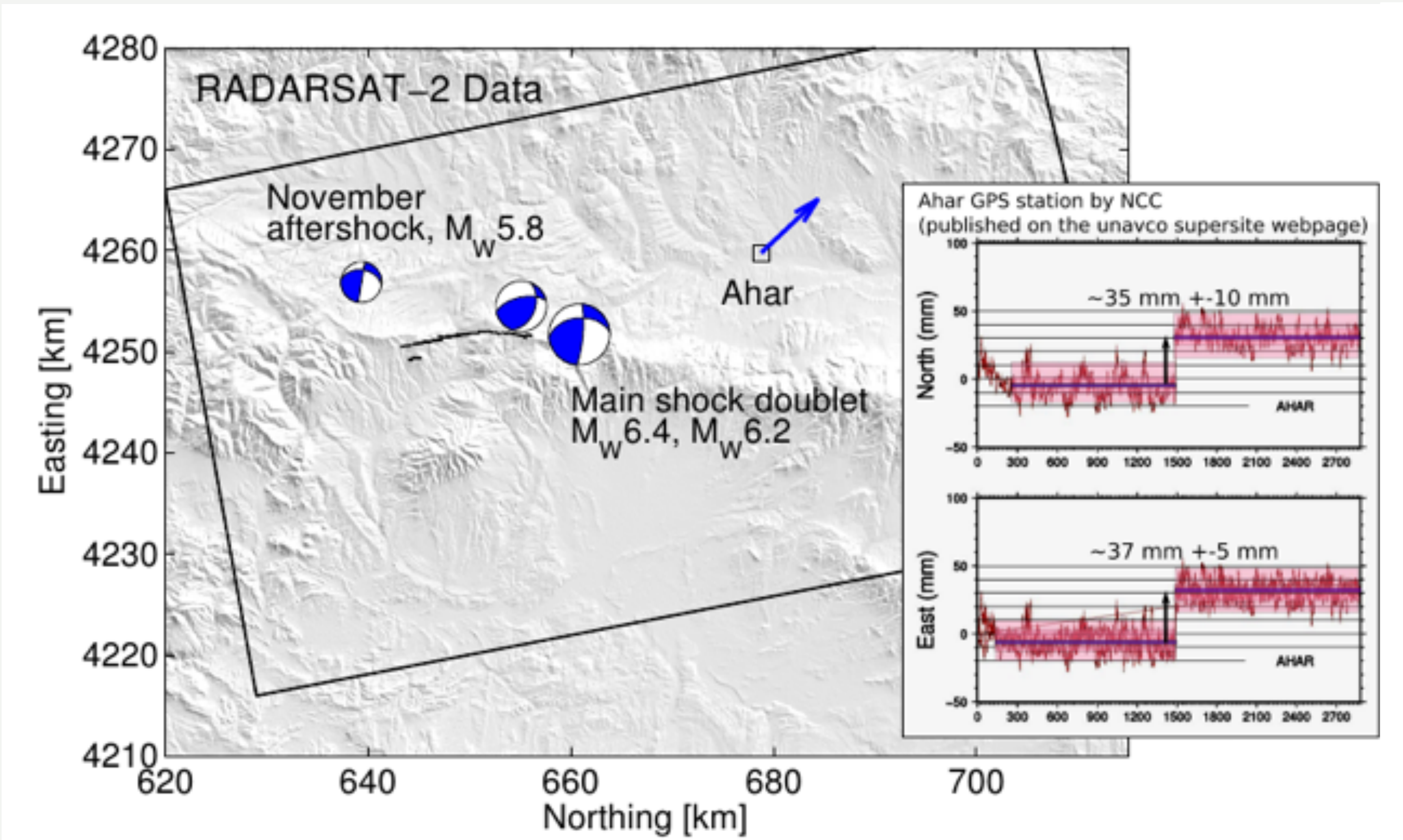
- *"differential" motion around a reference point (e.g. pendulum)*
- *first seismometers were pure (mostly horizontal) displacement sensors*
- *co-seismic displacement range from microns to dozens of meters*

Horizontal displacement sensor (ca. 1905); amplitude of ground deformation mechanically amplified by factor 200



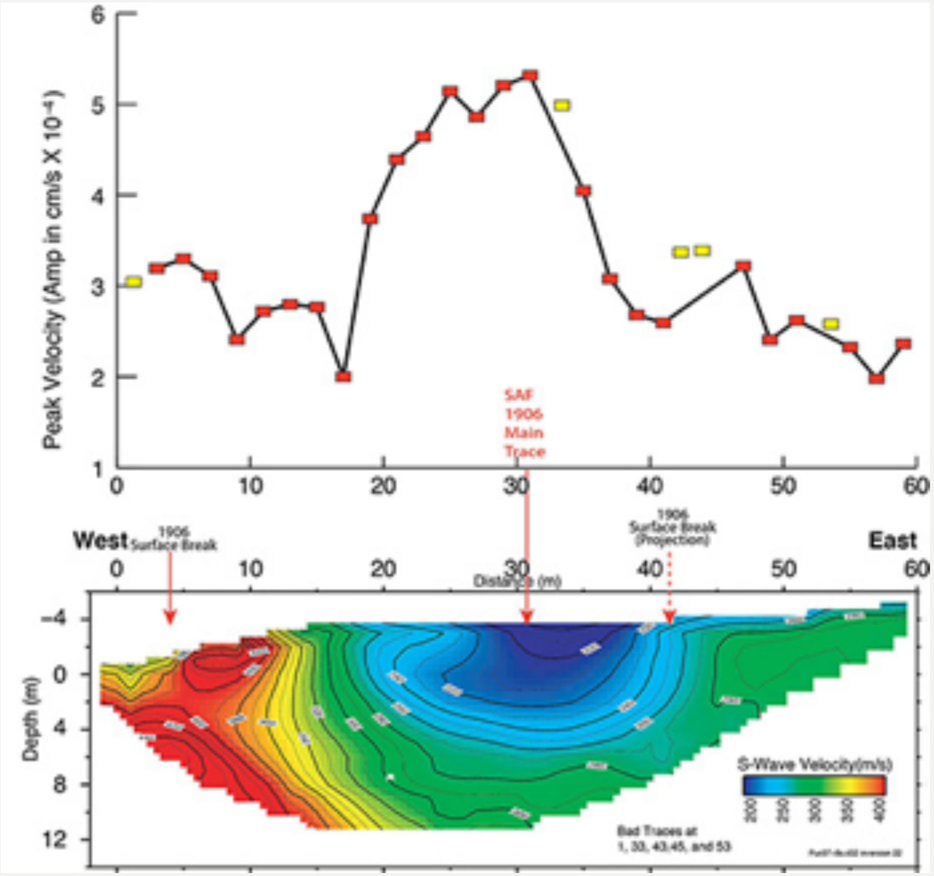
Translation – displacement

- *today displacement is measured by GPS sensors*



Translation – velocity

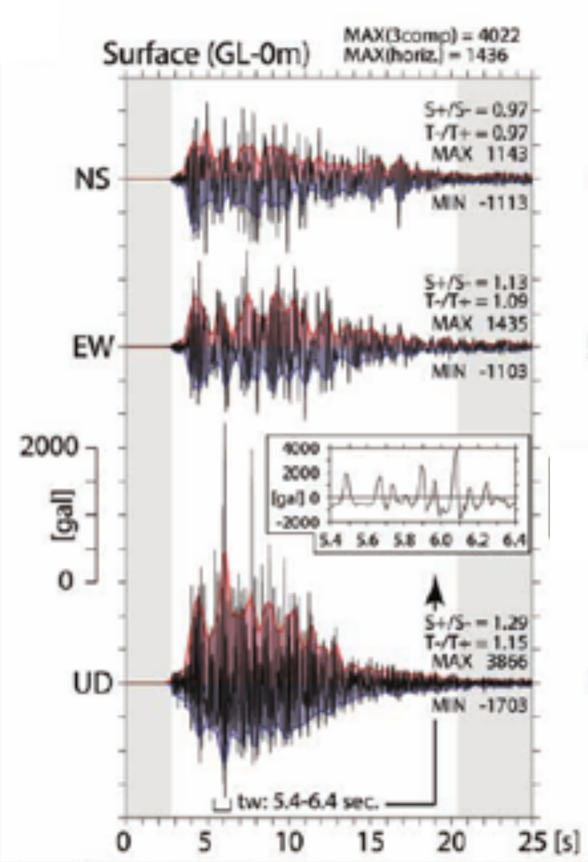
- *today most seismometers record velocity*
- *Reason: electro-mechanic principle*
  - *Earth shakes → coil moves in magnetic field → electric current generated*
  - *current ~ ground velocity*
- *velocities range between µm/s and m/s*



velocity amplification
in fault zone

Translation – acceleration

- *measure of strong motion (those getting close to or exceeding Earth's gravitational acceleration)*
- *common in earthquake engineering, aeroplanes, laptops, ipods, etc. ...*



Iwate Miyagi earthquake

Mw 6.9

14.06.2008

distance 3 km

PGA_{max} ***40 m/s² !!!***

Aoi et al. (2008)

Question

How to switch between displacement, velocity and acceleration?

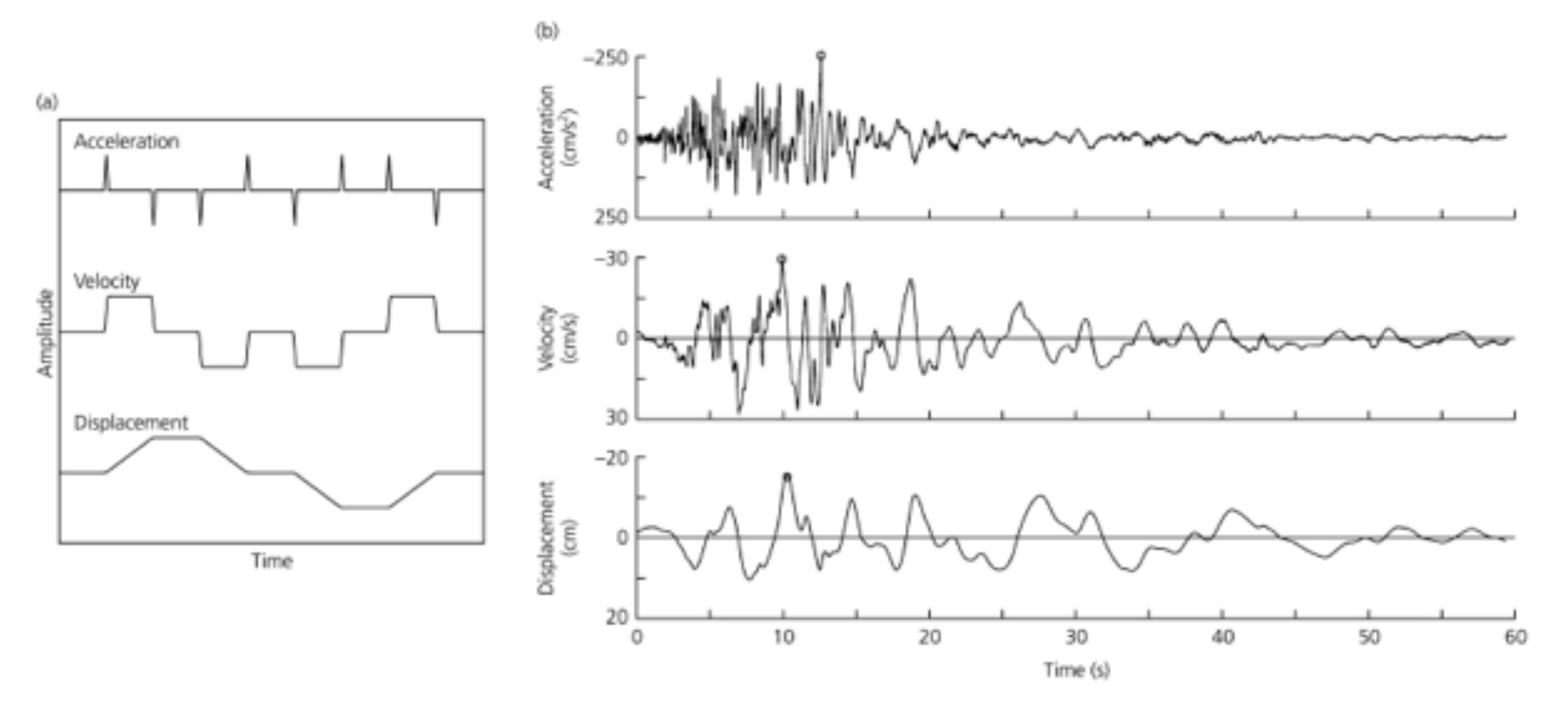
Answer

How to switch between displacement, velocity and acceleration?

… via integration and differentiation with respect to time ...

$$v(x,t) = \partial_t u(x,t) = \dot{u}(x,t)$$

$$a(x,t) = \partial_t^2 u(x,t) = \ddot{u}(x,t)$$



Strain

- *tensor with 6 independent linear combinations of the spatial derivatives of the displacement field*
- *purely geometric quantity without dimension*

Strain Tensor:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Stress (σ) and strain (ε) can be related via Young's Modulus (E).

