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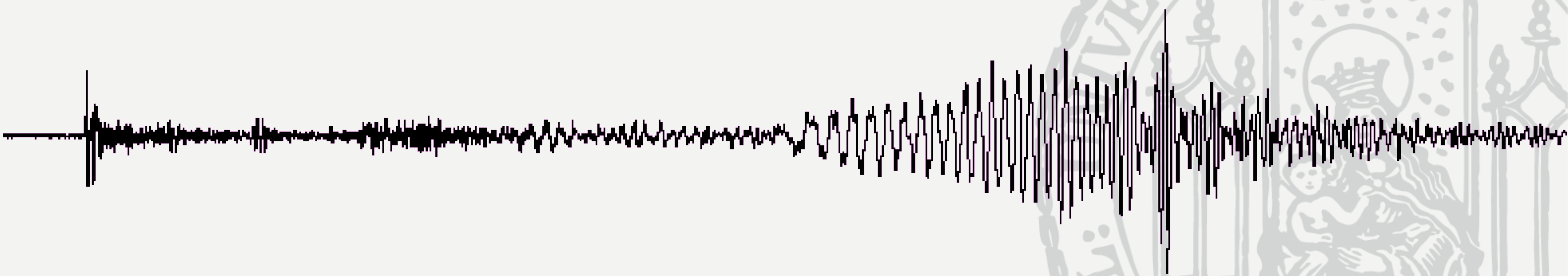
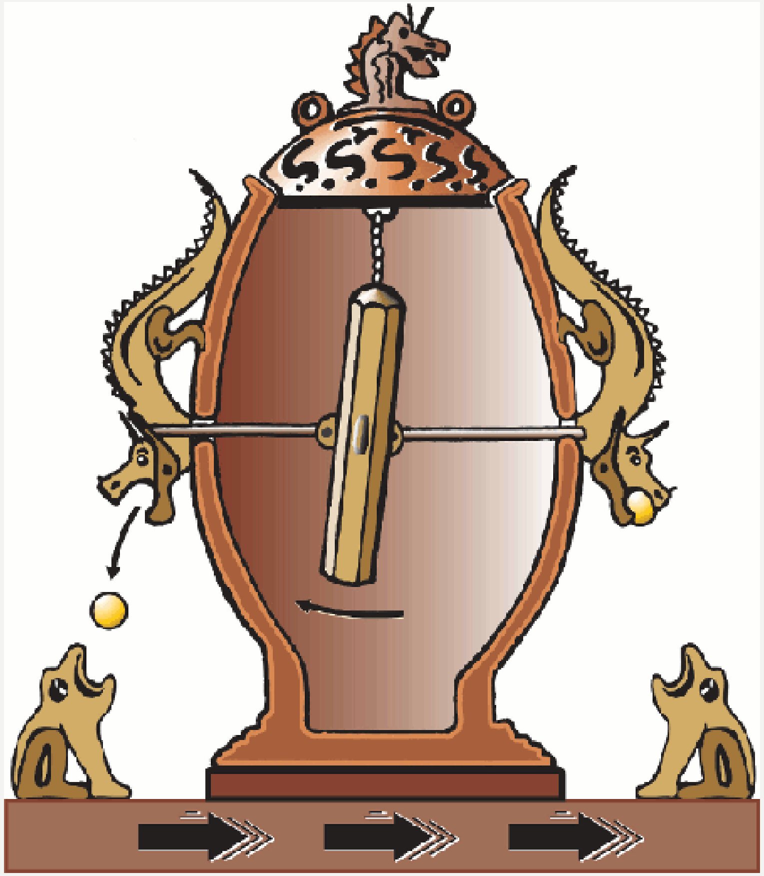
Stefanie Donner

Geophysical Data Analysis

L10 – Seismometry



Some history



Zhang Heng, China

132 AD

"instrument for measuring the seasonal winds and the movements of the Earth"

Some history

1755 – the Great Lisbon earthquake

- Subsequent fires and a tsunami destroyed the city
- Probably Mw 8.5 to 9.0
- Between 10.000 and 100.000 death
- First systematic analysis of earthquake damages with questionaires

Some history

First “modern” seismograph (1880-1885)

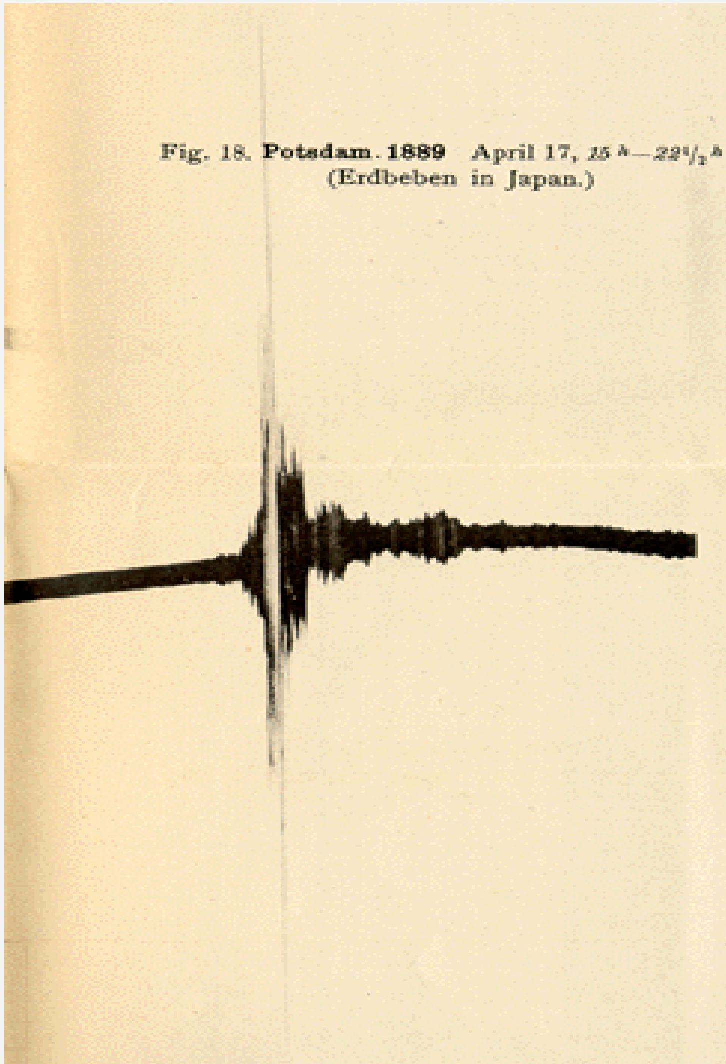
Horizontal pendulum

Pioniers: John Milne, Sir James Alfred Ewing and Thomas Gray

John Milne with Boris Galitzin and his wife, 1914

https://www.e-education.psu.edu/earth520/node/1784

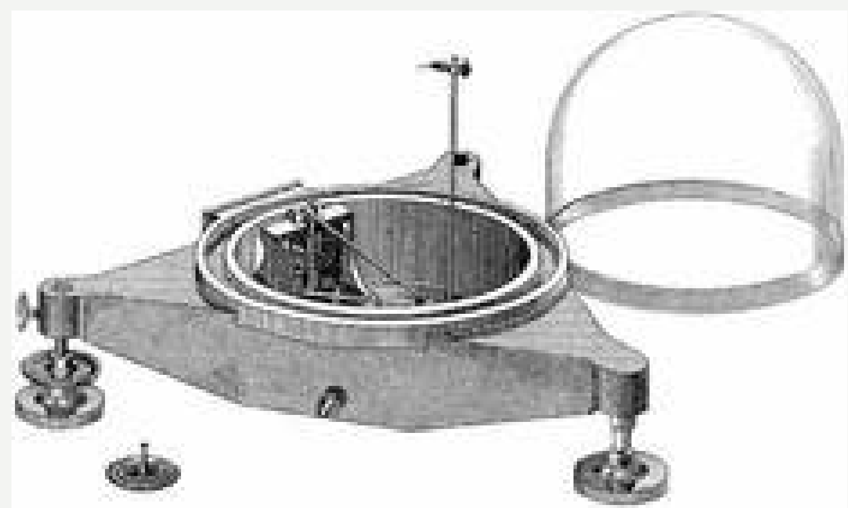
Some history



Ernst Ludwig August von Rebeur-Paschwitz

(1861-1895)

- Worked on improvement of horizontal pendulum for astronomical studies
- By chance registered first earthquake recording ever (Japan, 54° distance)

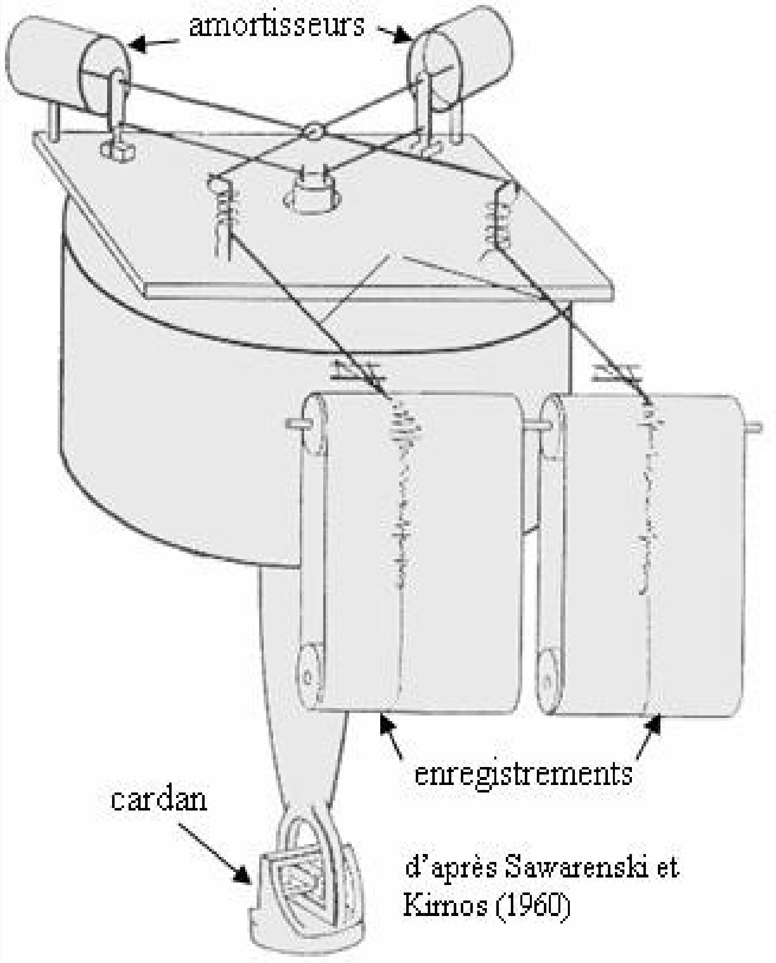


mass 200g; period 12s

Some history

Emil Wiechert (1861-1928)

