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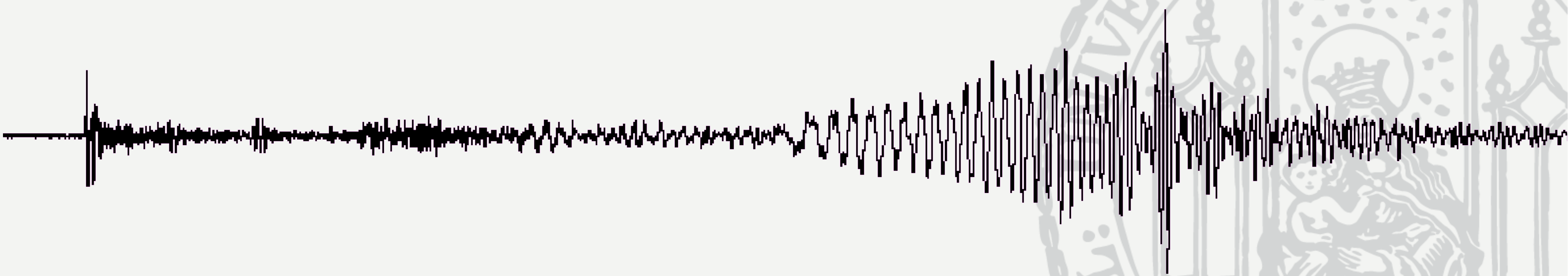
UNIVERSITÄT
MÜNCHEN

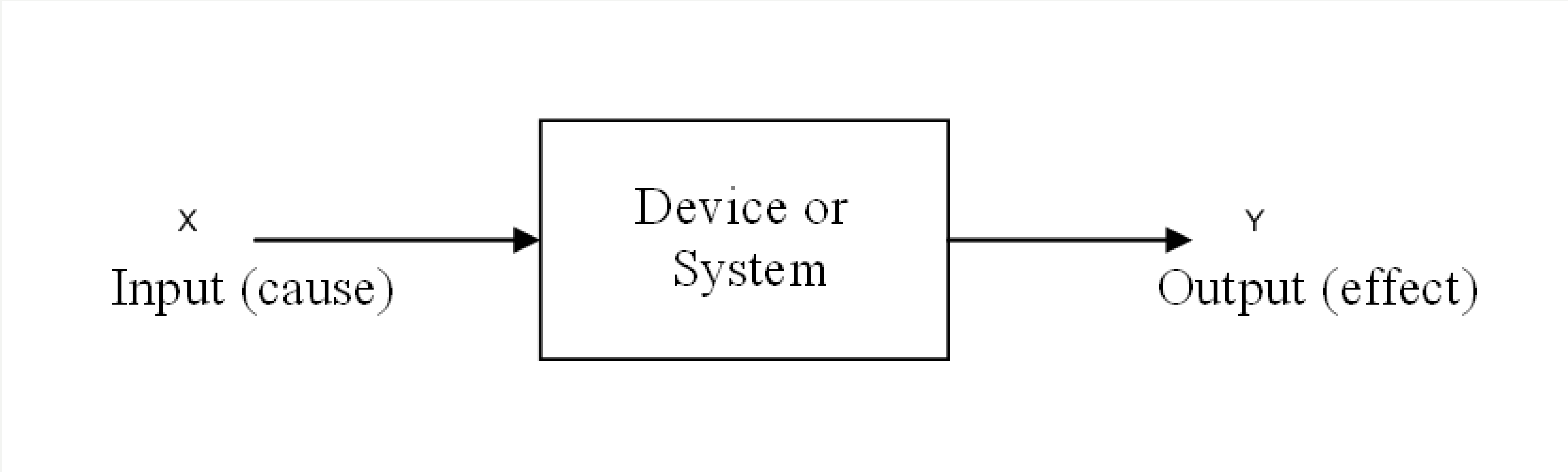
Céline Hadziioannou

Geophysical Data Analysis

L06 – Convolution

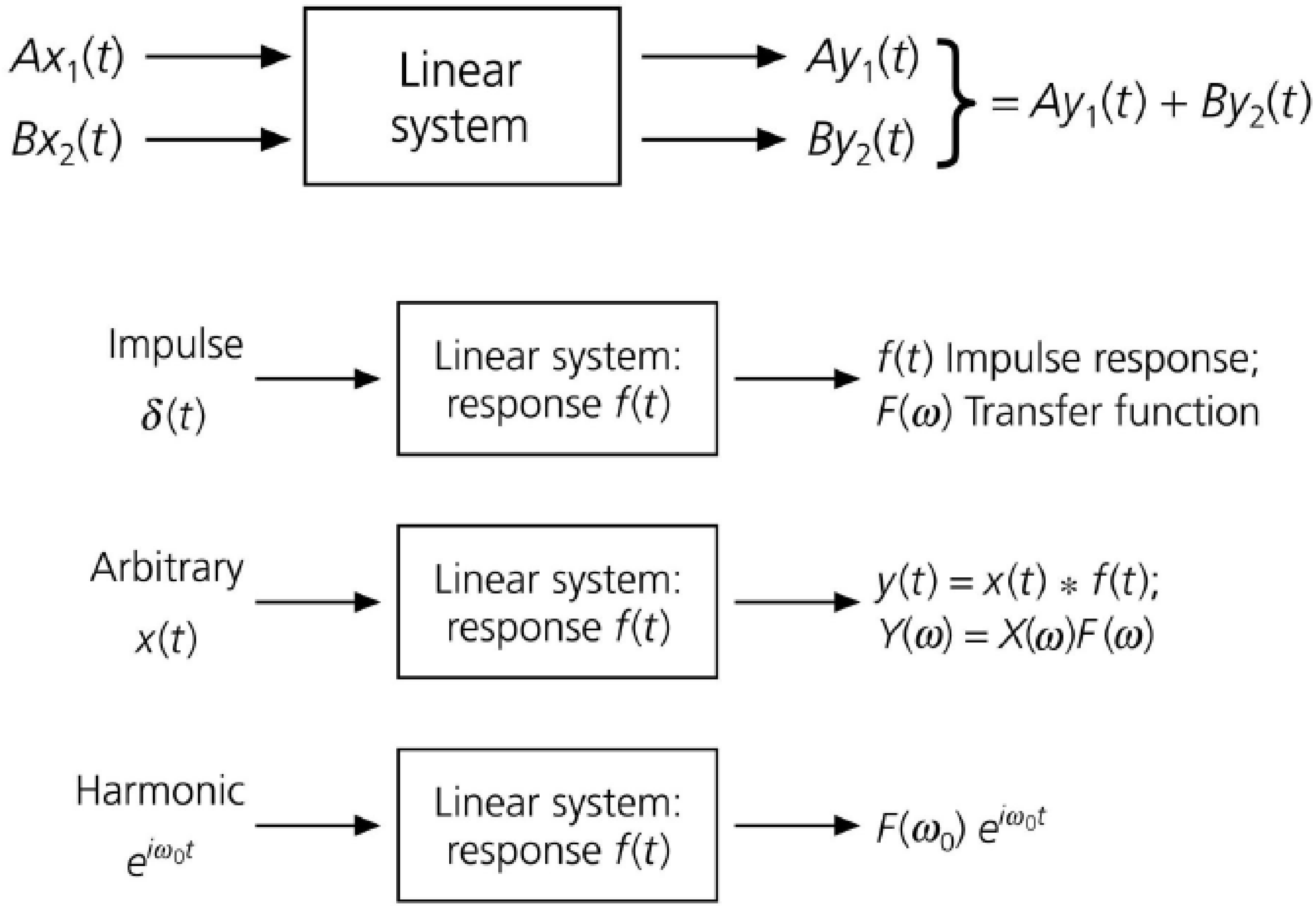


Linear systems



Linear systems

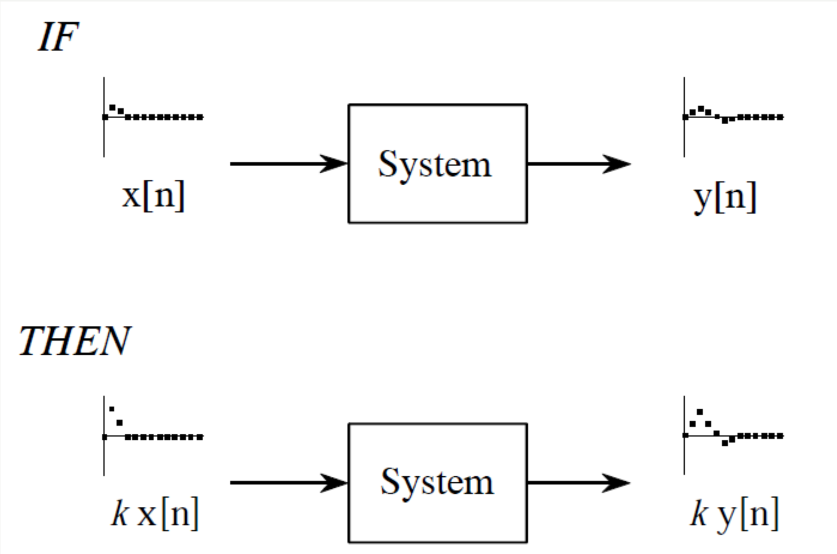
Figure 6.3-1: Definition of a linear system.



Linear systems

Properties:

Homogeneity: change in input signal amplitude → change in output amplitude



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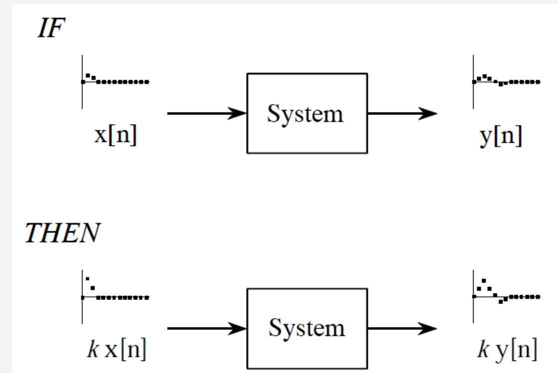
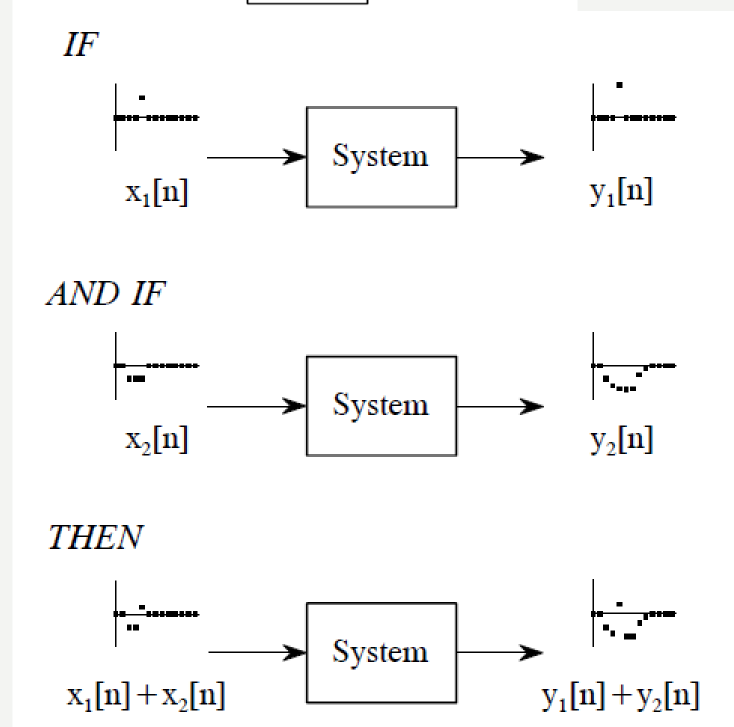


Additivity: signals added at the input produce signals that are added at the output.