This is a consequence of Hooke's Law

$$\vec{\sigma} = \overline{\overline{E}} \cdot \vec{\epsilon}$$

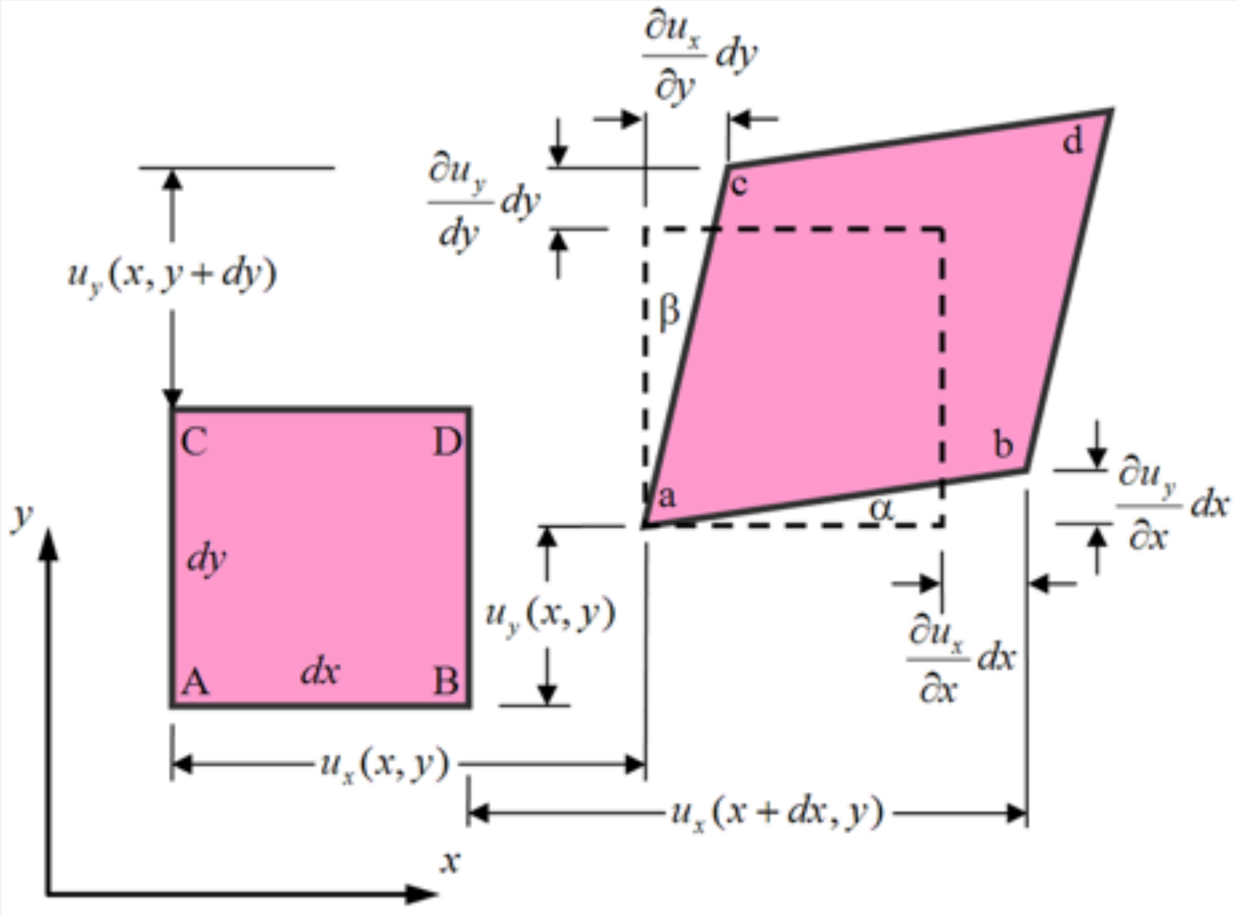
Strain components

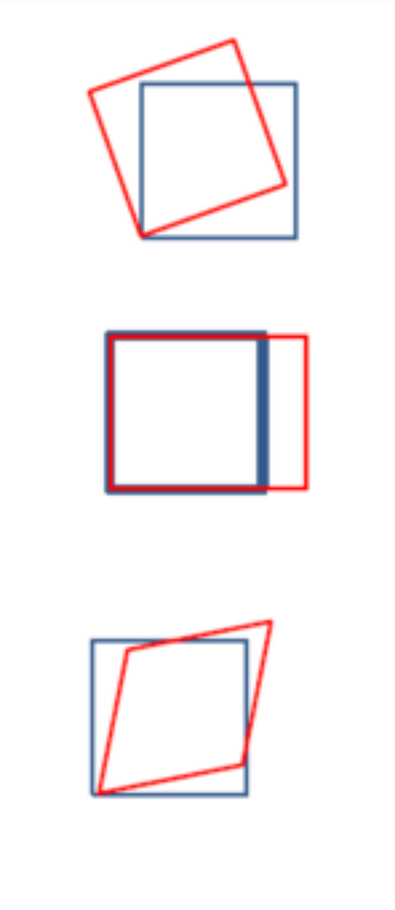
$$\epsilon_{ij} = \begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{1}{2}\cdot\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right) & \frac{1}{2}\cdot\left(\frac{\partial u_1}{\partial x_3}+\frac{\partial u_3}{\partial x_1}\right) \\ \frac{1}{2}\cdot\left(\frac{\partial u_2}{\partial x_1}+\frac{\partial u_1}{\partial x_2}\right) & \frac{\partial u_2}{\partial x_2} & \frac{1}{2}\cdot\left(\frac{\partial u_2}{\partial x_3}+\frac{\partial u_3}{\partial x_2}\right) \\ \frac{1}{2}\cdot\left(\frac{\partial u_3}{\partial x_1}+\frac{\partial u_1}{\partial x_3}\right) & \frac{1}{2}\cdot\left(\frac{\partial u_3}{\partial x_2}+\frac{\partial u_2}{\partial x_3}\right) & \frac{\partial u_3}{\partial x_3} \end{bmatrix}$$

Question

What is the meaning of the elements of the strain tensor?

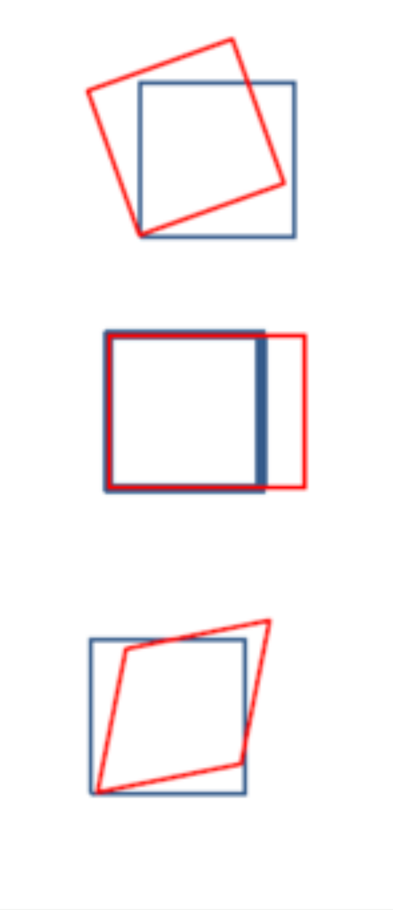
Answer


$\frac{\partial u_x}{\partial y}dy$
$\frac{\partial u_y}{dy}dy$
$u_y(x, y+dy)$
d
c
β
b
a
α
$\frac{\partial u_y}{\partial x}dx$
C
D
dy
y
$u_y(x, y)$
$\frac{\partial u_x}{\partial x}dx$
A
dx
B
$u_x(x, y)$
$u_x(x+dx, y)$
x



Answer

What is the meaning of the elements of the strain tensor?



Simple geometric interpretation:

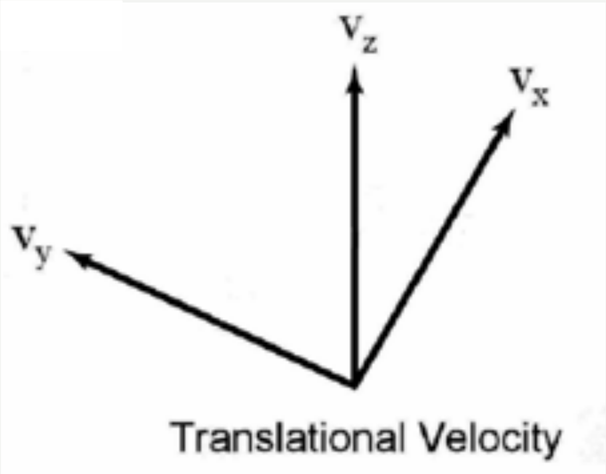
i = j (diagonal elements) → longitudinal deformation along axes of coordinate system

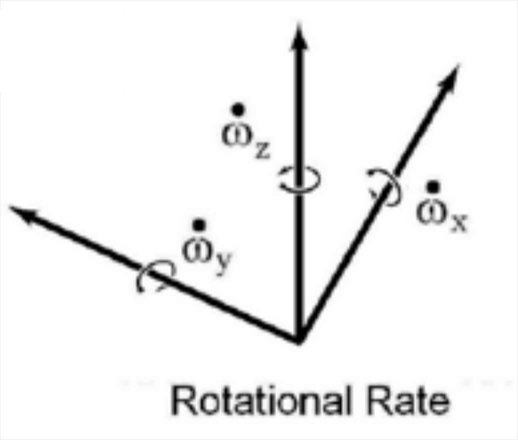
stretching

i ≠ j (off-diagonal elements) → variations in angle between directions of axes of deformed and undeformed states; shear strain

deforming

Rotations





Usually to the order of: *10e-6 to 10e0 m/s* *10e-11 to 10e-1 rad/s*

Rotations

Rotation vector (v) represents the angle of rigid rotation and can be expressed as a linear combination of spatial derivatives of the vector of translational displacement (**u**):

$$\boldsymbol{\omega} = \frac{1}{2}\nabla \times \mathbf{u} = \frac{1}{2}\begin{pmatrix} \partial_y u_z - \partial_z u_y \\ \partial_z u_x - \partial_x u_z \\ \partial_x u_y - \partial_y u_x \end{pmatrix}$$

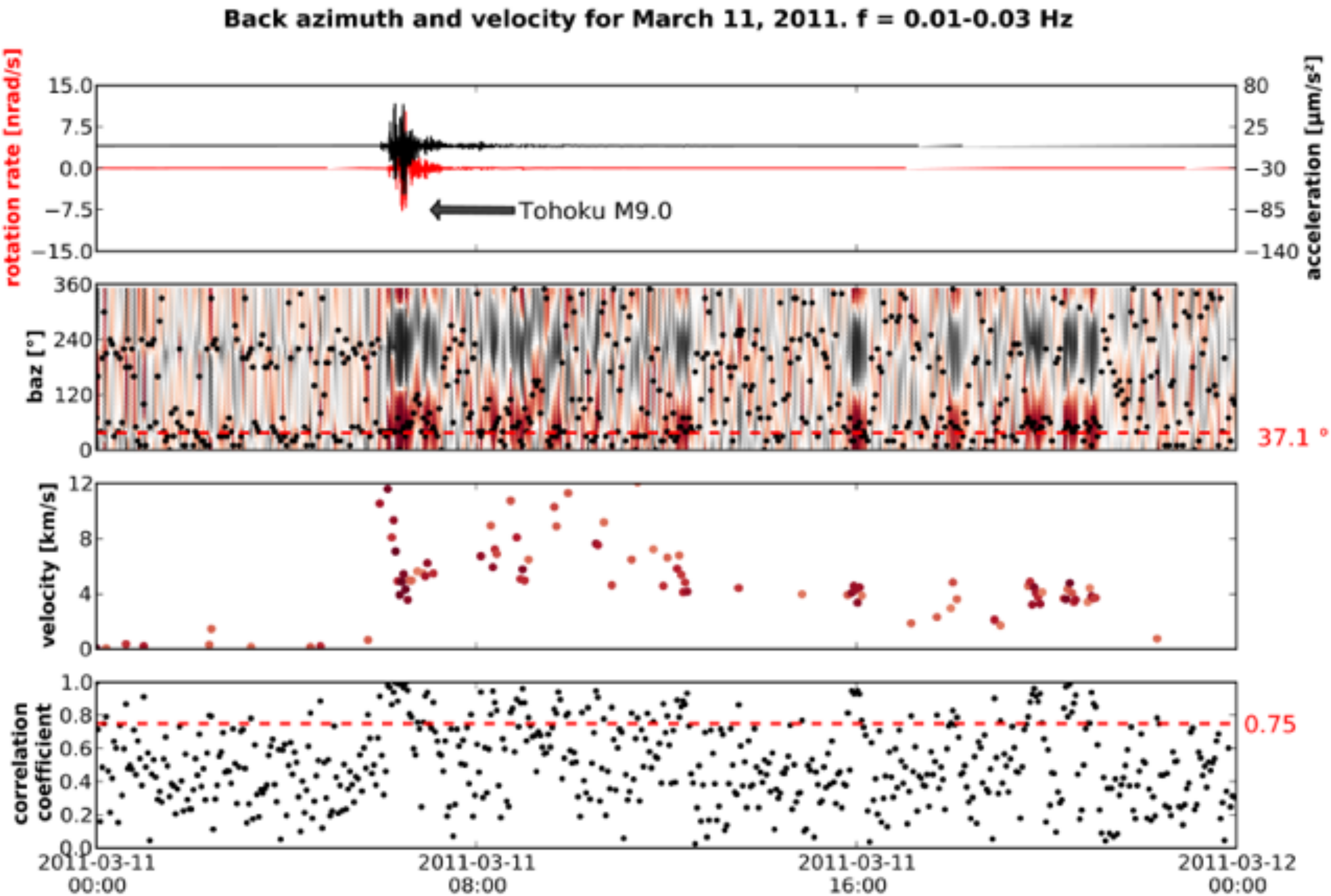
Rotations

- *Rotation is a vectorial quantity*
- *At the Earth's surface rotation is the same as tilt*
- *Rotations are only started to be recorded*
- *Rotations are likely to contribute to structural damage*

Monument in Chatak, India

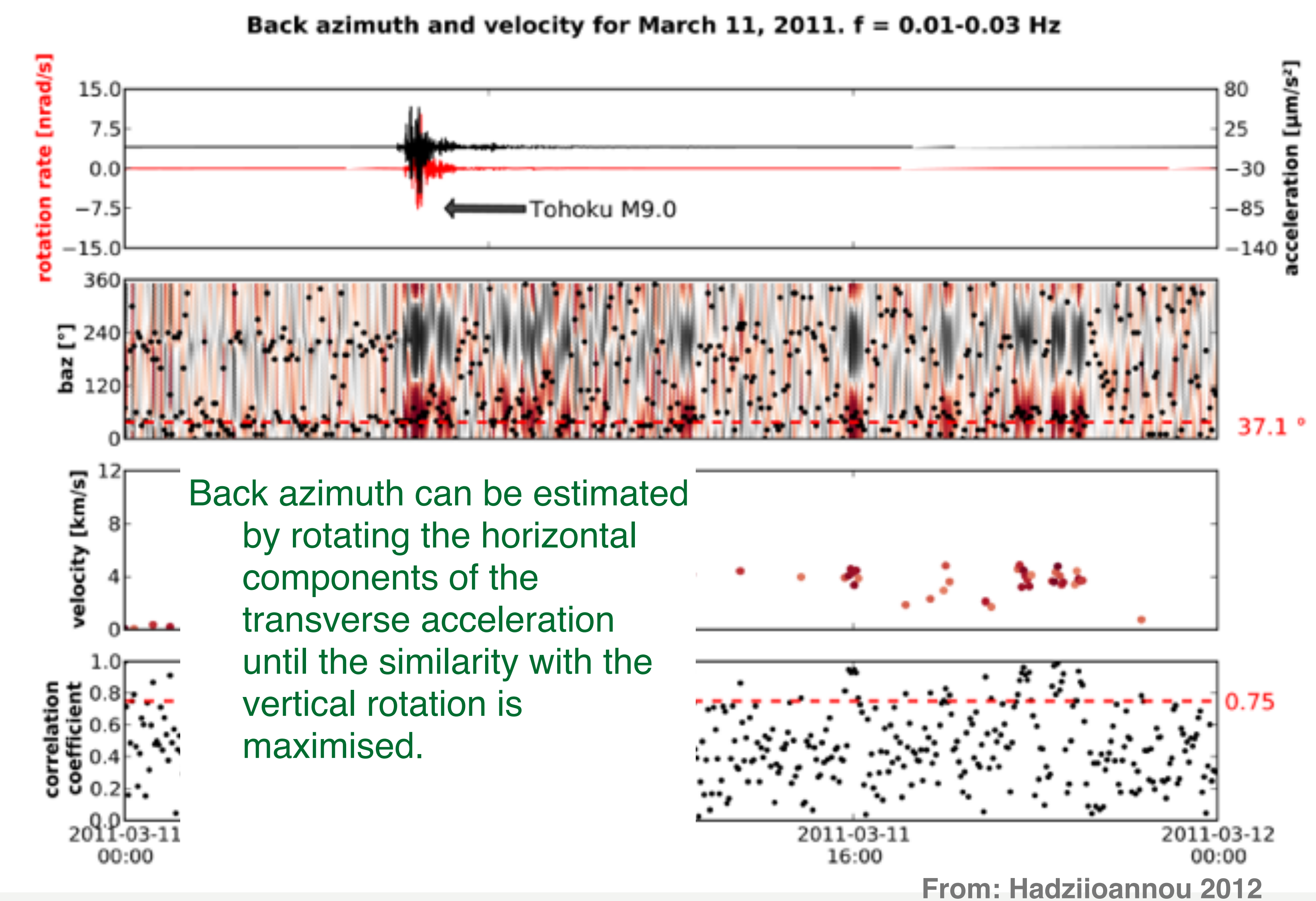
After the 1897 Great Shillong earthquake

Rotations



From: Hadziioannou 2012

Rotations



Back azimuth can be estimated by rotating the horizontal components of the transverse acceleration until the similarity with the vertical rotation is maximised.

From: Hadziioannou 2012

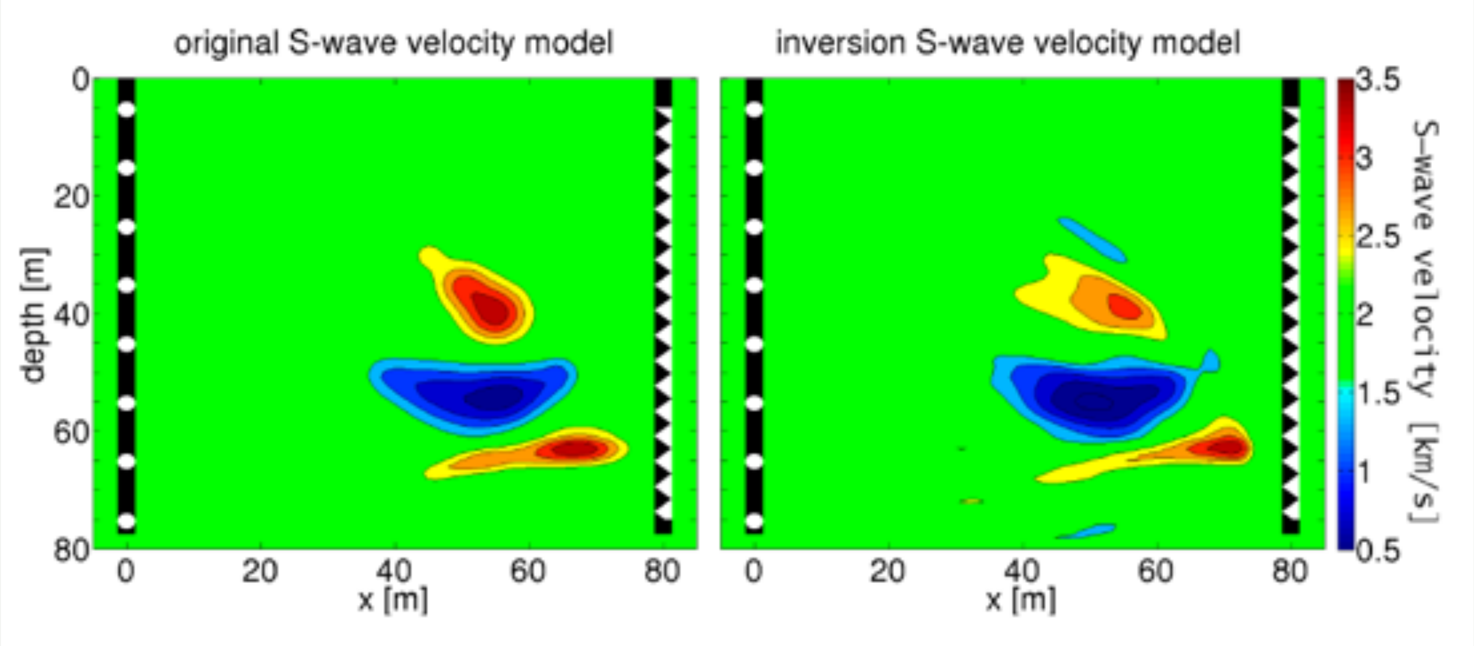
Rotations

Rotation rate and acceleration should be in phase.

