Stefanie Donner

Geophysical Data Analysis

L07 – Filtering I

Applications of the Convolution

Separate signal from noise.

Understand the effects of filtering on time series.

Definition of a filter

Filters or systems are, in the most general sense, devices (in the physical world) or algorithms (in the mathematical world) which act on some input signal to produce a – possibly different – output signal.

Of Poles and Zeros (2001) F. Scherbaum

Brainstorming ...

Why filtering?

... on the board.

Highlight signals of certain, wanted frequencies.

Get rid of unwanted frequencies.

Model a special instrument.

Why filtering?

Avoid aliasing effects.

Correct for instrument characteristic.

Prepare for down-sampling.

Identify harmonic signals.

→ ***Separation of signals that have been combined***

→ ***Restoration of signals that have been distorted in some way***

Classification

There is no simple hierarchical classification.

Let's try anyway.

Classification

linear ⇔ *non-linear*

(output contains frequencies not present in input)

Classification

linear ⇔ ~~*non-linear*~~

~~*(output contains frequencies not present in input)*~~

analog ⇔ *digital*

(elect. elements: circuits, resistors, conductors)

(logical components and signal processors)

Classification

linear ⇔ ~~*non-linear*~~

~~*(output contains frequencies not present in input)*~~

analog ⇔ *digital*

(elect. elements: circuits, resistors, conductors) *(logical components and signal processors)*

continuous ⇔ *discrete*

Classification

linear ⇔ ~~*non-linear*~~
~~*(output contains frequencies not present in input)*~~

analog ⇔ *digital*
(elect. elements: circuits, resistors, conductors) *(logical components and signal processors)*

continuous ⇔ *discrete*

IIR ⇔ *FIR*
(Infinite Impulse Response) *(Finite Impulse Response)*

recursive ⇔ *non-recursive*

Classification

linear ⇔ ~~*non-linear*~~

~~*(output contains frequencies not present in input)*~~

analog ⇔ *digital*

(elect. elements: circuits, resistors, conductors)

(logical components and signal processors)

continuous ⇔ *discrete*

IIR ⇔ *FIR*

(Infinite Impulse Response)

(Finite Impulse Response)

recursive ⇔ *non-recursive*

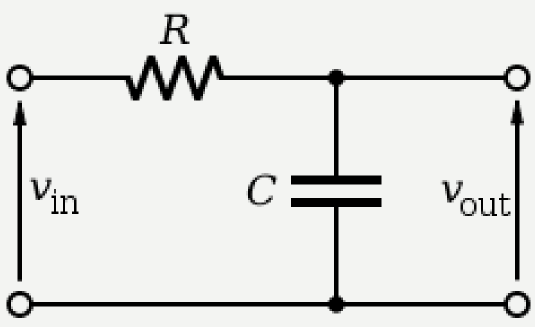
Examples of filters:

(analog or digital)

- *Butterworth*
- *Chebyshev*
- *Bessel*
- *Wiener (deconvolution)*

The RC filter

... is the most simplest filter.



- *input voltage x(t) produces current I(t) flowing through resistor R and capacitor C*

overall voltage balance: $$RI(t) + y(t) = x(t)$$

current controlled by capacitance C: $$I(t) = C\dot{y}(t)$$

$$\Rightarrow \quad RC\dot{y}(t) + y(t) - x(t) = 0$$

The RC filter

- *For a zero input signal $x(t) = 0$ we obtain*

$$RC\dot{y}(t) + y(t) = 0$$

with the solution:

$$y(t) = -\frac{1}{RC}\, e^{-t/(RC)}$$

What happens if we have an arbitrary input signal?

The RC filter

Propose the classical trial solution.

$$y(t) = A_0 e^{j\omega t} \qquad \dot{y}(t) = j\omega\, A_0 e^{j\omega t}$$

$$\Longrightarrow \qquad A_0 e^{j\omega t}(RCj\omega + 1) = A_i e^{j\omega t}$$

Now, we have defined the (complex) ***frequency response function***

$$\frac{A_0}{A_i} = \frac{1}{RCj\omega+1} = T(j\omega)$$

in polar form:

$$T(j\omega) = \frac{1}{\sqrt{1+(RC\omega)^2}} e^{j\Phi(\omega)}$$ with phase $$\Phi(\omega) = \arctan(-\omega RC) = -\arctan(\omega RC)$$

The RC filter

With $A_i(j\omega)$ *= the harmonic component of the input signal's Fourier spectrum and* $A_0(j\omega)$ *= the harmonic component of the output signal's Fourier spectrum, we can state:*

$$T(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

Once we know the ***frequency response function****, we can predict a filter's output for any given input signal.*

The RC filter

With $A_i(j\omega)$ = the harmonic component of the input signal's Fourier spectrum and $A_0(j\omega)$ = the harmonic component of the output signal's Fourier spectrum, we can state:

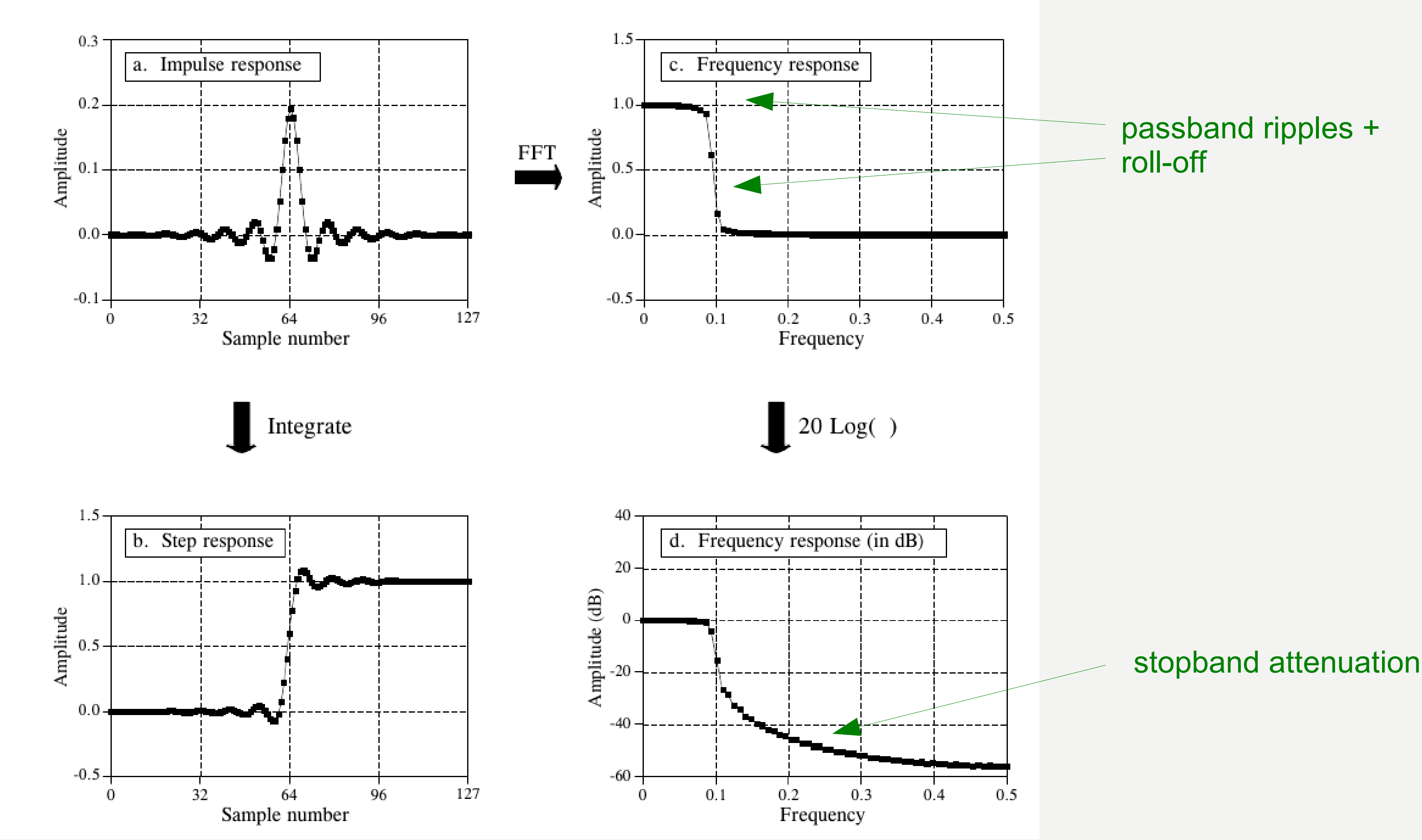
$$T(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

Once we know the frequency response function, we can predict a filter's output for any given input signal.

*The **frequency response function** is the Fourier transform of the **impulse response function**, defined as the response of a filter to an impulsive (delta function) input signal.*

Defining a filter

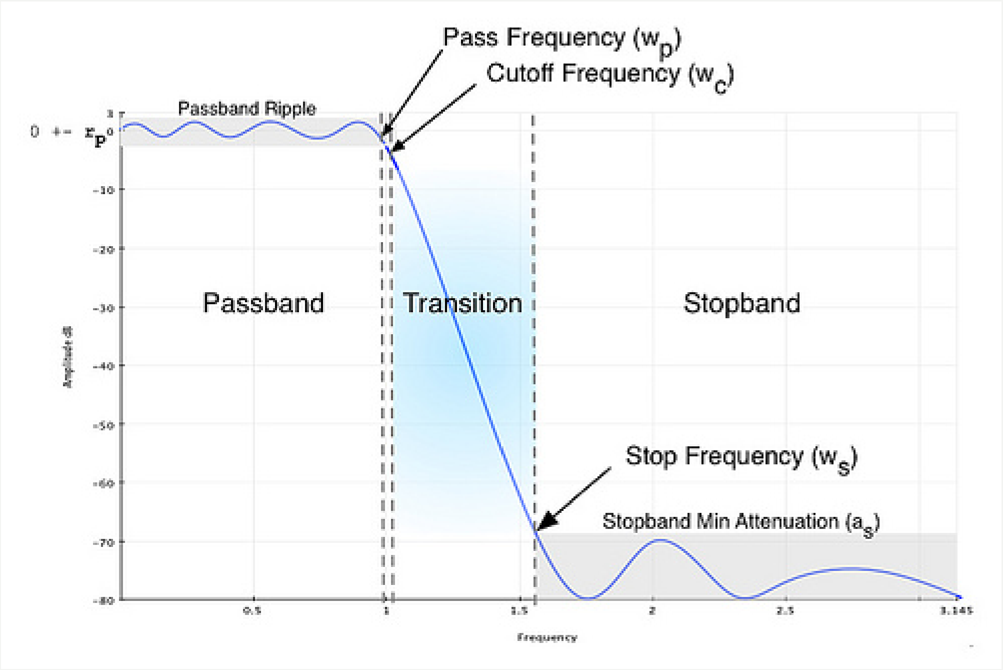
- *Via impulse response, step response, or frequency response*



- *All responses contain complete information on the filter*

Terminology

- *Cut-off frequency: frequency beyond which the filter will not pass signals*
- *Roll-off: rate at which attenuation increases beyond the cut-off frequency*
- *Order of the filter: degree of approximating polynomial; increasing order increases roll-off and brings filter closer to ideal response <u>but</u> ...*
- *Transition band: usually narrow band of frequencies between pass- and stopband*
- *Ripple: periodic variation of insertion loss with frequency (Gibbs)*