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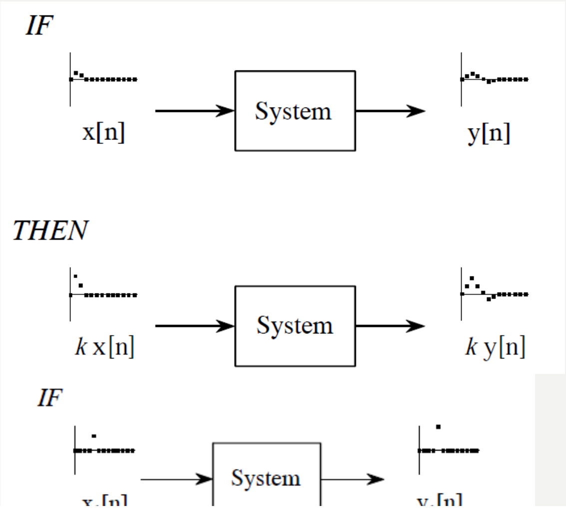
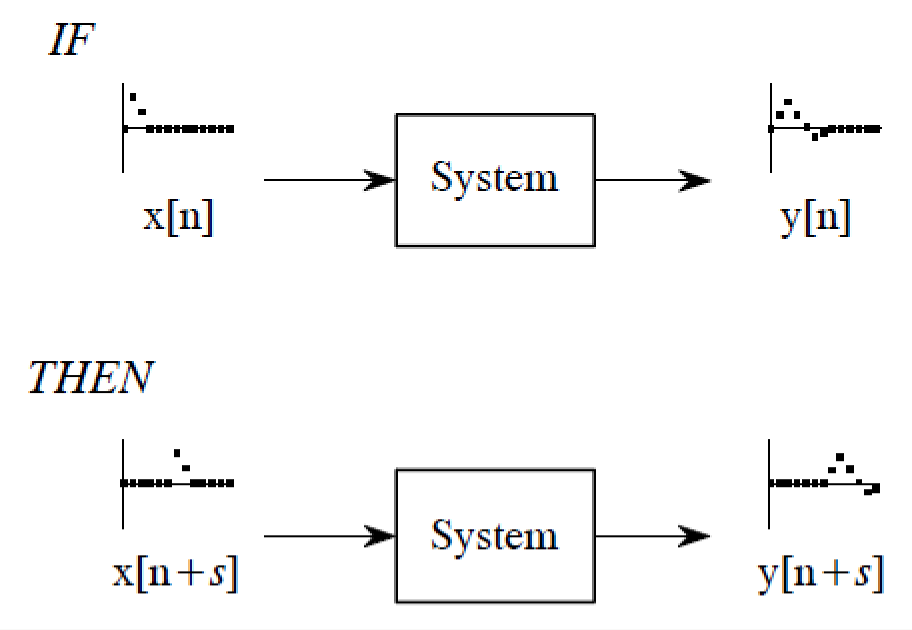


Additivity: signals added at the input produce signals that are added at the output.

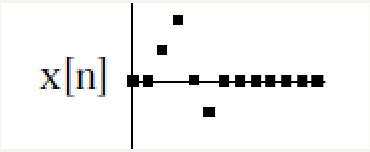
Shift invariance: shift in the input signal will result in nothing more than an identical shift in the output signal



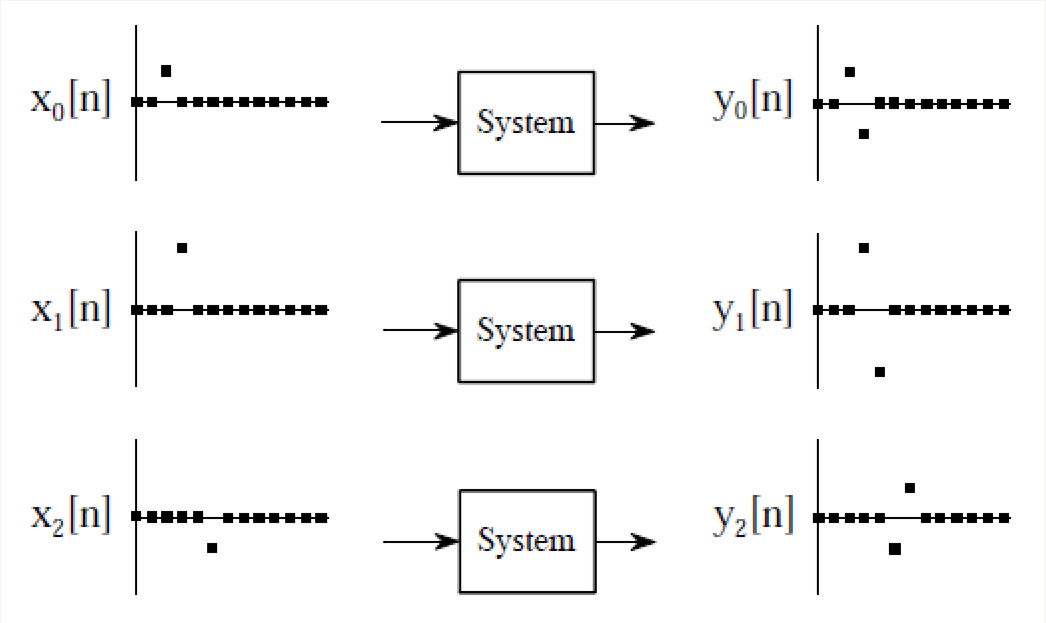
Linear systems

Superpostion

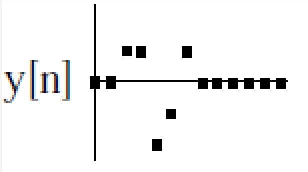
input



Decomposition



output



Example: impulse decompostion

Convolution theorem

*The output of a linear system is the **convolution** of the input and the impulse response*

Impulse response: the signal produced by a system when the input is a delta function

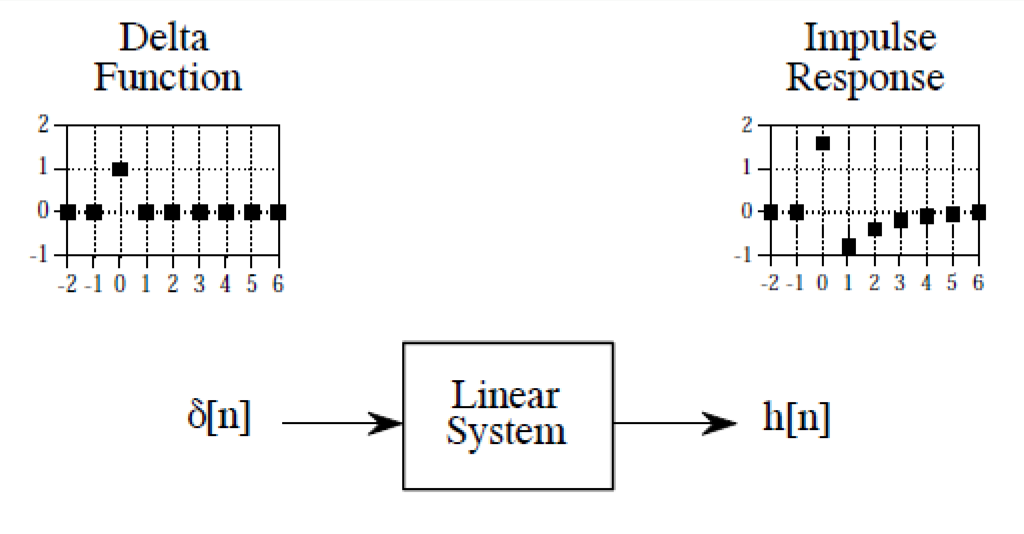


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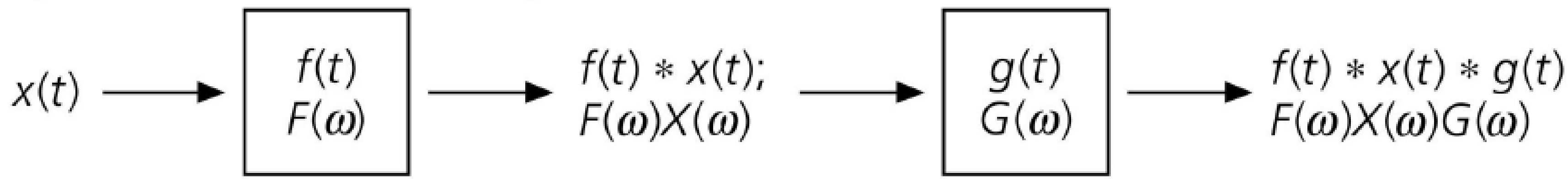
Figure 6.3-4: Two linear systems in succession.



A convolution in the time domain corresponds to a multiplication in the frequency domain.

...And vice versa ...

Convolution (Faltung)

*The convolution operation is at the **heart of linear systems**.*

$$y(t) = h(t) * x(t) \equiv \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

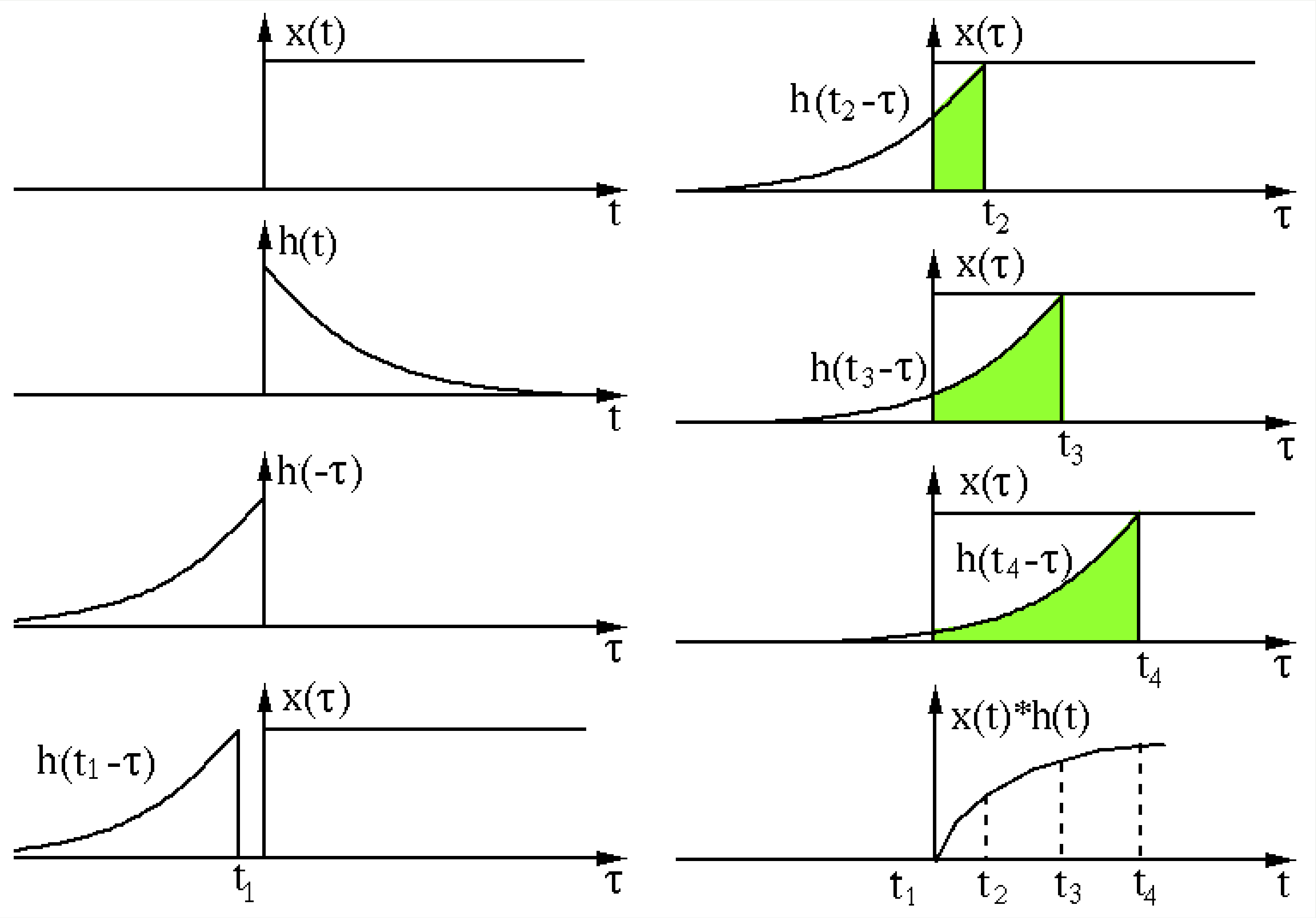
$$= \int_{-\infty}^{\infty} x(t-\tau)\, h(\tau)\, d\tau = x(t) * h(t)$$