The ratio between the transverse acceleration and vertical rotation rate:

$$\frac{a_T}{\dot{\omega}_z} = -2c$$

We can estimate apparent phase velocity and backazimuth if we measure rotation and translation at the same place!



Rotation is sensitive to local structure

Tilt

angle of the surface normal to the local vertical

→ *rotation around two horizontal axes*

in 3D anisotropic media all parts of the seismic wave field may produce tilt

$$\Theta(x,t) = \partial_x u_z$$

Other causes of tilt:

- *Earth tides*
- *Atmospheric pressure changes*
- *Soil deformation (water content)*
- *Temperature effects*
- *Mass movements*

Cross-axis sensitivity

Translation sensors (e.g. standard seismometers) are also sensitive to rotational motions (especially tilt).

This can affect data quality.

When?

Cross-axis sensitivity

Translation sensors (e.g. standard seismometers) are also sensitive to rotational motions (especially tilt).

This can affect data quality in the following situations:

- strong ground motions

- ocean bottom seismometers

- long period seismology

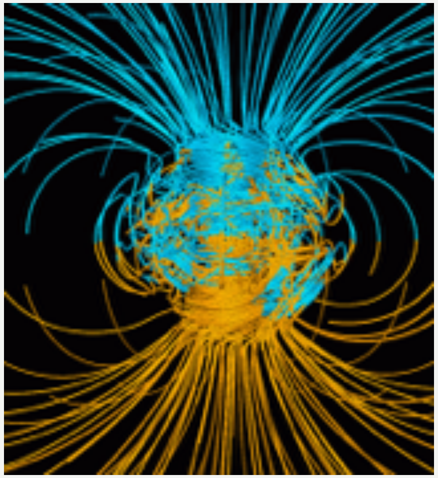
Cross-axis sensitivity

Similarly, tiltmeters are sensitive to transverse accelerations.

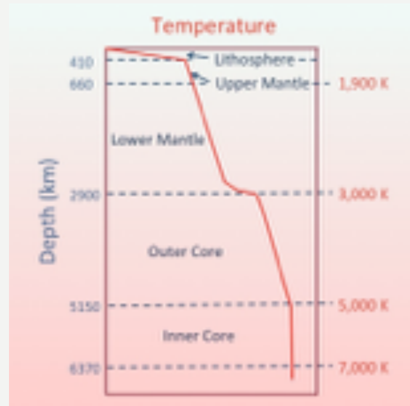
And the vertical component of ring laser measurements is sensitive to local tilts (tilt-ring laser coupling).

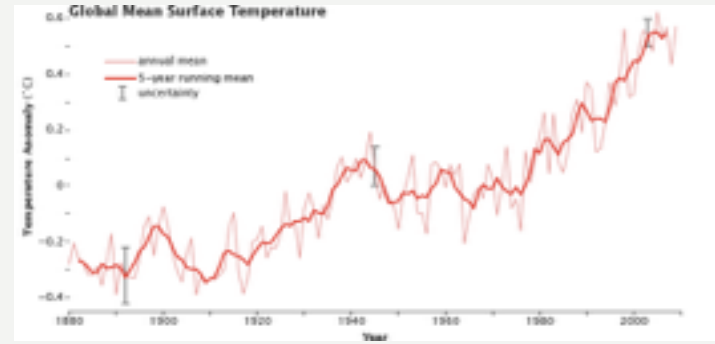
Beyond seismology ...

Geomagnetic field

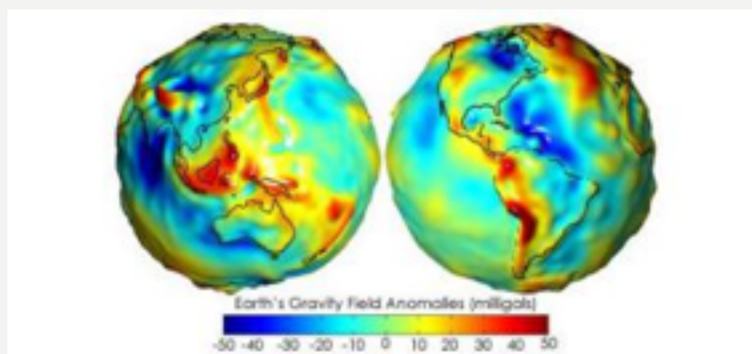


Temperature

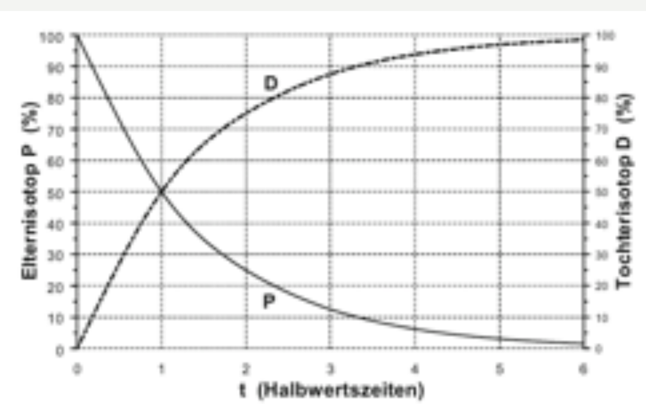




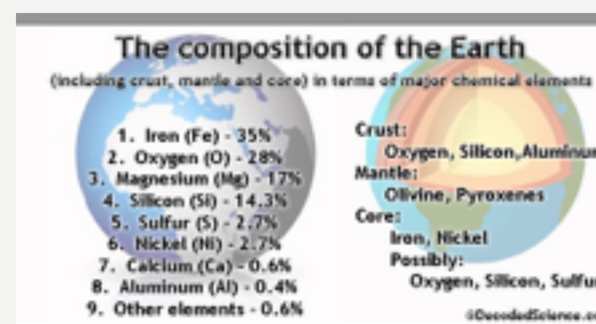
Gravimetric field



Radioactivity



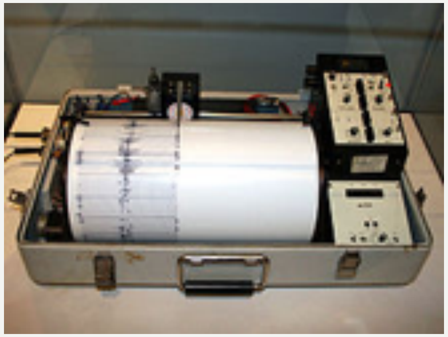
Chemistry



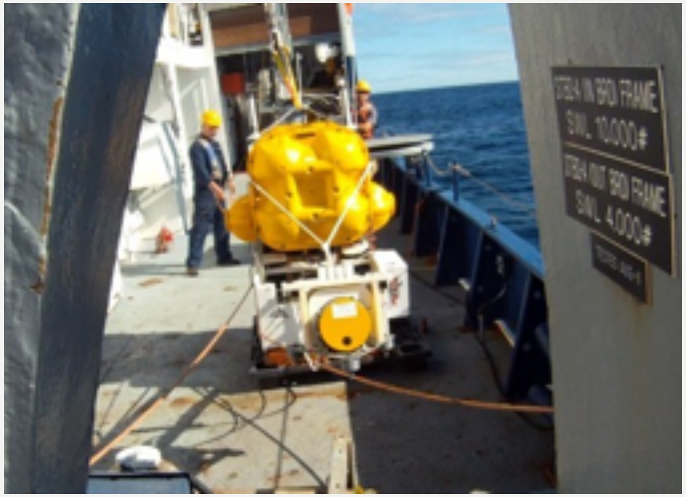
… and much more ...

Summary: observables

- ***Translations*** *are the most fundamental and widely observed quantity*
- *Translation sensors are sensitive to* ***rotations****! (but not vice versa)*
- ***Tilt*** *measurements are sensitive to translations*
- *Entire* ***field of motion contains 12 quantities*** *(collocated observation of translation, rotation, and strain)*
- *There are* ***many more observables in geosciences*** *in general (not necessarily with time dependence)*

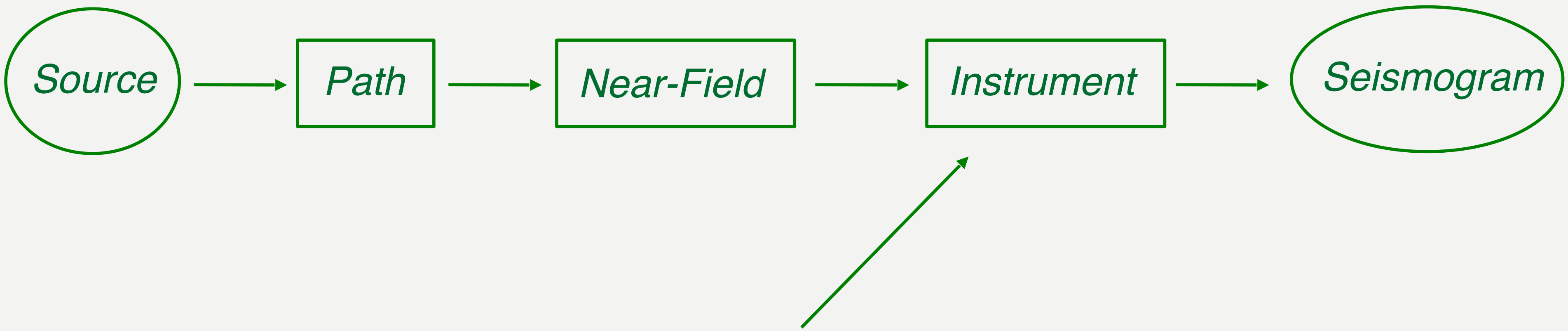


Instruments



Recording seismic data

… several influencing factors alter the seismogram ...



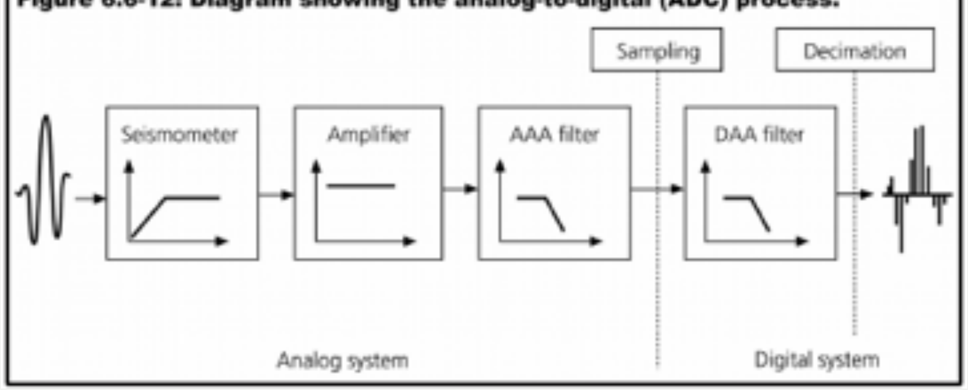
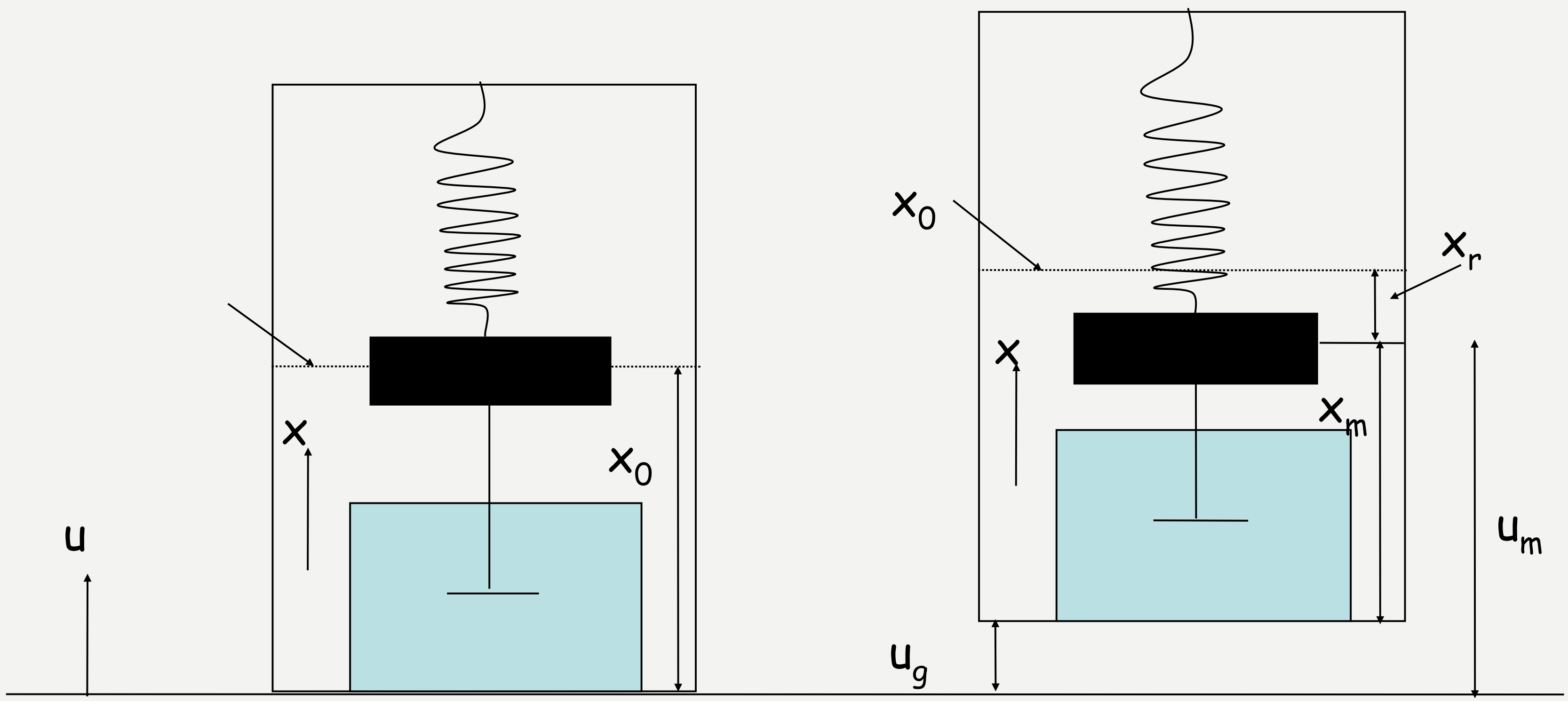


Figure 6.6-12: Diagram showing the analog-to-digital (ADC) process.