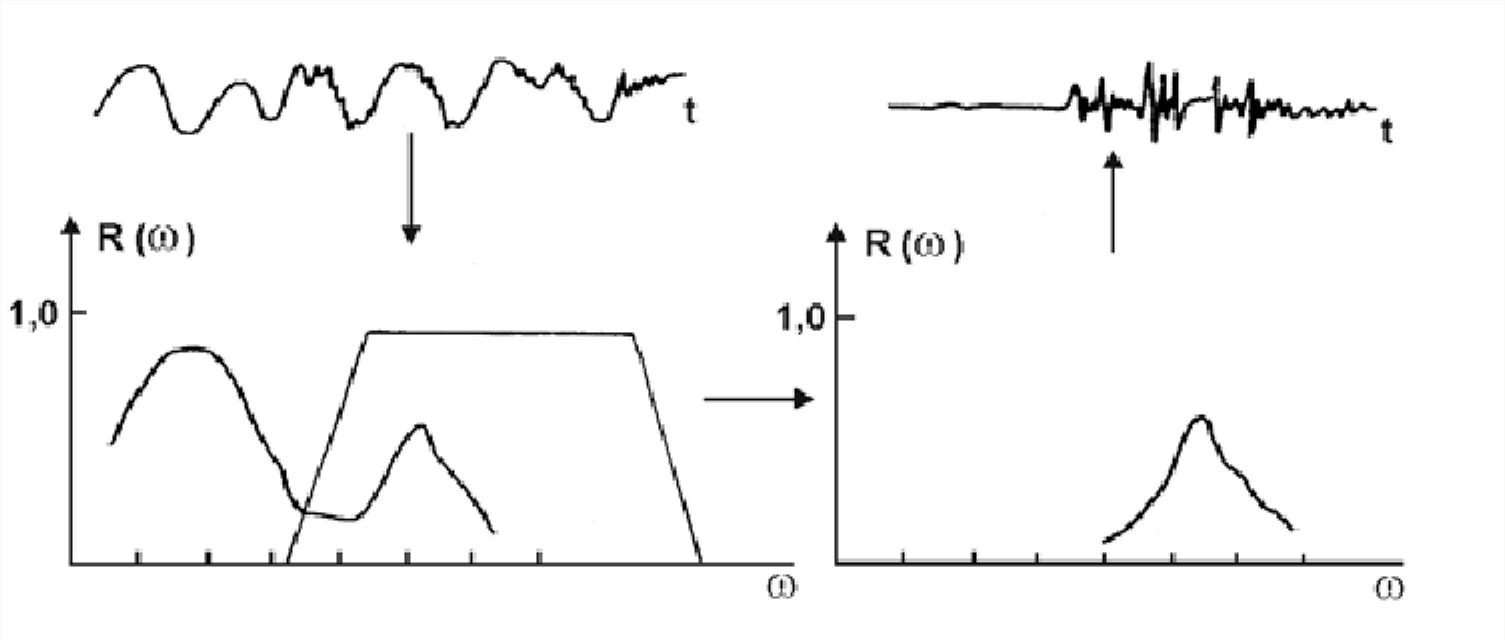
frequency

Principle of filtering

→ *FFT to frequency-domain*

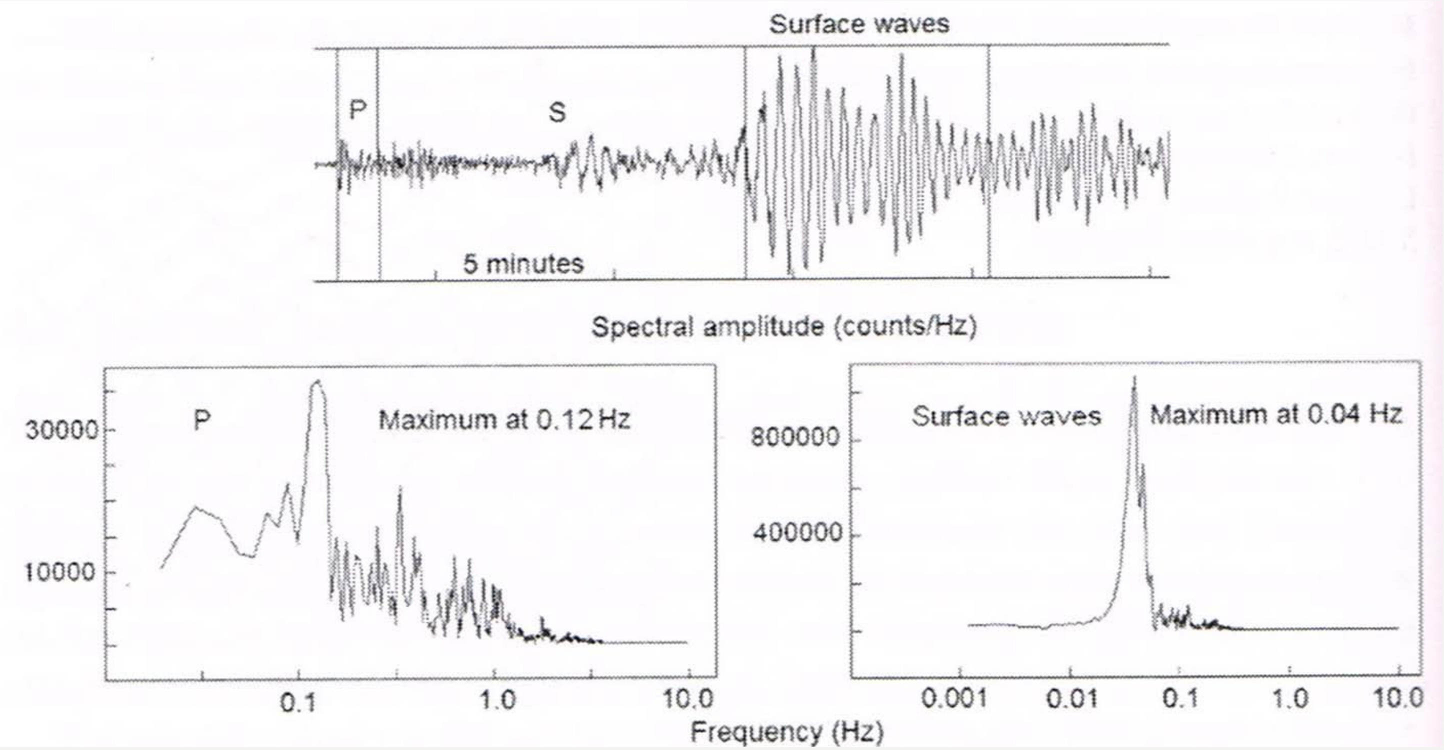
→ *convolution (i.e. ... what? ...) with impulse response of appropriate filter*

→ *back-FFT to time-domain*

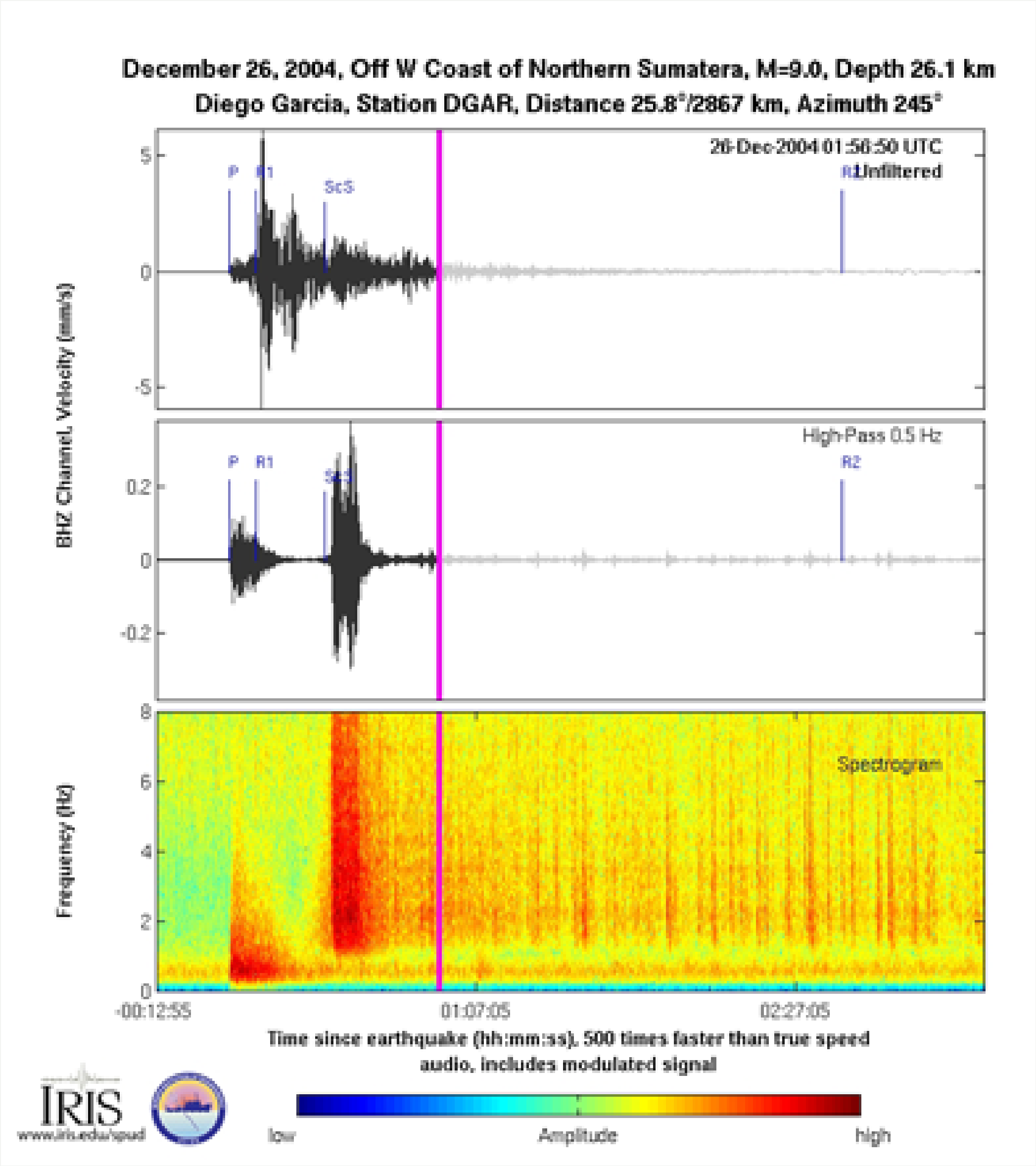


Example of application

... separate different seismic phases – not only by filtering but also by chosing appropriate time windows