It is the weighted mean of x(t) with h(t) as the weighting.

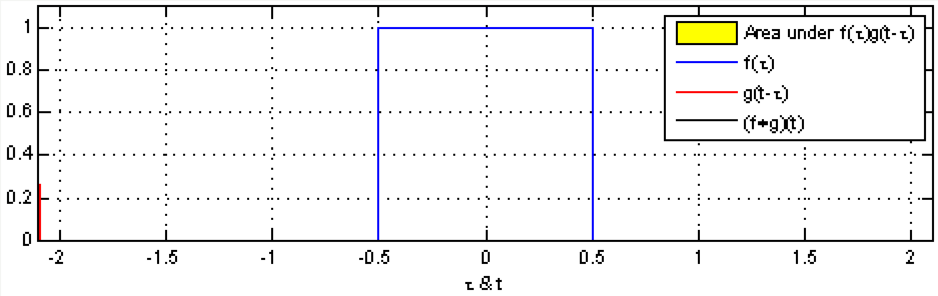
⇨ *superposition of x(t) with a mirrored and shifted version of h(t)*

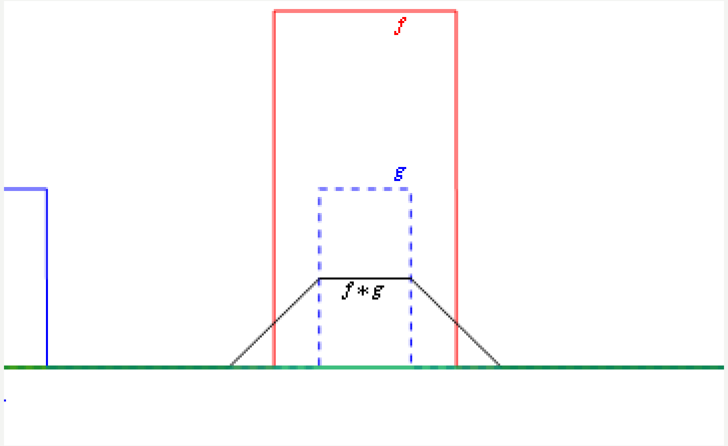
⇨ *flip, shift, multiply, and add*

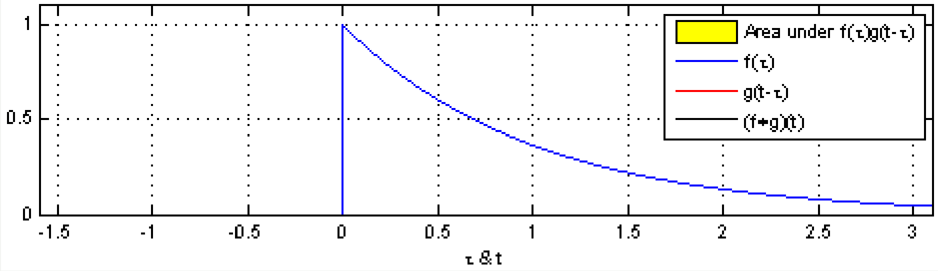
Convolution - Example

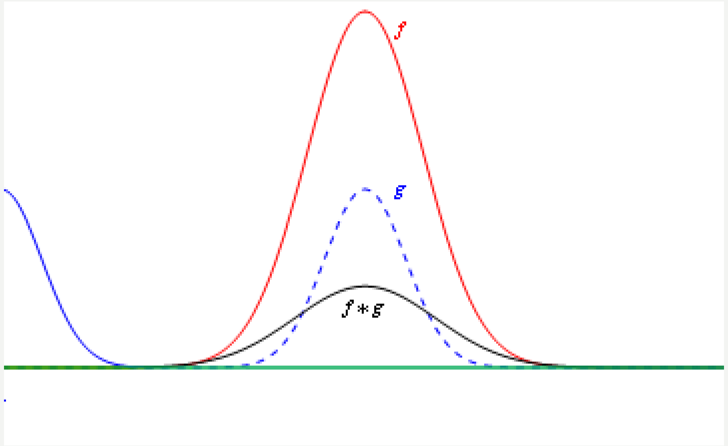


Convolution - Examples









Discrete convolution

$$(x * h)[k] \equiv \sum_{i=0}^{m} x[i]\, h[k-i]$$

$x_i \qquad i = 0, 1, 2, \ldots, m$

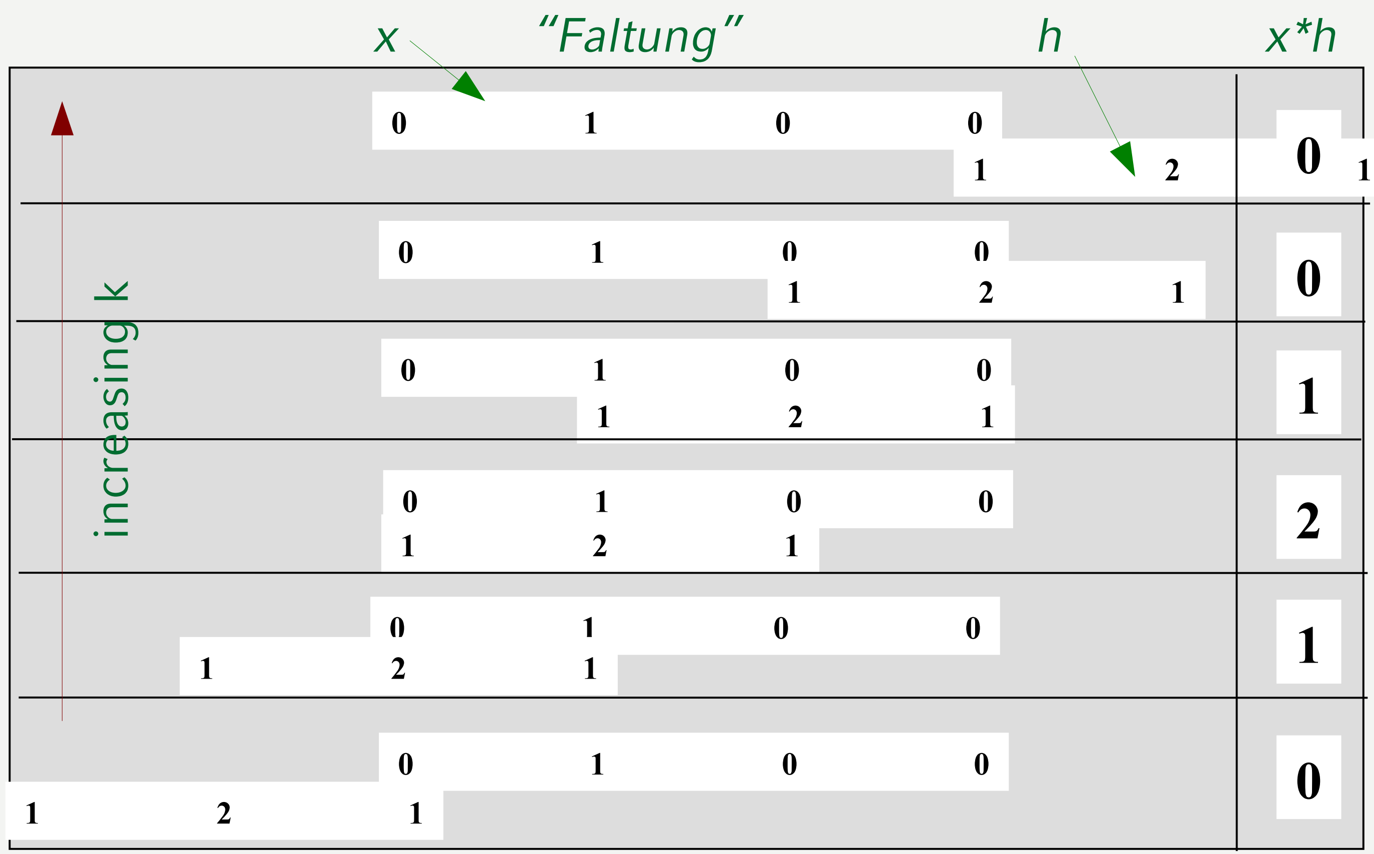
Length M

$h_j \qquad j = 1, 2, \ldots, n$

Length N

$k = 0, 1, 2, \ldots, m + n$ *-1*

Length: M+N-1



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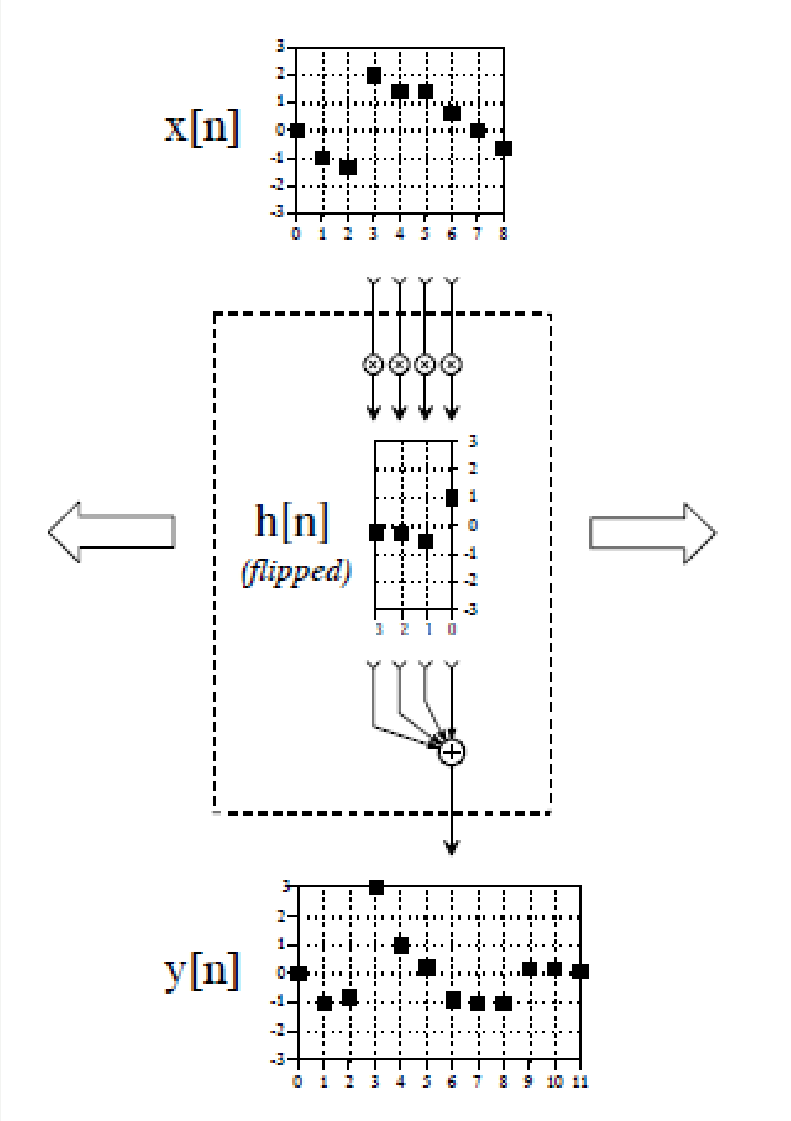