Seismometer
The basic principles

A seismometer is a mechanical pendulum



u	ground displacement
x_r	displacement of seismometer mass
x_0	mass equilibrium position

Seismometer
The basic principles

The motion of the seismometer mass as a function of the ground displacement is given through a differential equation resulting from the equilibrium of forces:

$$F_{spring} + F_{friction} + F_{gravity} = 0$$

$$F_{spring} = -k\dot{x}$$

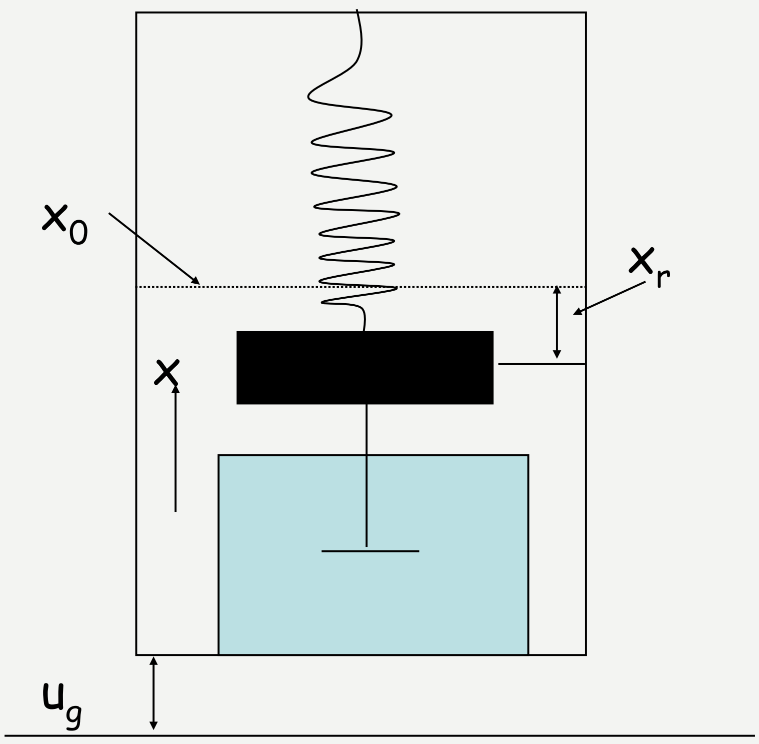
k – spring constant

$$F_{friction} = -D\ddot{x}$$

D – friction coefficient

$$F_{gravity} = -mu$$

m – seismometer mass

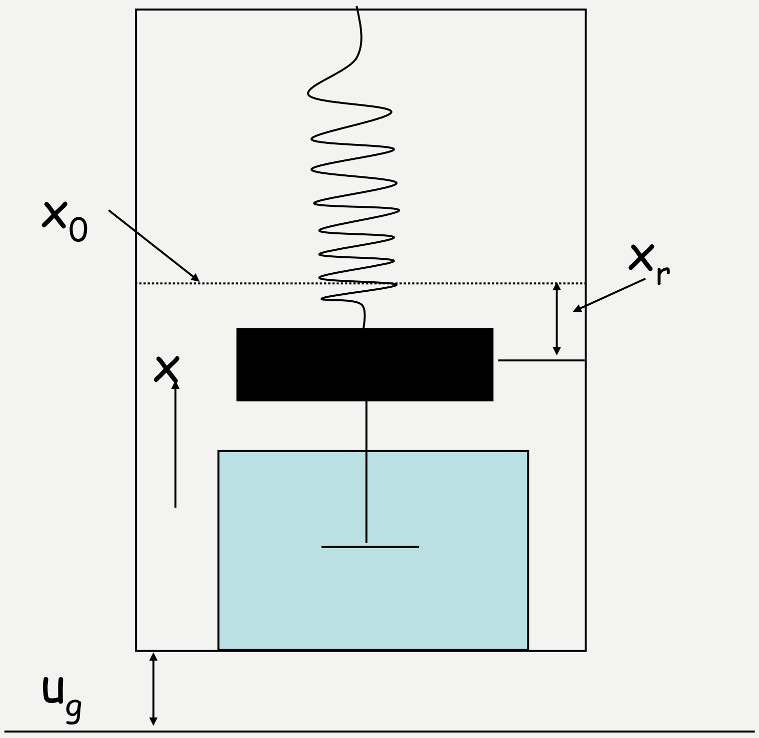


Seismometer
The basic principles

The equation of motion for the mass is then:

$$\ddot{x}_r(t) + 2\epsilon\dot{x}_r(t) + \omega_0^2 x_r(t) = -\ddot{u}_g(t)$$

$$\epsilon = \frac{D}{2m} = h\omega_0 \qquad \omega_0^2 = \frac{k}{m}$$



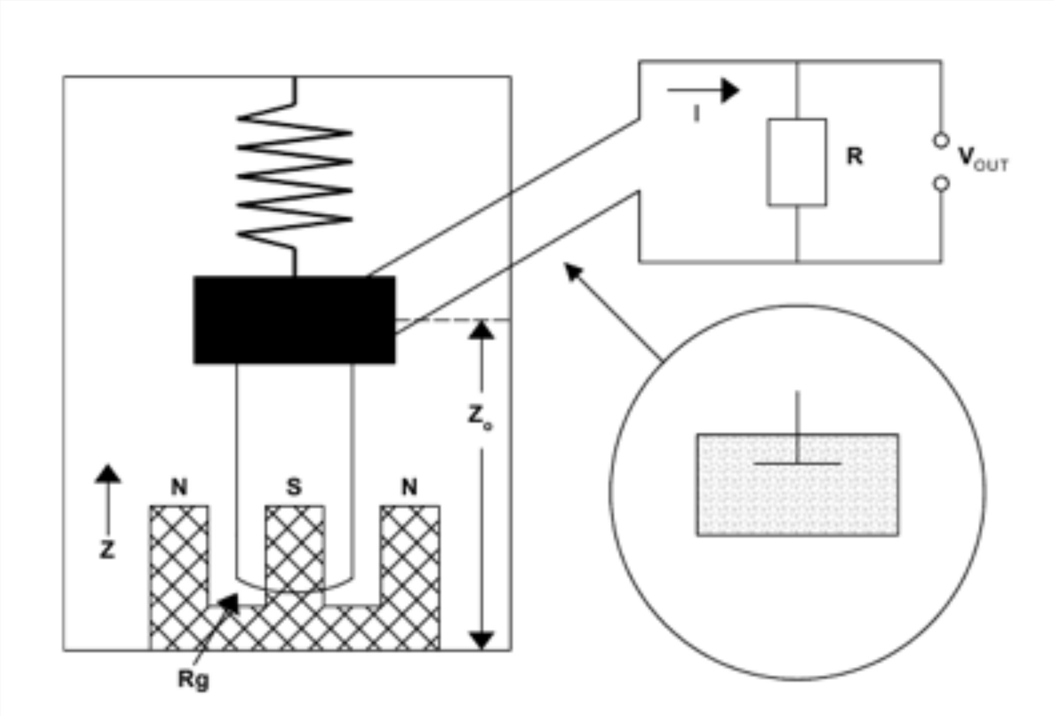
From this equation, we learn:

1. *slow movements → acceleration & velocity become negligible; seismometer record ground acceleration*

2. *fast movements → acceleration dominates; seismometer records ground displacement*

The electromagnetic seismometer

*Modern, electromagentic seismometers always measure **ground velocity**.*



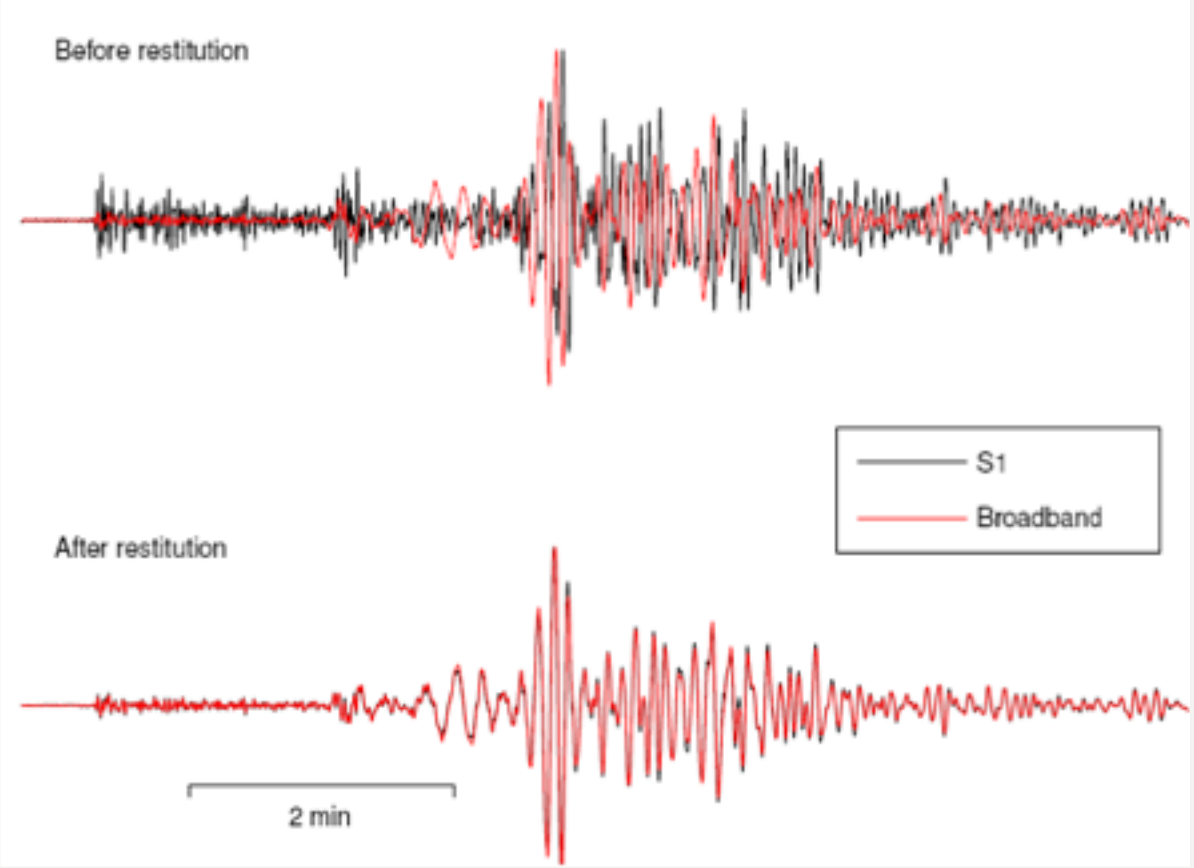
- *Damping through a coil moving in a permanent magnetic field*
- *Movement induces voltage*
- *Voltage ~ velocity*

seismometer by Boris B. Golizyn (1862-1916)
H: mass 7 kg, period 12 s
V: mass 10 kg, period 24 s

Seismometer = Filter

Seismometers act as a filter on the recorded data.

Filter-effect needs to be corrected by restitution (removing instrument response)



Seismometer bandwidth

The type of construction defines which frequency range can be discovered

→ scientific question defines the choice of instrument

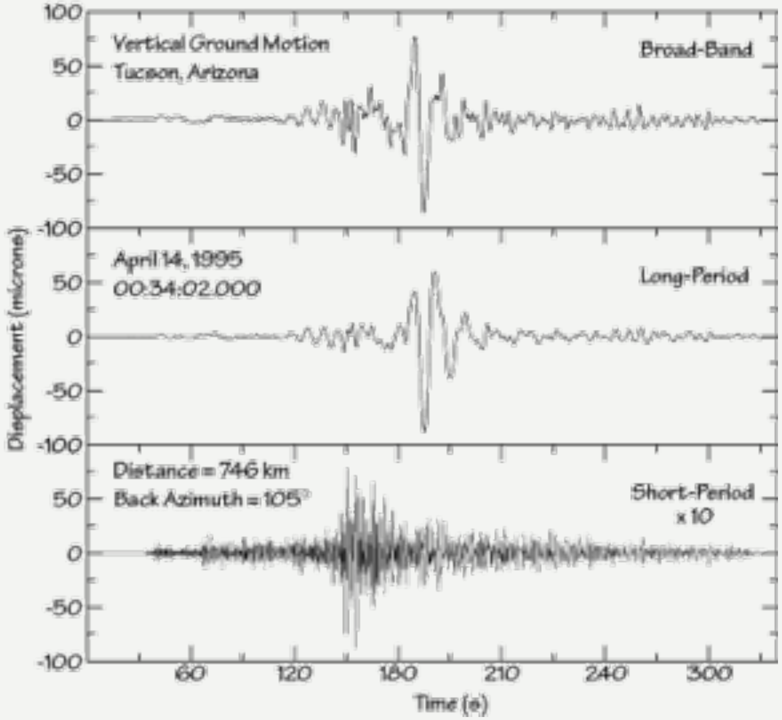
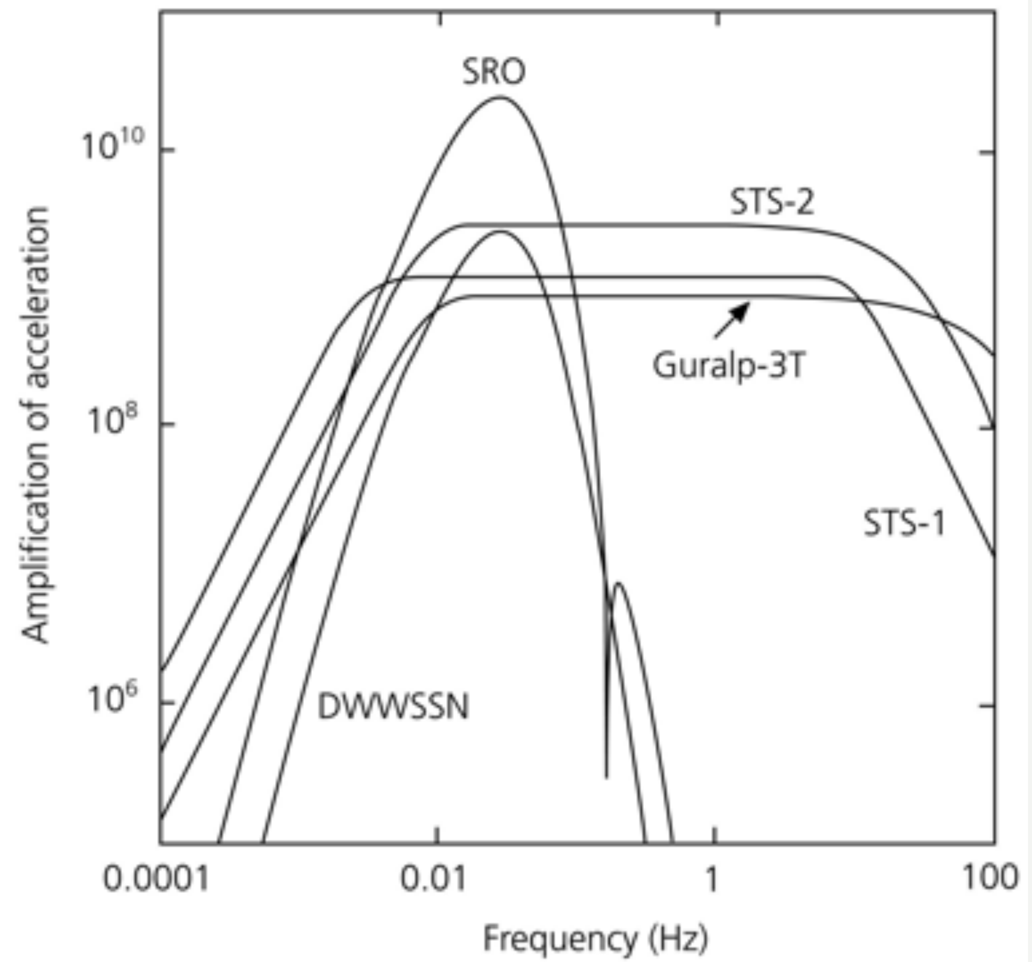


Figure 6.6-8: Instrument responses for several types of seismometers.