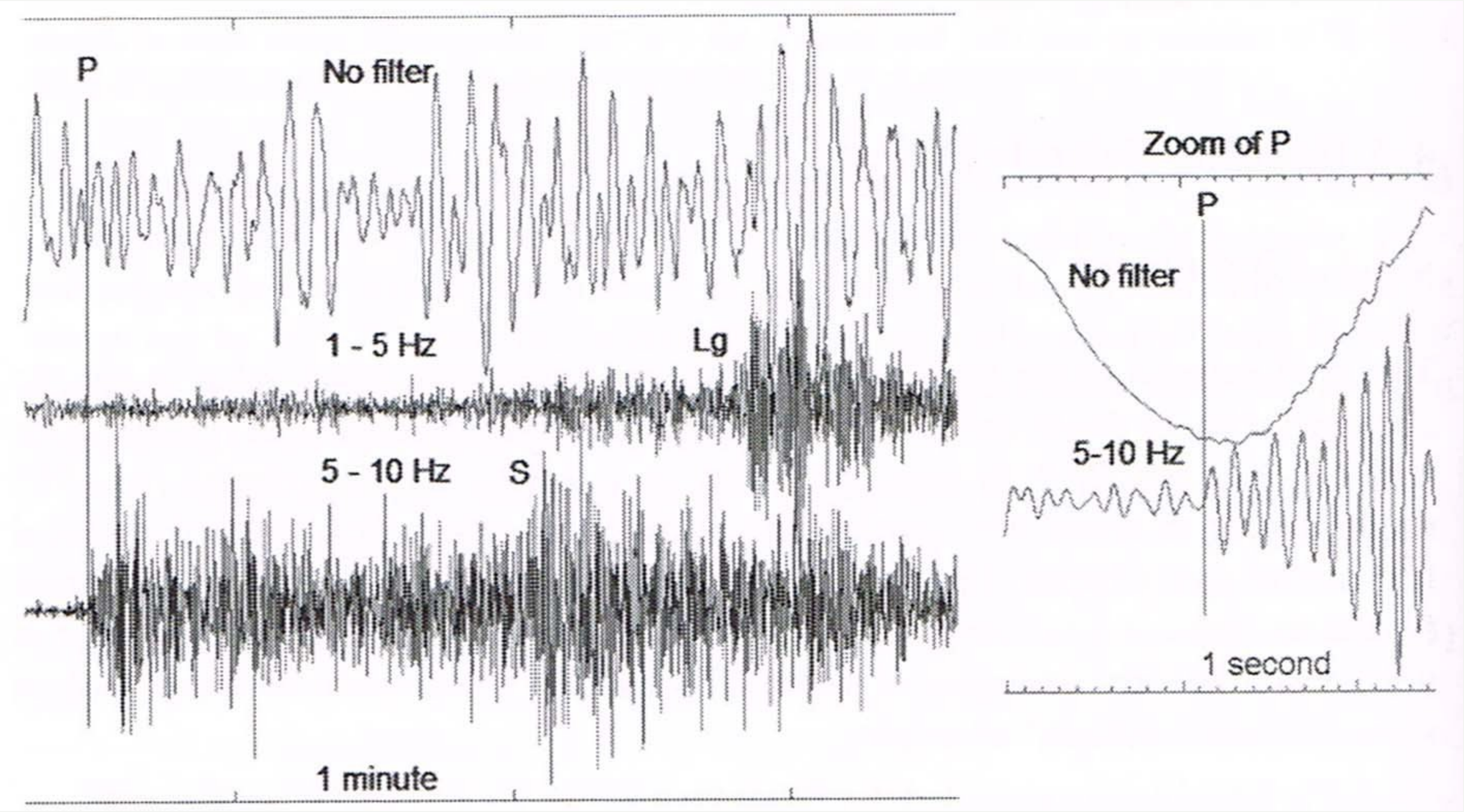


Example of application



Example of application

... identify P-wave first onset precisely



Filtering

Often a recorded signal contains a lot of information that we are not interested in (noise). To get rid of this noise we can apply a filter in the frequency domain.

The most important filters are:

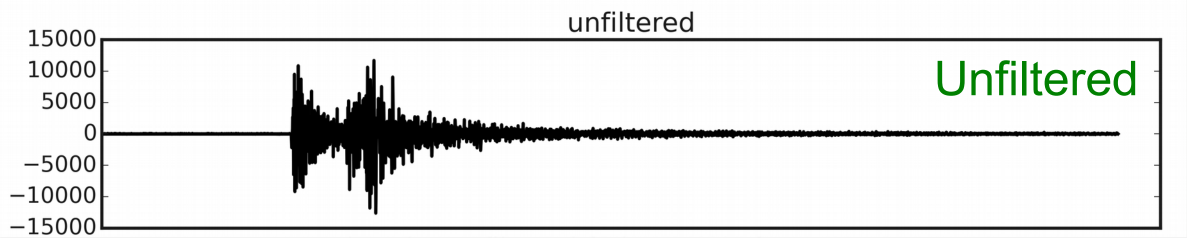
High pass: keeps/passes high frequencies, cuts out low frequencies

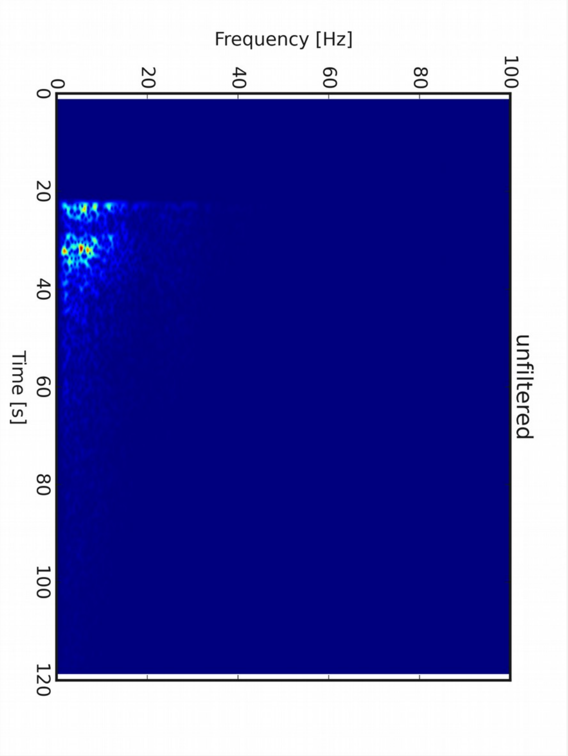
Low pass: keeps low frequencies, cuts out high frequencies

Band pass: cuts out both high and low frequencies and leaves a band of frequencies

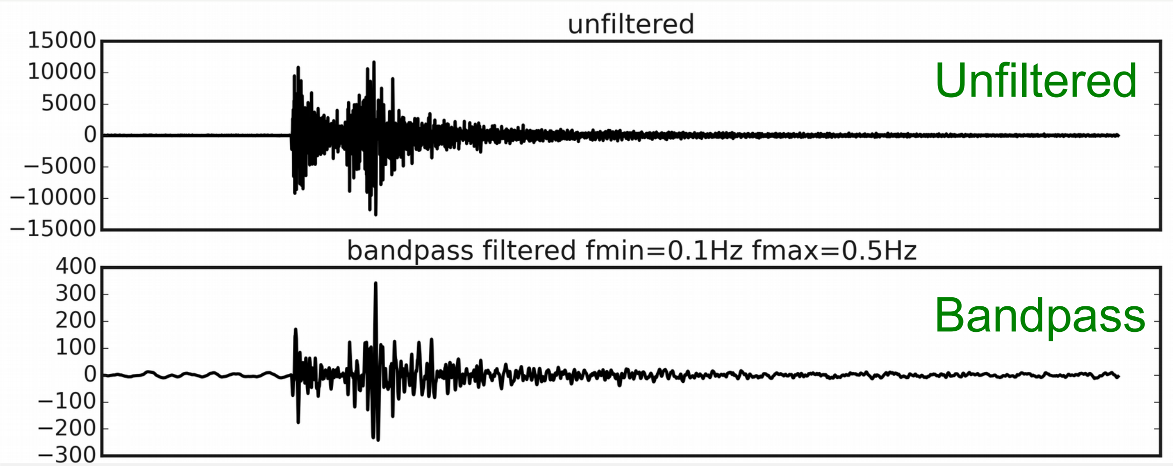
Band reject: cuts out certain frequency range and leaves all other frequencies