Exercise

Calculate the convolution of the vectors

x = (0 1 2 3 0)

and

h = (0 0 1 1)

Remember: flip, shift, multiply, and add

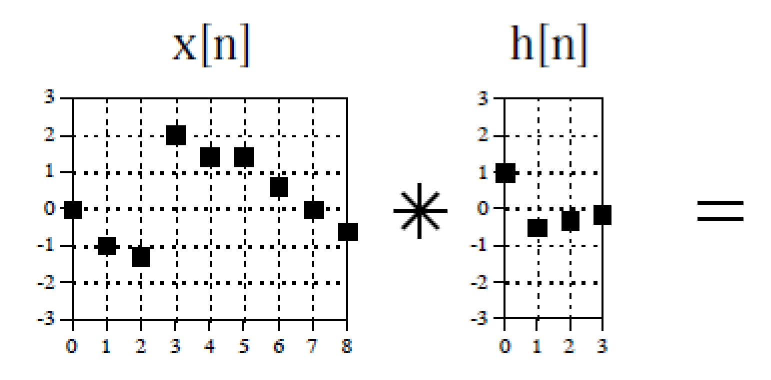
$$(x * h)[k] \equiv \sum_{i=0}^{m} x[i]\, h[k-i]$$

Convolution - Properties

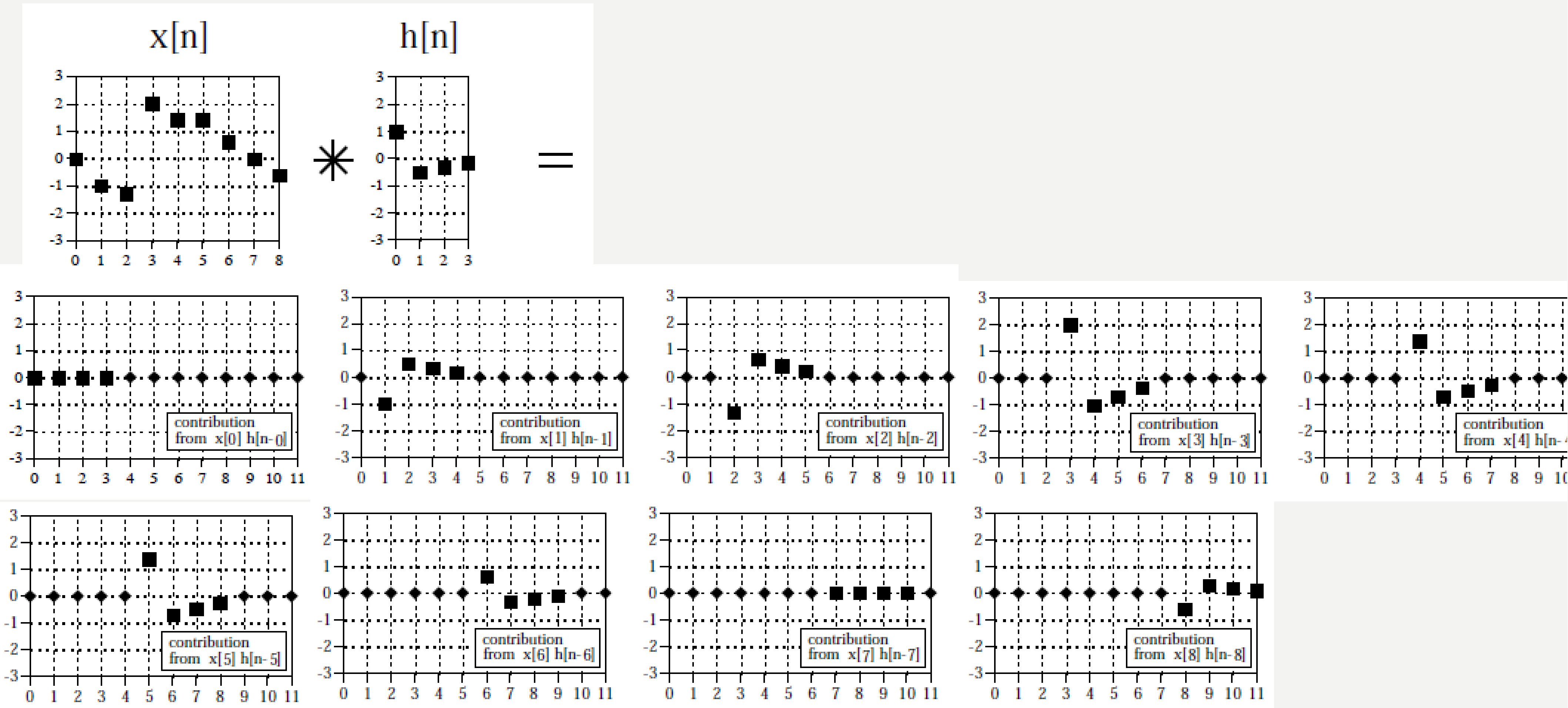
- *Commutative* $$x(t) * h(t) = h(t) * x(t)$$
- *Distributive* $$x(t) * [h(t) + g(t)] = [x(t) * h(t)] + [x(t) * g(t)]$$
- *Associative (with scalar)* $$x(t) * [h(t) * g(t)] = [x(t) * h(t)] * g(t)$$ $$a\,[x(t) * h(t)] = [a\,x(t)] * h(t)$$
- *Multiplicative identity* $$x(t) * \partial(t) = x(t)$$
-
- ***Convolution theorem*** $$\mathbf{F}\{\mathbf{x}(\mathbf{t}) * \mathbf{h}(\mathbf{t})\} = \mathbf{F}\{\mathbf{x}(\mathbf{t})\} \cdot \mathbf{F}\{\mathbf{h}(\mathbf{t})\}$$

A convolution in the time domain corresponds to a multiplication in the frequency domain.

Convolution - Examples

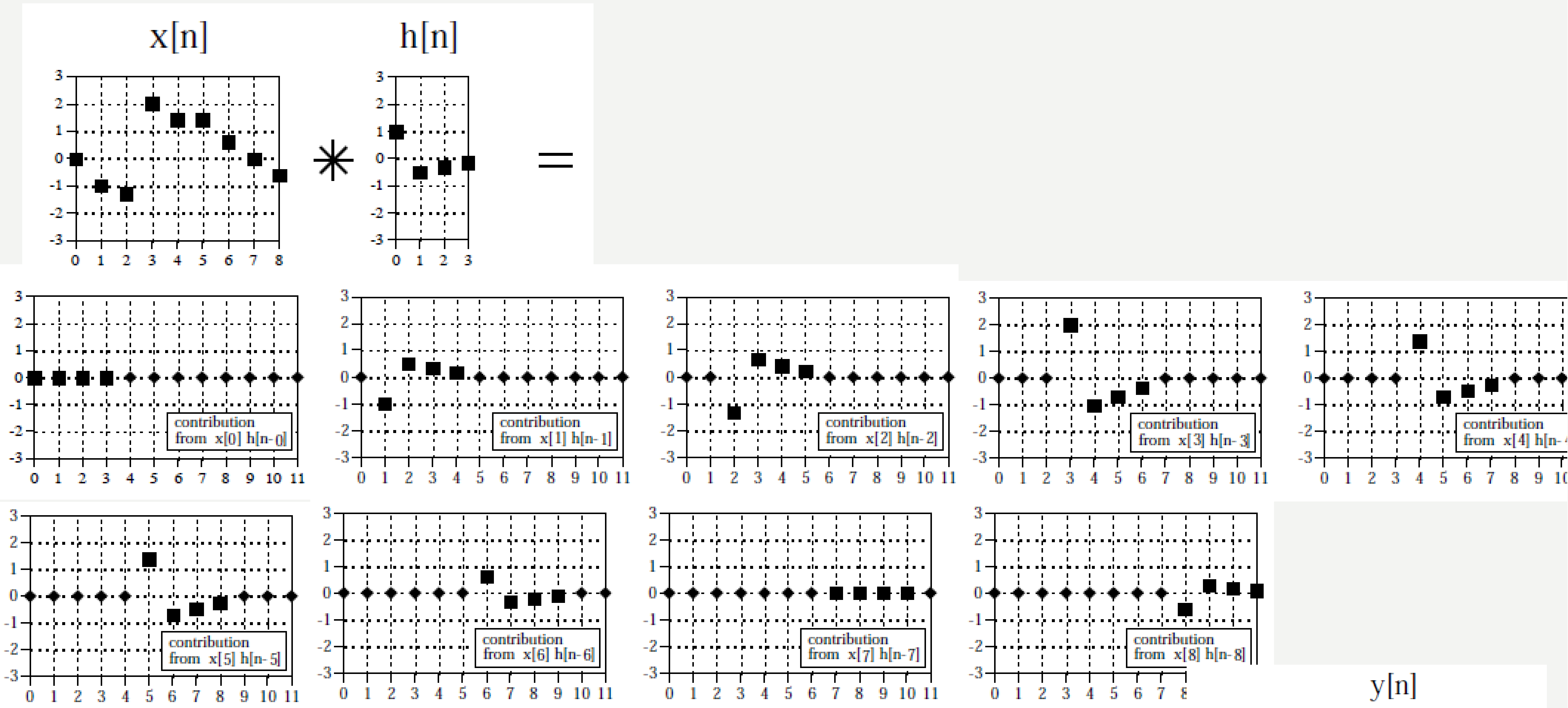


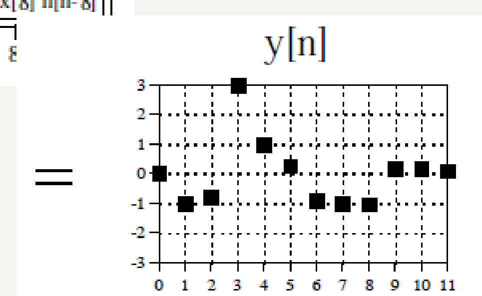
Convolution - Examples



Decompose into series of delta pulses, convolve with impulse response … and **add** to obtain output

