Midterm homework set

Homework should be handed in by the start of the lecture (13:00) on **May 30th, 2016**. The notebook exercise should be emailed to Stefanie Donner (donner@geophysik.uni-muenchen.de). The answers to Exercises 2 and 3 may either be emailed as well, or handed in on paper.
If you hand in handwritten answers, please make sure the writing is easily readable by others. For each step of the exercises, please show your work, and describe the step in words.

Exercise 1 (*Data centers and Observables*)

Exercise included in notebook "`MidTerm_notebook.ipynb`".
Please save a copy of the notebook with your answer. The code you write should be commented in such a way that it is clear what you are doing and why.
Make sure to include your name somewhere in the notebook and/or the notebook filename!

Exercise 2 (*Fourierseries, Fouriercoefficients, Fouriertransformation*)

a) What is the meaning of Fourier coefficients a_0, a_n, b_n?

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1} a_n \cos\left(\frac{\pi n}{L}x\right) + b_n \sin\left(\frac{\pi n}{L}x\right) \tag{1}$$

$$a_0 = \frac{2}{L}\int_0^L f(x)dx \tag{2}$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx \qquad (n \geq 1) \tag{3}$$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx \qquad (n \geq 1) \tag{4}$$

b) Using equation 4, calculate the coefficients for $n = 1, 2$ with $L = 1$ for the odd function $f(x) = x$ in the interval [-L,L]. Use the fact that:

$$\int x \sin ax = \frac{\sin ax}{a^2} - \frac{x\cos ax}{a} \tag{5}$$

Note that it is possible to find the general expression for all n.

c) Plot the original function $f(x)$ along with the approximation (for $N = 2$) in the same figure, on interval [0,1]. Use that:

$$f(x) \approx \sum_{n=1}^{N} b_n \sin\left(2\pi x\frac{n}{2L}\right) \tag{6}$$

d) What will happen for higher values of N?

Exercise 3 (*Fourier transform example*)

Calculate the Fourier transform of a Gaussian:

$$f(x) = e^{-x^2/a^2} \tag{7}$$