Seminar für Finanzökonometrie
Institut für Statistik
Akademiestr. 1/I
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Based on exercises from Steffen Unkel

Statistical Geophysics
Exercise Sheet 2

Exercise 1 A geophysicist declares that only by touching a stone he is able to discriminate whether it is made of Granit or not. He agrees to do an experiment. A person puts ten stones in a box. Which stone is made of Granit and which is not is randomized by the person and the geophysician is kept in the dark about which stone is made of Granit and which is not. Suppose the geophysician is just guessing for each stone with probability 1/2. What is the probability that he has at least eight successes?

Exercise 2 The discrete uniform distribution is a probability distribution whereby a finite number of values $x \in \{a, a+1, a+2, ..., b-2, b-1, b\}$ are equally likely to be observed; every one of n values has equal probability 1/n. Find the expectation and variance of this distribution.

Exercise 3 Assume that X is a continuous random variable that is uniformly distributed on the interval $[a, b]$, denoted by $X \sim \mathcal{U}(a, b)$. The pdf of the continuous uniform distribution is

$$f_X(x) = \begin{cases} \text{(b-a)}^{-1} & \text{for } a \leq x \leq b \\ 0 & \text{for x<a or } x > b \end{cases}.$$

Find the expectation and variance of the continuous uniform distribution on $[a, b]$.

Exercise 4 Let X be a continuous random variable with density $f_X(x)$. Assume that $X \sim \mathcal{U}(0, 1)$. Consider the transformation $Y = g(X)$ with $g(X) = -\ln(X)$. Derive the density $f_Y(y)$. How is Y distributed?

Exercise 5

a) Suppose that $X \sim \mathcal{N}(0,1)$. Show that $f_X(x)$ is a density, that is, show that $\int_{-\infty}^{\infty} f_X(x)dx = 1$. You may use the fact that for $a > 0$

$$\int_{-\infty}^{\infty} \exp(-a^2x^2)dx = \frac{\sqrt{\pi}}{a}.$$

b) Assume that $X \sim \mathcal{N}(0,1)$ and $Y \sim \mathcal{N}(0,1)$ with X and Y being independent. Consider $Z = X + Y$. Show that $Z \sim \mathcal{N}(0,2)$.

Exercise 6 Assume that X is an exponentially distributed random variable with parameter λ.

a) Prove the memoryless property of the exponential distribution, that is, show that X obeys the relation $P(X > s + x|X > s) = P(X > x)$ for all $s, x \geq 0$.

b) Show that $\mathrm{Var}(X) = \frac{1}{\lambda^2}$.

Exercise 7 Assume that $X \sim P_\lambda$ (Poisson distributed).
Calculate $\mathrm{Var}(X)$.