Convolution - Examples

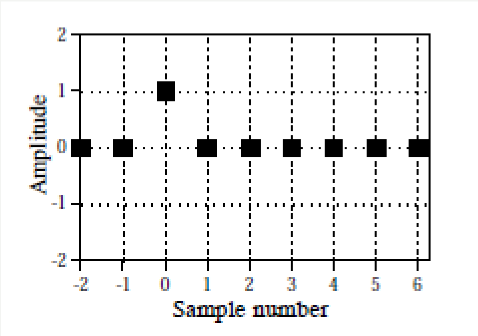




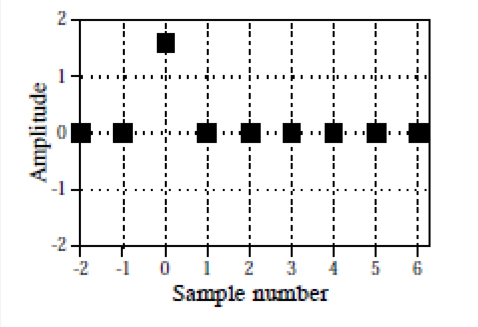
Decompose into series of delta pulses, convolve with impulse response … and **add** to obtain output

Example impulse responses

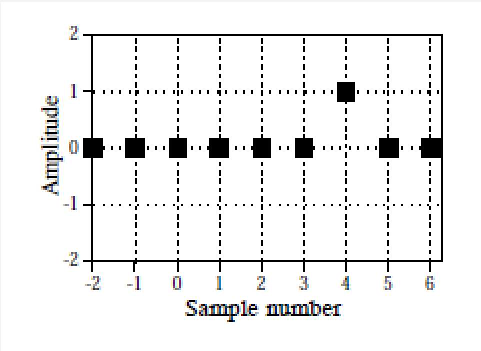
Identity



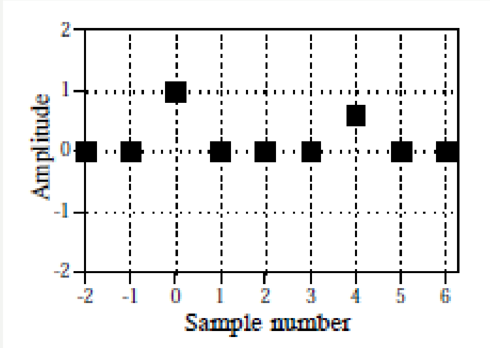
Amplification



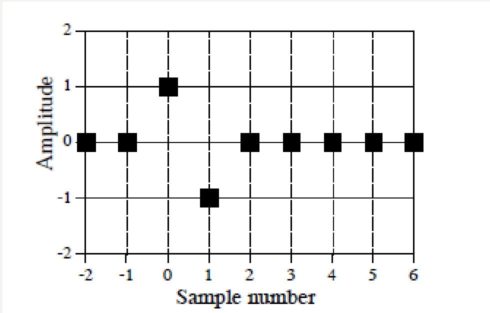
Delay/time shift



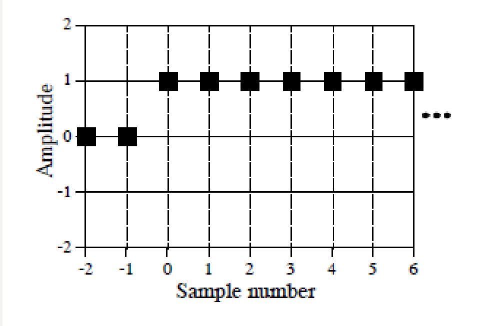
Echo



First difference

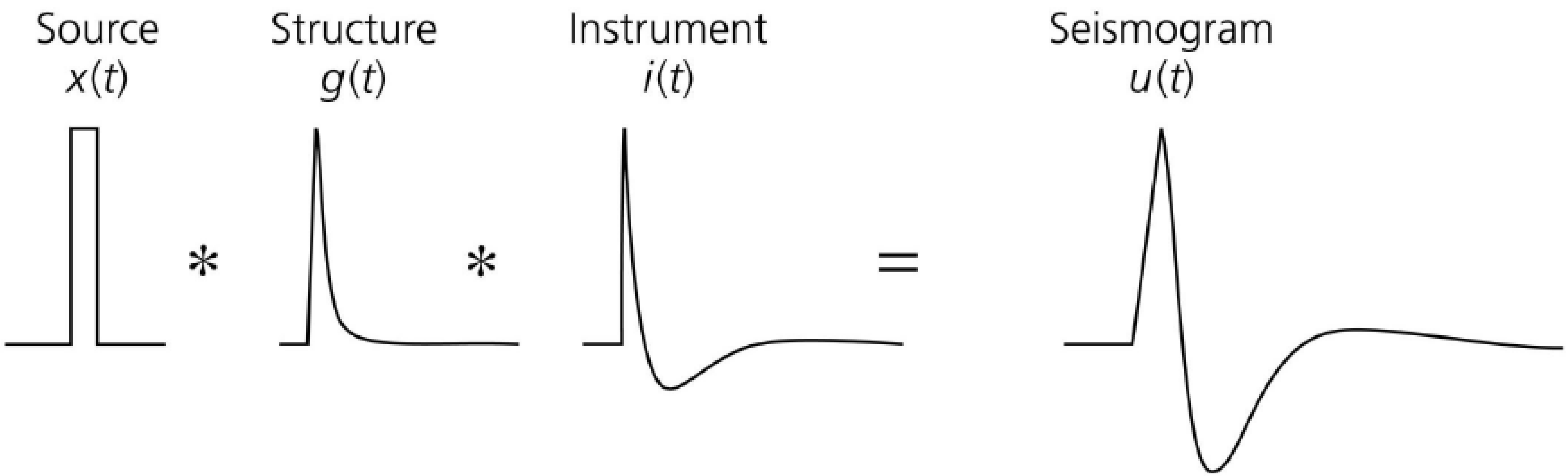


Running sum



Example: seismogram

Figure 6.3-5: Seismogram as the convolution of the source, structure, and instrument signals.



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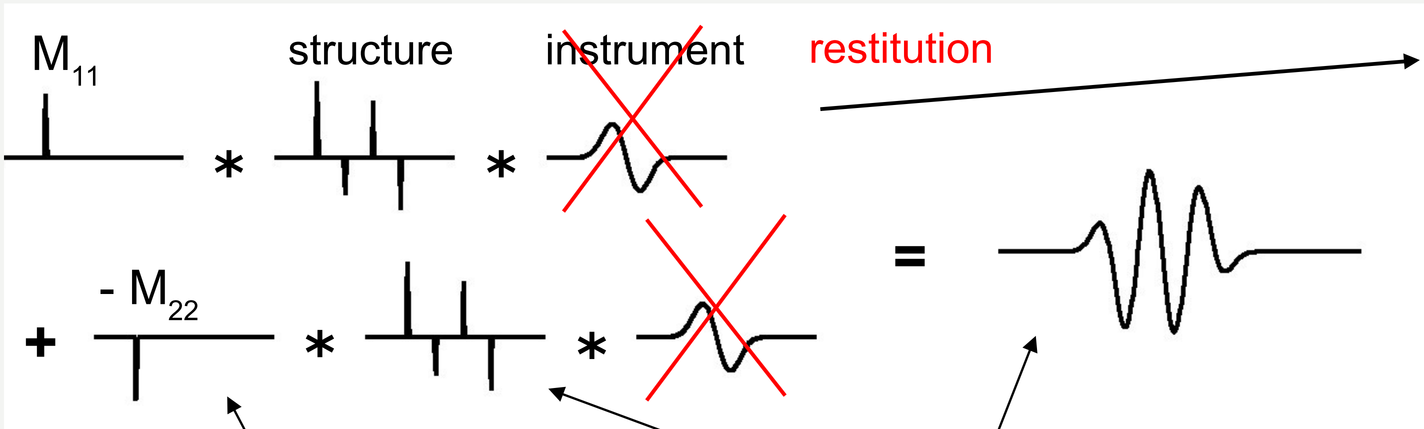
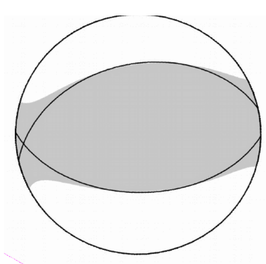
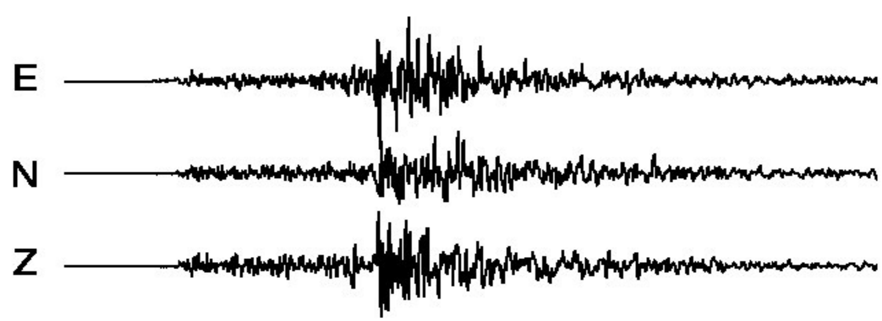
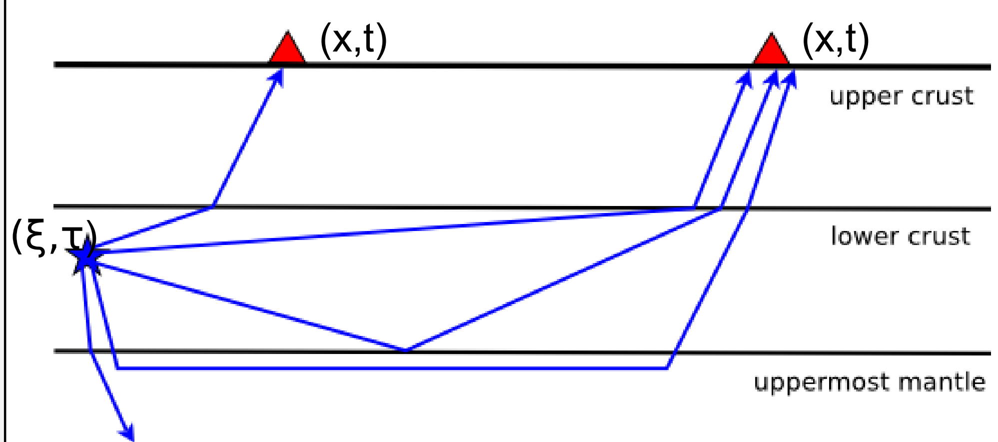



Courtesy of Daniel Vollmer



- mechanism (strike, dip, rake)
- centroid depth
- seismic moment (magnitude)