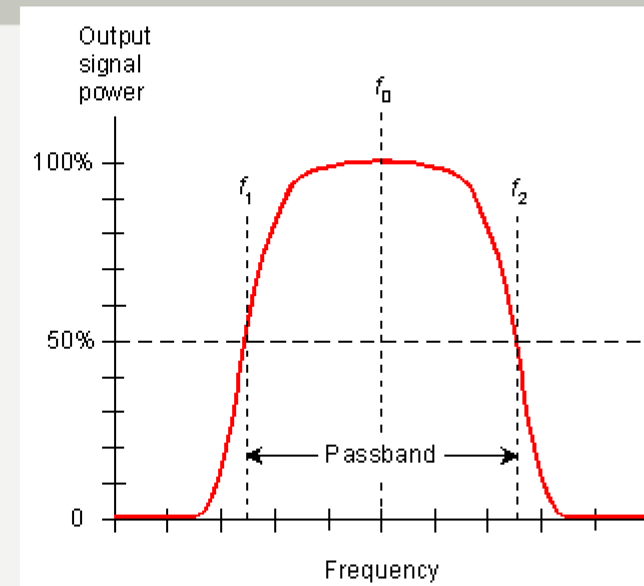
Filtering

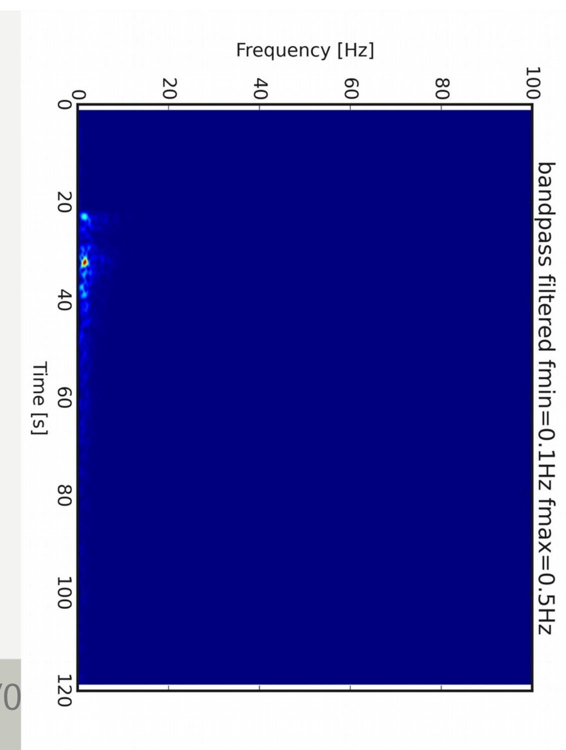

unfiltered
15000
10000
5000
0
−5000
−10000
−15000
Unfiltered


Frequency [Hz]
0
20
40
60
80
100
Time [s]
0
20
40
60
80
100
120
unfiltered

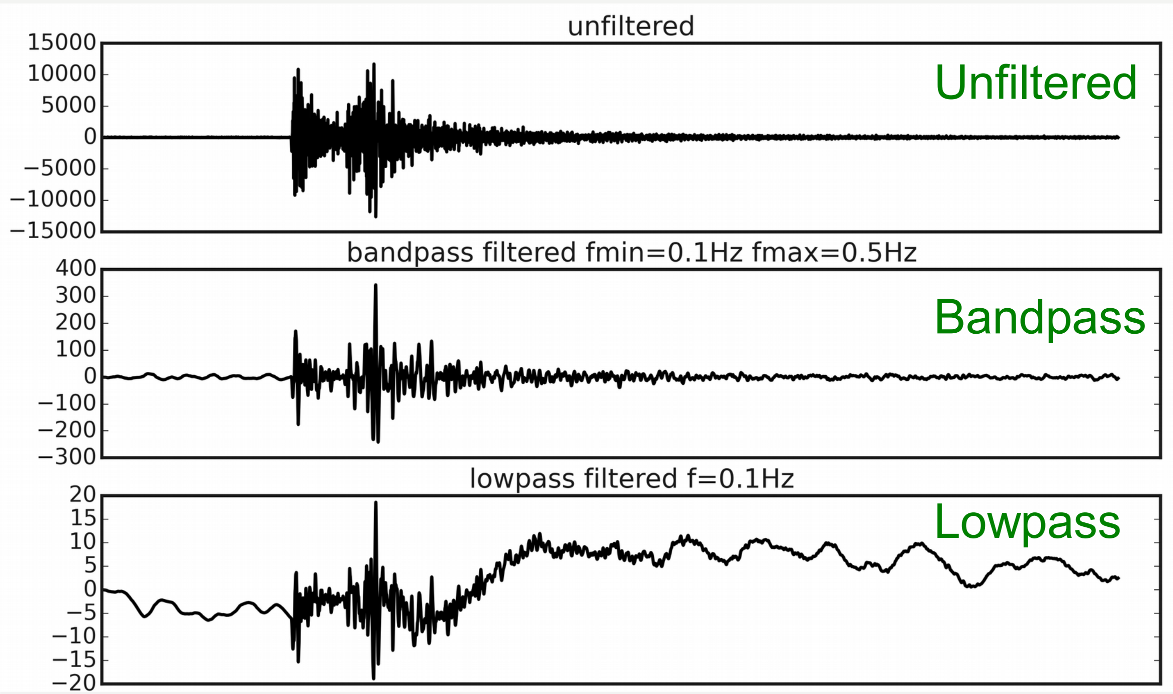
Filtering

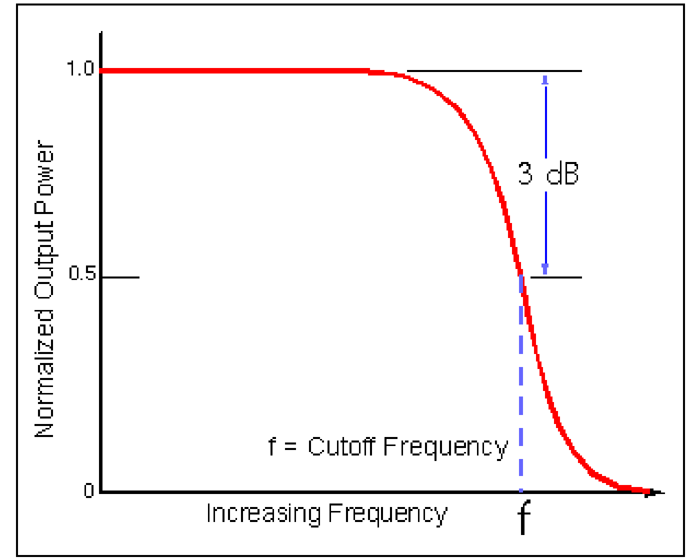

unfiltered
Unfiltered
15000
10000
5000
0
−5000
−10000
−15000
bandpass filtered fmin=0.1Hz fmax=0.5Hz
Bandpass
400
300
200
100
0
−100
−200
−300

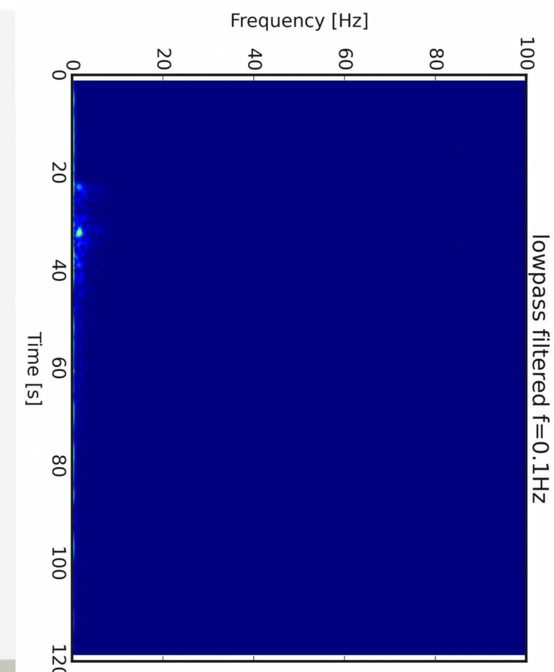

Output signal power
100%
50%
0
f_0
f_1
f_2
Passband
Frequency


Frequency [Hz]
0
20
40
60
80
100
Time [s]
120
bandpass filtered fmin=0.1Hz fmax=0.5Hz

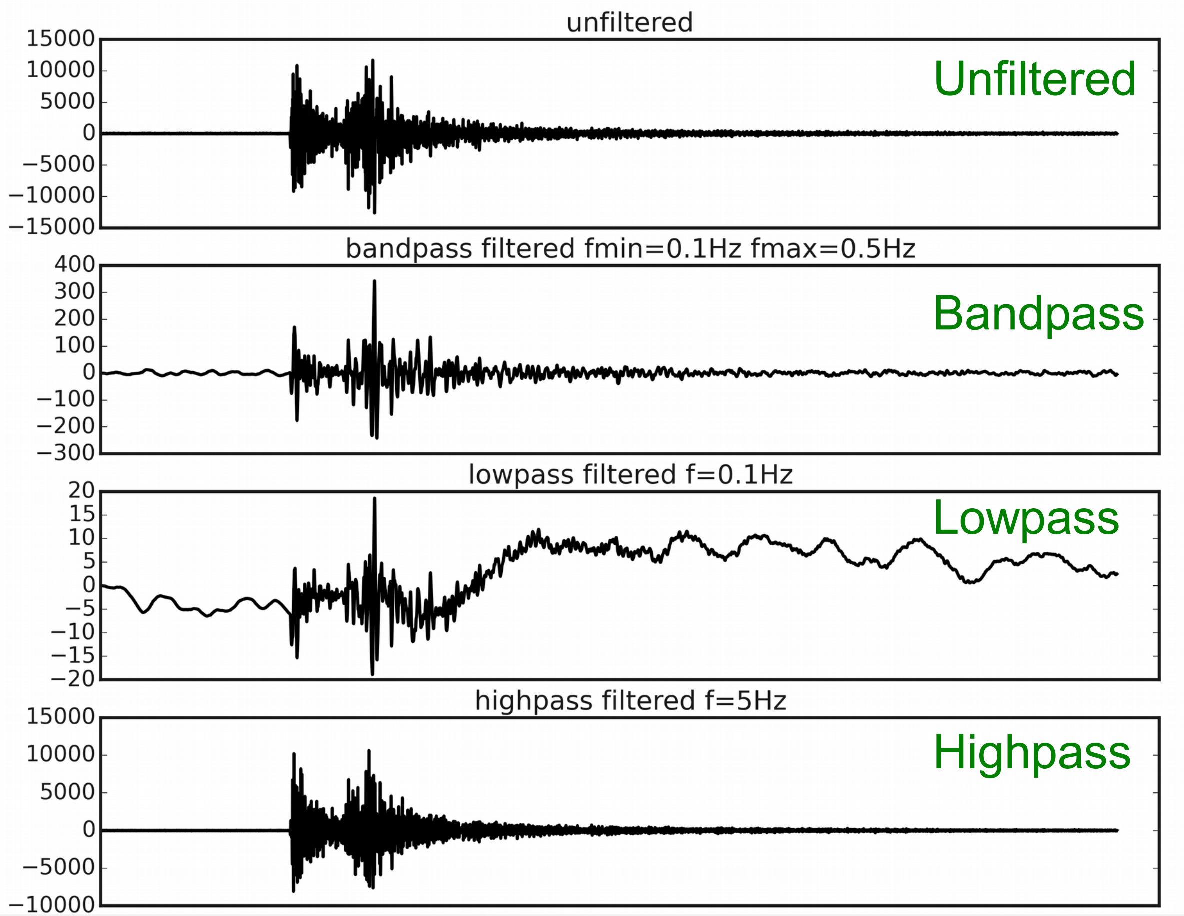
Filtering

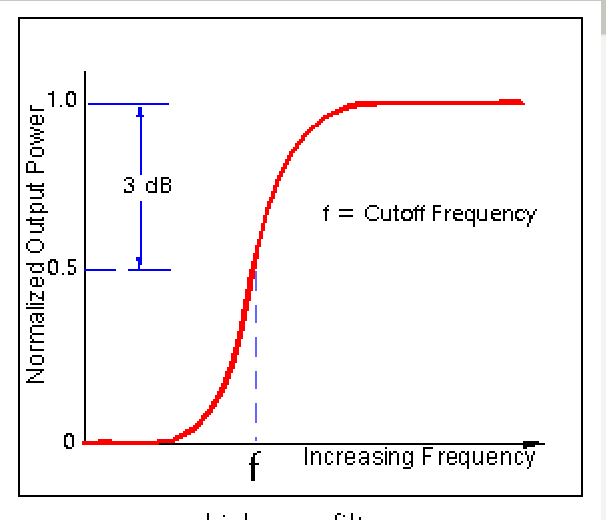

unfiltered
Unfiltered
bandpass filtered fmin=0.1Hz fmax=0.5Hz
Bandpass
lowpass filtered f=0.1Hz
Lowpass


Normalized Output Power
1.0
0.5
0
3 dB
f = Cutoff Frequency
Increasing Frequency
f