- Converted or astatic pendulum
- 1st seismograph with viscous damping → useful recordings for the entire duration of ground motion
- Founder of the 1st global seismic network
- One of the co-founder of the Association Internationale de Seismologie; today: International Association of Seismology and Physics of the Earth's Interior (IASPEI)



mass 1.000g

period 10s

Moxa

Quelle: http://musee-sismologie.unistra.fr/english/the-collection-seismology

Some history

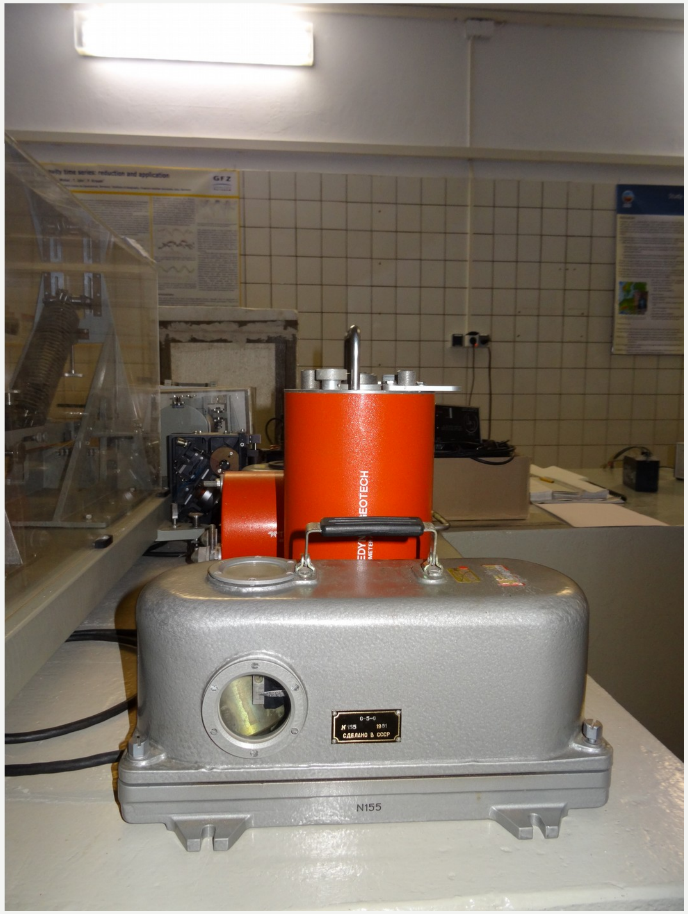
Boris Borissowitsch Golizyn (1862-1916)

- Invented 1910 first electrodynamic seismometer
- Damping replaced by coil, coil connected with mass, movement in permanent magnetic field
- Movement induces **voltage ~ velocity**

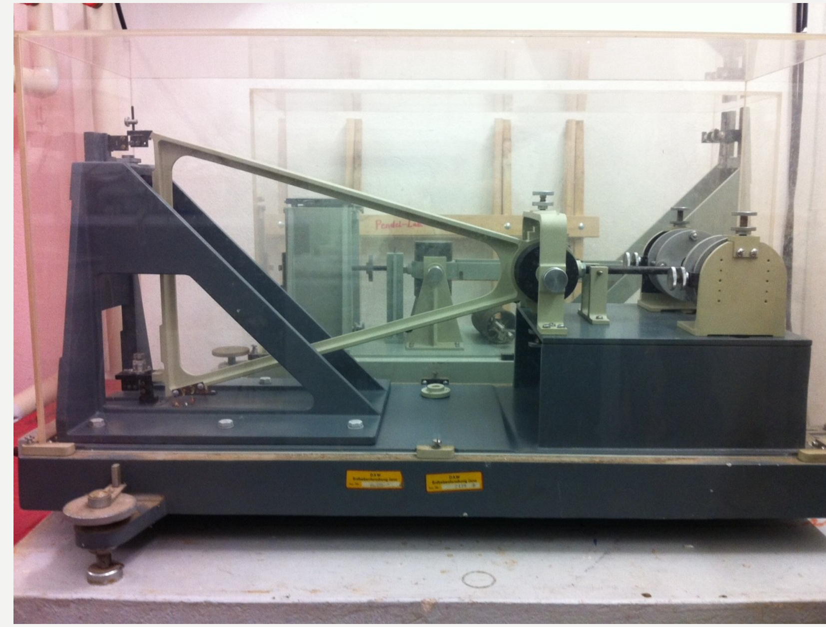
H: mass 7g, period 12s

V: mass 10kg, period 24s

Historic instruments



Jena



Moxa

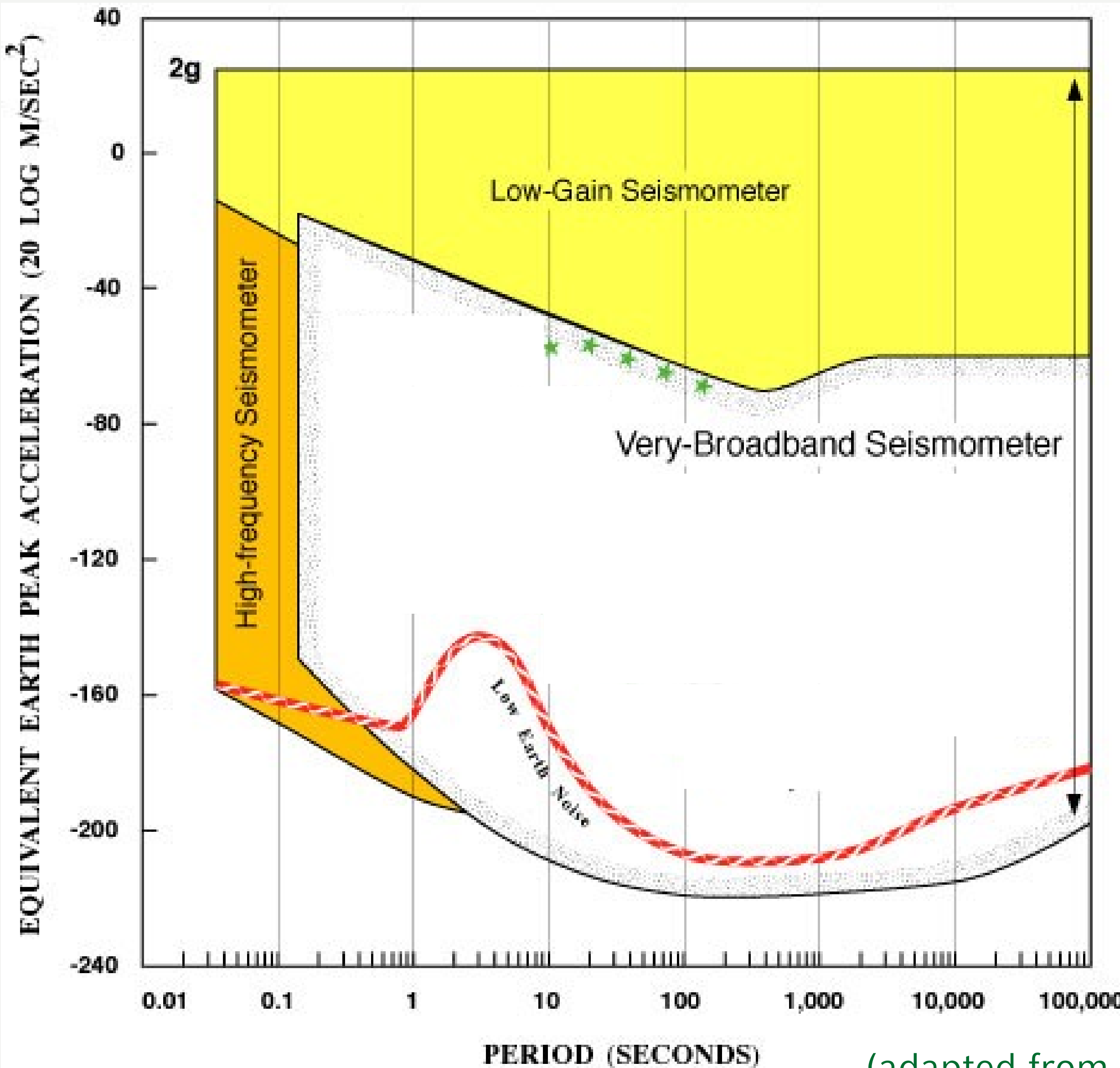
We want to measure ...

Ground motion!

Caused by ...

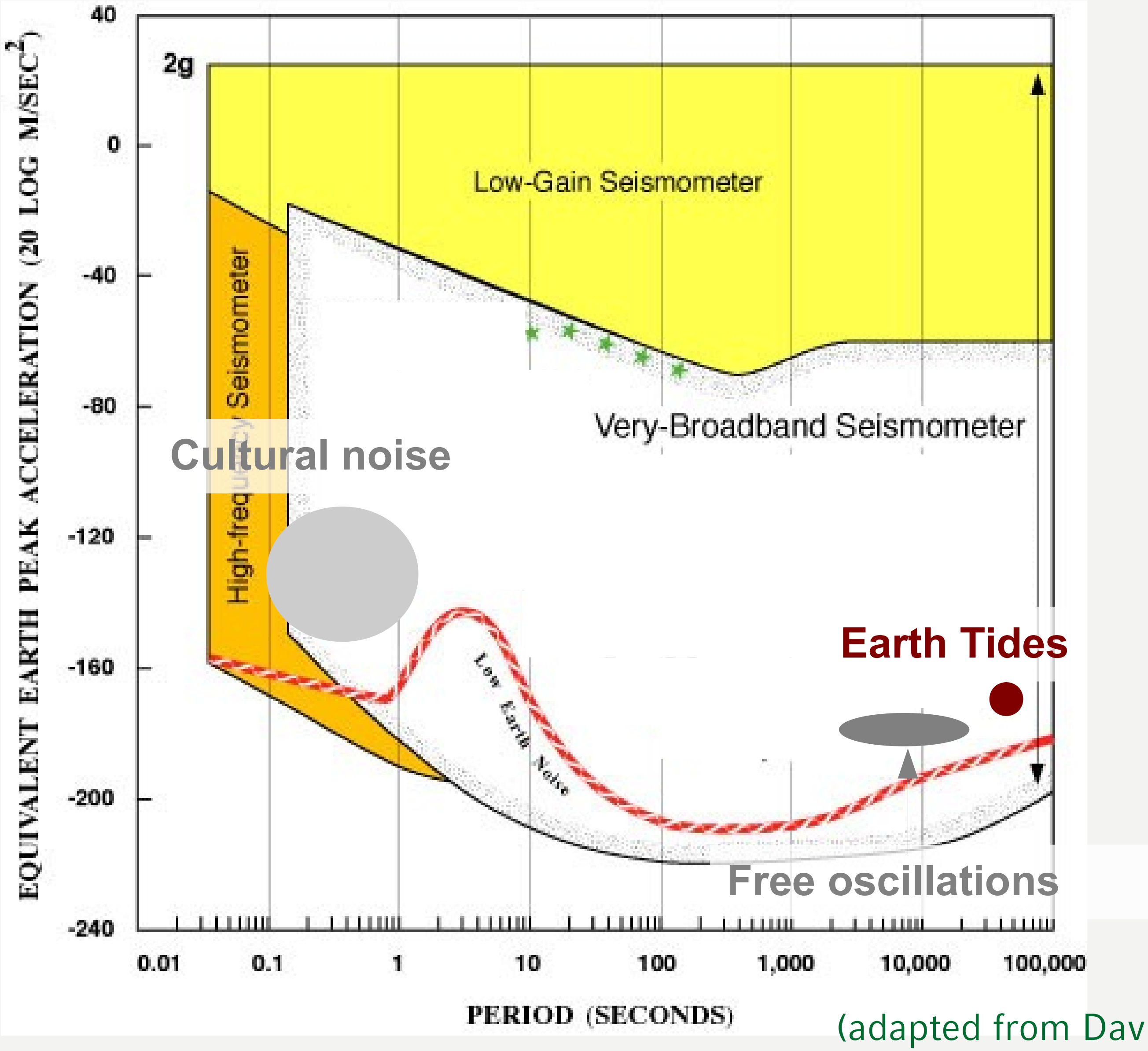
+ Earthquakes
  - Surface waves
  - Body waves
  - Free oscillations
  - Large & small
+ Cultural noise
+ Ocean noise
+ Tides
+ ...

We want to measure ...