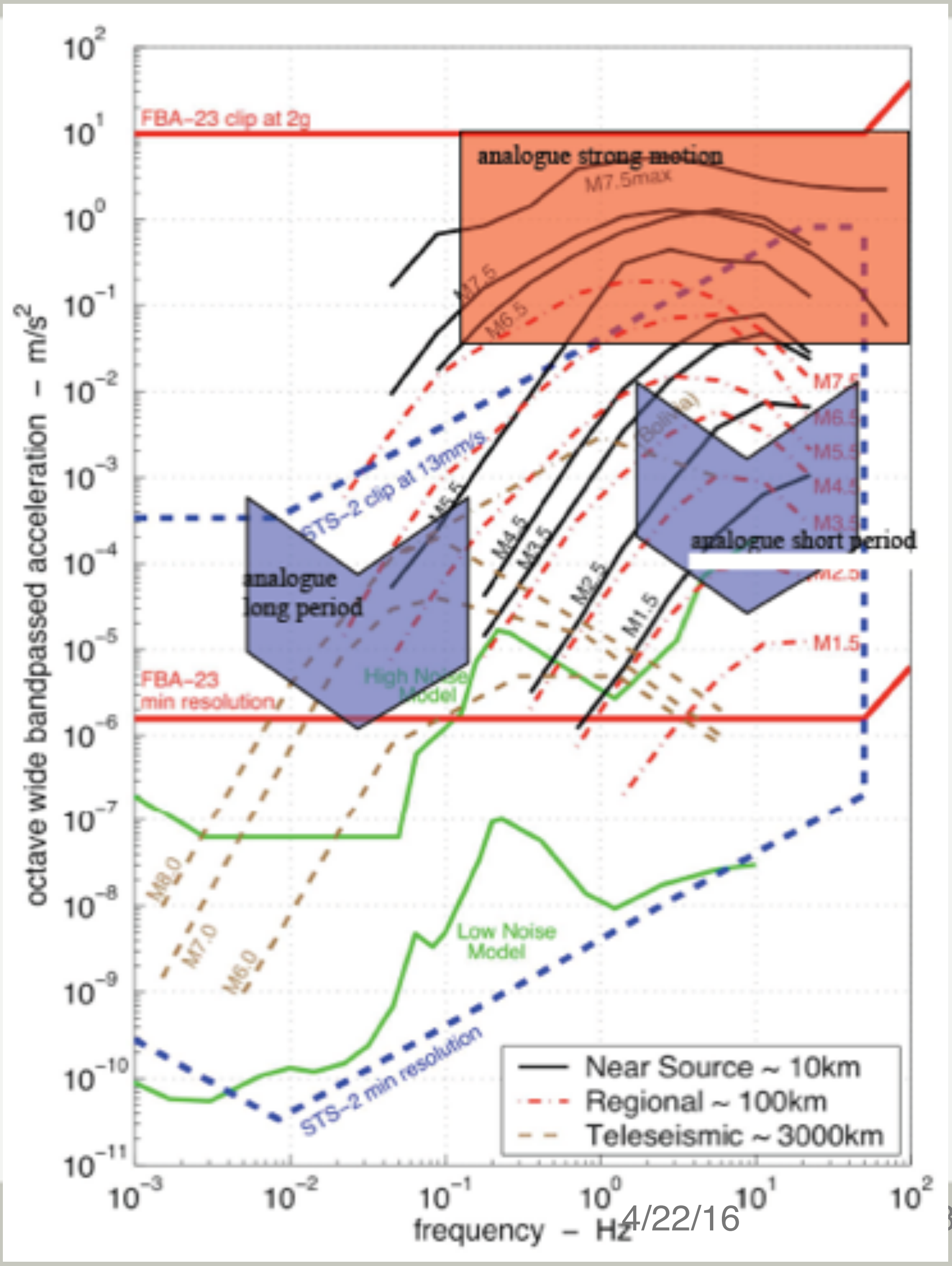


Observed amplitudes

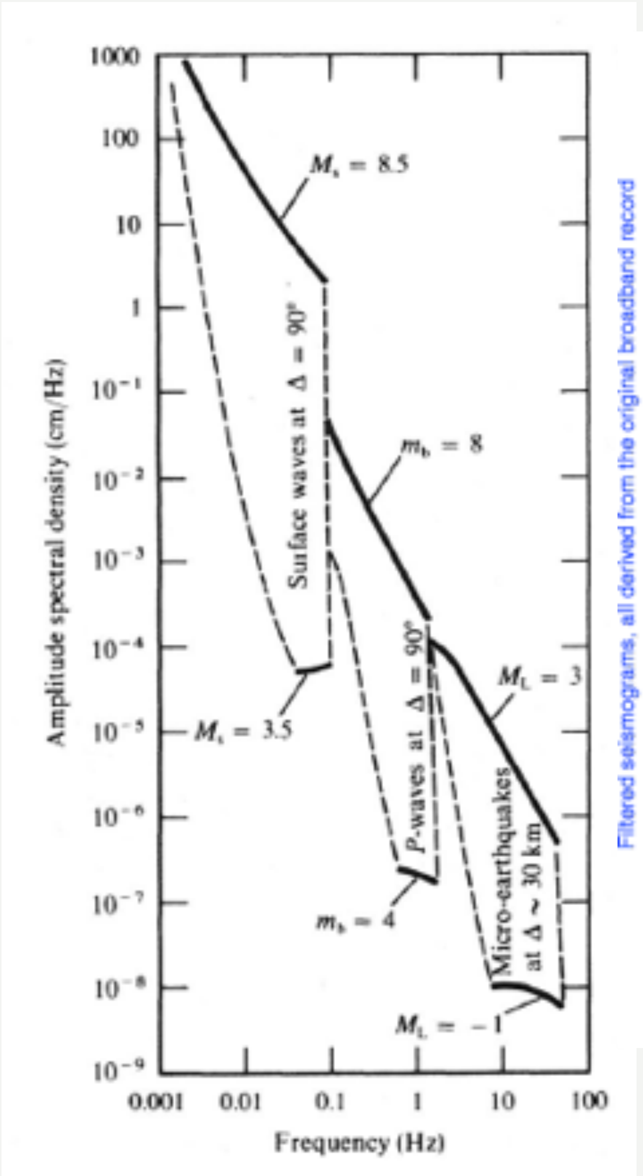
1. Earthquake magnitudes
   - –––– local events
   - - - - regional events
   - - - - teleseismic events

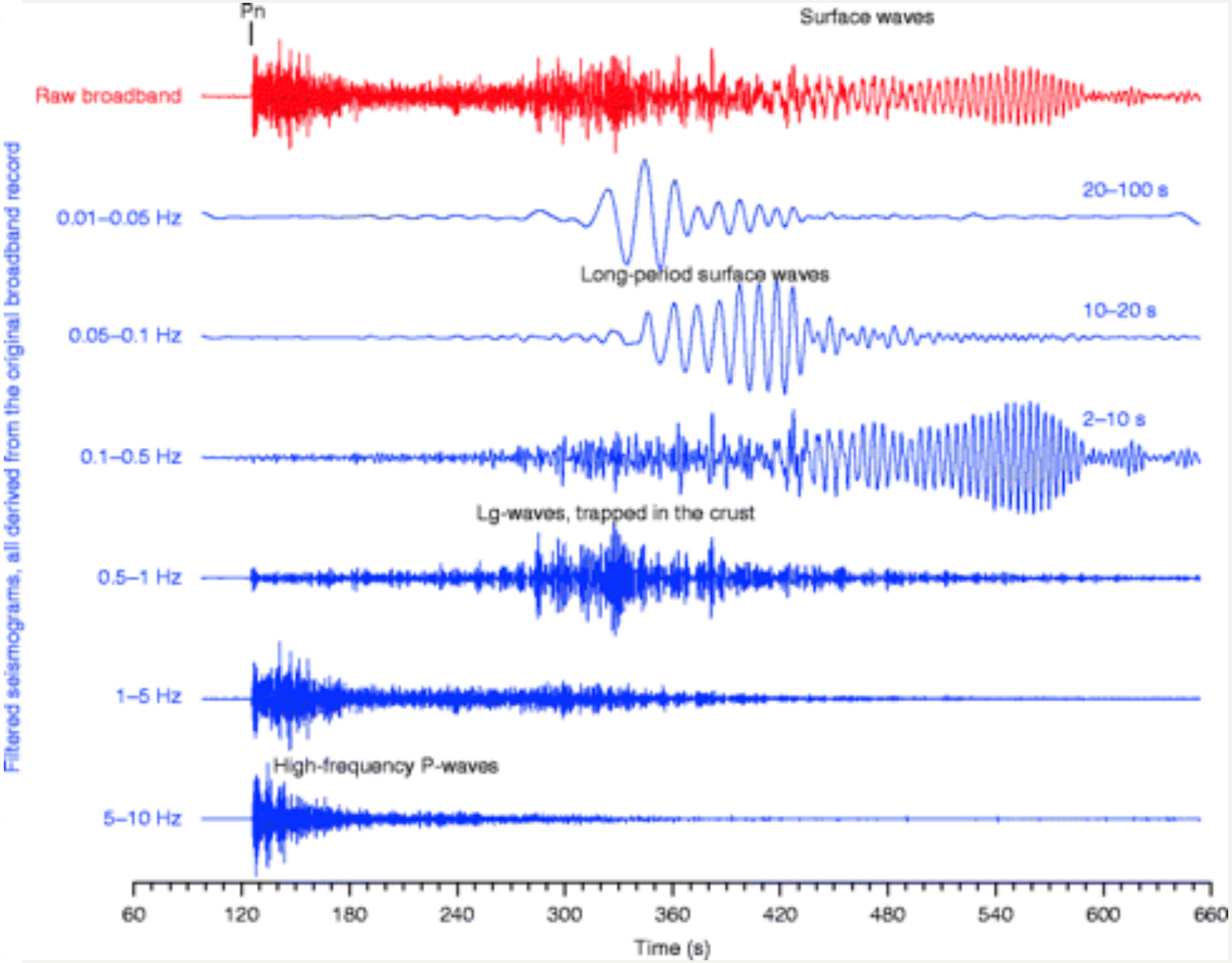
2. instrument recording ranges
   - ----- FBA-23 (accelerometer)
   - - - - STS-2
   - ----- analogue long & short period
   - ----- analogue strong motion

3. Noise models

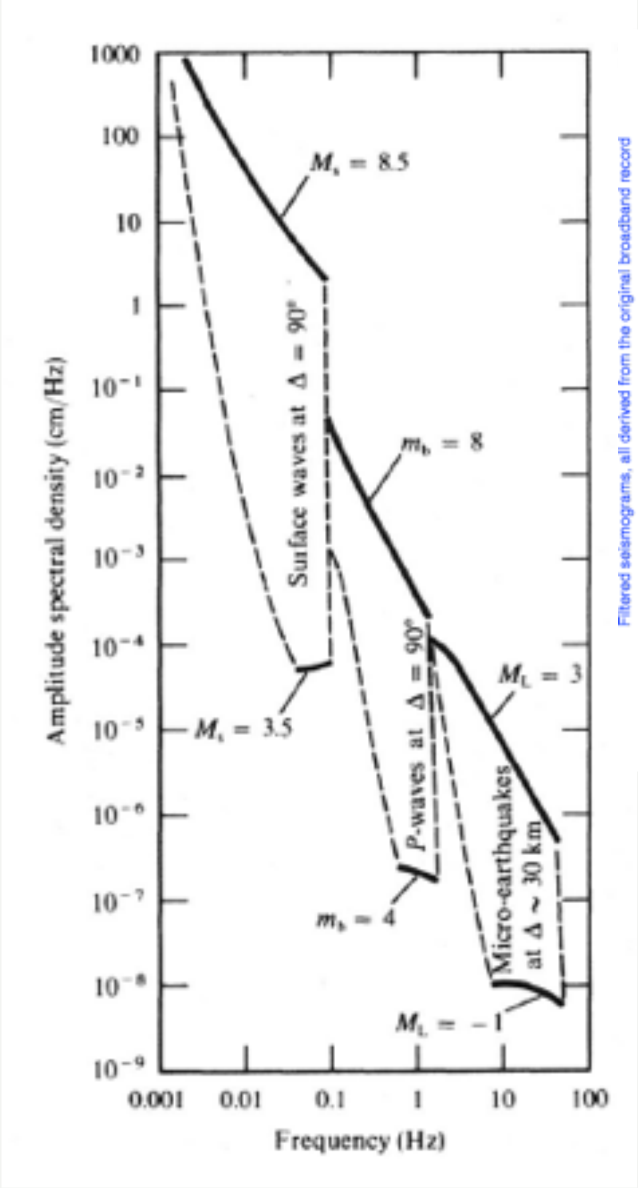


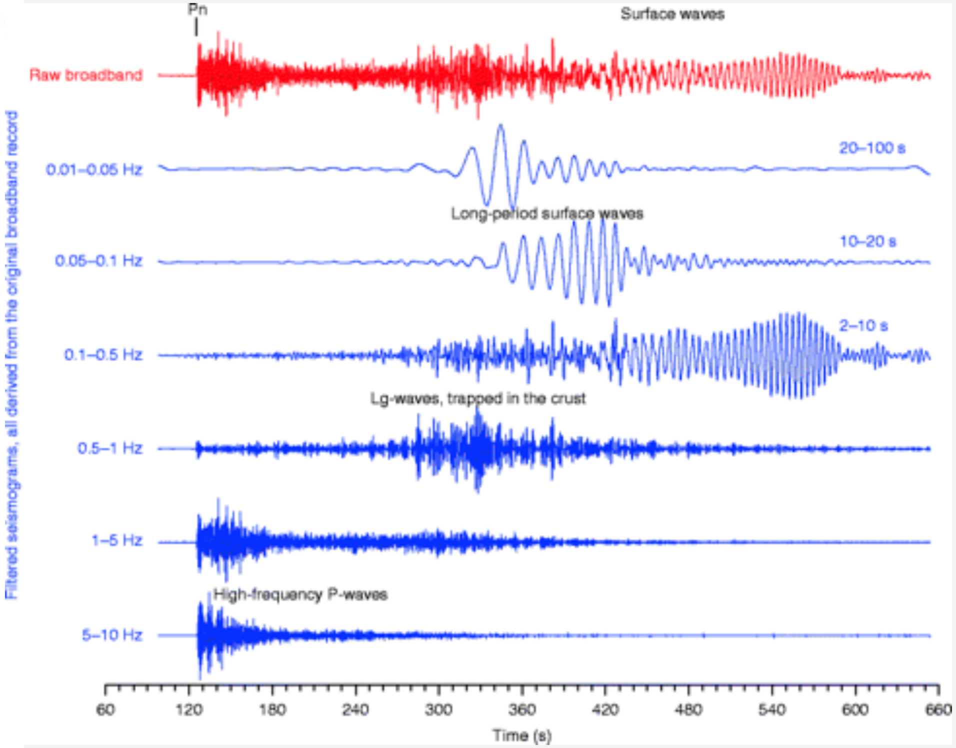
Observed amplitudes





Observed amplitudes





Seismic Monitoring of Nuclear Explosions, Figure 1

Pn Any P wave bottoming in the uppermost mantle or an up-going P wave from a source in the uppermost mantle.
Lg A wave group observed at larger regional distances and caused by superposition of multiple S-wave reverberations and SV to P and/or P to SV conversions inside the whole crust.

Quiz

What do modern seismometers measure?

Quiz

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*Modern, electromagentic seismometers always measure **ground velocity**.*

Quiz

What do modern seismometers measure?

*Modern, electromagentic seismometers always measure **ground velocity**.*

How do we translate this to ground acceleration?

Quiz

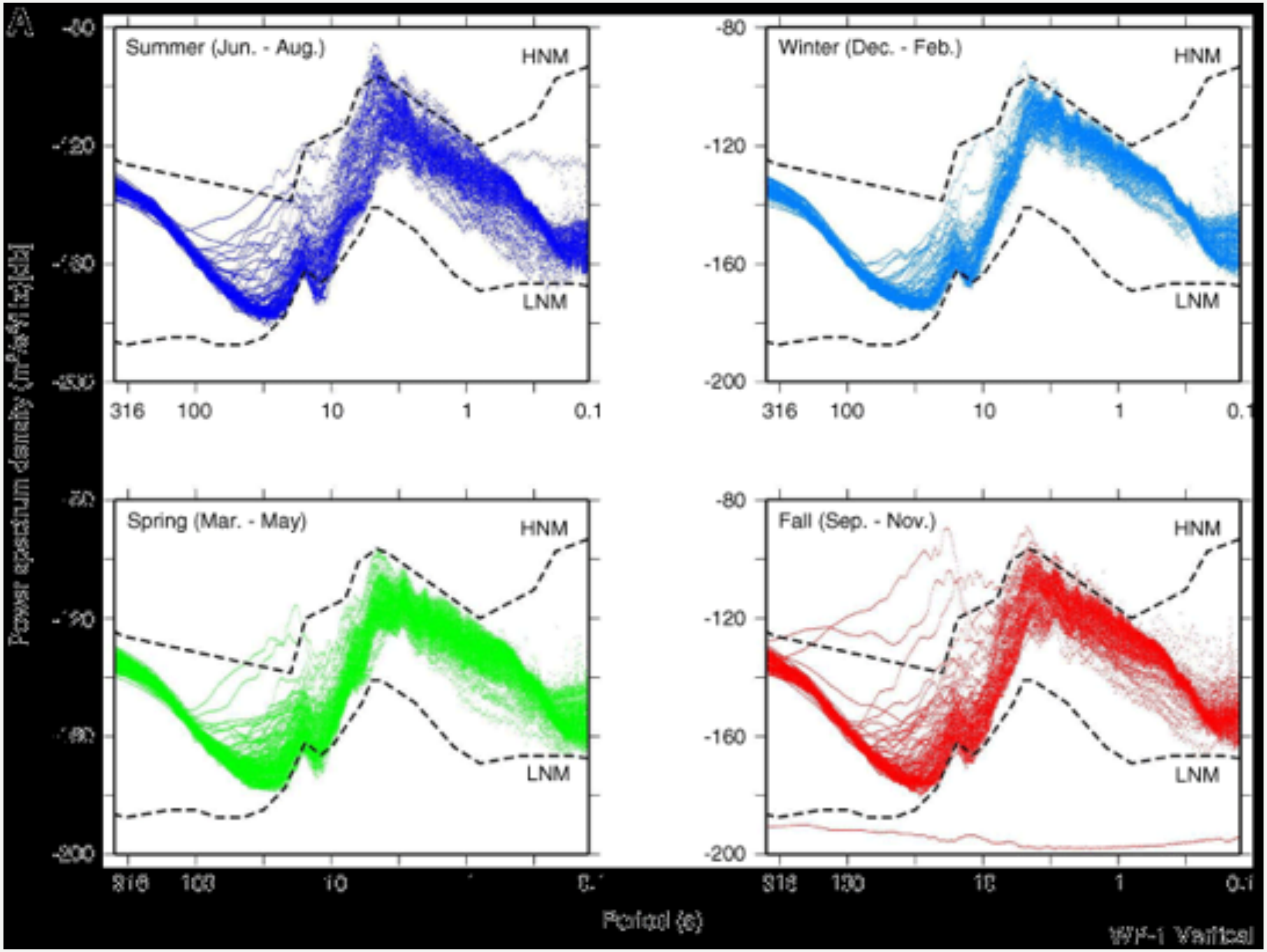
What do modern seismometers measure?

