Seminar für Finanzökonometrie
Institut für Statistik
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Based on exercises from Steffen Unkel

Statistical Geophysics
Exercise Sheet 5

Exercise 1 Consider a normally distributed population of diamonds. In order to deduce the mean purity of the diamonds we evaluate the following sample of size $n = 5$.

$$1 \quad 2 \quad 4 \quad 2 \quad 2$$

Calculate a 95%-confidence interval for the mean μ.

Proposal for solution Calculate the sample mean:

$$\bar{x} = \frac{1+2+4+2+2}{5} = 2.2$$

Calculate the sample variance:

$$s^2 = \frac{1}{5-1}((1-2.2)^2 + (2-2.2)^2 + (4-2.2)^2 + (2-2.2)^2 + (2-2.2)^2) = 1.2$$

We get:

$$s = \sqrt{1.2}$$

Since we our sample size is $\leqslant 30$, we use the t-distribution.
Now we can calculate our confidence interval:

$$\bar{x} \pm t_{1-\frac{\alpha}{2}}(n-1)\frac{s}{\sqrt{n}}$$

We get:

$$2.2 \pm t_{0.975}(4)\frac{\sqrt{1.2}}{\sqrt{5}} \tag{1}$$

So our confidence interval is

$$[0.84, 3.56].$$

Exercise 2 Consider a normally distributed population of natural tunnels at a digging site. In order to deduce the variance of the diameters of the holes we evaluate the following sample of size $n = 5$.

$$1 \quad 1 \quad 2 \quad 2 \quad 4$$

Calculate a 95%-confidence interval for the variance σ^2.

Proposal for solution Compute the sample mean:

$$\bar{x} = \frac{1+1+2+2+4}{5} = 2$$

Calculate the sample variance:

$$s^2 = \frac{1}{5-1}((1-2)^2 + (1-2)^2 + (2-2)^2 + (2-2)^2 + (4-2)^2) = 1.5$$

Now we can calculate our confidence interval:

$$[\frac{(n-1)s^2}{\chi^2_{1-\frac{\alpha}{2}}(n-1)}, \frac{(n-1)s^2}{\chi^2_{\frac{\alpha}{2}}(n-1)}]$$

We get:

$$[\frac{6}{\chi^2_{0.975}(4)}, \frac{6}{\chi^2_{0.025}(4)}] = [0.54, 12.39]$$

(Remember, the χ^2-distribution is not symmetric!)

Exercise 3 A group of geophysicists walks into an antiquity shop and wonders about the mean gold content in these antiquities, and submits to an independent laboratory a random sample of 12 pieces of jewellery for analysis. The percentage of gold in each of the pieces is as follows.

$$21 \quad 18 \quad 19 \quad 16 \quad 18 \quad 24 \quad 22 \quad 19 \quad 24 \quad 14 \quad 18 \quad 15$$

The shop owner claims that the mean gold content of his jewellery is exactly 20%. Assuming percentage gold content to be normally distributed, carry out an appropriate hypothesis test in order to advise the group of geophysicians as to the validity of the shop owner's claim using a significance level of $\alpha = 0.05$.

Exercise 4 It has long been known that offenders often commit crimes under the influence of stones. A geophysicist wants to examine the stone of choice for stone-involved offenders who committed crimes for which they were arrested while under the influence. A total of 50 stone-using arrestees are sampled and each was asked to indicate which stone was in their hand at the time of their offense/arrest. The following data indicate how many arrestees were using any given category of stone:

Amethyst	Diamond	Sapphire	Ruby
18	17	7	8

Test the hypothesis

H_0 :In the general population of stone-using arrestees, there is no difference for the stone of choice at the time of arrest.

VS

H_1 :In the general population of stone-using arrestees, one or more of the stones is preferred as evidenced by their use at the time of arrest,

with $\alpha = 0.05$.

Exercise 5 Trace metals in drinking water affect the flavor and an unusually high concentration can pose a health hazard. Ten pairs of data were taken measuring zinc concentration in bottom water (1) and surface water (2).
We assume both populations to follow a normal distribution.

Zinc concentration in bottom water (1)	.1	.1	.2	.1	.3	.4	.2	.3	.2	.1
Zinc concentration in surface water (2)	.4	.2	.3	.2	.1	.3	.1	.1	.3	.2

Perform a suitable test with $\alpha = 0.05$ to check if $\mu_1 \neq \mu_2$, where μ_1 is the mean of population (1) and μ_2 is the mean of population (2).

Exercise 6 A worldwide association of geophysicians is searching for diamonds for 12 days in a row and suspects that amount of diamonds found per day is affected by the amount it rains.

Measuring the amount of rainfall and the amount of diamonds found, we get:

	1	2	3	4	5	6	7	8	9	10	11	12
Diamonds found p. d.	215	325	185	332	406	522	412	614	544	421	445	408
Rainfall	14.2	16.4	11.9	15.2	18.5	22.1	19.4	25.1	23.4	18.1	22.6	17.2

a) Carry out a linear regression by calculating the regression coefficients.

b) Assume all conditions for hypothesis testing are satisfied and check whether $H_0 : \beta_1 = 0$ can be rejected at a 5%-significance level.

c) State the confidence interval for the hypothesis test.