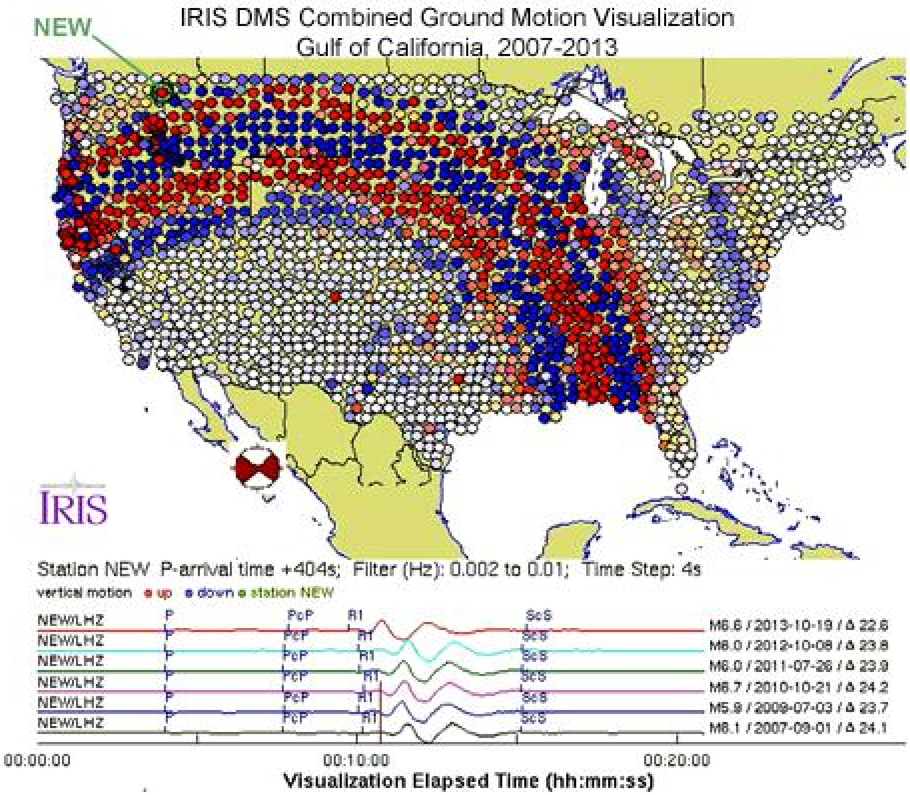
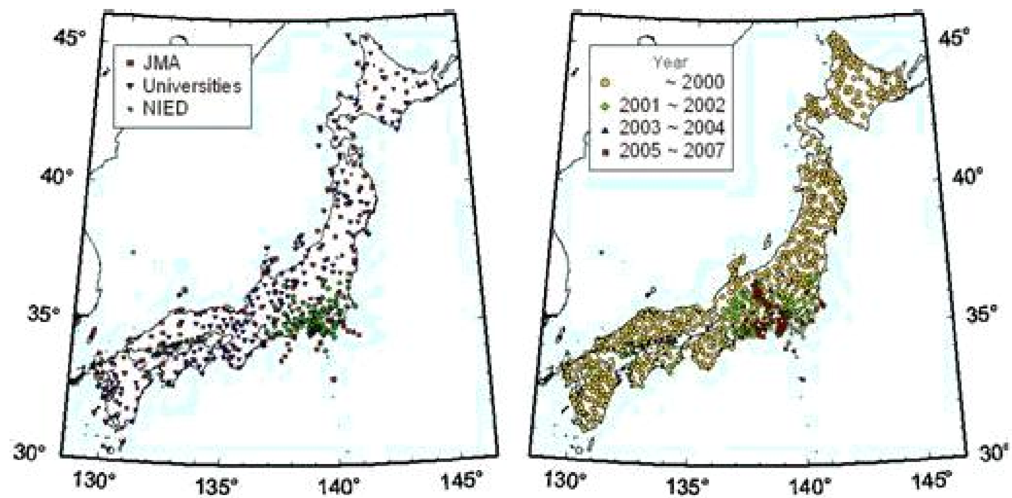
Large Seismic Networks

- USArray (http://www.usarray.org/)
- Hi-net (http://www.hinet.bosai.go.jp/), one of the densest, distance ~20 km between receivers (*Okada et. al., 2004*)



courtesy: http://www.iris.edu/



courtesy: http://www.hinet.bosai.go.jp/

Seismic Gradiometry - Introduction

- New technique for treating a large array data to reconstruct and to characterize a seismic wavefield observed
- Original concept by Langston (2007a), to measure the spatial gradient of seismic waves observed by dense array. He showed that slowness could be directly estimated.
- Igel (2005) proposed a similar approach with rotational seismometers.
- Langston (2007b) SG applied to 2D problem
- Langston (2007c) improved its stability in time domain
- Langston & Liang (2008) SG applied to polarized waves
- Liang & Langston (2009) estimated Rayleigh waves with grid locations collocated with seismic stations (limited to the station location)
- Spudich (1995) performed a similar series expansion to estimate surface strain and stress tensors recorded by dense array
- Maeda (2016) used SG to reconstruct the seismic wavefield as a continuous 2D field irrespective of the station location