*Modern, electromagentic seismometers always measure **ground velocity**.*

How do we translate this to ground acceleration?

Acceleration is the derivative of velocity.

Seismic noise



(Relative) dynamic range

- *ratio between largest and smallest measurable amplitude*

$$DB = \frac{V_{max}}{V_{min}}$$

- *unit: bell … is the base 10 logarithm the ratio of two energies*

$$L = \log\left(\frac{E_1}{E_2}\right) B = 10 \log\left(\frac{E_1}{E_2}\right) dB$$

- *with dB is a 10th of B*

… in terms of amplitude:

$$L = 10 \log\left(\frac{A_1}{A_2}\right)^2 dB = 20 \log\left(\frac{A_1}{A_2}\right)$$

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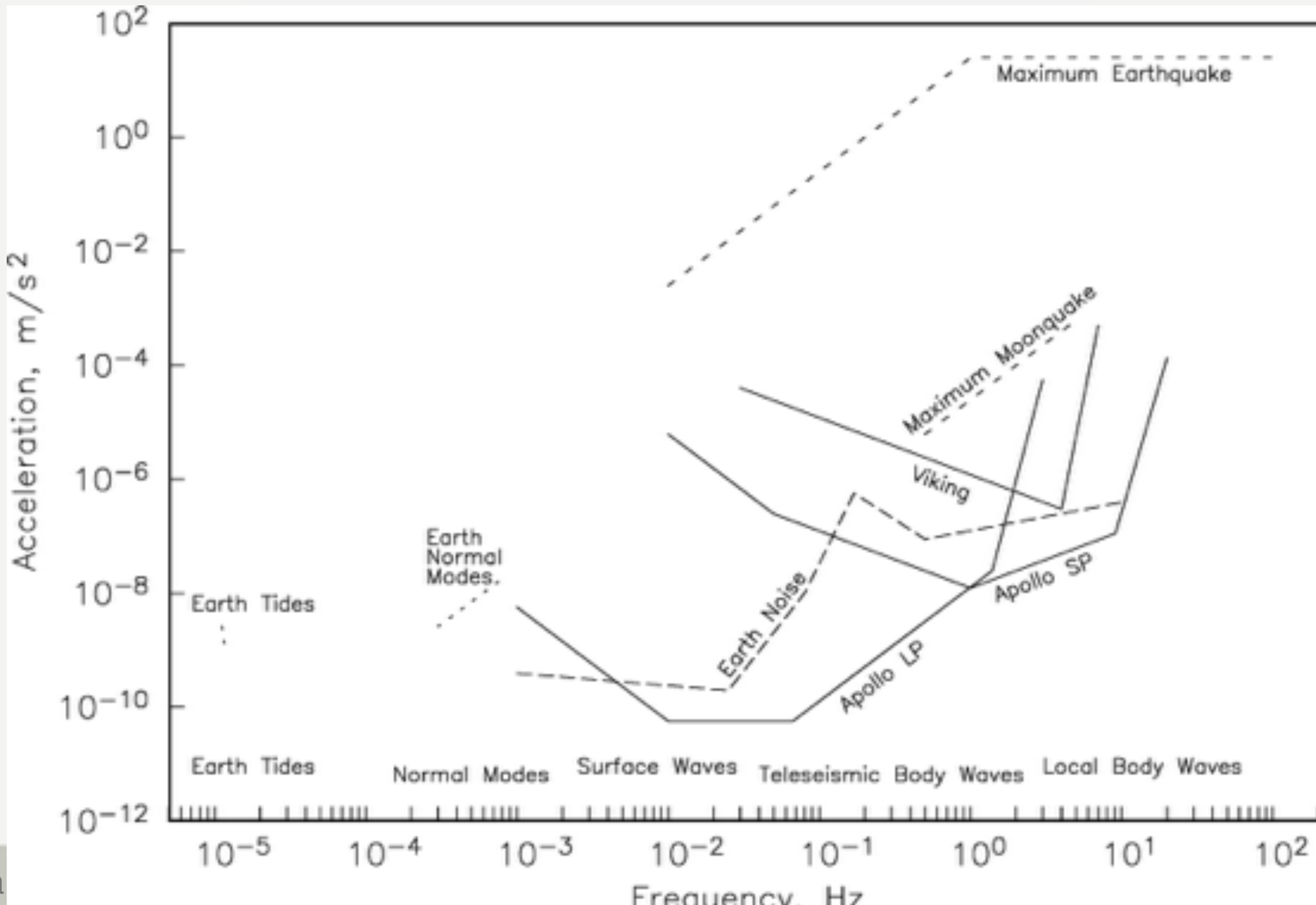
(Relative) dynamic range

Earth produces motions varying 10 orders of magnitude !!!

$$DR_{Earth} = 20 \log \left(10^{10}\right)\ dB = 200\, dB$$



Bits, counts, dynamic range

#bits	Dynamic Range *($2^{\#bits-1}$) counts*	DR_{dB} *((#bits-1)x6)*	Orders of Magnitude *(DR_{dB}/20)*
8	256/2	42	~2
12	4,096/2	66	~3
16	65,536 /2	90	~4.5
20	1,048,576/2	114	~6
24	16,777,216/2	138	~7

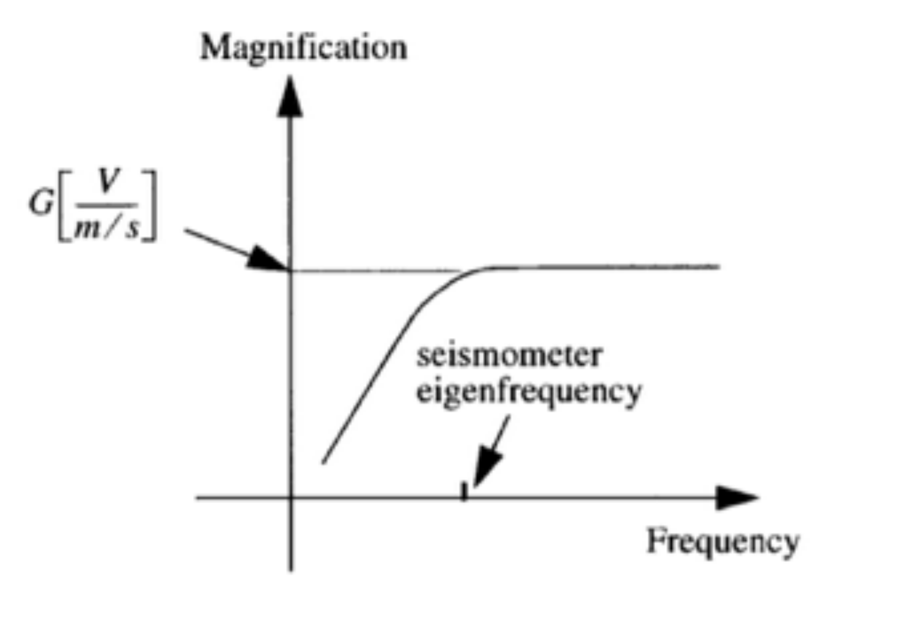
Seismometer
Mechanical : 100*dB*
Force-balance : 140-150*dB*

Telemetry
analogue phone : 50*dB*
digital : α bandwidth
(possibly ∞)

Recording
paper : 60*dB*
digital : $2^{\#bits-1}$counts
(#bits-1)x6 *dB*
(up to 144*dB*)

Generator Constant

For modern seismometers, the magnification for ground velocity is constant (above the seismometer eigenfrequency).



The magnification factor is known as the Generator Constant:

$$G\left[\frac{V}{m/s}\right]$$

From: Of Poles and Zeros, Frank Scherbaum

Exercise

1. Assume you have a 16-bit digitizer. What is its dynamic range?

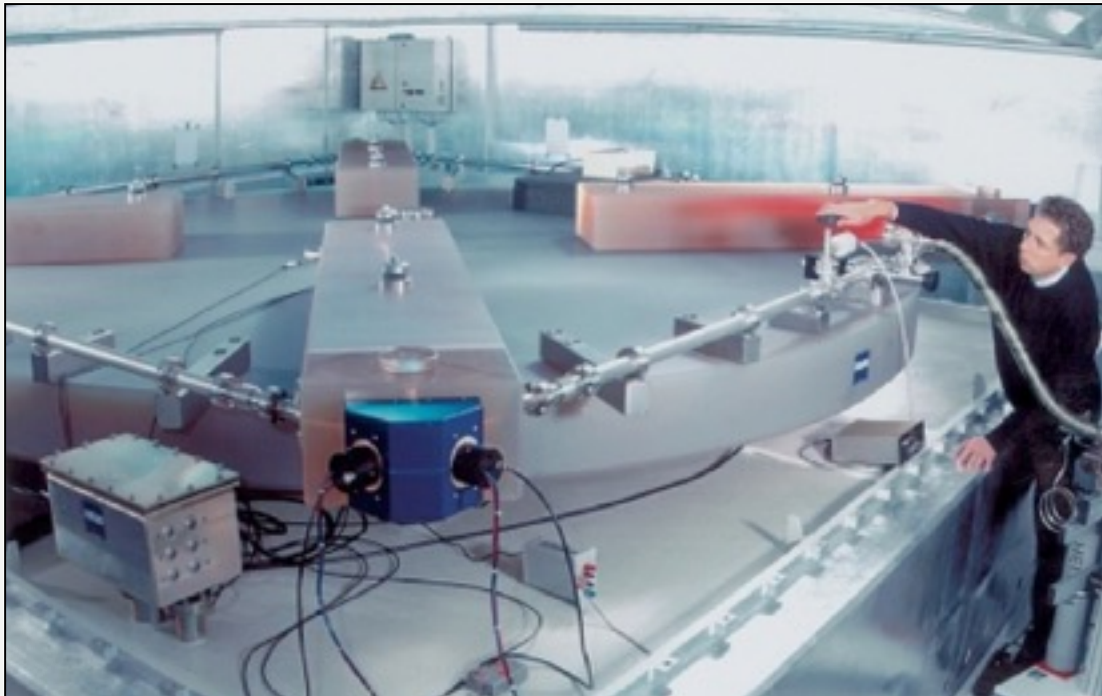
Exercise

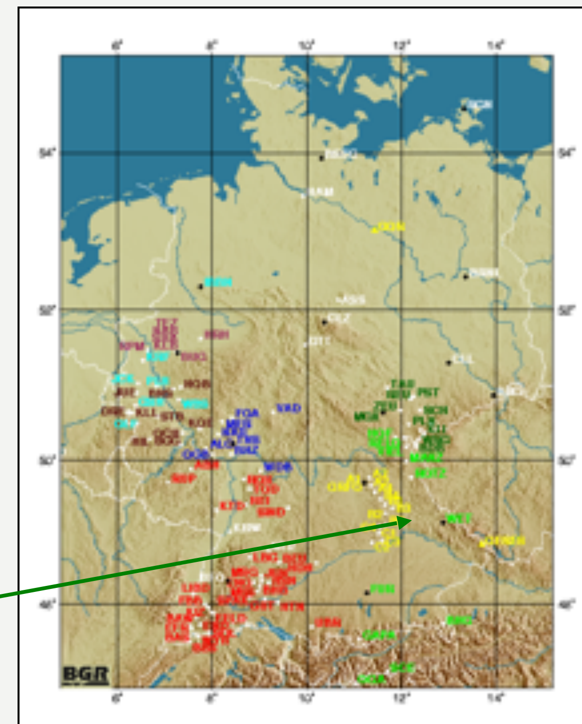
1. *Assume you have a 16-bit digitizer. What is its dynamic range?*

   $2^{\wedge(16-1)}$ = 32768 counts

Measuring rotations

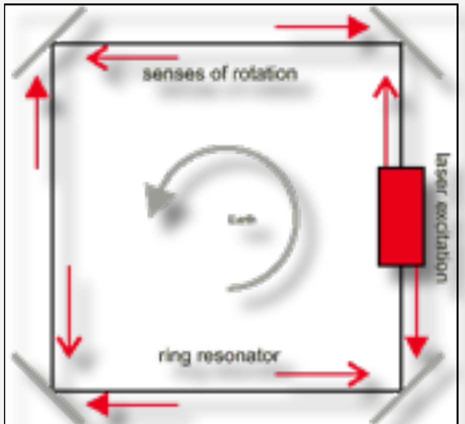
The ring laser at Wettzell

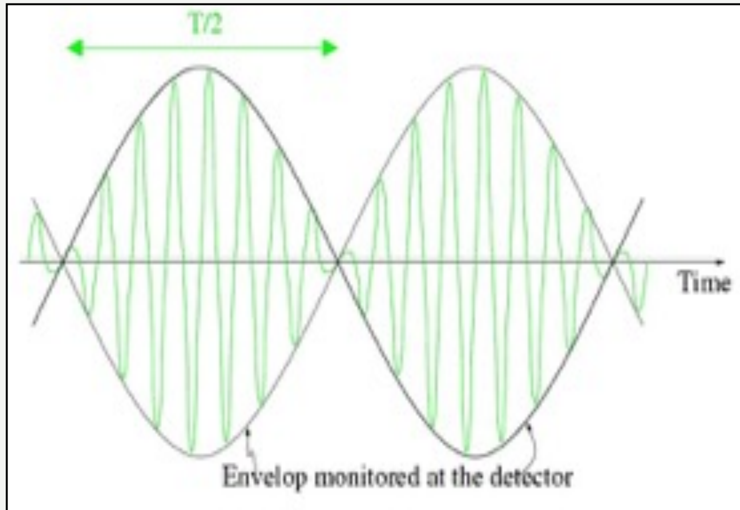


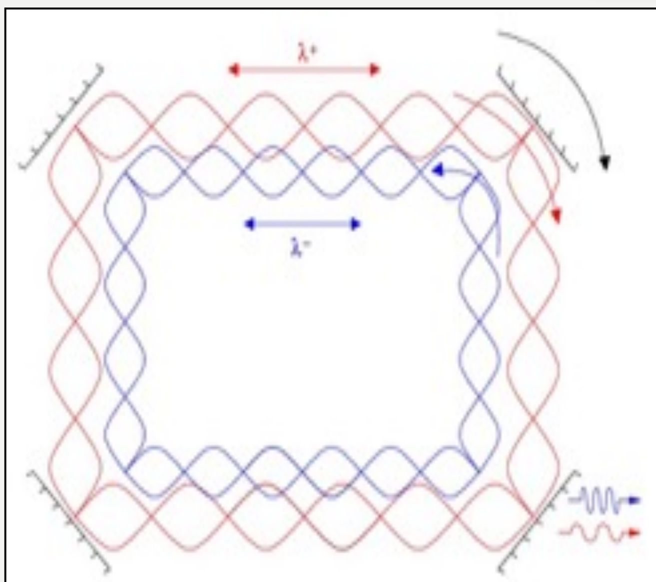


Ring laser

Laser principle





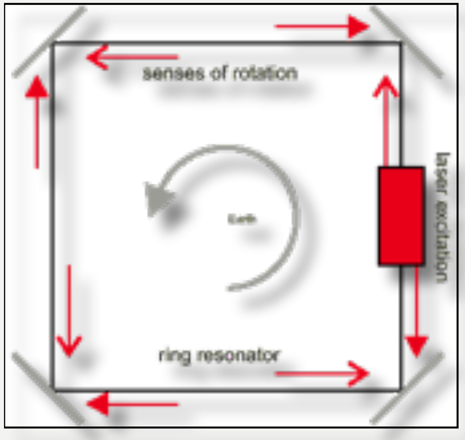


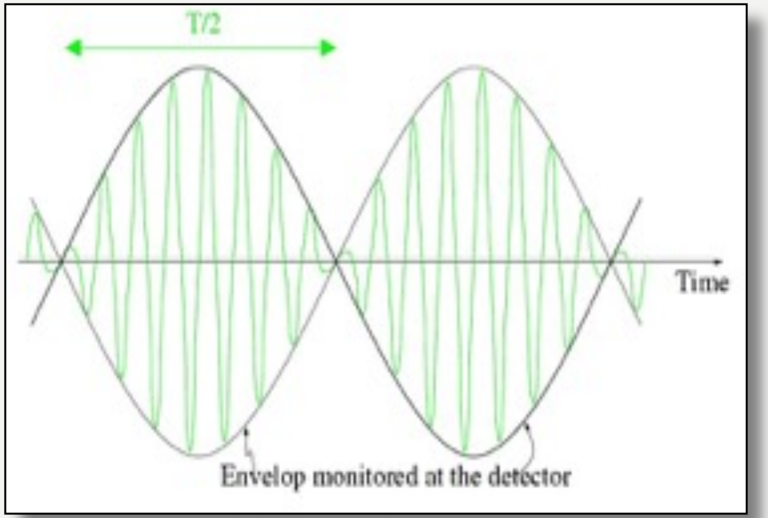
Ring lasers are based on the Sagnac effect which is a consequence of Special Relativity.