(adapted from Davis, IGPP)

We want to measure ...



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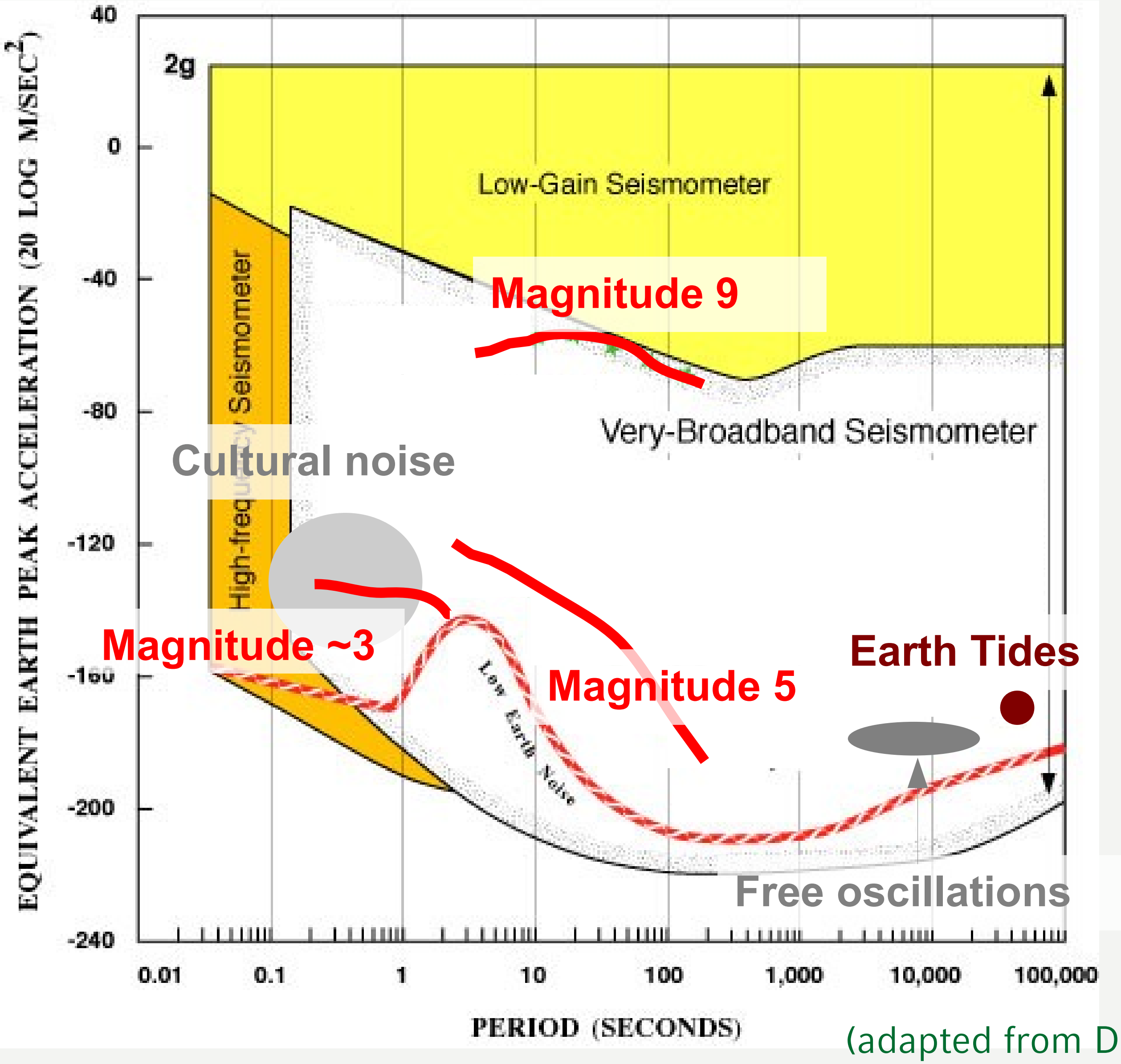
We want to measure …



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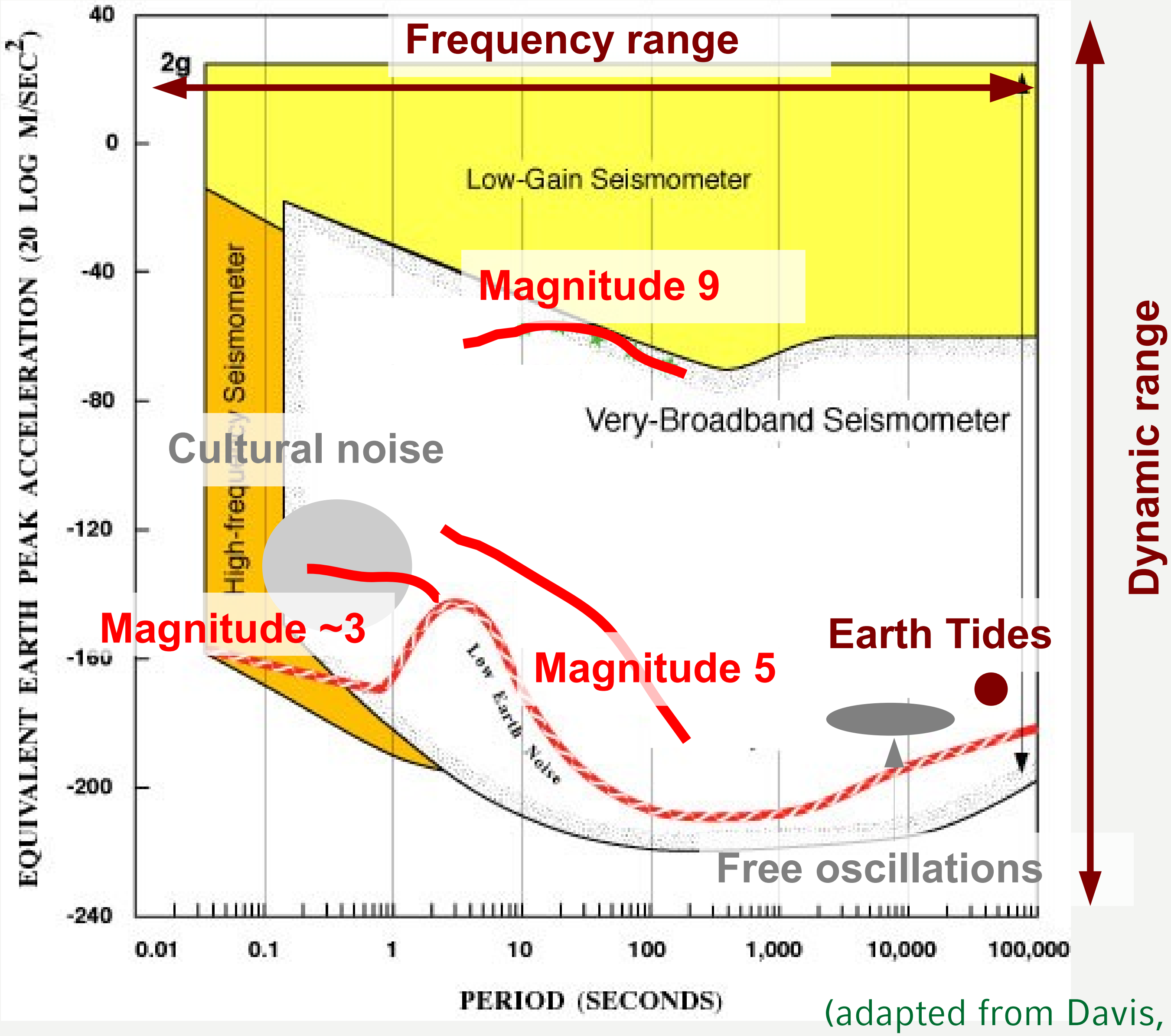
We want to measure ...



(adapted from Davis, IGPP)

Frequency range:

10 – 10e-4 Hz

Amplitude or dynamic range:

10e-1 – 10e-10 m

~ 180 dB

(Relative) dynamic range

- *ratio between largest and smallest measurable amplitude*

$$DB = \frac{V_{max}}{V_{min}}$$

- *unit: bell … is the base 10 logarithm the ratio of two energies*

$$L = \log\left(\frac{E_1}{E_2}\right) B = 10 \log\left(\frac{E_1}{E_2}\right) dB$$

- *with dB is a 10*th *of B*

  … in terms of amplitude:

$$L = 10 \log\left(\frac{A_1}{A_2}\right)^2 dB = 20 \log\left(\frac{A_1}{A_2}\right)$$

We want to measure ...

"... the Earth motion at a point with respect to this same point undisturbed."

Problems:

- measurement is done in a moving reference frame
  → displacement cannot be measured directly, can only observe the motion if it has an acceleration

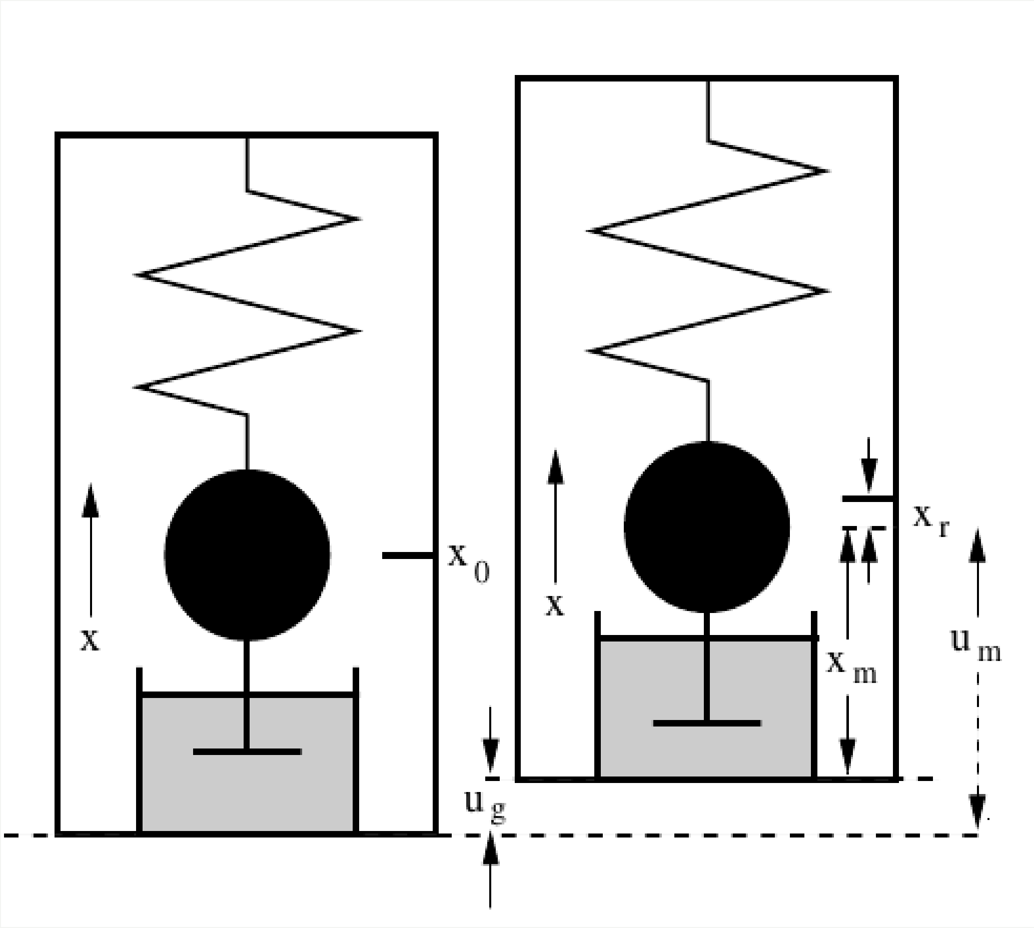
- amplitude and frequency range of seismic signals is very large
  → need several instruments to cover the full range in amplitudes and frequencies