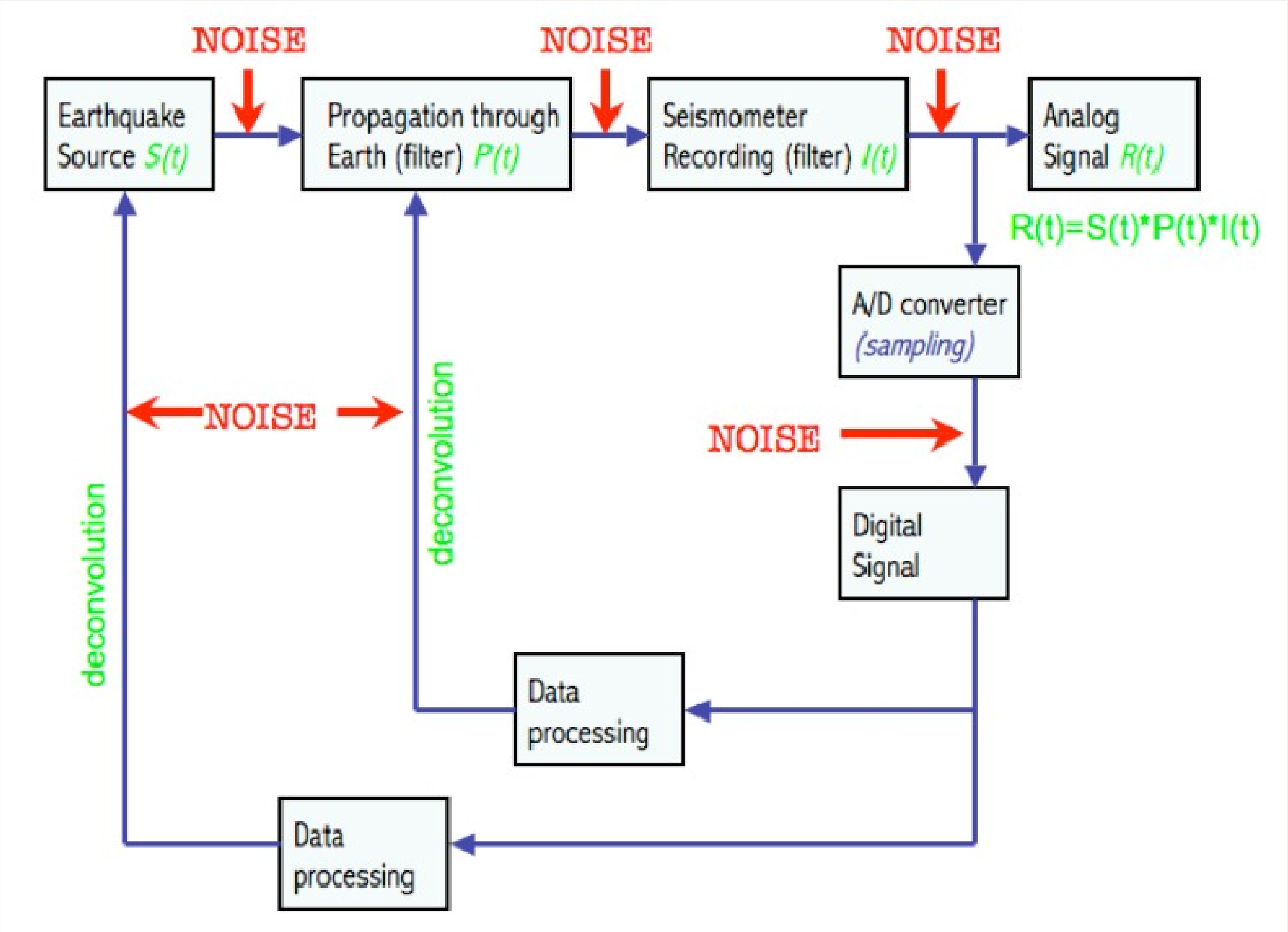




modeled by Green's functions: response of the medium to an impulsive excitation

$$\mathbf{G} = G(x,t;\xi,\tau)$$

The Earth as a filter



Deconvolution

… is the reverse operation to convolution.

*It is the **most important application** in seismic data processing, e.g. for removing the instrument response of a seismometer.*

How would you do it?

...remember the convolution theorem

Deconvolution

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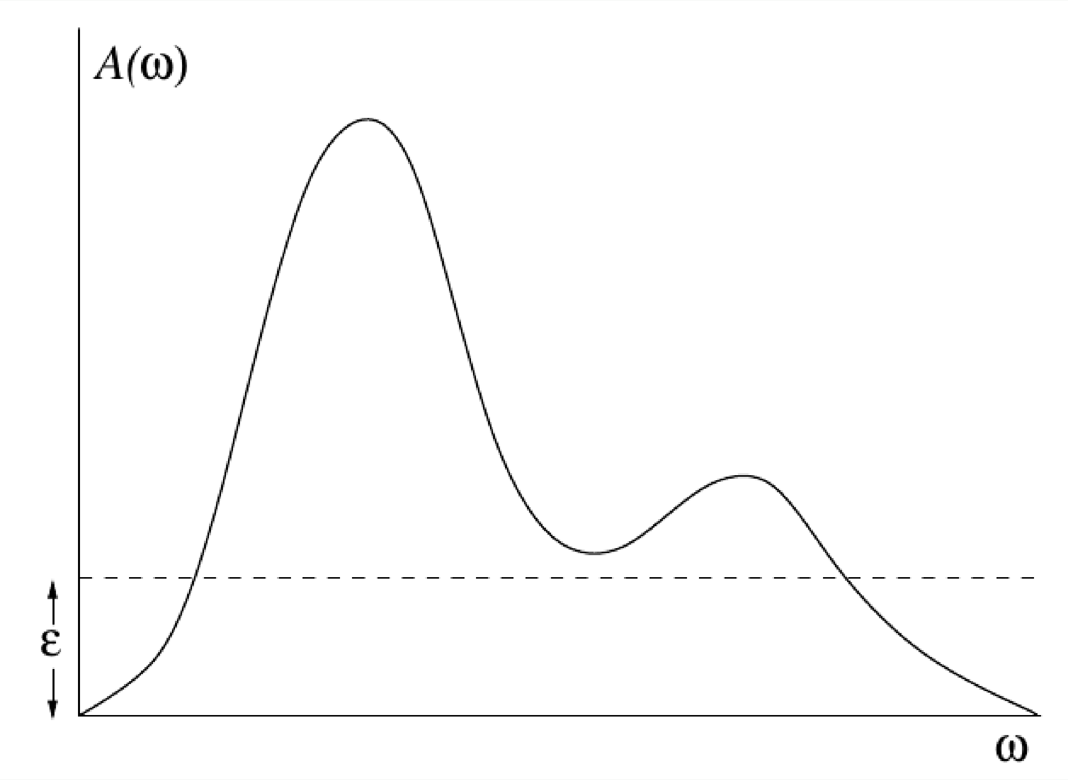

$$B(\omega) = \frac{C(\omega)}{A(\omega)}$$

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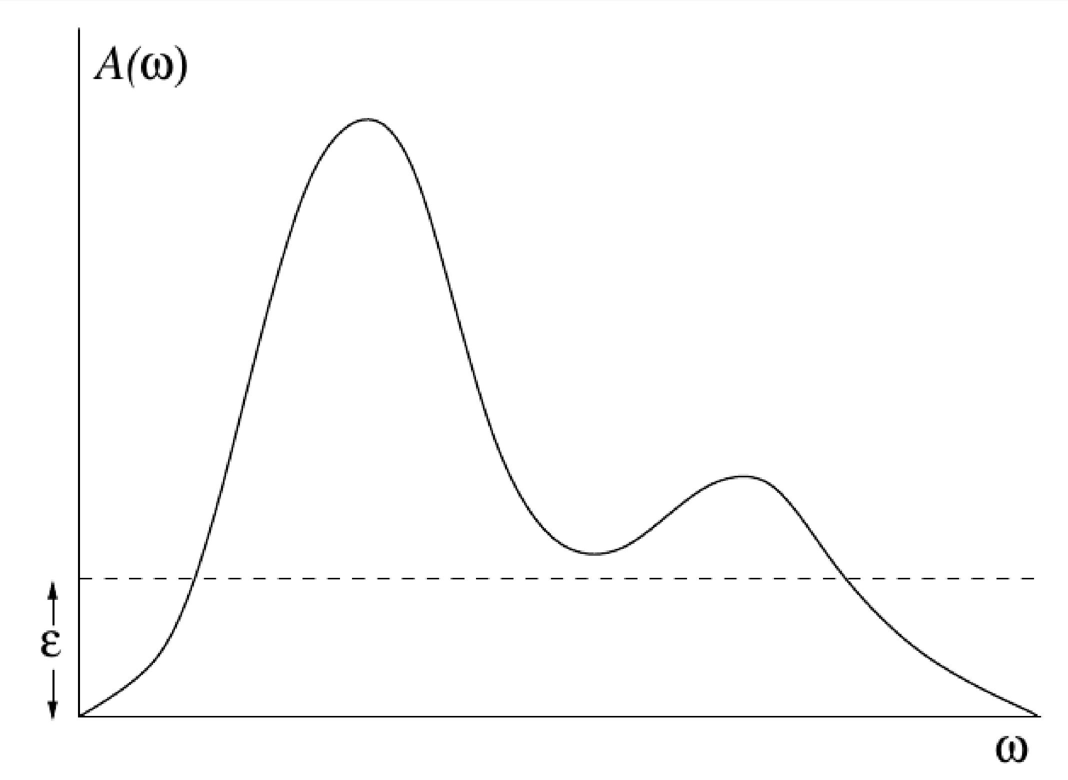

$$B(\omega) = \frac{C(\omega)}{A(\omega)}$$

Major problem:

A(ω) is zero or close to zero in the presence of noise.

Possible fix:

"waterlevel" method – basically adding white noise

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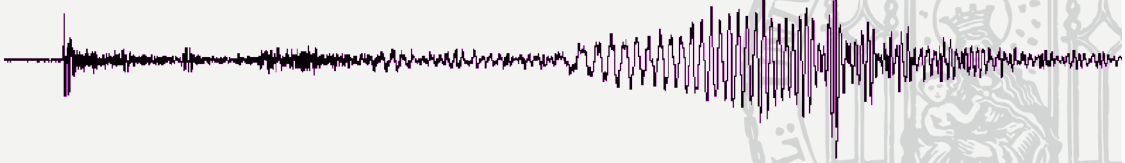
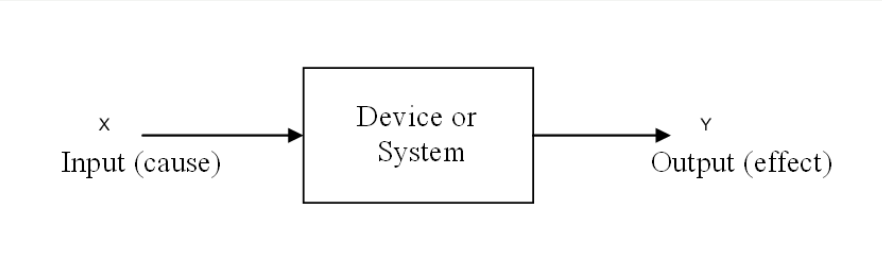
Geophysical Data Analysis