Because the velocity of light is fixed:

the co-rotating laser beam is effectively longer than the anti-rotating beam => Beating

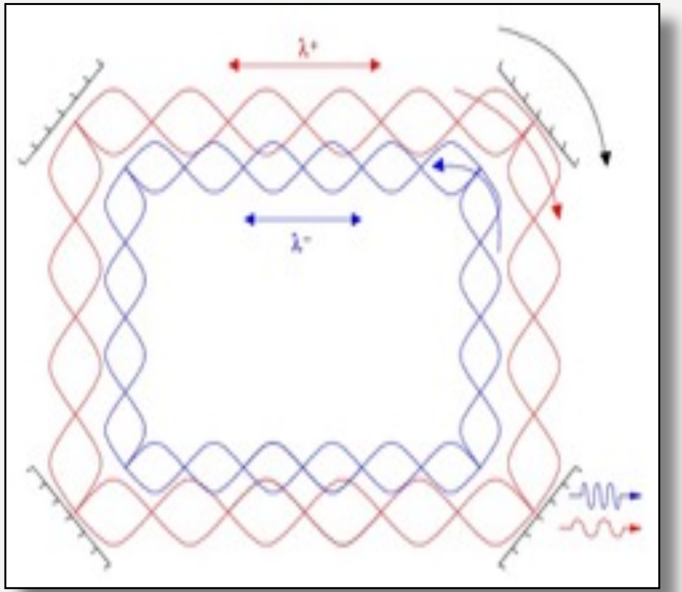
Laser principle





Technology developed at TUM and University of Christchurch, NZ

$$\delta f = \frac{4A}{\lambda L} \mathbf{n} \cdot \dot{\boldsymbol{\omega}}$$

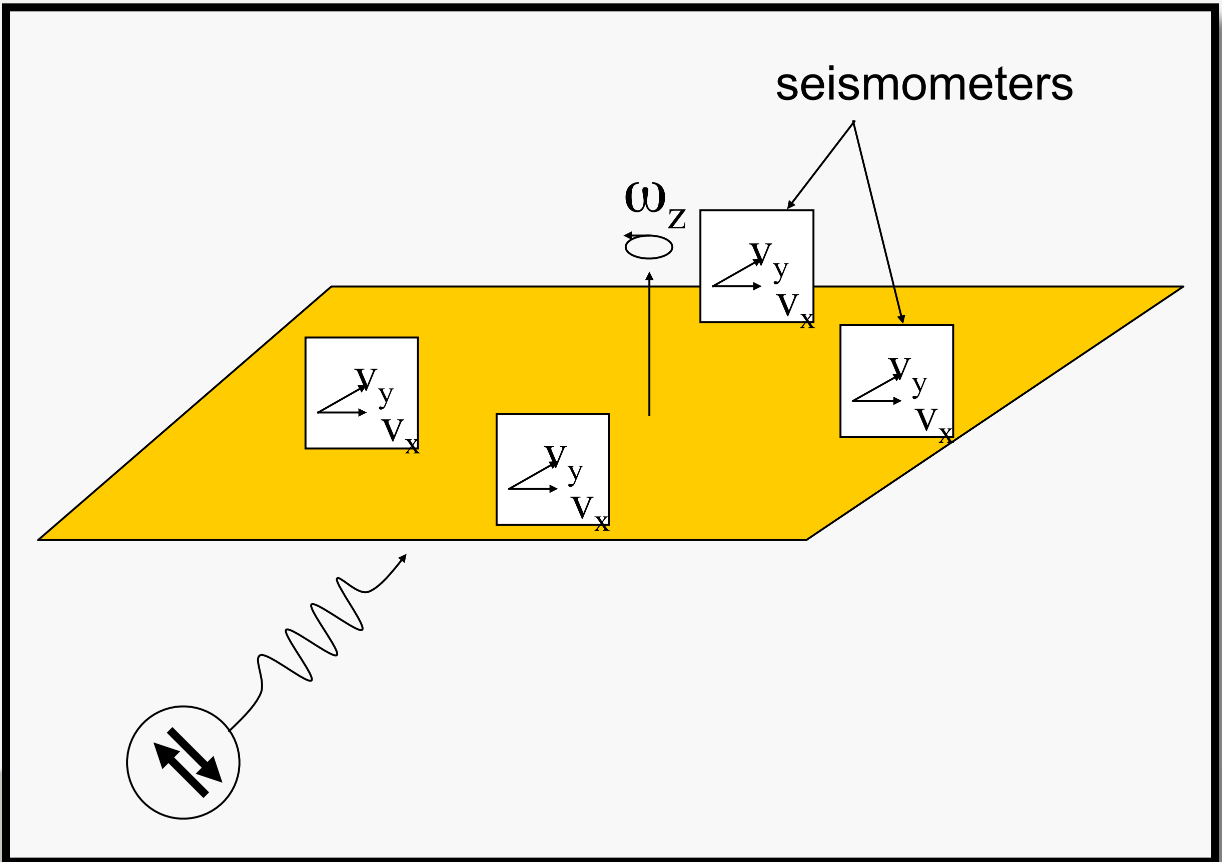


δf beating frequency
A area
L perimeter of cavity (e.g. 4-16 m)
n normal vector
λ laser wavelength (e.g. 633 nm)
$\dot{\boldsymbol{\omega}}$ imposed rotation rate (Earth's rotation + earthquake + …)

Rotations from **seismic arrays**

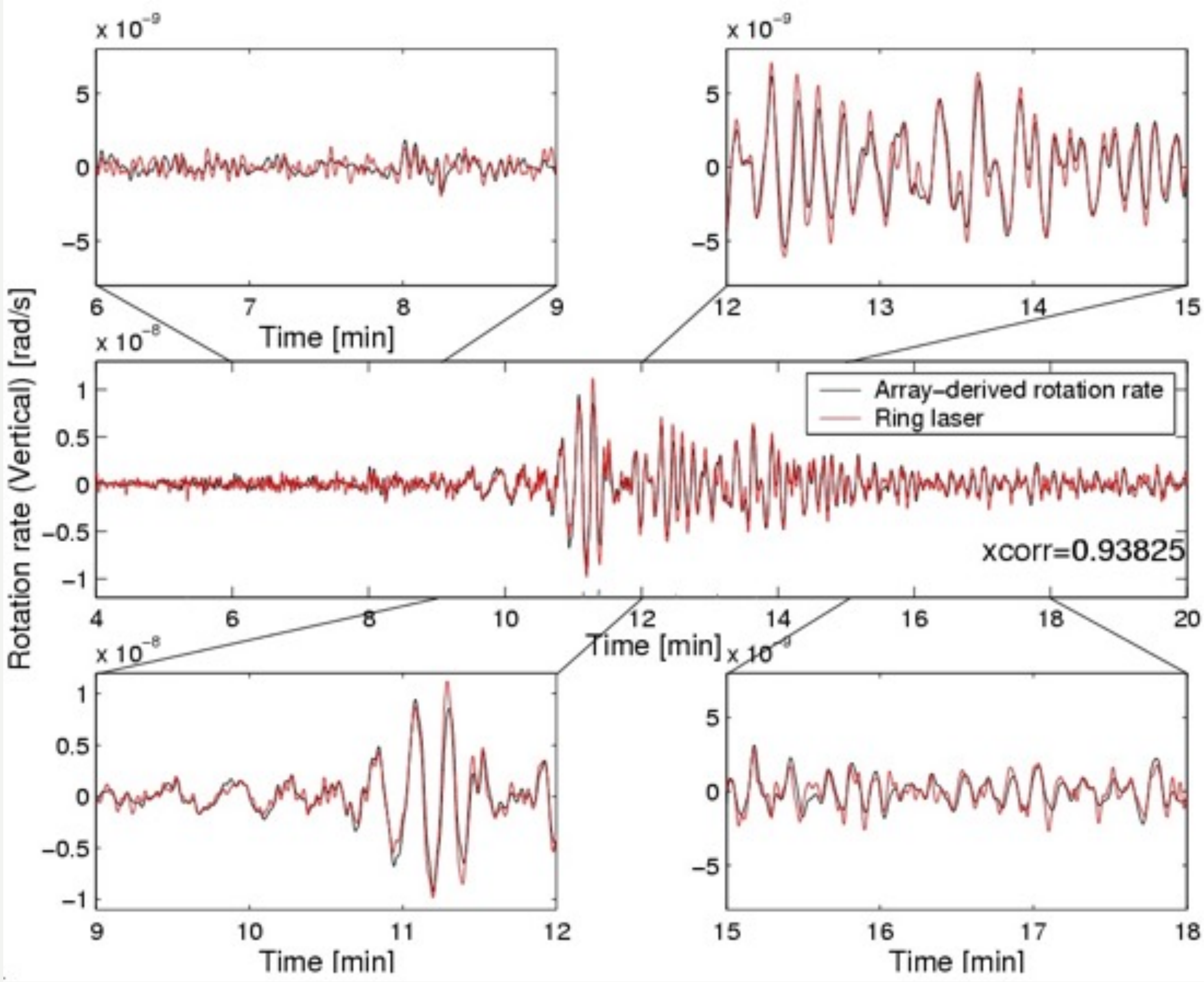
… the curl of the wave field …

$$\begin{pmatrix} \omega_x \\ \omega_y \\ \boxed{\omega_z} \end{pmatrix} = \frac{1}{2} \nabla \times v = \frac{1}{2} \begin{pmatrix} \partial_y v_z - \partial_z v_y \\ \partial_z v_x - \partial_x v_z \\ \boxed{\partial_x v_y - \partial_y v_x} \end{pmatrix}$$



curl - describes the infinitesimal rotation of a 3-D vector field

Direct vs. array derived rotation



Suryanto 2006

Principle of a strain-meter

Basic definition: $$\epsilon = \frac{\text{change in length}}{\text{initial length}} = \frac{L - L_0}{L_0}$$

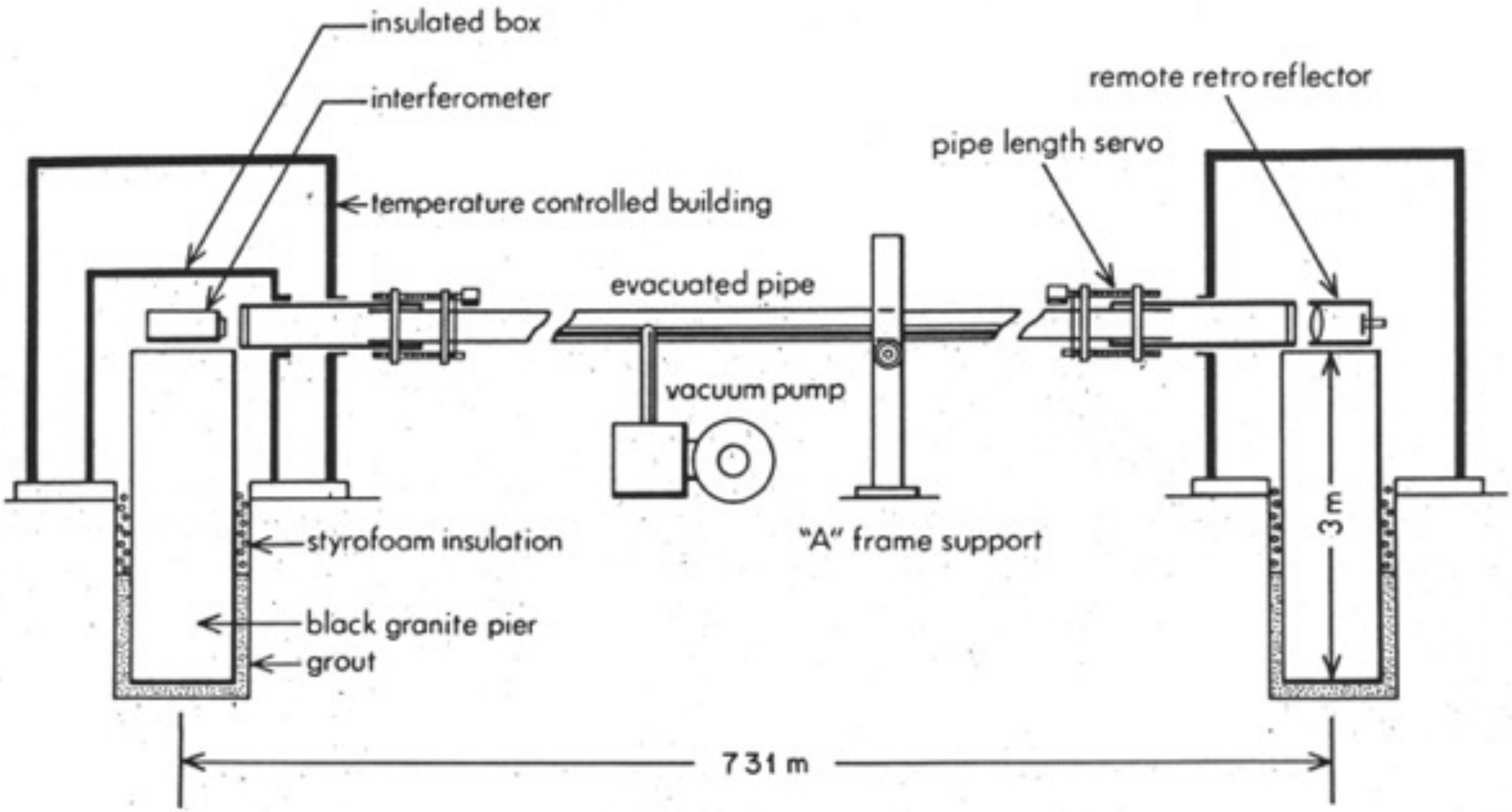
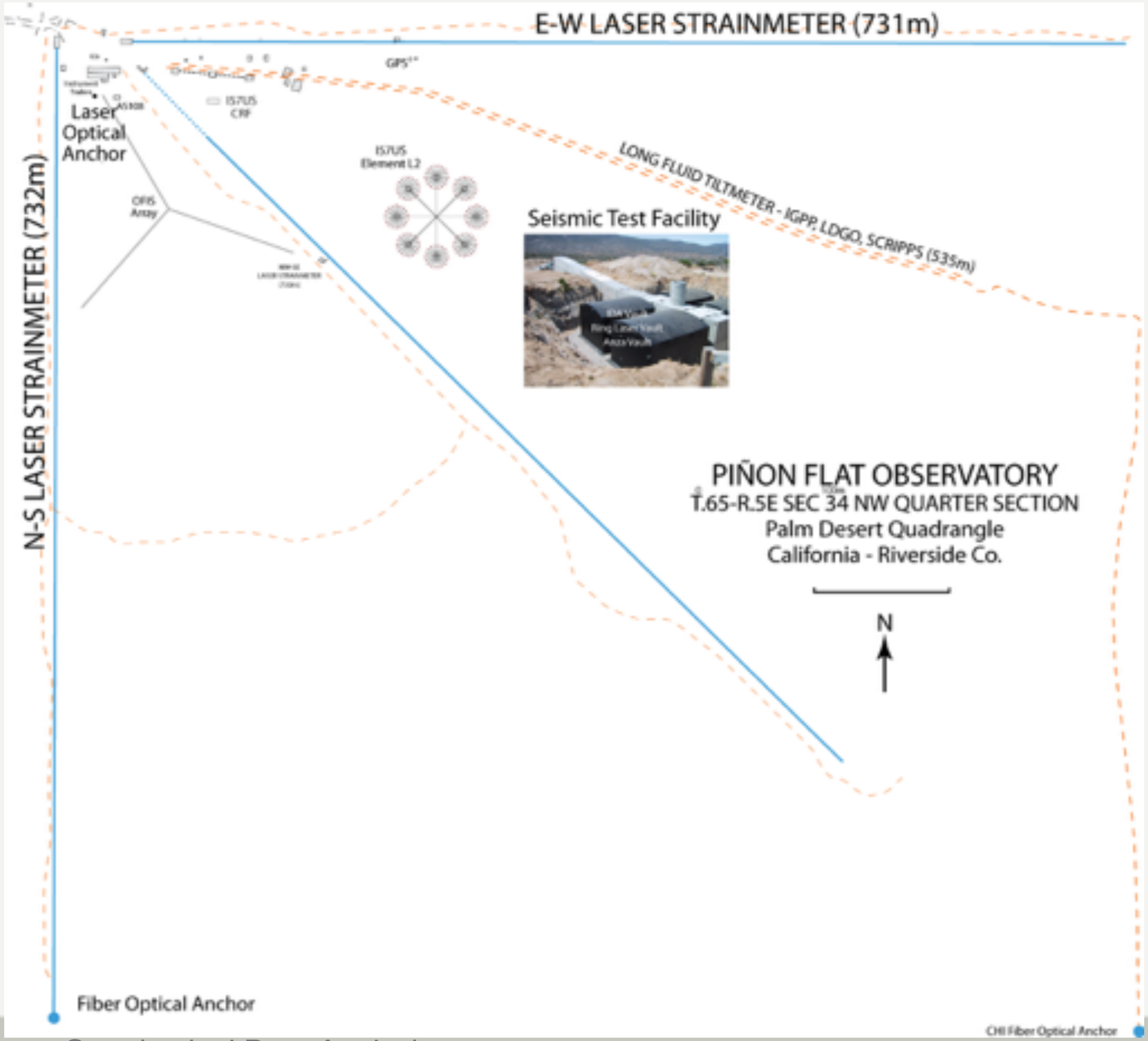