(Instrumentation in earthquake seismology, Havskov & Alguacil, 2002)

Mechanical pendulum

- movement of a inertal, suspended mass → damped harmonic oscillator
- tend to remain stationary in response to external motion

  → relative motion between mass and ground is a function of ground's motion

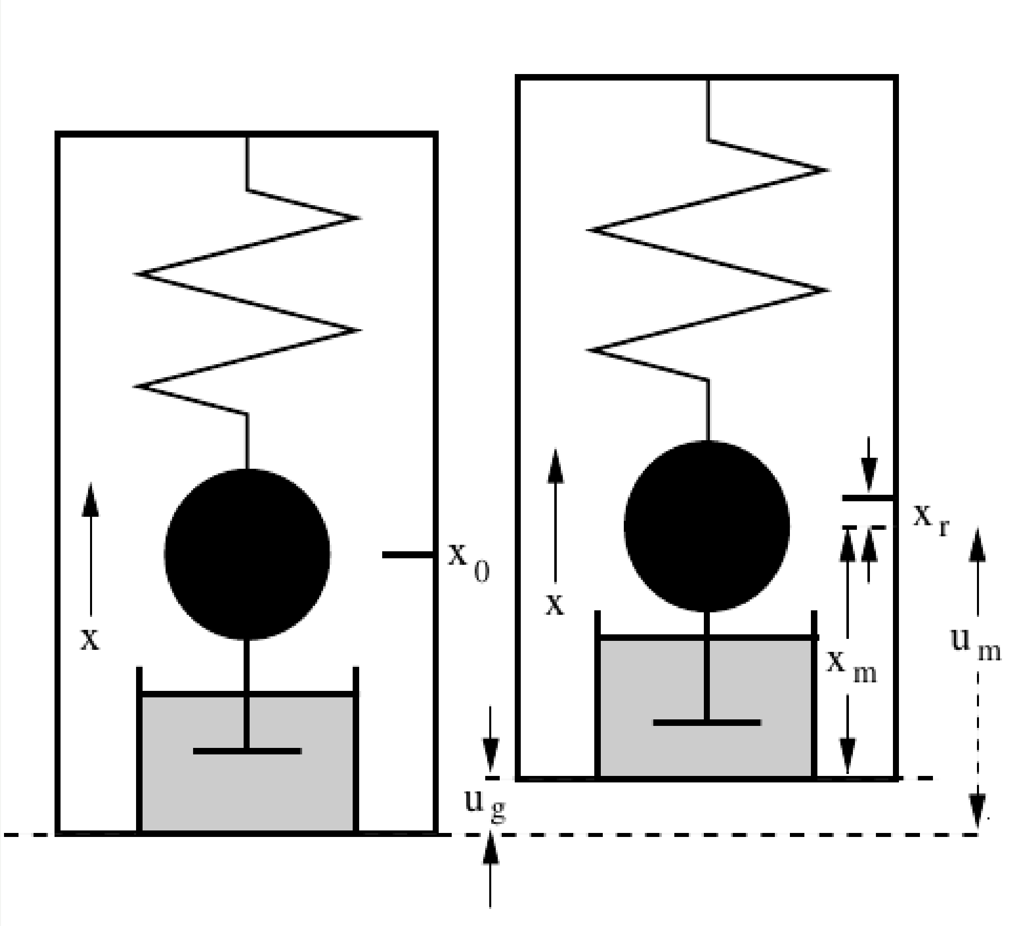


- *u_g – movement of case with ground*
- *x_0 – rest postion*
- *movement of mass measured in the reference system of the case*

$$
\begin{aligned}
x_r &= x_m - x_0 \\
x_m &= u_g - u_m
\end{aligned}
$$

Mechanical pendulum

forces acting on the system:



- inertia ~ mass m
  $$f_i = -m\ddot{u}_m(t)$$
- restoring force ~ elongation of spring
  $$f_{sp} = -kx_r(t)$$
- friction force ~ velocity of movement
  $$f_f = -D\dot{x}_m(t)$$

$\Downarrow$

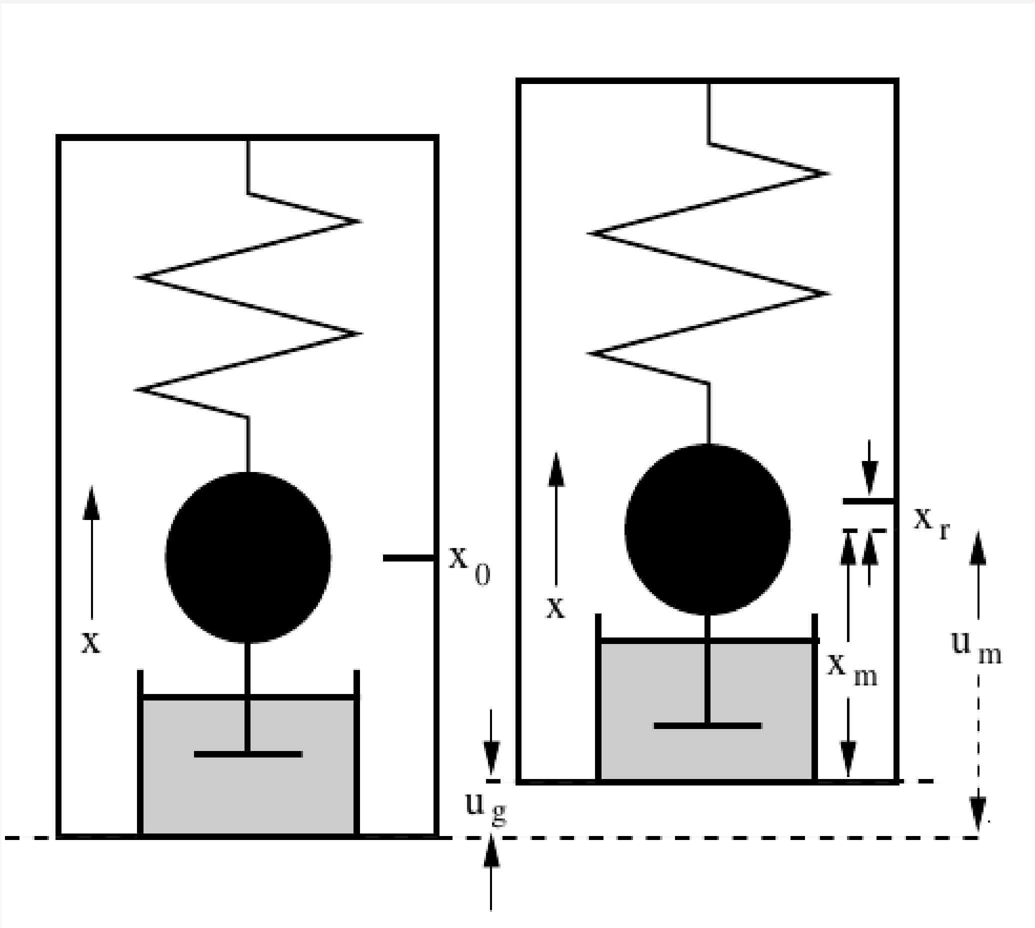
seismometer equation

(in equilibrium)

$$-m\ddot{u}_m(t) - D\dot{x}_m(t) - kx_r(t) = 0$$

Mechanical pendulum

Seismometer equation:



- Relation between u_g, u_m, and x_m

$$u_m(t) = u_g(t) + x_m(t)$$

$$\Downarrow$$

$$-m(\ddot{u}_g(t) + \ddot{x}_m(t)) - D\dot{x}_m(t) - kx_r(t) = 0$$

$$+$$

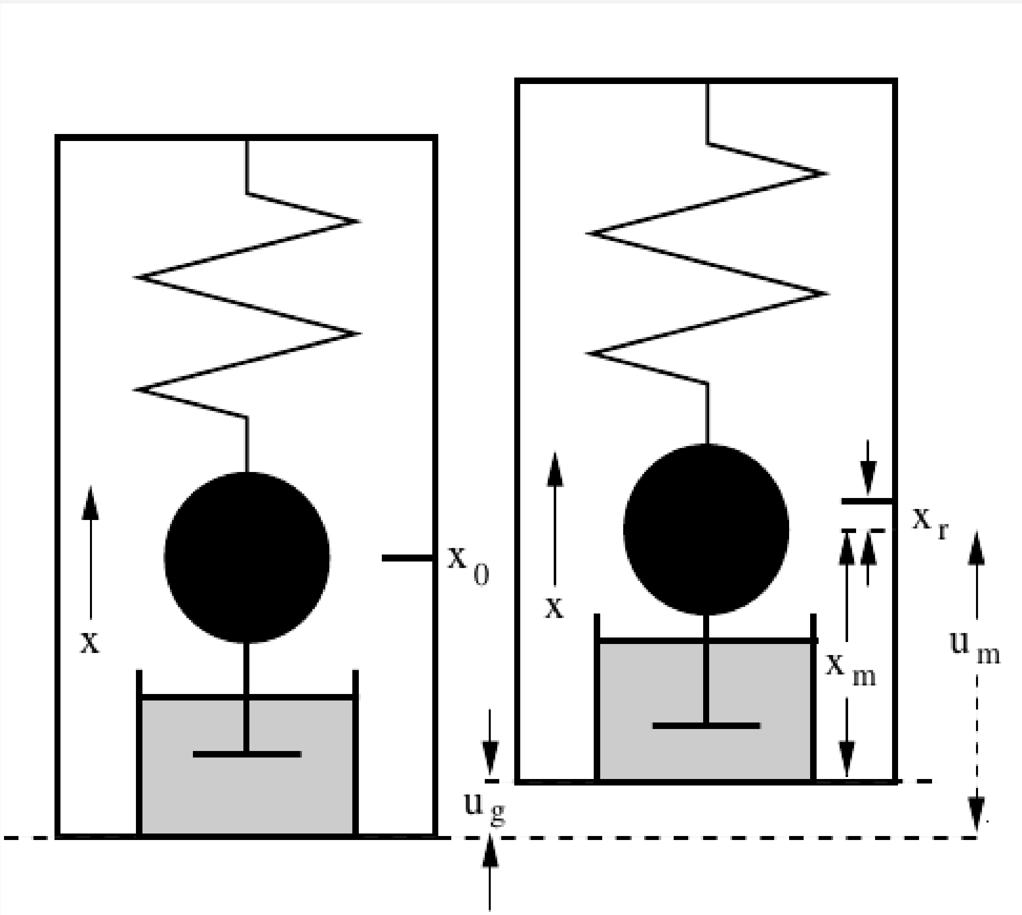
$$\dot{x}_m(t) = \dot{x}_r(t) \qquad \ddot{x}_m(t) = \ddot{x}_r(t)$$

$$\Downarrow$$

$$m\ddot{x}_r(t) + D\dot{x}_r(t) + kx_r(t) = -m\ddot{u}_g(t)$$

Mechanical pendulum

Seismometer equation:



$$m\ddot{x}_r(t) + D\dot{x}_r(t) + kx_r(t) = -m\ddot{u}_g(t)$$

- Divide this equation by m and insert

$$\omega_0^2 = \frac{k}{m} \qquad 2\epsilon = \frac{D}{m} = 2h\omega_0 \qquad h = \frac{\epsilon}{\omega_0}$$

eigenfrequency

damping constant

$$\Downarrow$$

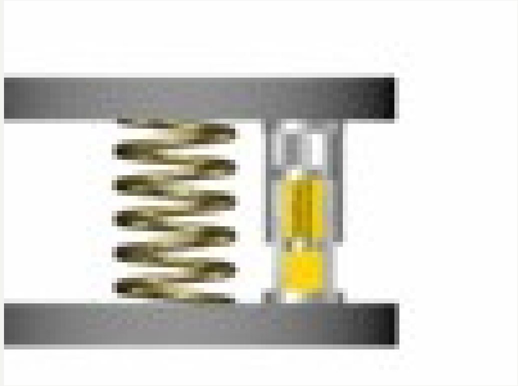
$$\boxed{\ddot{x}_r(t) + 2\epsilon\dot{x}_r(t) + \omega_0^2 x_r(t) = -\ddot{u}_g(t)}$$