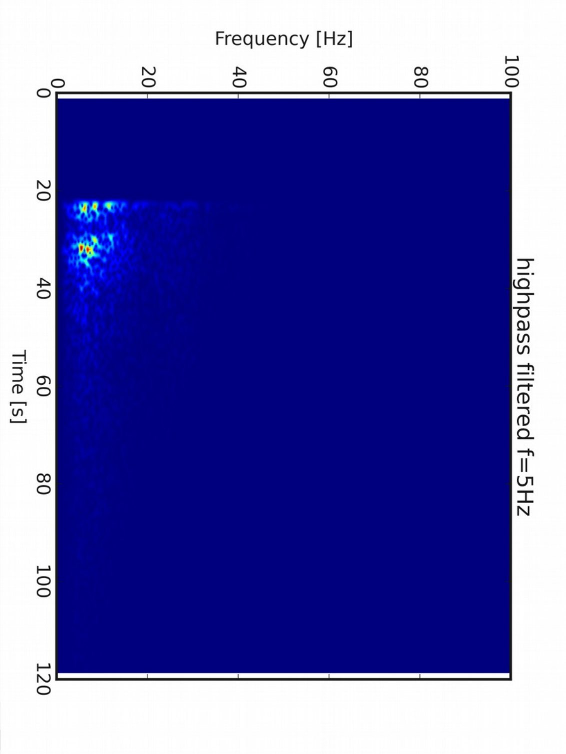

Frequency [Hz]
Time [s]
lowpass filtered f=0.1Hz

Filtering

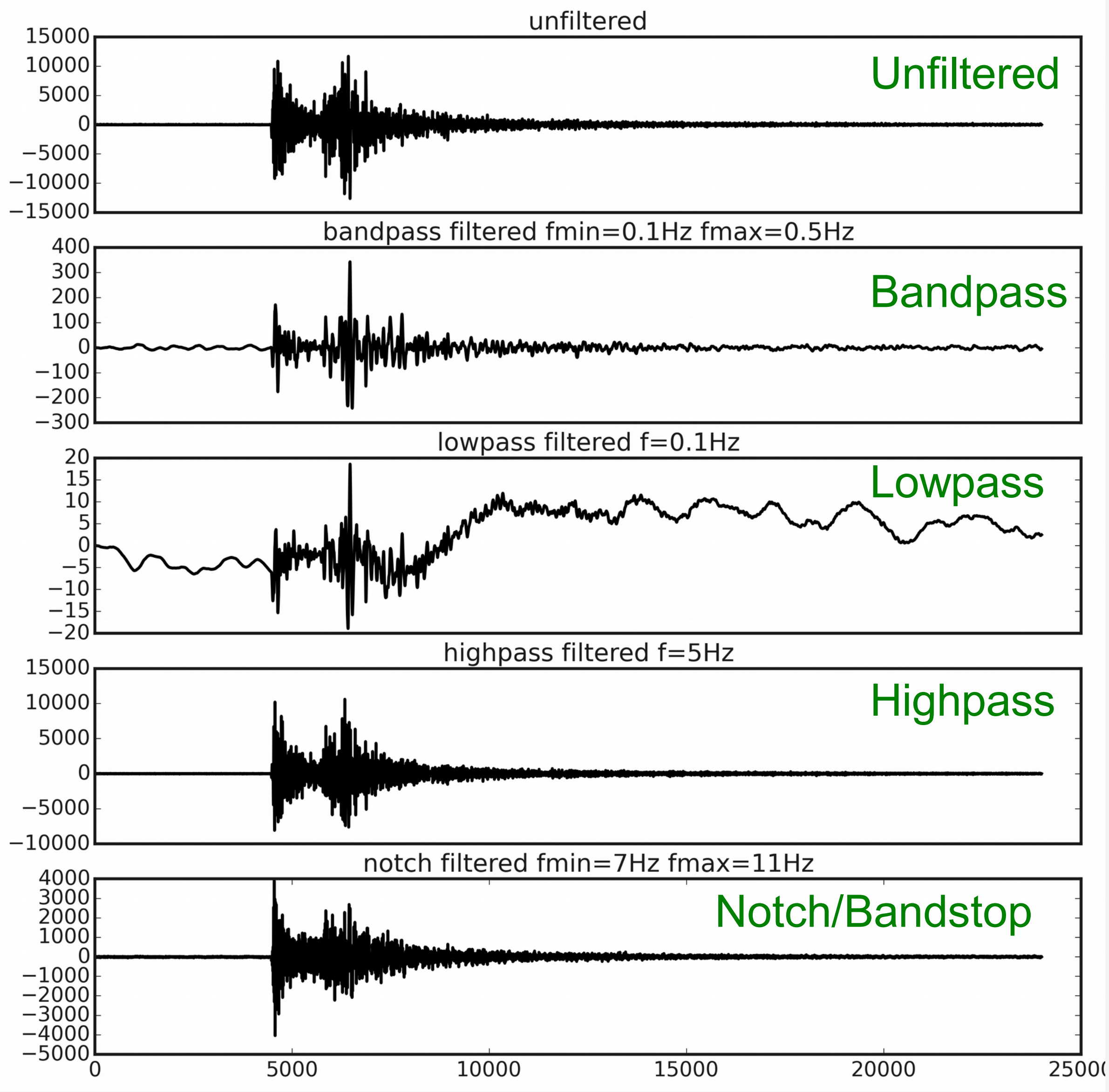


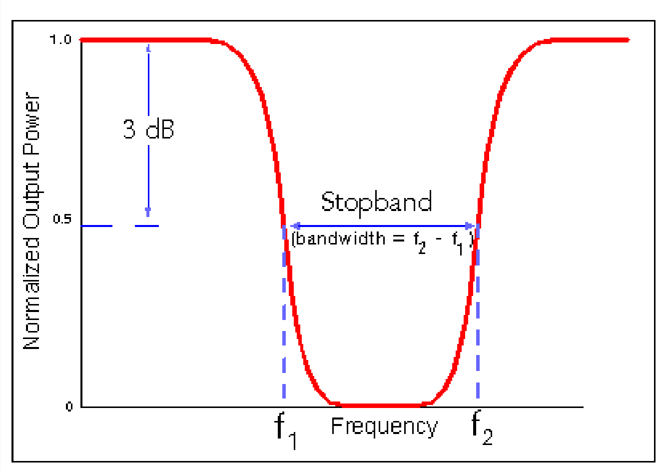


high-pass filter

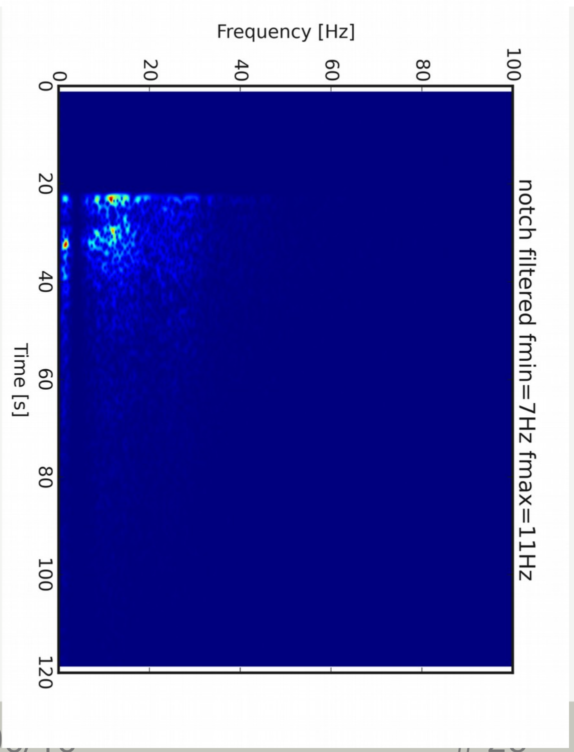


Filtering



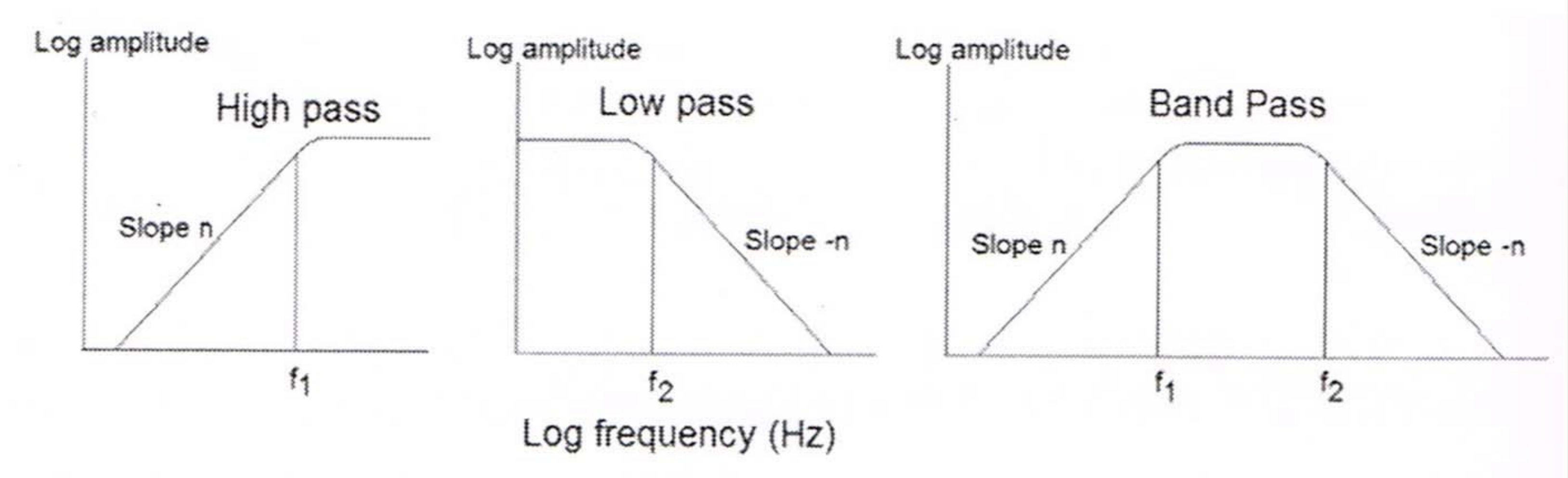


band-stop filter

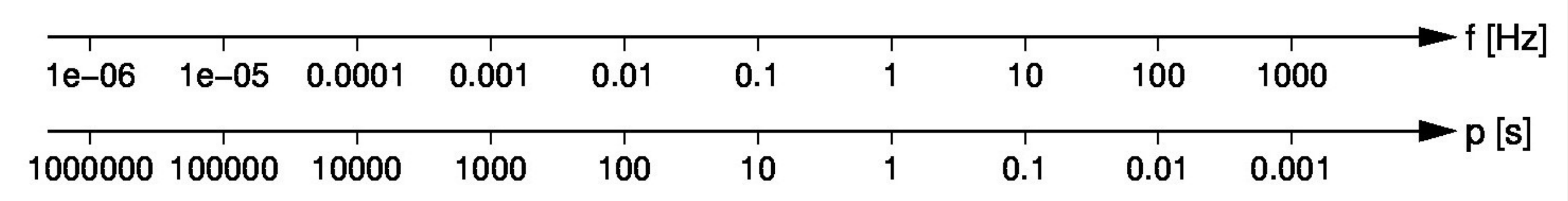


Passing frequency bands

Categorization according which frequencies are "let through"

