Seminar für Finanzökonometrie
Institut für Statistik
Akademiestr. 1/I
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Based on exercises from Steffen Unkel

Statistical Geophysics
Exercise Sheet 1

Exercise 1 Examining stones, 30% of the samples are tested to contain the radioactive element *Promethium*, 15% show signs of the element *Radon* and 5% contain both elements.
Now one of the stones is chosen by chance. Compute the probability that this stone

a) contains at least one of the two elements.
b) only contains *Promethium*.
c) showed signs neither of *Promethium* nor *Radon*.
d) was tested to contain exactly one of the two elements.

Exercise 2 Consider cards 2 to 10, the 3 „picture cards“ Jack (J), Queen (Q), King (K) and the Ace, one card each (we do not consider the color).
J,Q,K each count as 10 and Ace counts as eleven.
Let's assume a person draws a card from this set of cards. Consider the following events:

A) A picture card, with a male on it, is drawn.
B) A card, that is no Jack, with a value bigger than 5 is drawn.

What is the probability that this person draws

a) a picture card, with a male on it.
b) a card, that is no Jack, with a value bigger than 5.
c) a picture card, with a male on it, that is no Jack and has a value bigger than 5.
d) a picture card, with a male on it or a card that is no Jack, with a value bigger than 5.

Exercise 3 Use Kolmogorov's axioms to show that:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Exercise 4 A group of 60 geophysicians, who conduct excavations in contaminated areas, take part in a therapy (A =stationary, $\bar{A}$ =ambulatory). Moreover they take a test for contamination (B = contaminated, $\bar{B}$ = not contaminated). In doing so, 45 of the 60 persons turn out to be not contaminated and 15 persons turn out to be contaminated. 80% of the persons who are contaminated are in the stationary therapy while only 40% of the non-contaminated are in the stationary therapy.

a) Formulate the above information as probabilities.
b) One of the 60 geophysicians is chosen by chance. Calculate the probability that this person
   b1) takes part in the stationary therapy and is contaminated.
   b2) takes part in the stationary therapy and is not contaminated.
   b3) takes part in the stationary therapy.
c) Calculate $P(B|A)$.
d) How is $P(B|A)$ linked to $P(A)$ if A and B are independent?

Exercise 5 A geological institute developed a test to determine whether a stone sample is older than 5000 years. If the test suggests that the sample is older than 5000 years, we call this result a positive result, otherwise a negative result. It is known that 60% of the samples are older than 5000 years. It has been found out that

1) in 95% of the cases the test will show a positive result if the sample is indeed older than 5000 years,
2) in 97% of the cases the test will show a negative result if the sample is not older than 5000 years.

What is the probability that a sample is over 5000 years old if the test shows a positive result?

Exercise 6 Assume the following probability mass function:

x	-1	1	2
$P(X = x)$	0.1	0.4	0.5

1) Plot the cumulative probability distribution function of X.
2) Compute the expectation and standard deviation of X.

3) Determine the probability mass function of $Y = 3 + 5X$ and plot the distribution function of Y.

4) Evaluate the expectation and standard deviation of Y.