Mechanical pendulum

$$\boxed{\ddot{x}_r(t) + 2\epsilon\dot{x}_r(t) + \omega_0^2 x_r(t) = -\ddot{u}_g(t)}$$

- *For slow movements (low frequency)*

$$x_r(t) \gg \dot{x}_r(t), \ddot{x}_r(t) \quad \Rightarrow \quad \omega_0^2 x_r(t) \approx -\ddot{u}_g$$



movement of mass ~ ground acceleration

→ measure ground acceleration

Mechanical pendulum

$$\boxed{\ddot{x}_r(t) + 2\epsilon\dot{x}_r(t) + \omega_0^2 x_r(t) = -\ddot{u}_g(t)}$$

- *For fast movements (high frequency)*

$$\ddot{x}_r(t) \gg \dot{x}_r(t), x_r(t) \quad \Rightarrow \quad \ddot{x}_r(t) \approx -\ddot{u}_g$$

movement of mass ~ ground displacement

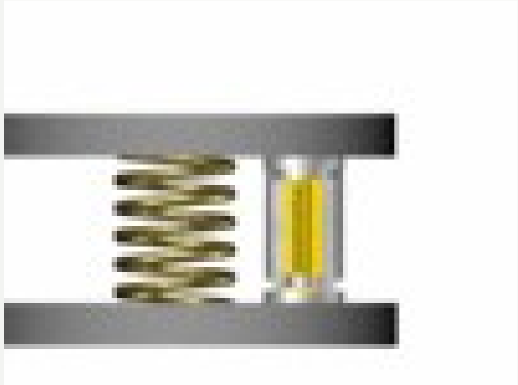
→ measure ground displacment

Mechanical pendulum

$$\boxed{\ddot{x}_r(t) + 2\epsilon\dot{x}_r(t) + \omega_0^2 x_r(t) = -\ddot{u}_g(t)}$$

- *Natural frequency*

  new push at exact the „right time“, i.e. when mass is at extreme position



movement of mass >> ground displacement

→ amplitudes get larger and larger (gain > 1)

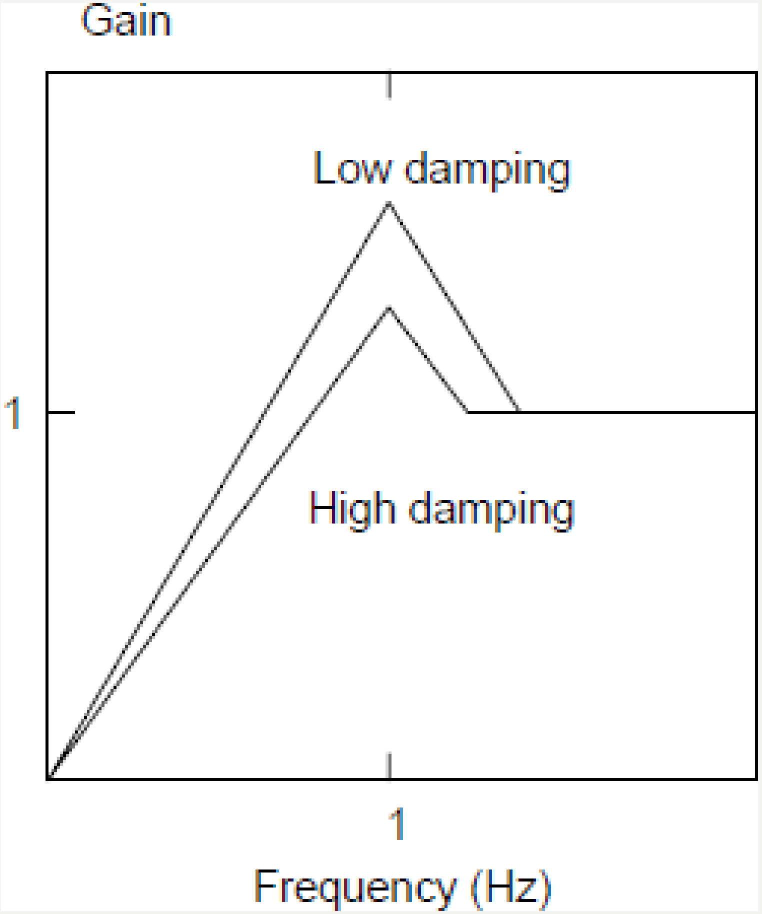
Amplitude response

- **Low frequency:**
  mass displacement ~ 0

- **Resonance:**
  mass displacement >> ground displacement

- **High frequency:**
  mass displacement ~ ground displacement

for $\omega_0 = 1$ Hz



And arbitrary input?

- Superposition of harmonic functions → Fourier series ….

$$\mathcal{F}\left[x(t)\right] = X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt \Longrightarrow X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

- So, let's start with $u_g(t) = A_i e^{j\omega t}$ and therefore $\ddot{u}_g(t) = -\omega^2 A_i e^{j\omega t}$

⇨ $$\ddot{x}_r(t) + 2\epsilon \dot{x}_r(t) + \omega_0^2 x_r(t) = \omega^2 A_i e^{j\omega t}$$

And arbitrary input?

$$\ddot{x}_r(t) + 2\epsilon\dot{x}_r(t) + \omega_0^2 x_r(t) = \omega^2 A_i e^{j\omega t}$$

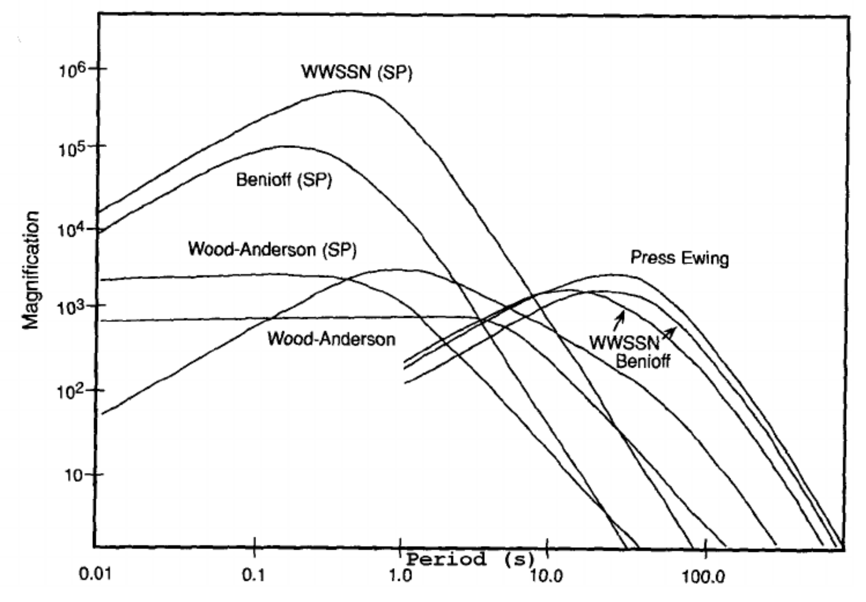
- Solution with exponential ansatz:
  (consider: A_i and A_0 are complex)

$$\begin{aligned} x_r(t) &= A_0 e^{j\omega t} \\ \dot{x}_r(t) &= j\omega A_0 e^{j\omega t} \\ \ddot{x}_r(t) &= -\omega^2 A_0 e^{j\omega t} \end{aligned}$$

$$\Rightarrow \quad -\omega^2 A_0 + 2\epsilon j\omega A_0 + \omega_0^2 A_0 = \omega^2 A_1$$

- Examine relation of output to input:

$$\frac{A_0}{A_i} = \frac{\omega^2}{\omega_0^2 - \omega^2 + 2j\epsilon\omega} = T(j\omega)$$



Voila, the frequency response function

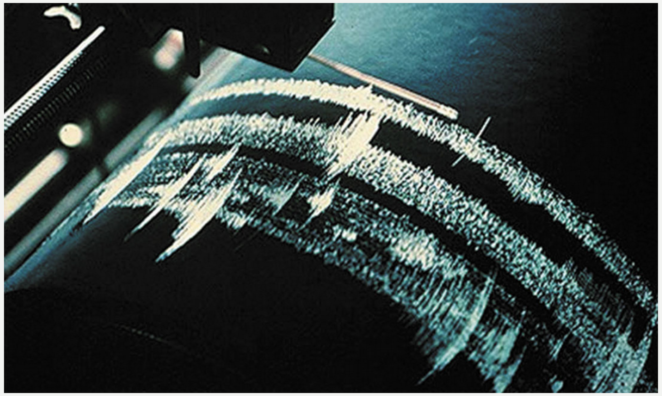
of a seismometer. (YES, seismometers are filters!)

Seismometer → seismograph

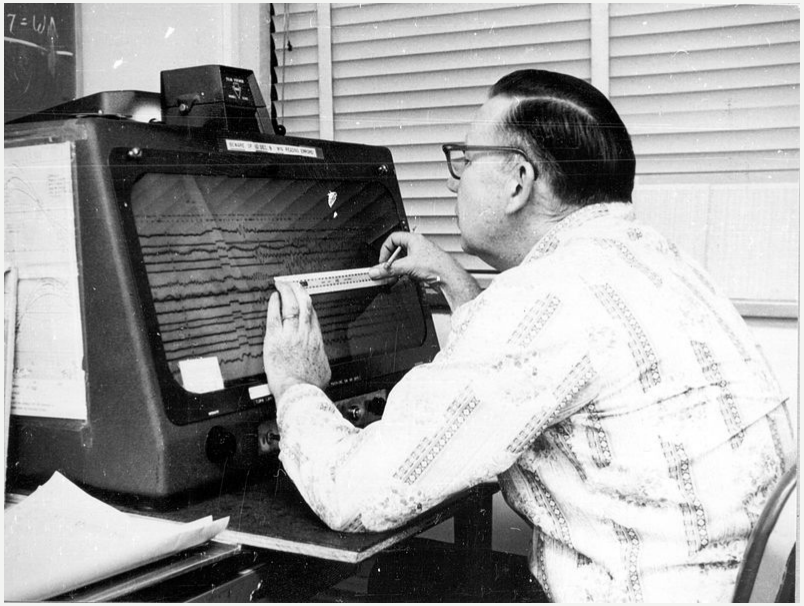
Fine!

But how to measure the movement of the mass?