L06 – Convolution



Linear systems



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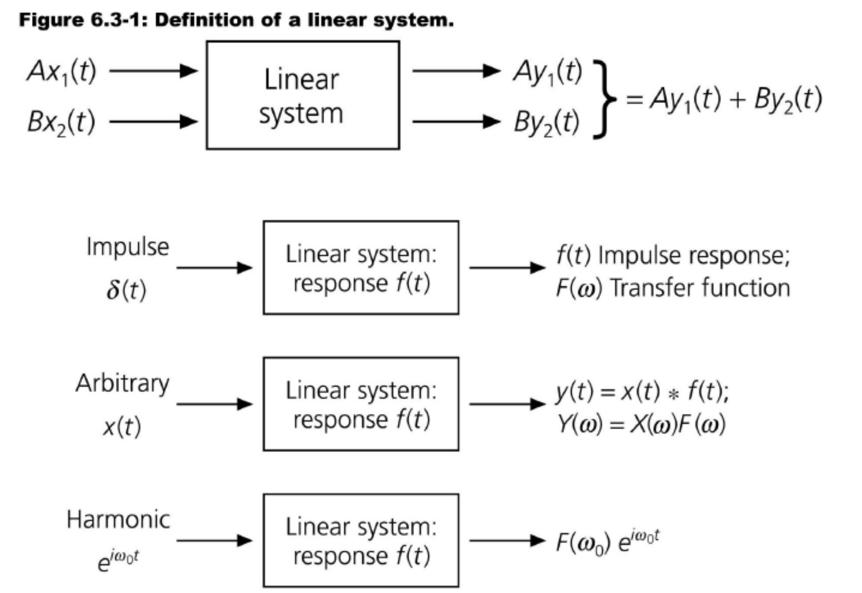

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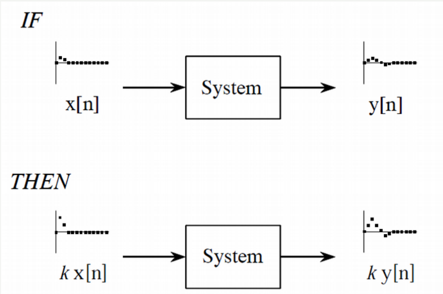

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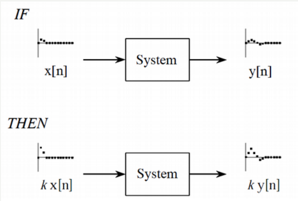

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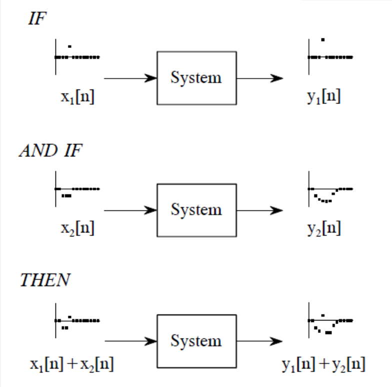

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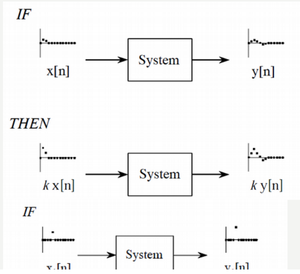

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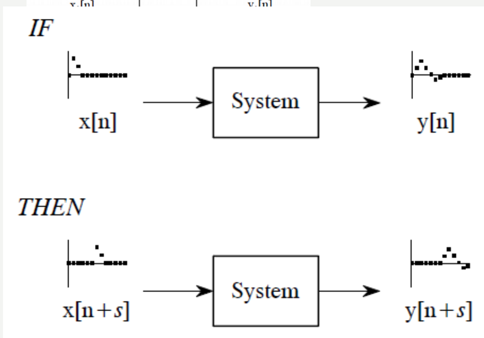
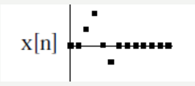


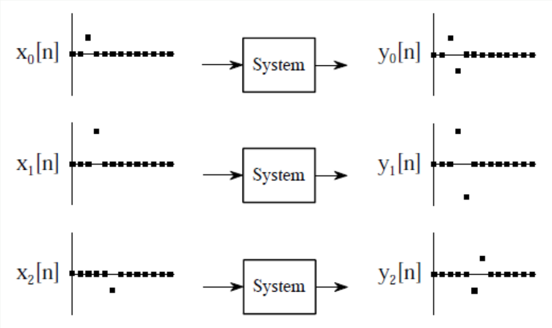
Linear systems

Superpostion

input

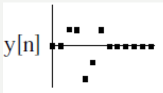


Decomposition



Example: impulse decompostion

output



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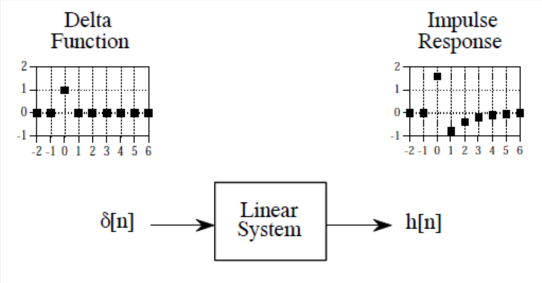

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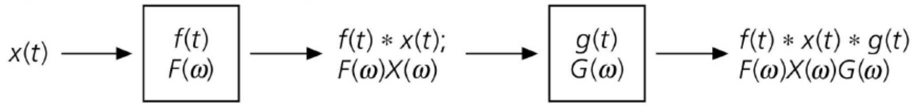

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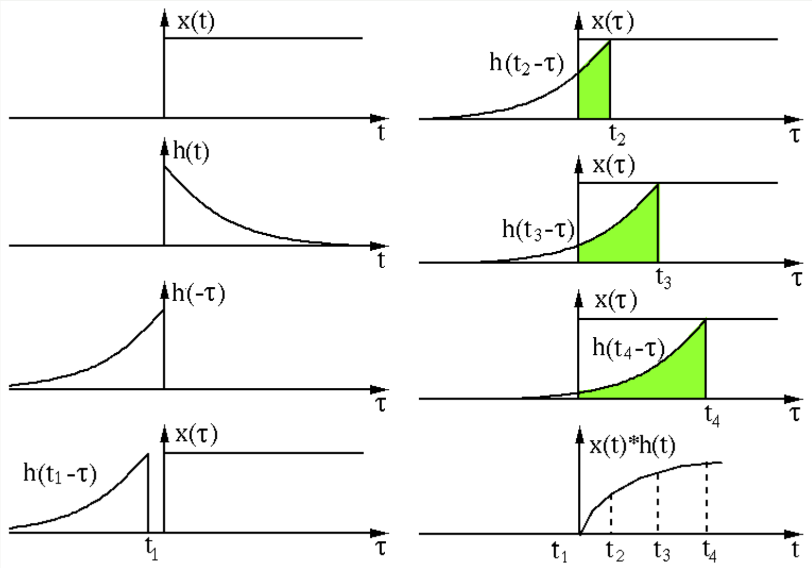
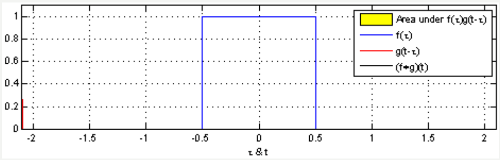
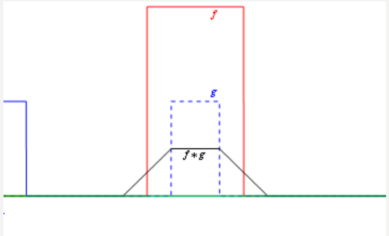
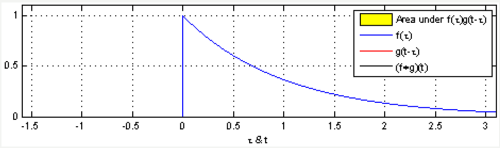
Convolution - Example

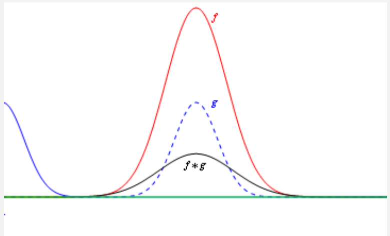


Convolution - Examples









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Length M

$h_j \qquad j = 1, 2, \ldots, n$
Length N

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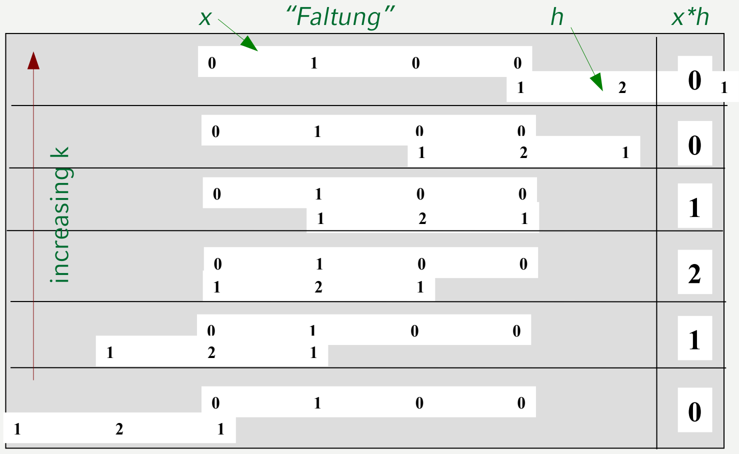
Length: M+N-1



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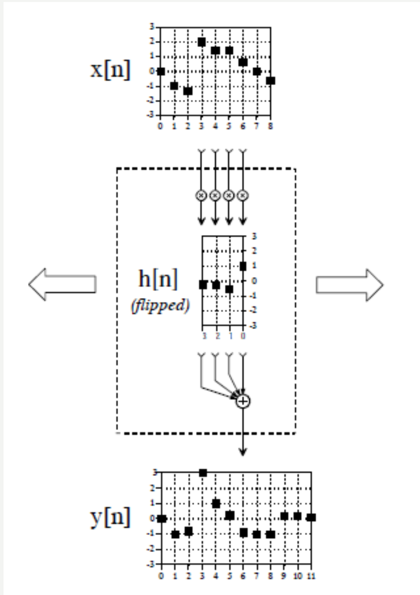
$x_i \qquad i = 0, 1, 2, \ldots, m$

$h_j \qquad j = 1, 2, \ldots, n$

$k = 0, 1, 2, \ldots, m + n$ -1



Exercise

Calculate the convolution of the vectors

x = (0 1 2 3 0)

and

h = (0 0 1 1)

Remember: flip, shift, multiply, and add

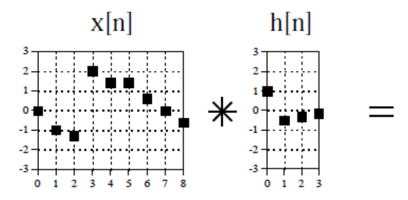
$$(x * h)[k] \equiv \sum_{i=0}^{m} x[i]\, h[k-i]$$

Convolution - Properties

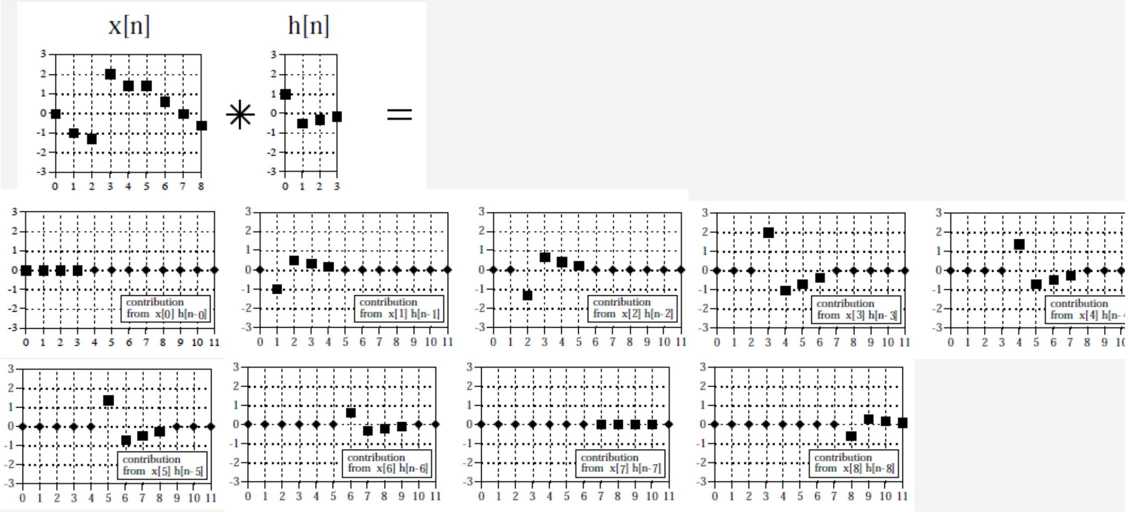
- *Commutative* $$x(t) * h(t) = h(t) * x(t)$$
- *Distributive* $$x(t) * [h(t) + g(t)] = [x(t) * h(t)] + [x(t) * g(t)]$$
- *Associative (with scalar)* $$x(t) * [h(t) * g(t)] = [x(t) * h(t)] * g(t)$$ $$a\,[x(t) * h(t)] = [a\,x(t)] * h(t)$$
- *Multiplicative identity* $$x(t) * \partial(t) = x(t)$$
- ***Convolution theorem*** $$\mathbf{F\{x(t) * h(t)\} = F\{x(t)\} \cdot F\{h(t)\}}$$

A convolution in the time domain corresponds to a multiplication in the frequency domain.