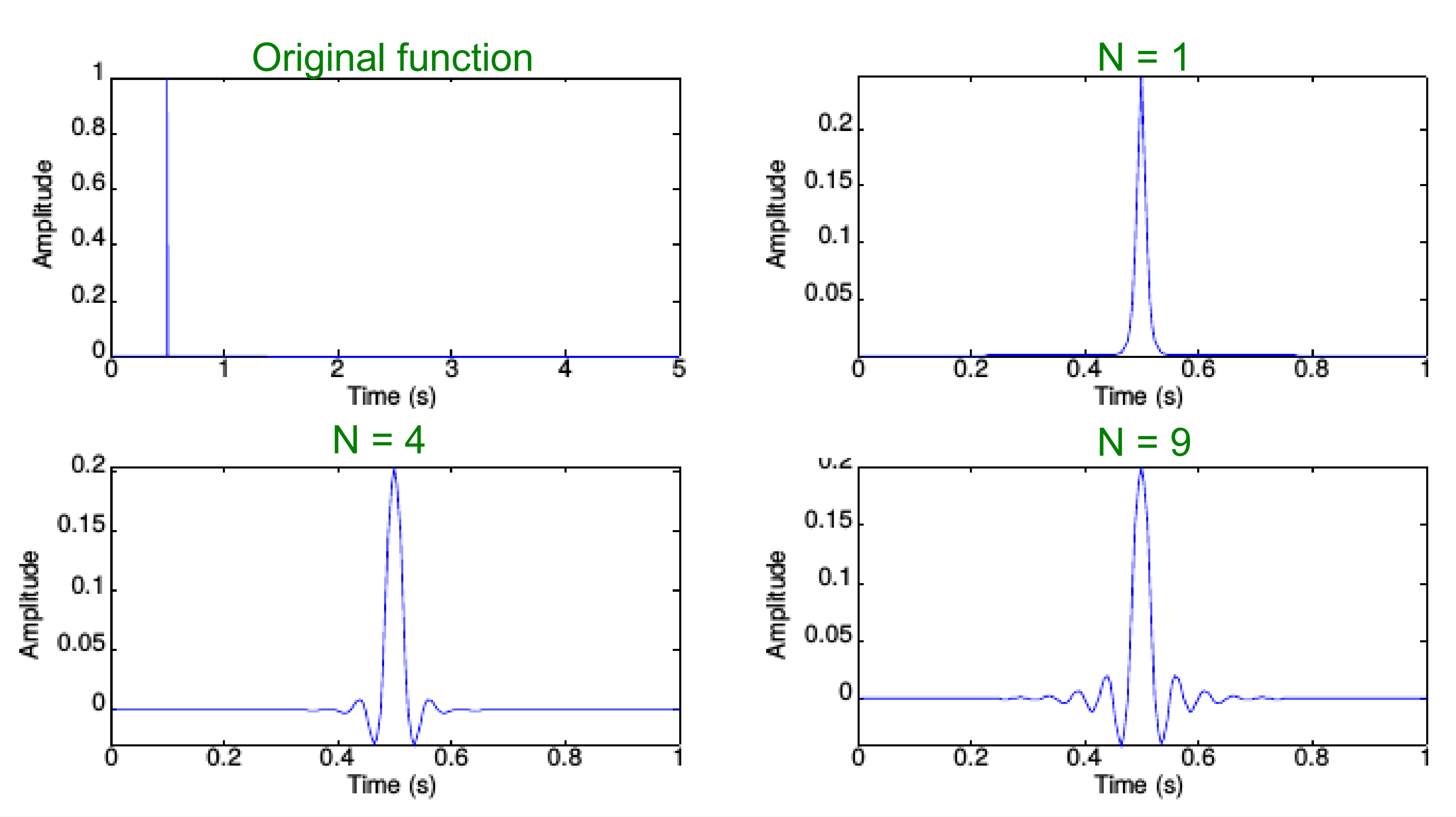


***Be aware**: "high"- and "low"-pass refers to the unit frequency !!!!*



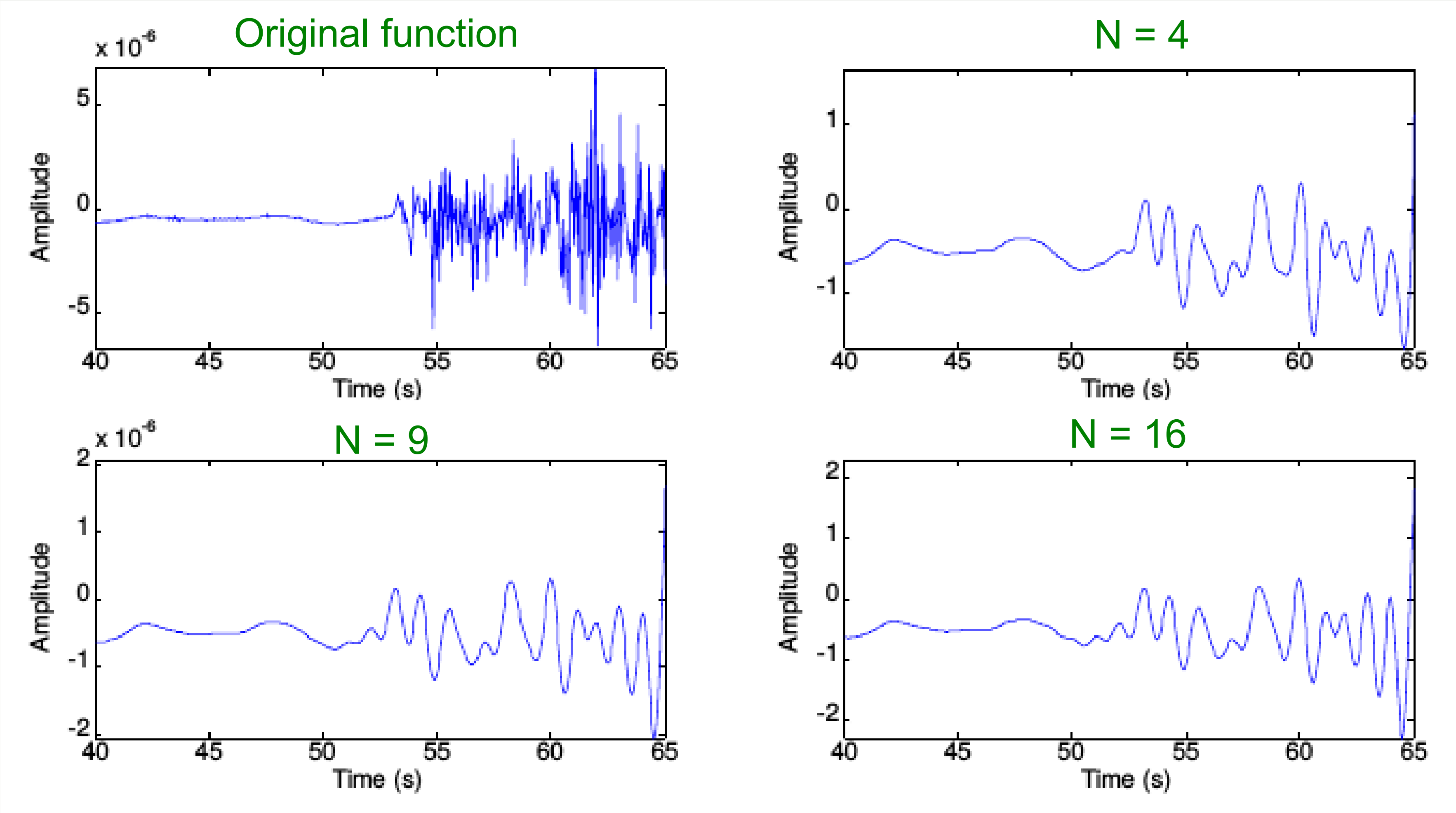
Filter effects – order of filter

LP at 20 Hz on a spike



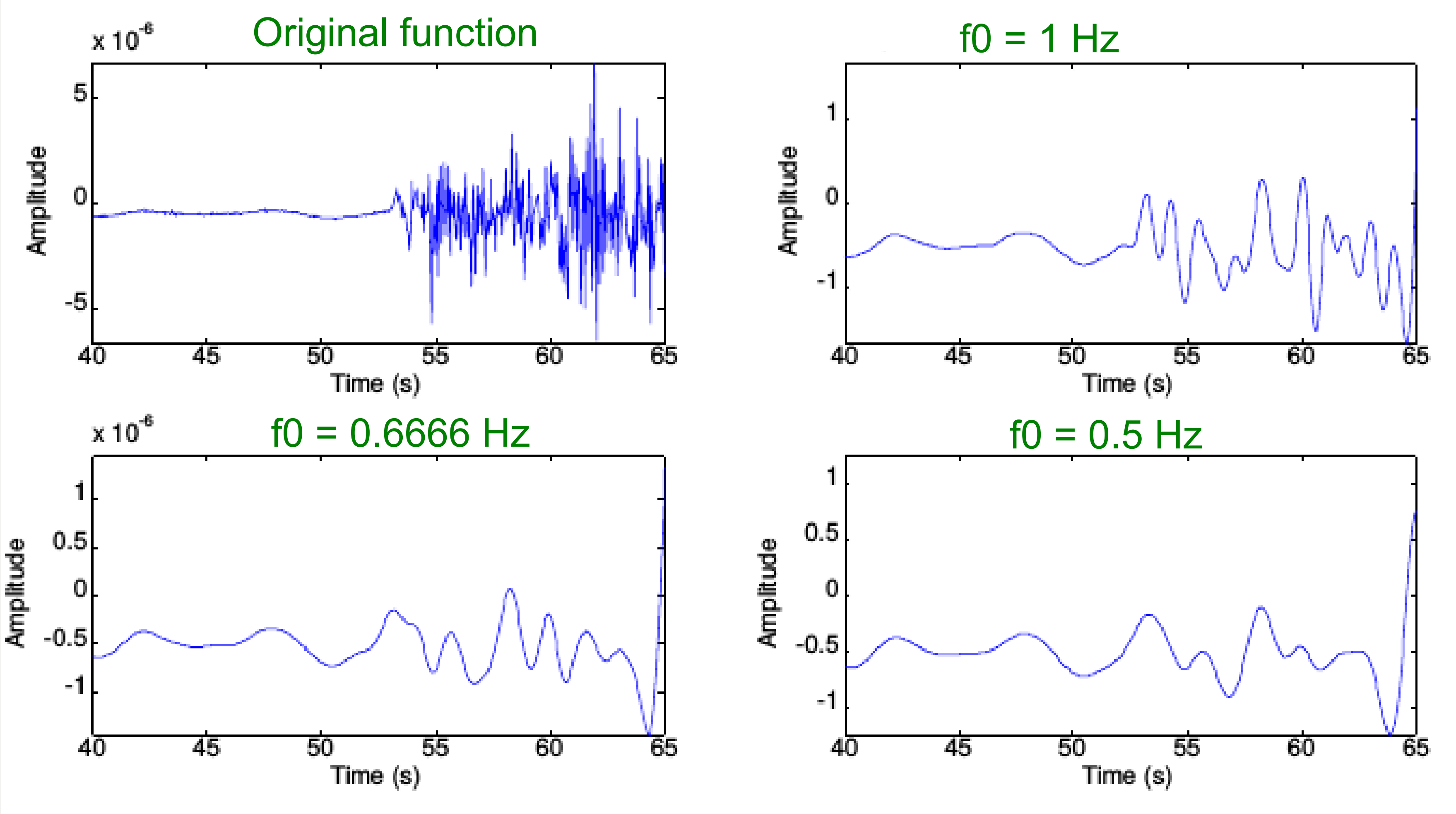
Filter effects – order of filter

LP at 1Hz on a seismogram



Filter effect – cut-off frequency

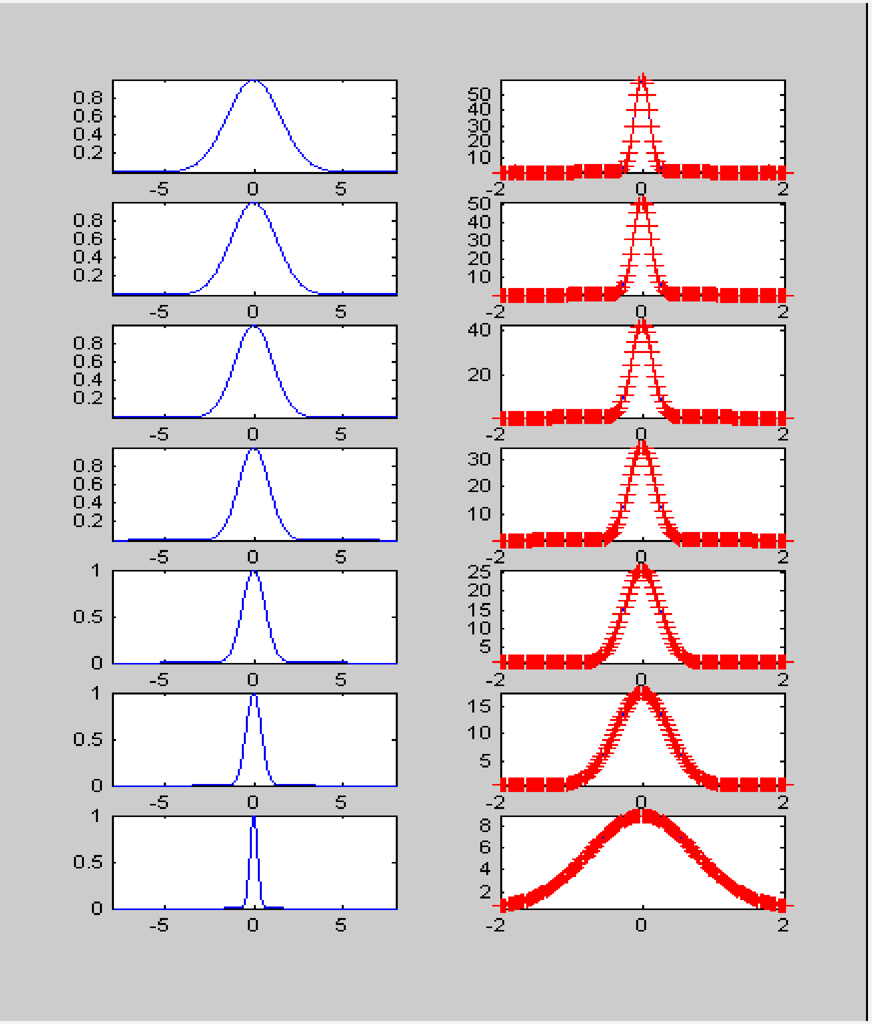
4th order LP on a seismogram



Pulse-width & Frequency Bandwidth

time(space) *spectrum*

Narrowing physical signal



Widening frequency band

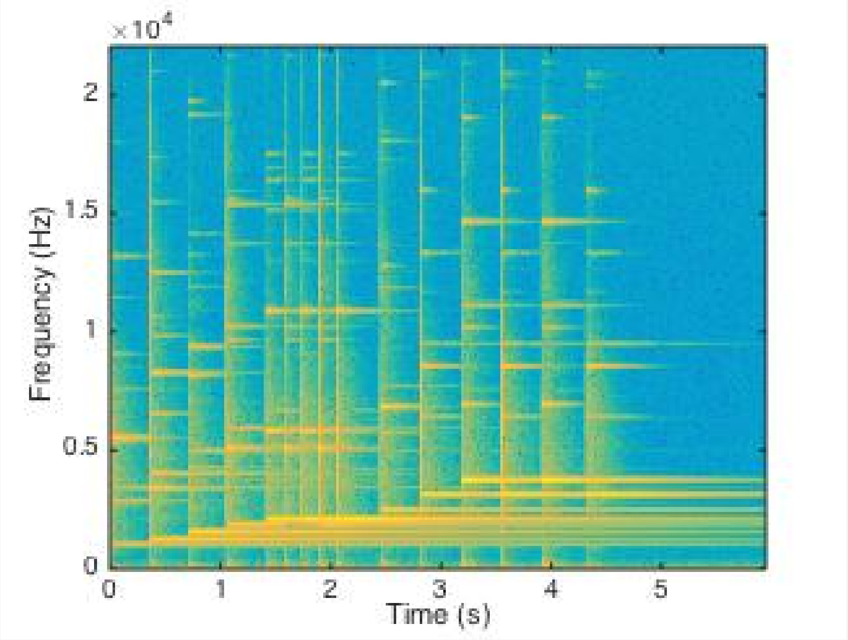
The uncertainty principle

There is a trade-off between the precision of the representation in the time- and frequency-domain.

Werner Karl Heisenberg (1901 – 1976)

Example: recording of a Glockenspiel playing several notes

short time window used

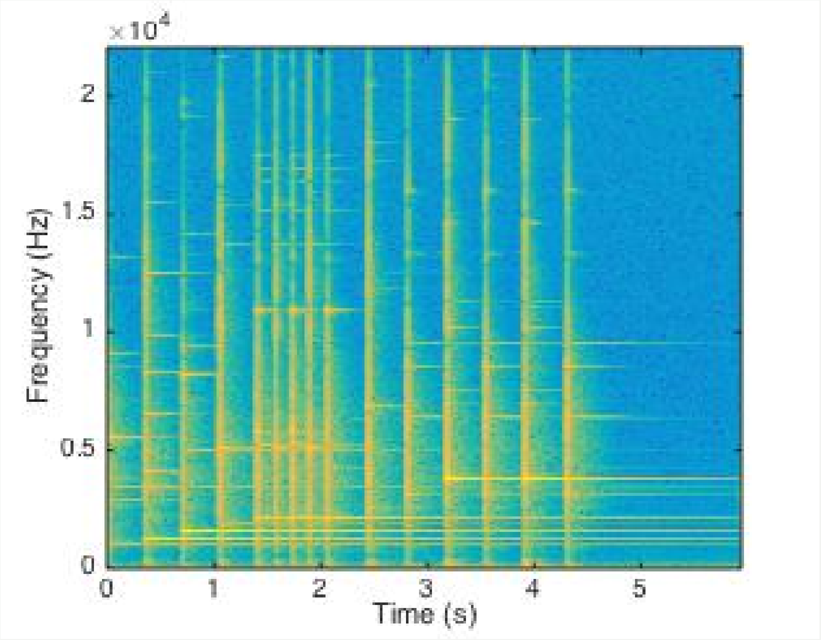


single notes = thin & sharp (vertical)