Variations of recording

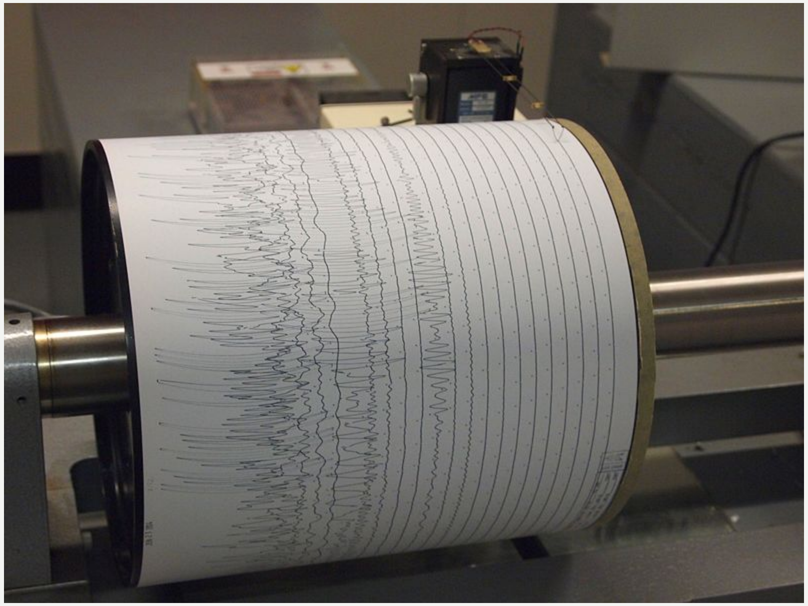


On smoked paper



On film or foto paper

With ink on paper

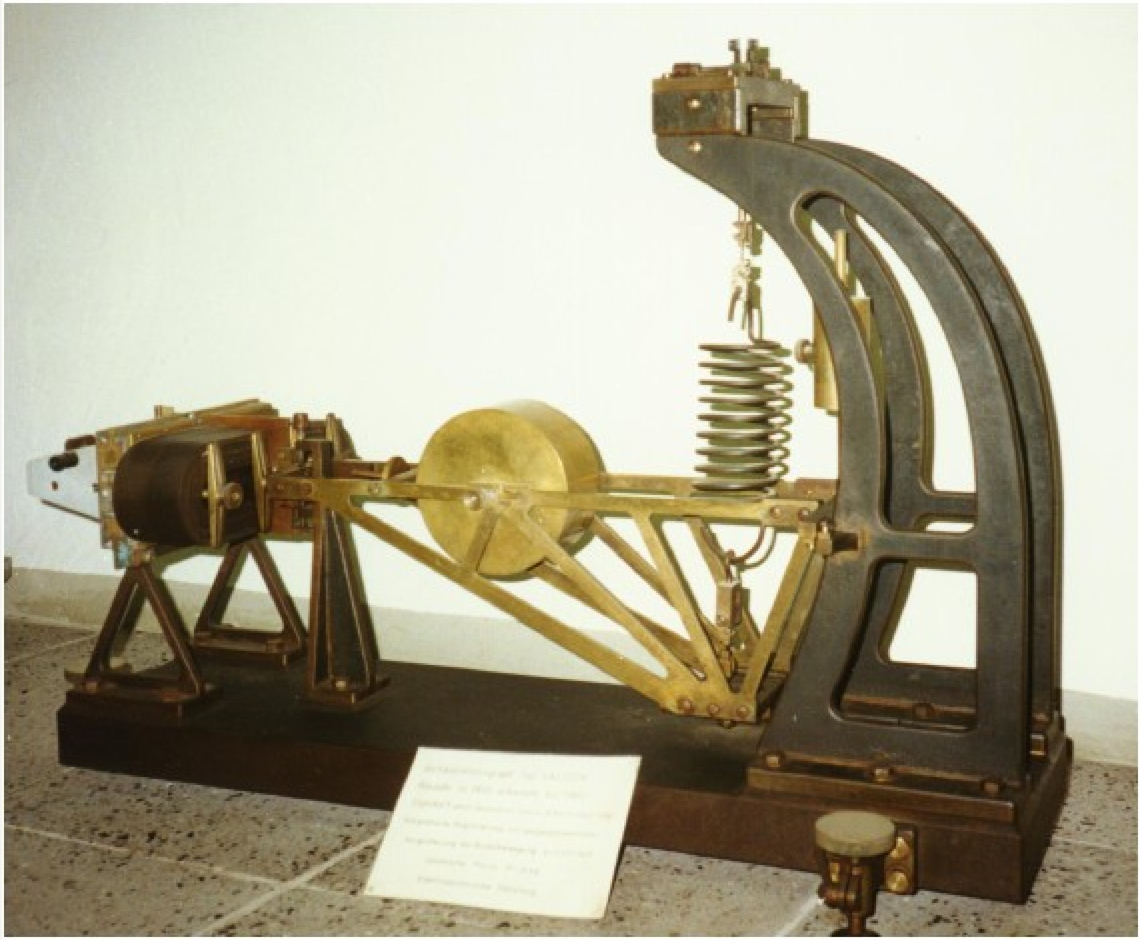


Later: magnetic tapes

Today: digital (since about 1970')

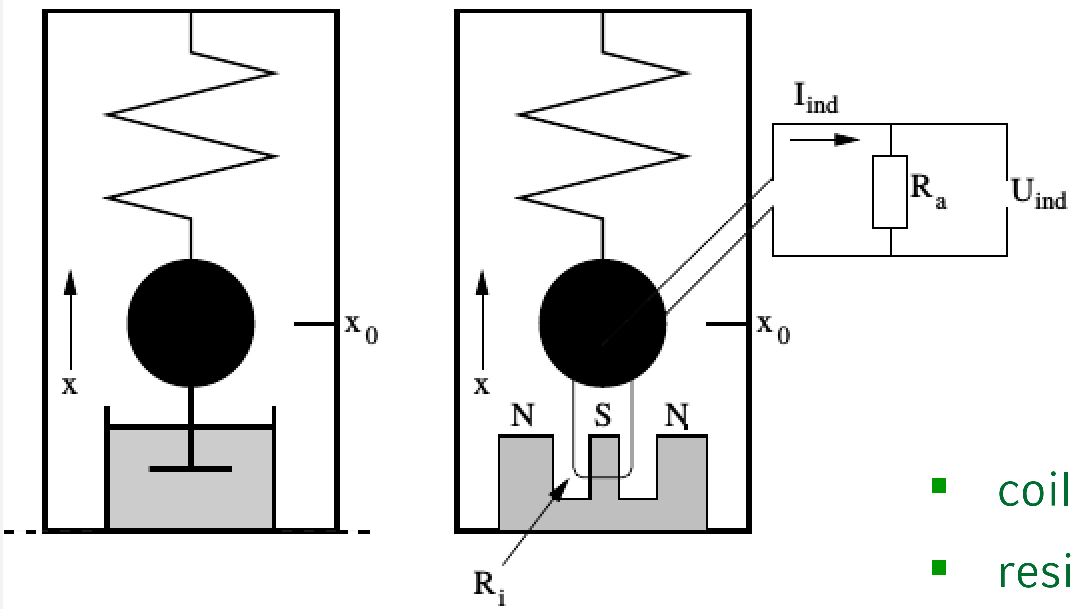
Breakthrough ...

... change the mechanical movement into electrical voltage!



Galitzin, 1904

Electrodynamic seismometer



$$I_{ind} = \frac{U_{ind}}{R_a + R_i}$$

- coil instead of dashpot
- resistor connected in parallel to coil induces voltage

  → permanent magnetic field
- coil is connected with mass and moves in magentic field

  → movement induces **voltage ~ velocity**
- magnetic field slows the coil down

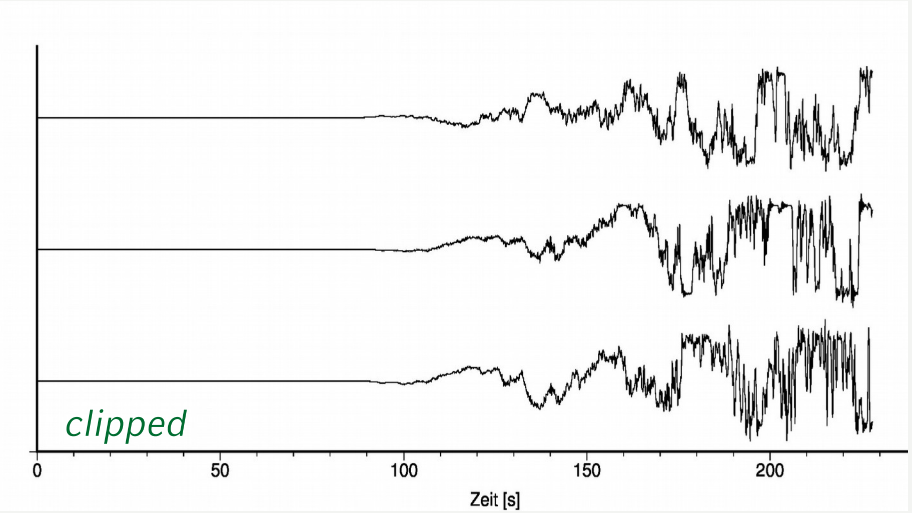
electrodynamic damping factor:

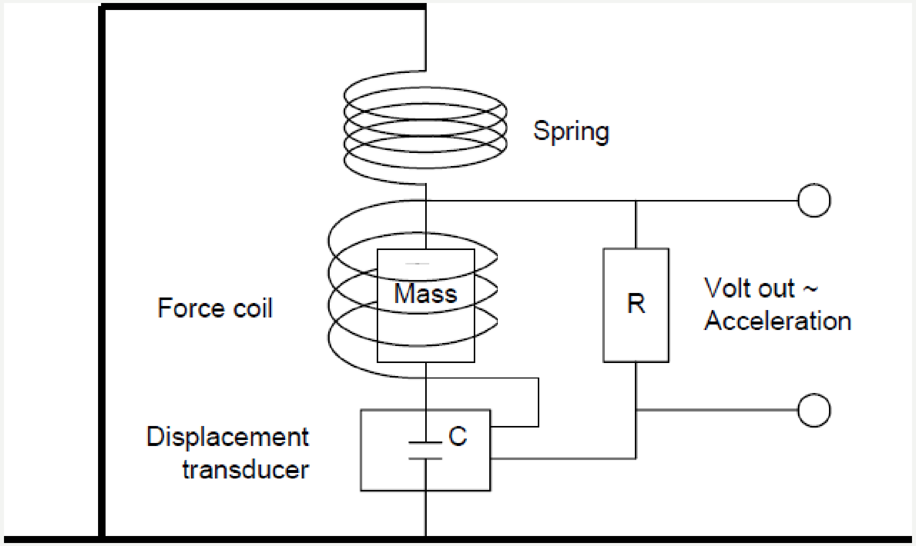
$$\epsilon_c \sim \frac{1}{R_a + R_i}$$

Problem: Nonlinearity

- Instruments are **non**linear when:
  + mass moves out of measurement range (or coil)
  + clipping
  + spring changes/ages
  + large spring extension