Seismic Gradiometry - Benefits

- SG has the advantage that it estimates slowness as a spatially varying value, whereas other array methods ie. semblance method (Neidell & Taner, 1971) which assume a homogeneous plane-wave incidence and utilize phase differences to estimate slowness
- SG models both phase and amplitude of the observed wavefield
- Possibility to separate non-plane wave characteristics, ie. the radiation pattern and geometrical spreading
- SG is a tool for reconstructing a spatially continuous seismic wavefield
- Divergence (div) and Rotation (rot) of 3C seismic wavefield can be estimated by SG and used to characterize seismic wave propagation features

SG Method - Theory

- Uses Taylor series expansion (1st order) of the seismic wavefield in 2D horizontal space

$$\boxed{u^{\text{obs}}(x_S, y_S; t)} \cong \boxed{u(x_G, y_G; t)} \boxed{+ \frac{\partial u(x_G, y_G; t)}{\partial x}(x_S - x_G) + \frac{\partial u(x_G, y_G; t)}{\partial y}(y_S - y_G)}$$

observed location

nearby grid location

1st order Taylor expansion

- Using several observations from stations near target grid to construct an observation equation (N is the number of stations used for estimation)

$$\boldsymbol{u}^{\text{obs}} = \boldsymbol{G}\boldsymbol{m}$$

displacement

$$\begin{pmatrix} u^{\text{obs}}(x_{S1}, y_{S1}; t) \\ u^{\text{obs}}(x_{S2}, y_{S2}; t) \\ \vdots \\ u^{\text{obs}}(x_{SN}, y_{SN}; t) \end{pmatrix} = \begin{pmatrix} 1 & x_{S1} - x_G & y_{S1} - y_G \\ 1 & x_{S2} - x_G & y_{S2} - y_G \\ & \vdots & \\ 1 & x_{SN} - x_G & y_{SN} - y_G \end{pmatrix} \begin{pmatrix} \boxed{u(x_G, y_G; t)} \\ \boxed{\begin{matrix} \partial_x u(x_G, y_G; t) \\ \partial_y u(x_G, y_G; t) \end{matrix}} \end{pmatrix}$$

spatial derivatives

SG Method - Theory (continued)

- If input data 3 or more stations then it is over-determined problem, regularization by smoothing or damping is not necessary, we use least-squares (LSQ) method to estimate the wave amplitude and the spatial gradients
- Computational cost reduced by using a previously computed kernel matrix for LSQ calculation because the inverse problem depends only on the station configuration (K is a weighting factor and W is a diagonal weight matrix eg. Menke, 2012)

$$\boldsymbol{m} = \boxed{\left(\boldsymbol{G}^T \boldsymbol{W} \boldsymbol{G}\right)^{-1} \boldsymbol{G}^T \boldsymbol{W}} \boldsymbol{u}^{\mathrm{obs}}$$

$$\equiv \underbrace{\boxed{\boldsymbol{K}}}_{\text{weighting factor}} \boldsymbol{u}^{obs}$$

- Decomposed as follow

$$\boldsymbol{m} = \begin{pmatrix} \boldsymbol{k}_{\mathrm{u}} \cdot \boldsymbol{u}^{\mathrm{obs}} \\ \boldsymbol{k}_{\partial x} \cdot \boldsymbol{u}^{\mathrm{obs}} \\ \boldsymbol{k}_{\partial y} \cdot \boldsymbol{u}^{\mathrm{obs}} \end{pmatrix}$$

SG Method - Theory (continued)

- Weighting done by empirically adopted the Gaussian function with variance and cutoff distance

$$\sigma^2 = \Delta_0^2/10 \qquad \Delta_0 = 50 \text{ km}$$

- The quality of the estimation of ***m*** (the displacement and the spatial gradients) depends on locations of the stations relative to the grid point.
- The quality of the estimation of ***m*** improves as the number of stations used increases
- However, the use of more distant stations may under-estimate of the spatial derivatives of the waves, therefore Maeda (2016) used only stations within 50 km with smoothly decreasing weighting factor

Love & Rayleigh Wave Decomposition

Slowness Estimation

Proposed SG Notebook Project for Skience 2018

- Simple 2D horizontal space
- Rock properties (?)
- SG analytical solution
- FDM + PML (or else like SPECFEM2D, SALVUS, SOFI2D, OpenSWPC, etc.)
- Moment tensor (simulate an earthquake, P-SV or SH, Rayleigh wave), f = 10-15 Hz
- 3C or 6C (?), around 20-50 receivers, random spreading with distance ~20-30 m
- Compare numerical solution (FDM + PML) with seismic gradiometry (SG) solution

References

- Davis, B. (2011). *The Basic Theory of Wave Gradiometry*. GEOP 523D.
- Igel, H., Schreiber, U., Flaws, A., Schuberth, B., Velikoseltsev, A., Cochard, A. (2005). *Rotational motions induced by the M8.1 Tokachi-oki earthquake, September 25, 2003.* Geophys. Res. Lett. 32(8):L08309.
- Langston, C., & Lang, C. (2009). *Wave gradiometry for USArray: Rayleigh waves*. Journal of Geophysical Research, vol. 114.
- Langston, C., & Liang, C. (2008). *Gradiometry for polarized seismic waves*. Journal of Geophysical Research, vol. 113.
- Liu, Y., & Holt, W. (2015). *Wave gradiometry and its link to Helmholtz equation solutions applied to USArray in the eastern U.S.* Journal of Geophysical Research: Solid Earth.
- Maeda, T., Nishida, K., & Obara, K. (2016). *Reconstruction of a 2D seismic wavefield by seismic gradiometry.* Progress in Earth and Planetary Science.
- Wielandt, E. (1993). *Propagation and structural interpretation of non-plane waves.* Geophys. J. Int., 113.