harmonic oscillations = long

(window too localised for long osc.)

long time window used



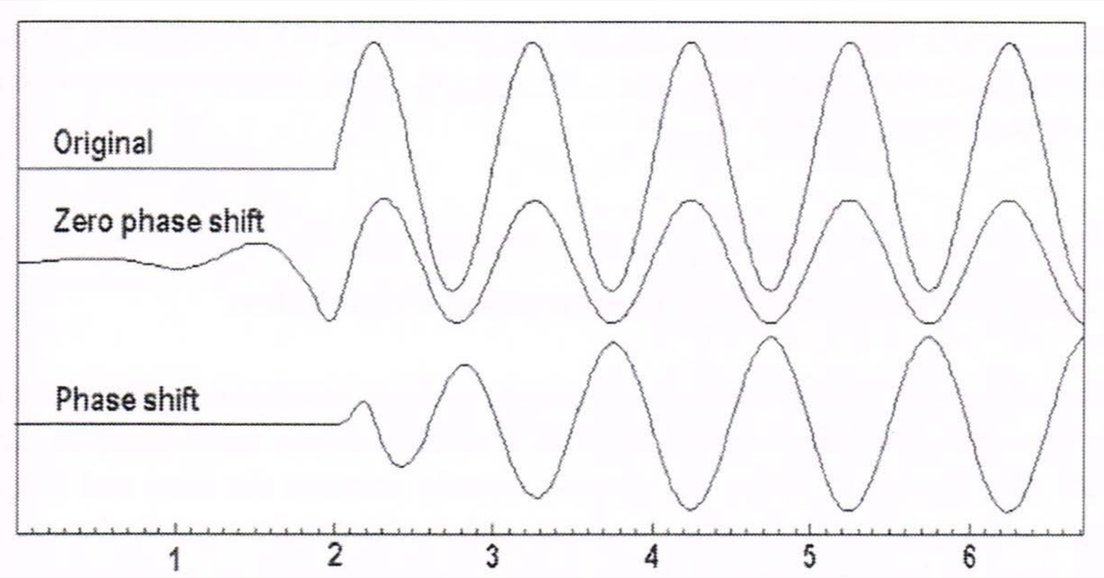
single notes = widened (vertical)

harmonic oscillations = more precise (horizontal)

Phase-shift

Filters do not only reduce the amplitude of the signal outside the pass-band but also introduce a phase-shift.

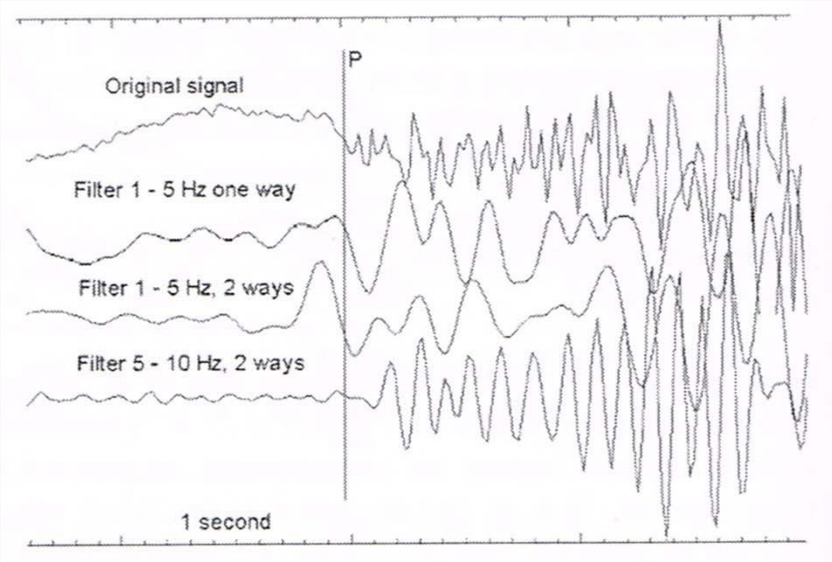
This can have serious implications for reading arrival times of seismic phases.



Solution → zero-phase filter

Zero-phase filter

1. Convolve the signal with the chosen filter.
2. Time reverse the filter and convolve again.



The first operation is a multiplication with $F(\omega)$, the second with $F^*(\omega)$.

The net multiplication is thus $|F(\omega)|^2$

→ *also called "two-pass filter"*

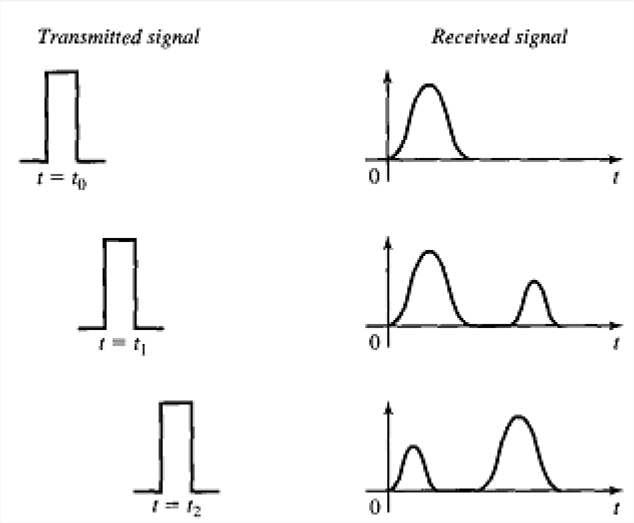
Time-invariant filters

... the output does NOT depend in time.