Solution:
force feedback seismometer



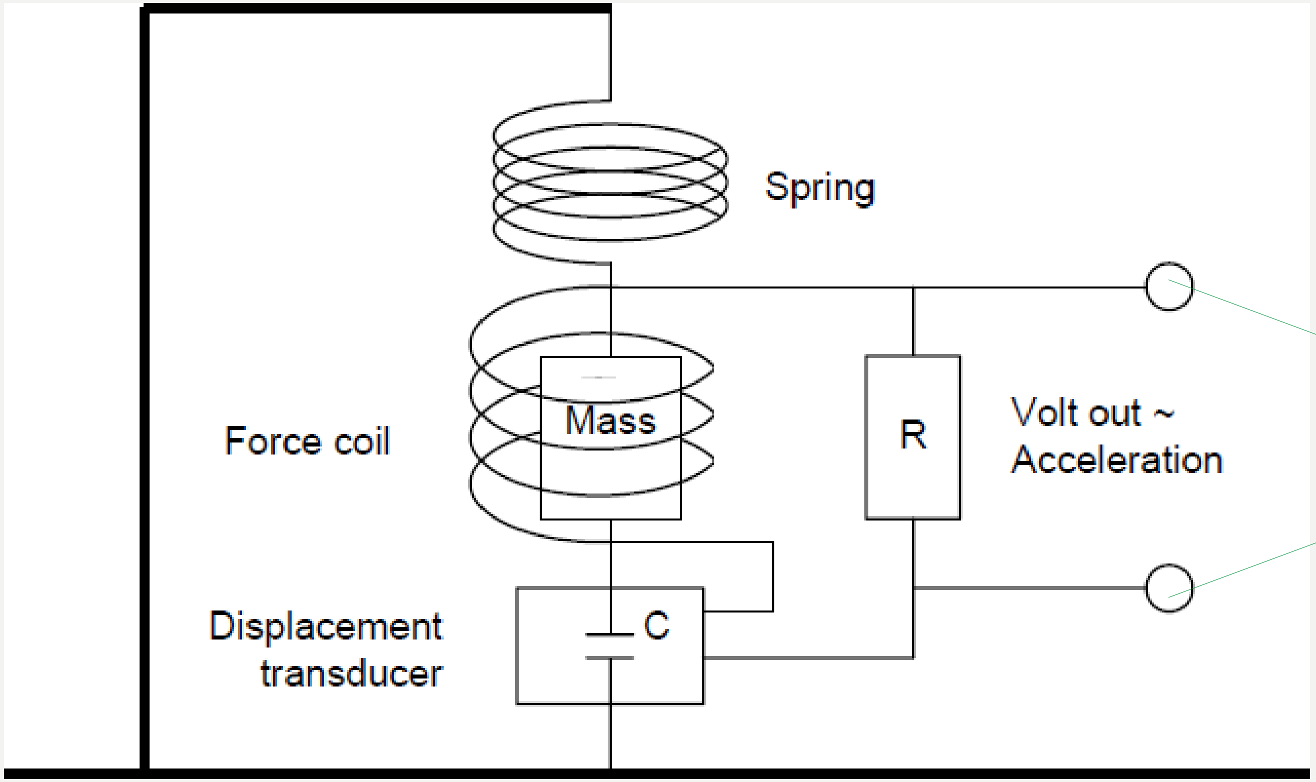


Don't let the mass move!

Force feedback seismometer

+ Don't let the mass move!

→ detect movement and **push back**



Measure:

Current needed to generate opposing force

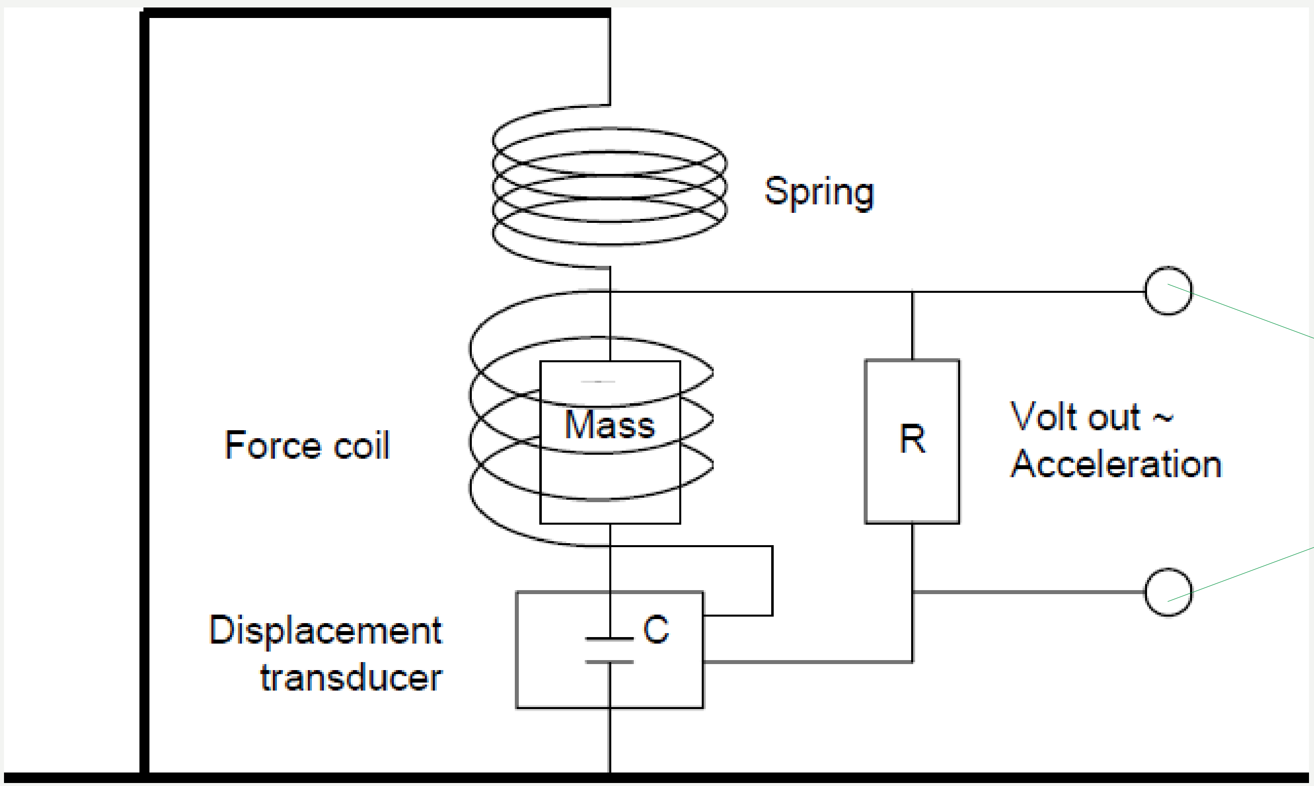
Force feedback seismometer

+ Don't let the mass move!

→ detect movement and **push back**

Ideal seismometer:

+ sensitive
+ high dynamic range
+ broadband
+ linear



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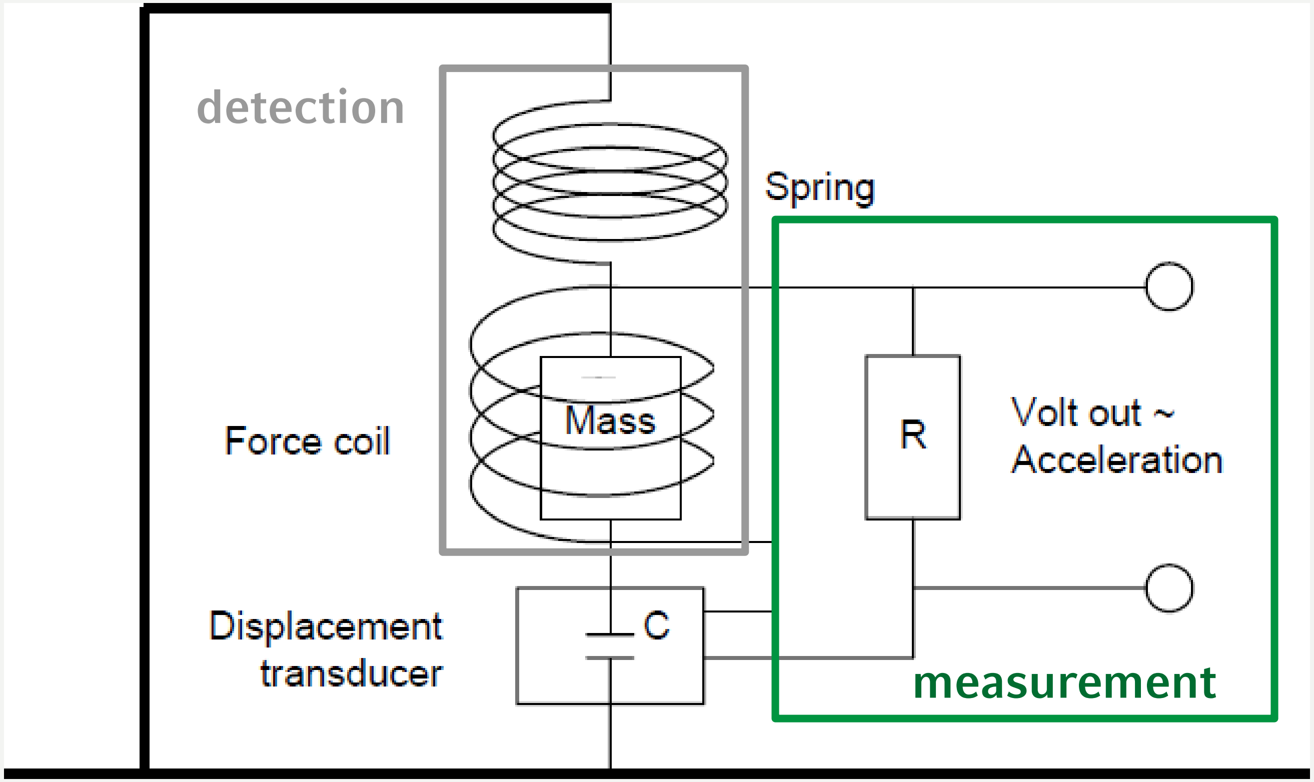
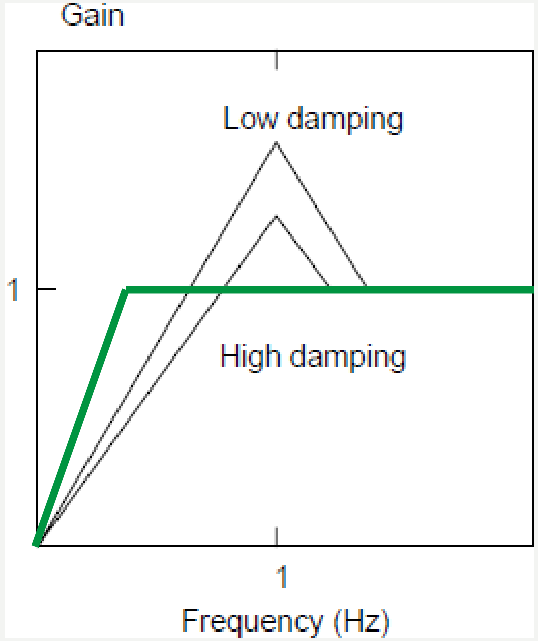




And arbitrary input?

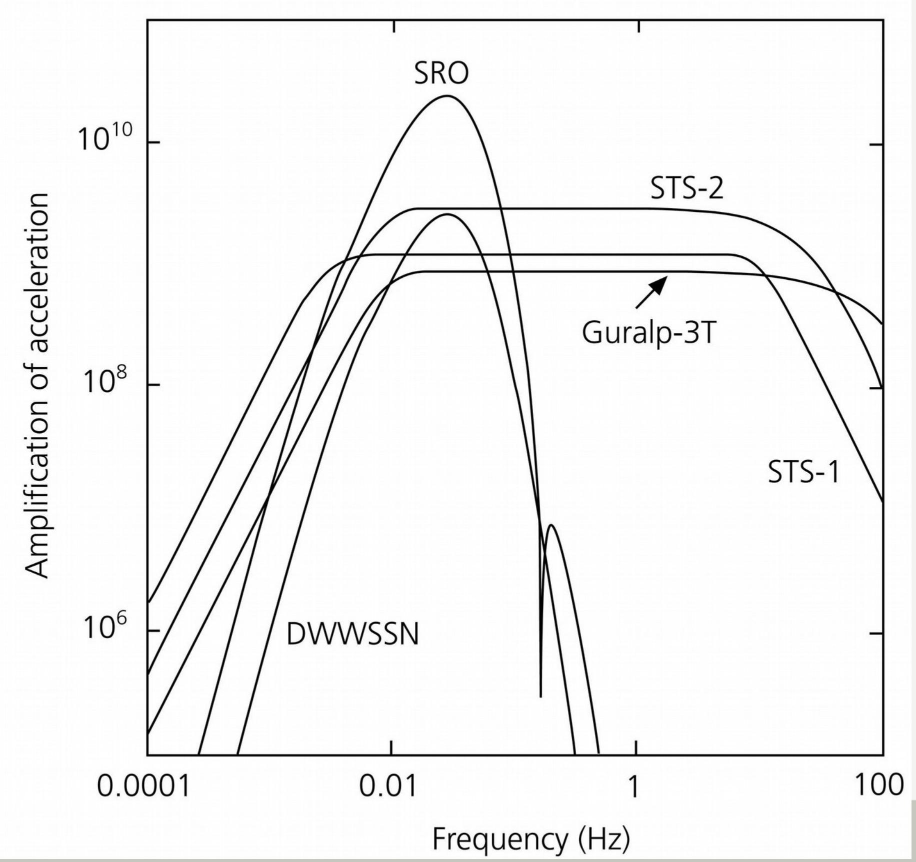
- Amplitude frequency response of an electrodynamic system:

$$|T(j\omega)| = G\frac{\omega^3}{\sqrt{(\omega_0^2-\omega^2)^2+4\epsilon^2\omega^2}}$$

with generator constant G

$$G=\frac{\text{output voltage}}{\text{ground velocity}}$$

describing the gain of the seismometer.
The unit of G is [V/(m/s)] or [Vs/m].