Convolution - Examples

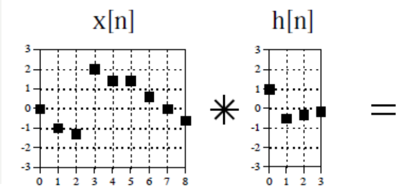


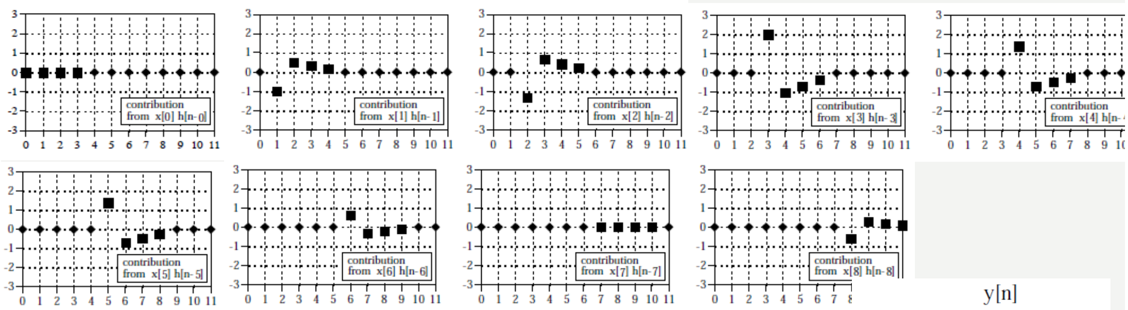
Convolution - Examples



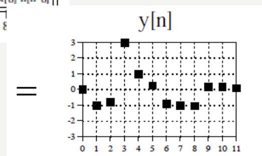
Decompose into series of delta pulses, convolve with impulse response … and **add** to obtain output

Convolution - Examples



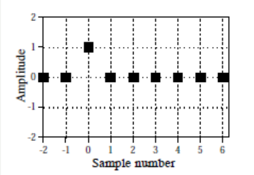


Decompose into series of delta pulses, convolve with impulse response … and **add** to obtain output

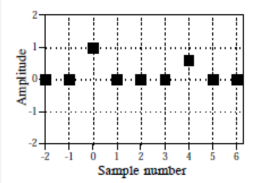


Example impulse responses

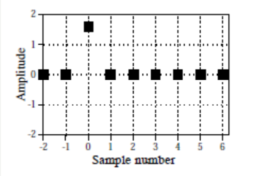
Identity



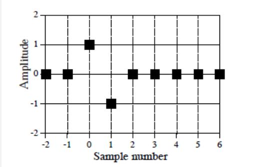
Echo



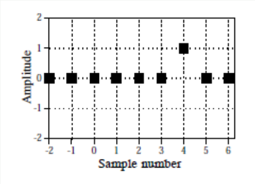
Amplification



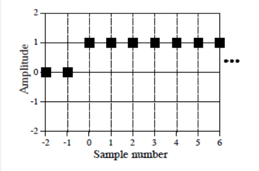
First difference



Delay/time shift

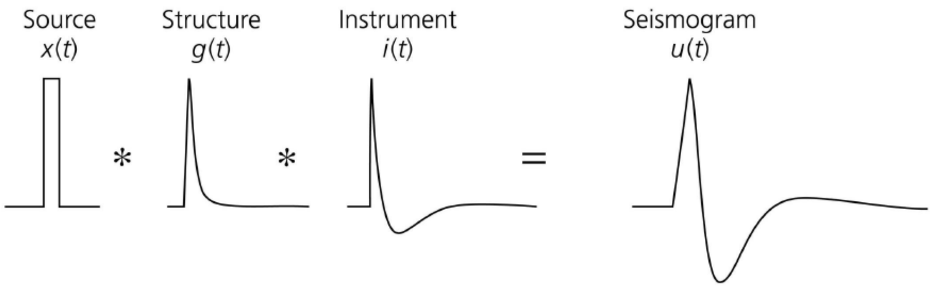


Running sum

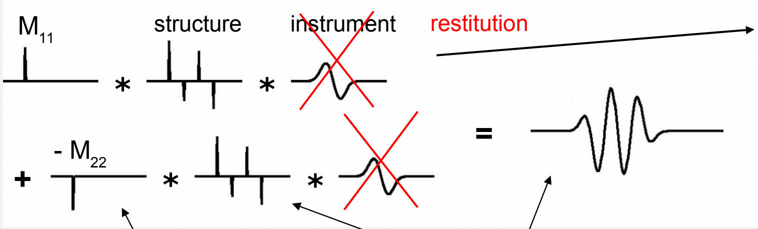


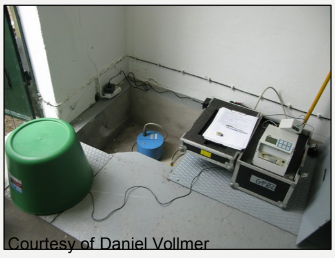
Example: seismogram

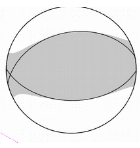
Figure 6.3-5: Seismogram as the convolution of the source, structure, and instrument signals.



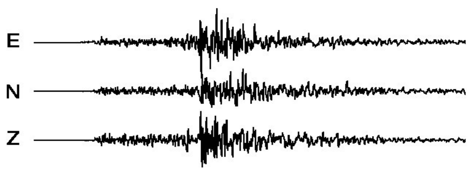
Example: seismogram

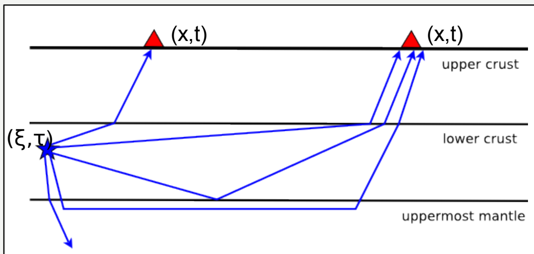






- mechanism (strike, dip, rake)
- centroid depth
- seismic moment (magnitude)

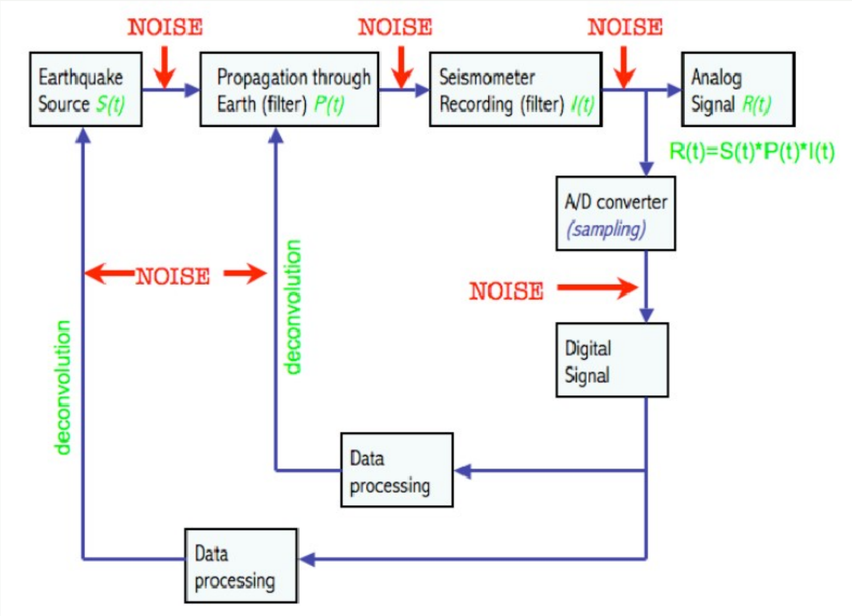




modeled by Green's functions: response of the medium to an impulsive excitation

G = G(x,t;ξ,τ)

The Earth as a filter



Deconvolution

… is the reverse operation to convolution.

*It is the **most important application** in seismic data processing, e.g. for removing the instrument response of a seismometer.*

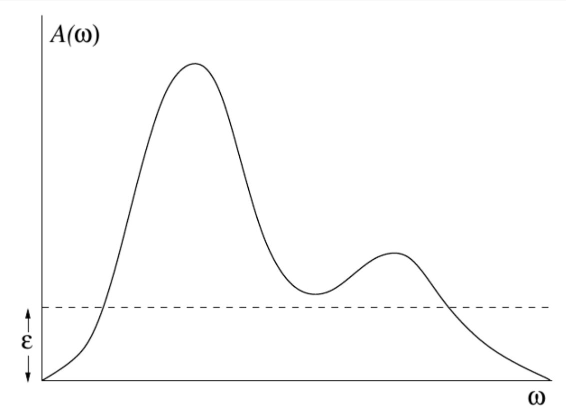
How would you do it?

...remember the convolution theorem

Deconvolution

… is the reverse operation to convolution.

*It is the **most important application** in seismic data processing, e.g. for removing the instrument response of a seismometer.*

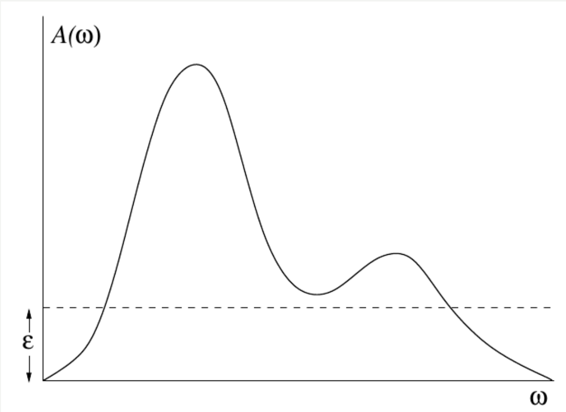


$$B(\omega) = \frac{C(\omega)}{A(\omega)}$$

Deconvolution

… is the reverse operation to convolution.

It is the ***most important application*** *in seismic data processing, e.g. for removing the instrument response of a seismometer.*



$$B(\omega) = \frac{C(\omega)}{A(\omega)}$$

Major problem:

A(ω) is zero or close to zero in the presence of noise.

Possible fix:

"waterlevel" method – basically adding white noise