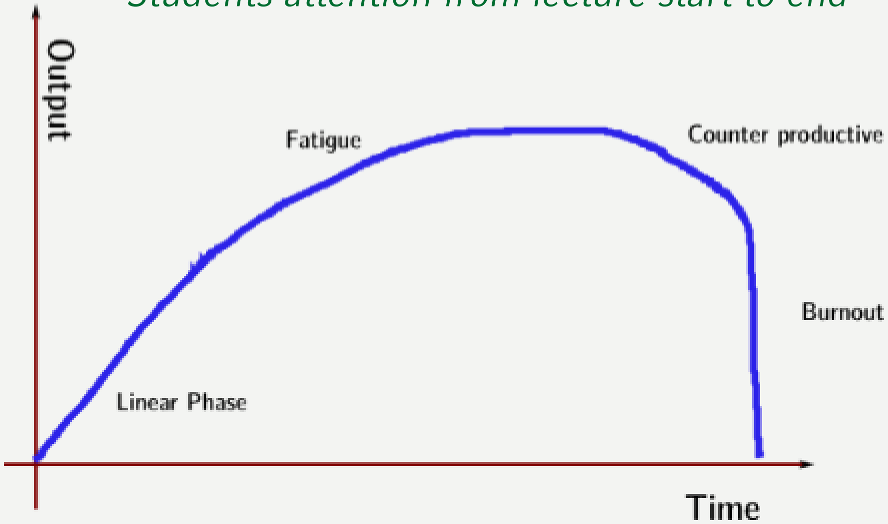
If the input signal $x(t)$ *produces an output* $y(t)$ *then any time-shifted input* $x(t+\delta)$ *will result in a time-shifted output* $y(t+\delta)$

$$x(t+1) = \delta(t+1) * x(t)$$

*Examples for **time-variant** systems:*



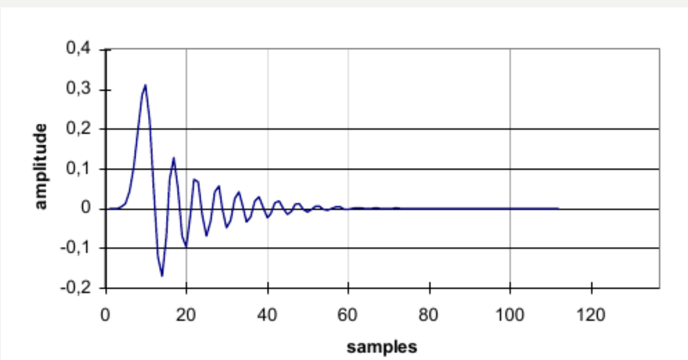
Students attention from lecture start to end

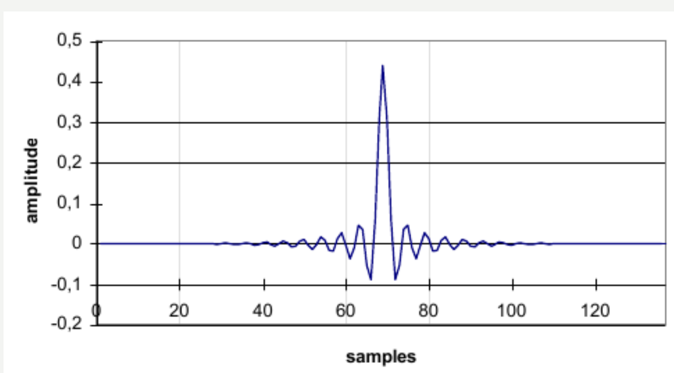


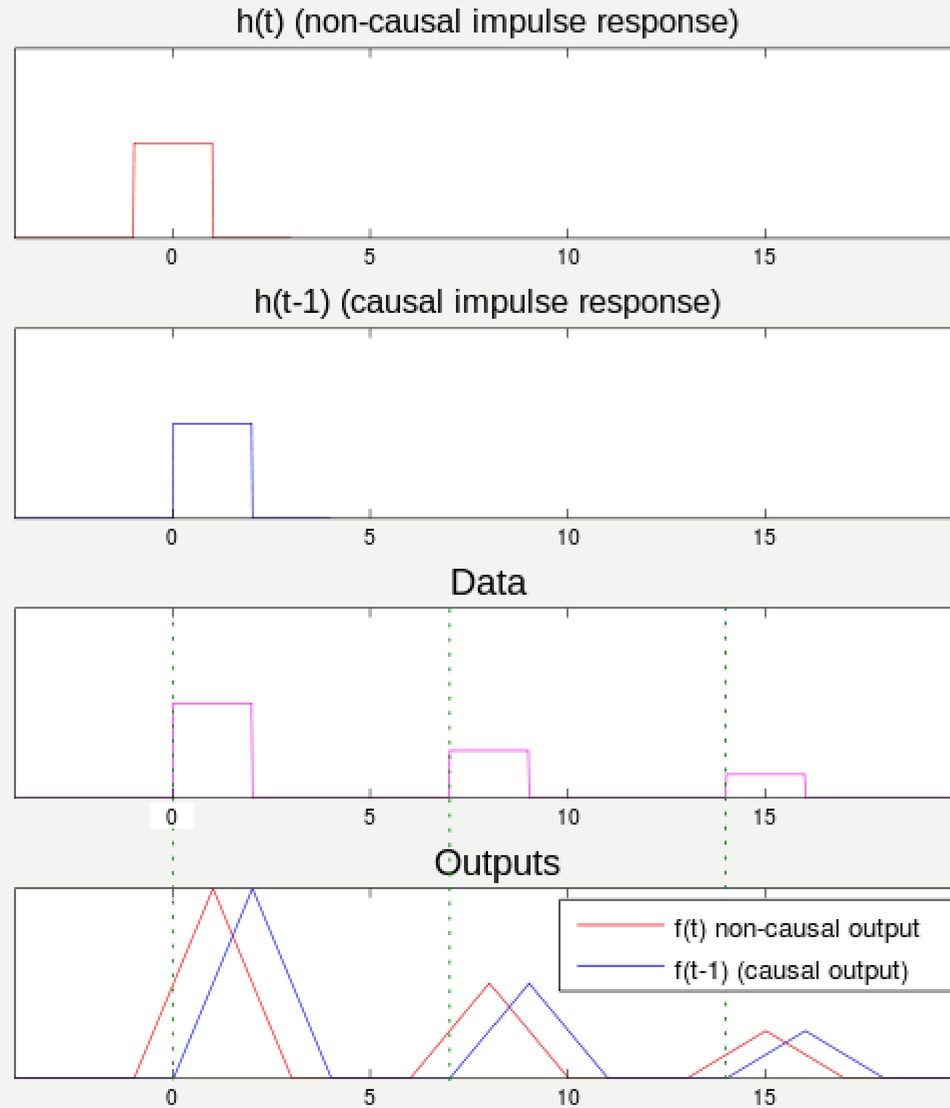
Causality

Causal: filter output ***only*** *depends on past and present input.*

Acausal filters are always symmetric, causal ones not.

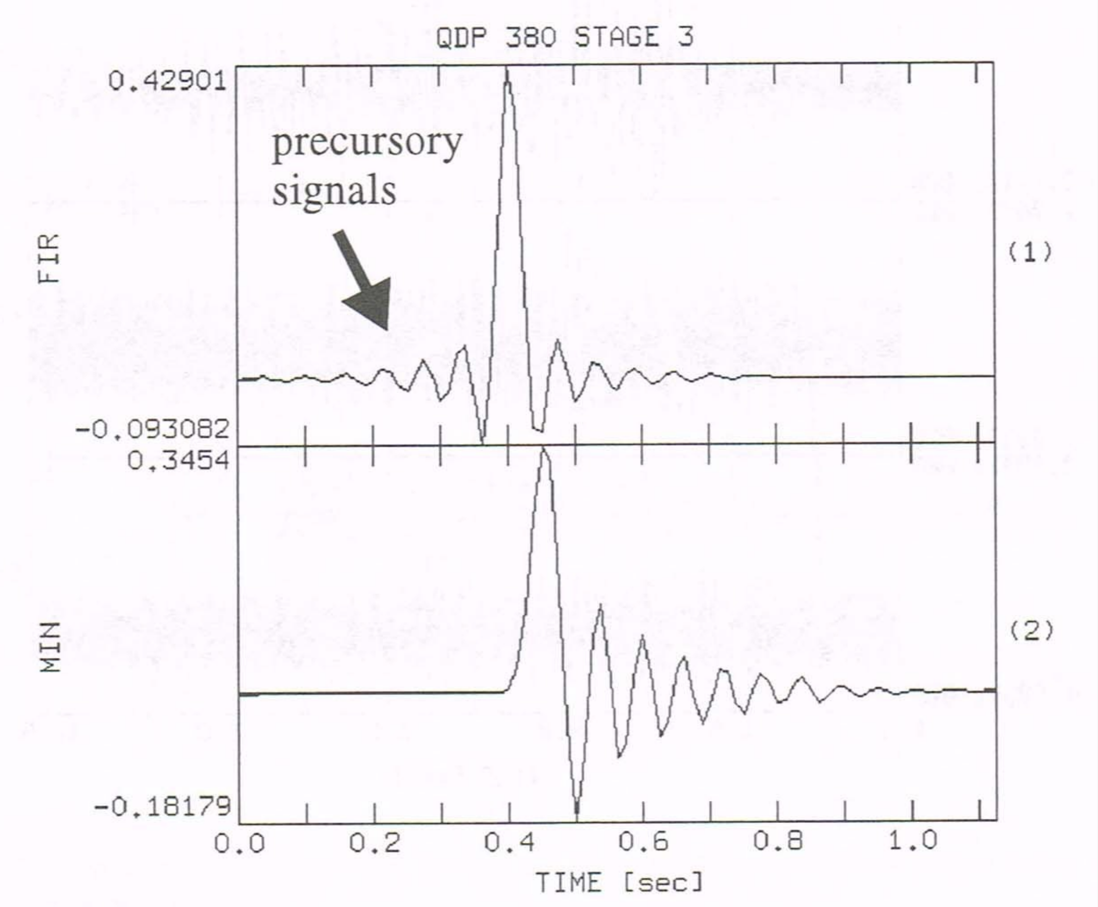






Effect of an acausal filter

... ringing !!! ...



Question

Think about different applications in seismology

where you would use causal or acausal filters.

Answer

Causal: picking of phase onsets

Acausal: analysis of polarisation or amplitudes

Summary

- *Filtering is not necessarily straight forward; even the fundamental operations (LP, HP, BP, etc.) require some thinking before application to data.*
- *The form of the filter decides upon the changes to the waveform of the time series you are filtering.*
- *For seismological applications filtering might drastically influence observables such as travel times or amplitudes.*
- *"Windowing" the signal in the right way is fundamental to obtain the desired filter sequence.*