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Take a breath!

Questions?

Some energy left? ....

Calibration

- provides knowledge of relationship between input (ground motion) and output (electric voltage)

- To be considered:

  + seismic noise

  + tilt ⇨ performed in basement room

  + etc. …

- Parameters to be checked:

  + natural frequency and damping

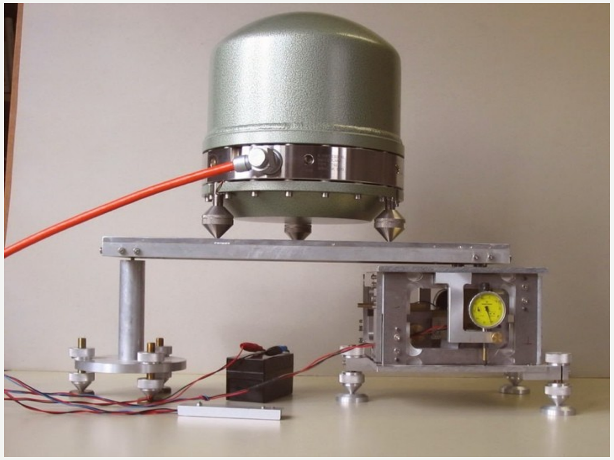
  + „critical damping resistance“ and open circuit damping (adjust desired damping value)

  + sensitivity

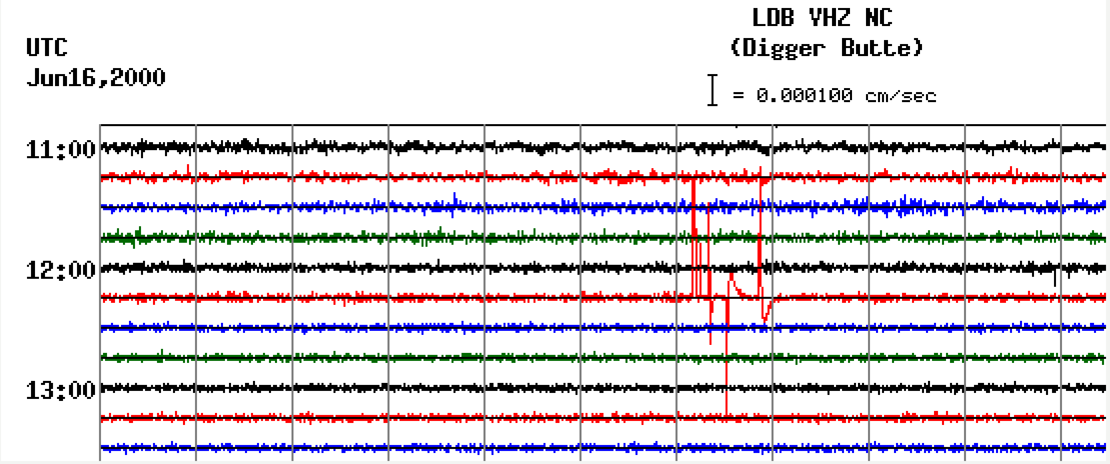
Calibration - principle

- *feed in a simple and know signal , measure output signal, and derive transfer function via*

shaker table



$$T(j\omega)=\frac{output}{input}$$



USGS

Registration chain

Each step between earthquake and recording alters (i.e. filters) the signal!

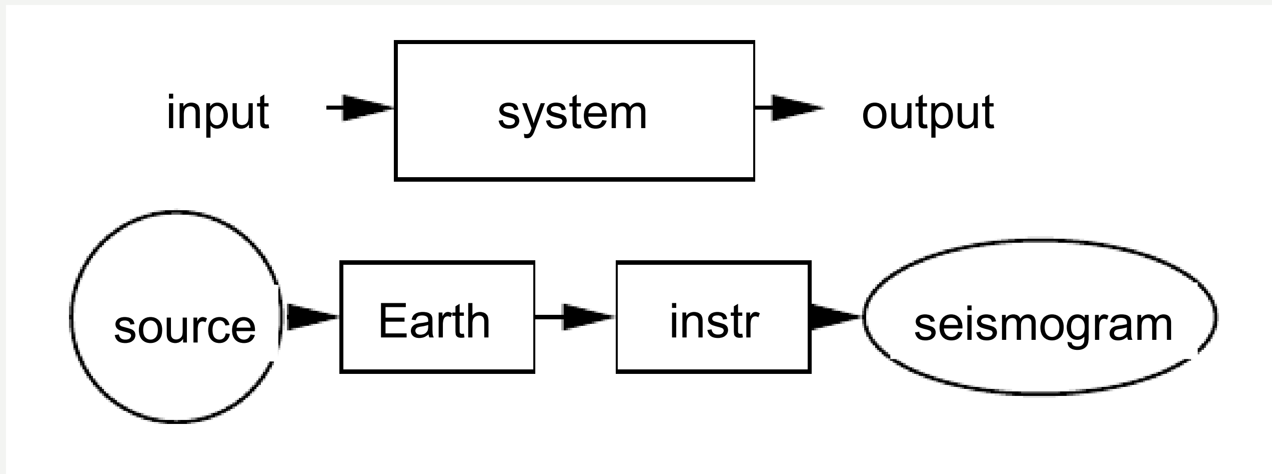
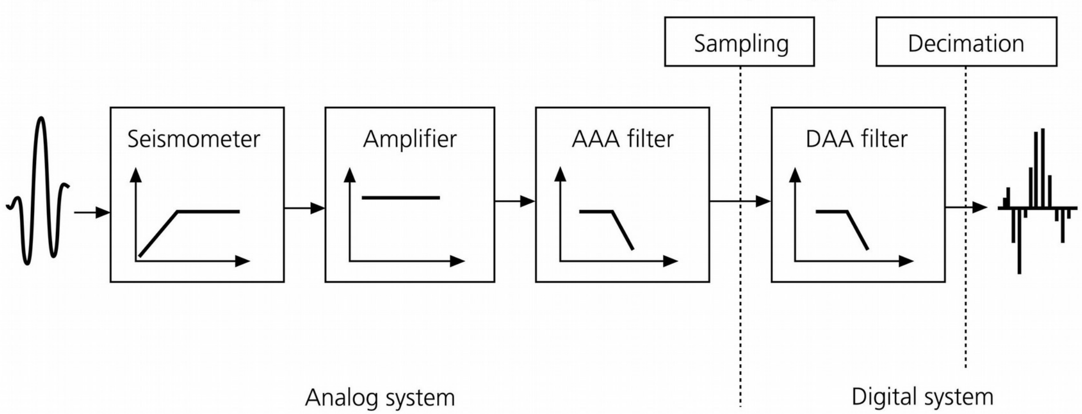


Figure 6.6-12: Diagram showing the analog-to-digital (ADC) process.



Registration chain

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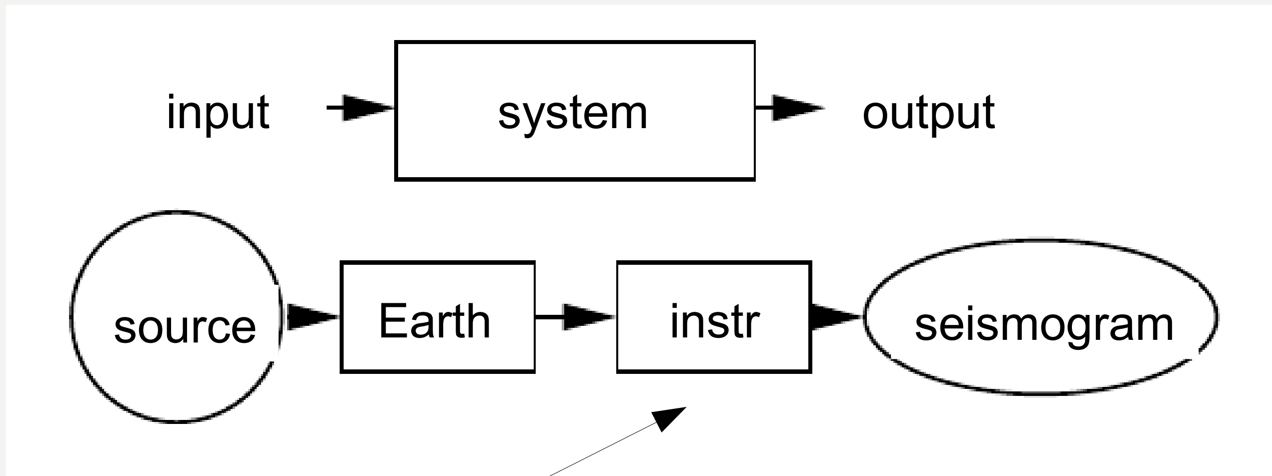
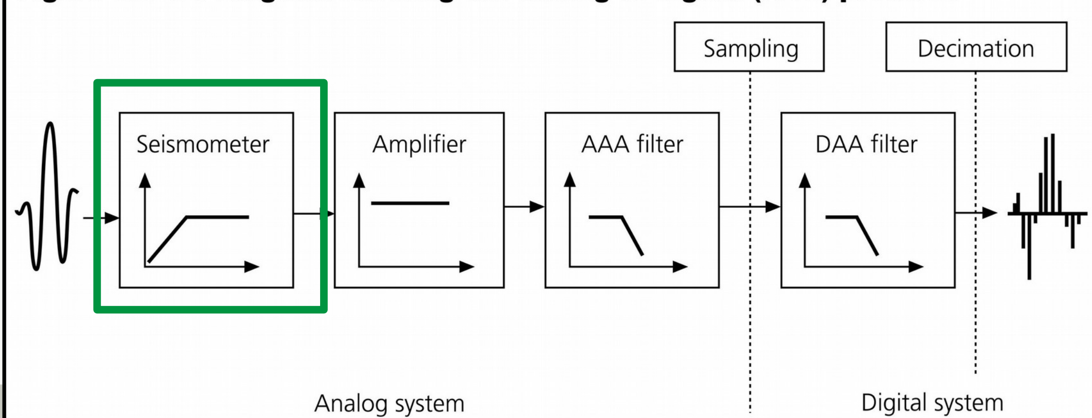


Figure 6.6-12: Diagram showing the analog-to-digital (ADC) process.



Acutally, there would be much more to say ...