Fig. 21. Mechanical design of the UCSD laser strainmeter. The two endpoints are tall piers of dimension stone sunk in the ground. These, and the optics they carry, are inside temperature-controlled enclosures in air-conditioned buildings. The measurement path is inside a vacuum pipe except at the very ends; telescopic joints keep the length of the air paths constant.

Piñon Flat, CA



Piñon Flat, CA

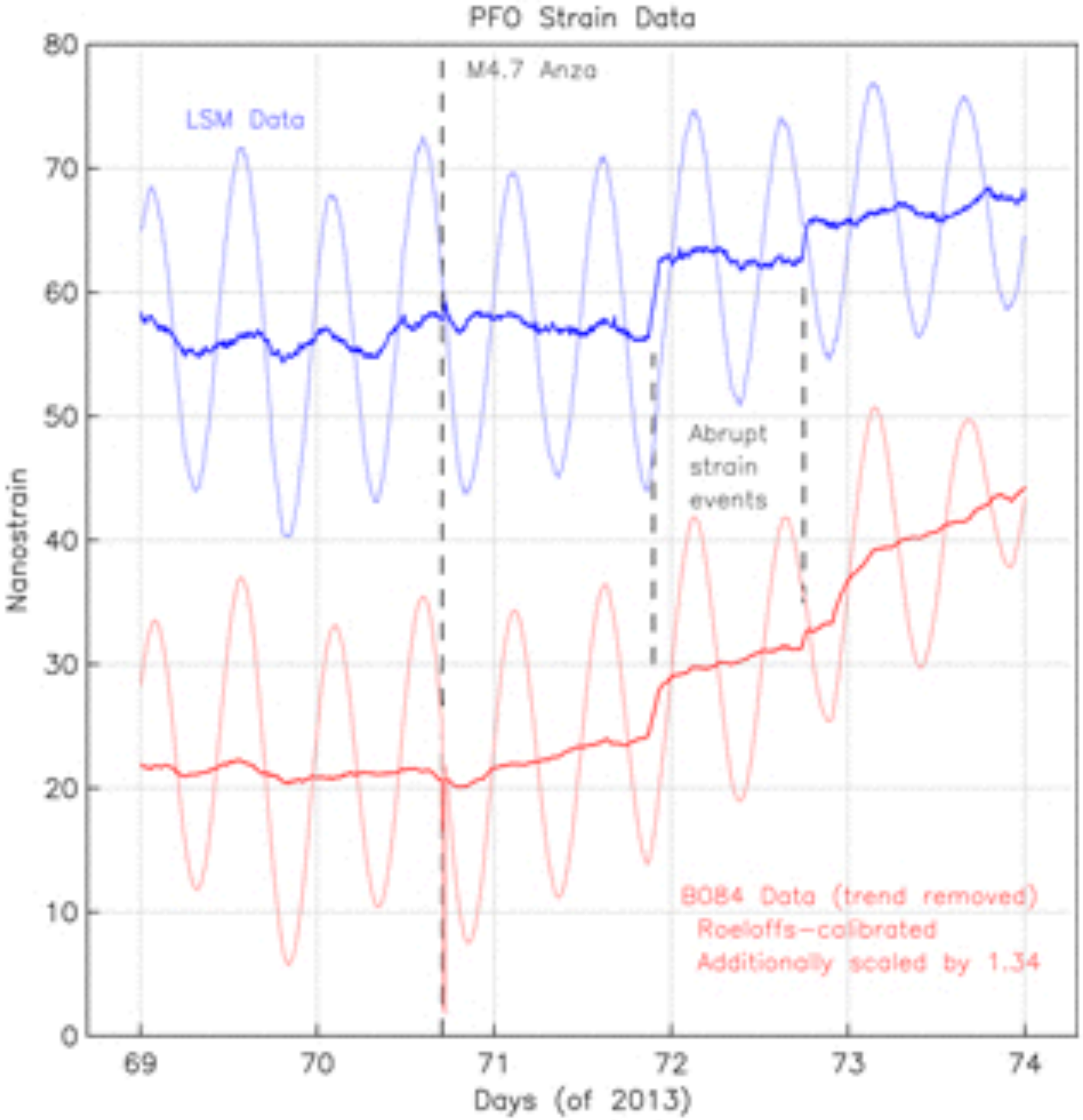
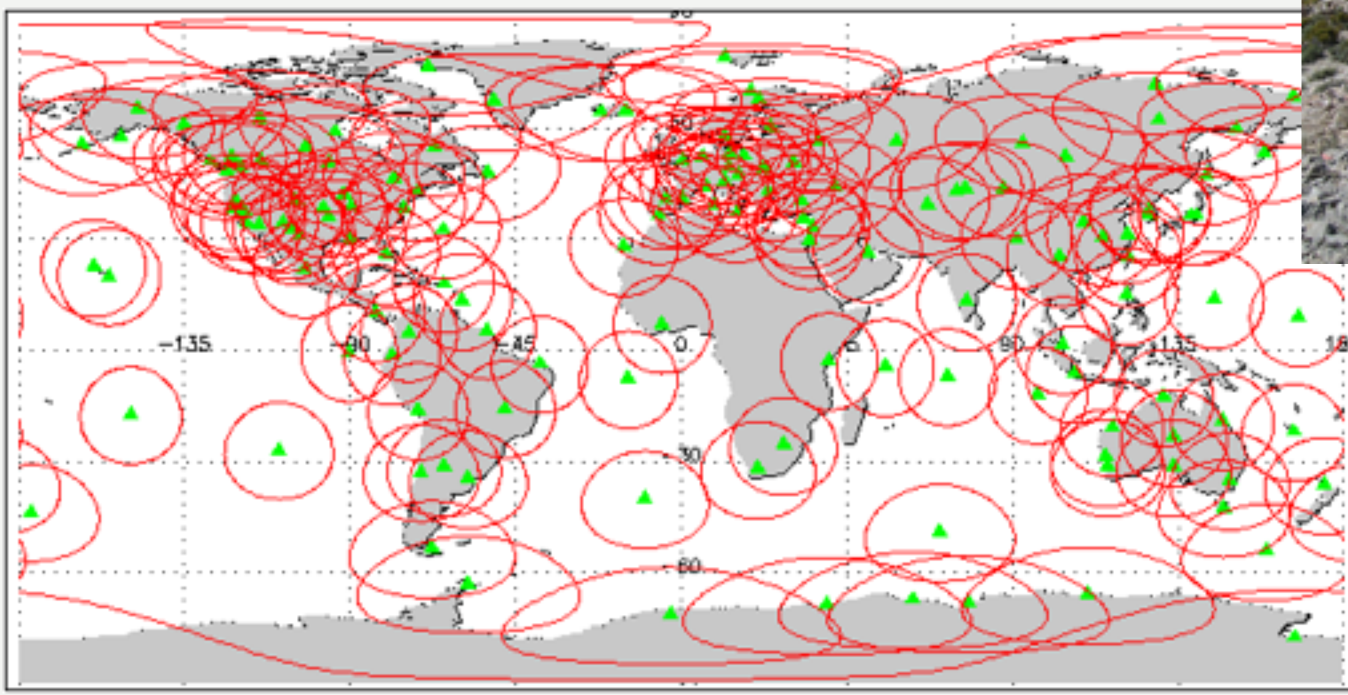


Figure 4. Both the borehole and laser strainmeters at PFO detected small strain events on the 12th and 13th March 2013. Figure prepared by Billy Hatfield Scripps Institution of Oceanography, University of California San Diego.
Figure from https://www.unavco.org

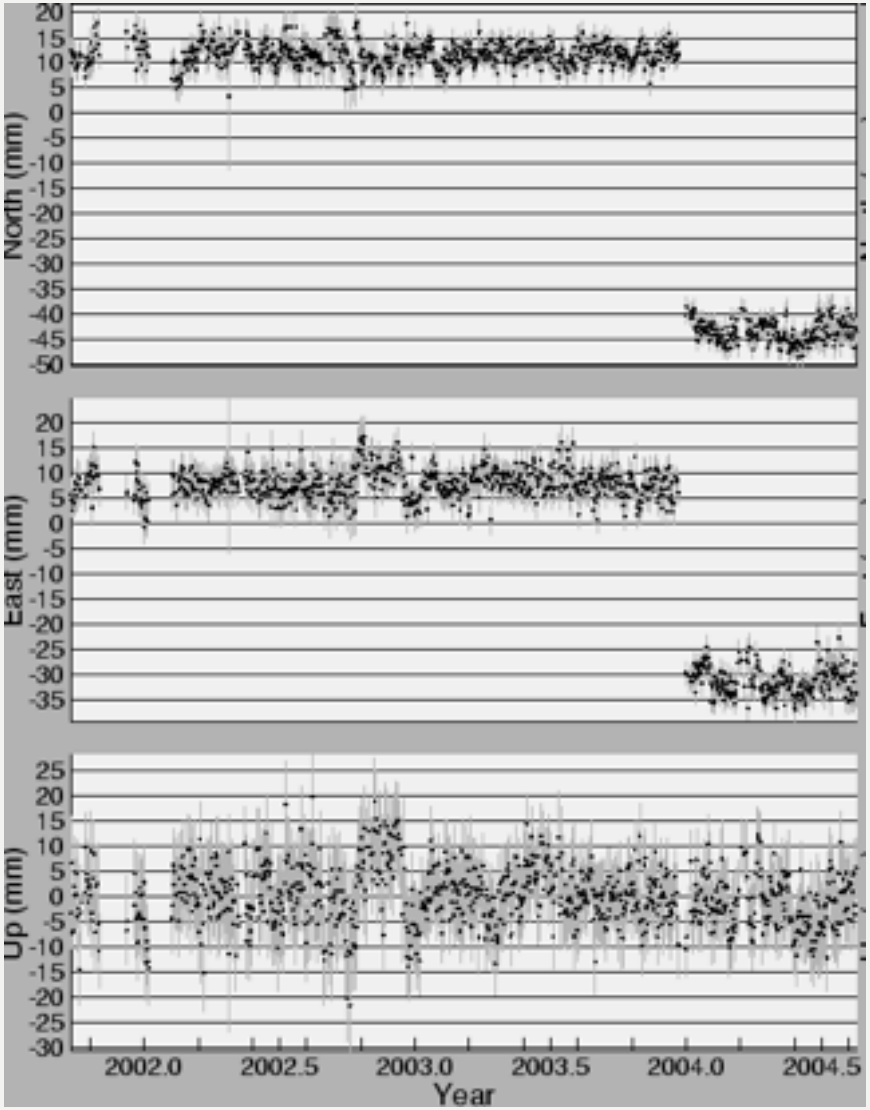
Global GPS network

… coverage at ionospheric heights ...

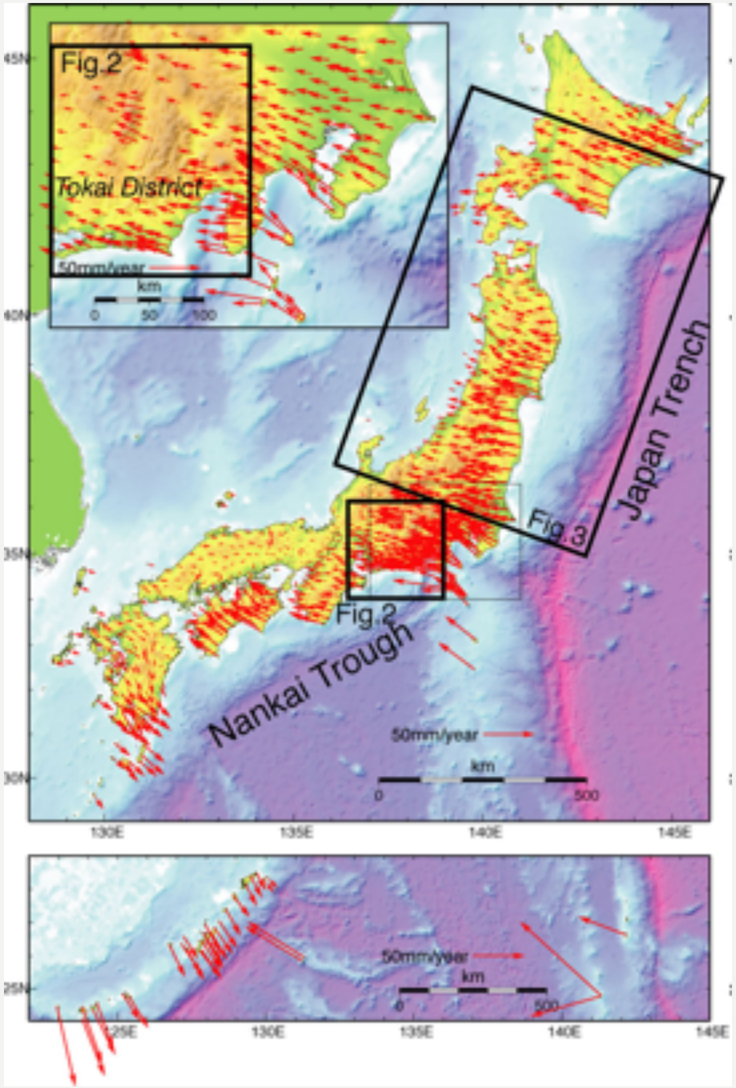


GPS measurements

… Californian earthquake ...



… Japan earthquake ...



Summary: instruments

- *Seismometers are forced oscillators*
- *Seismograms need to be corrected for the instrument*
- *Design of instrument defines the measurable frequency and amplitude range*
- *Rotations can be measured with either an array of seismometers or ring laser.*
- *Cross-axis sensitivity is an important issue*