Wavefield Reconstruction using Seismic Gradiometry:

A brief introduction, discussion, and Jupyter notebook :)

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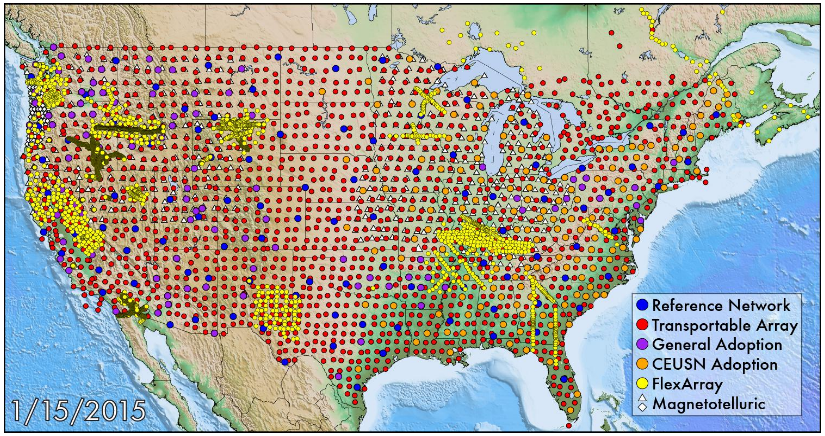
Overview

- Introduction
- Methodology / basic theory
- Case examples
- Discussion
- Jupyter notebook with simple model

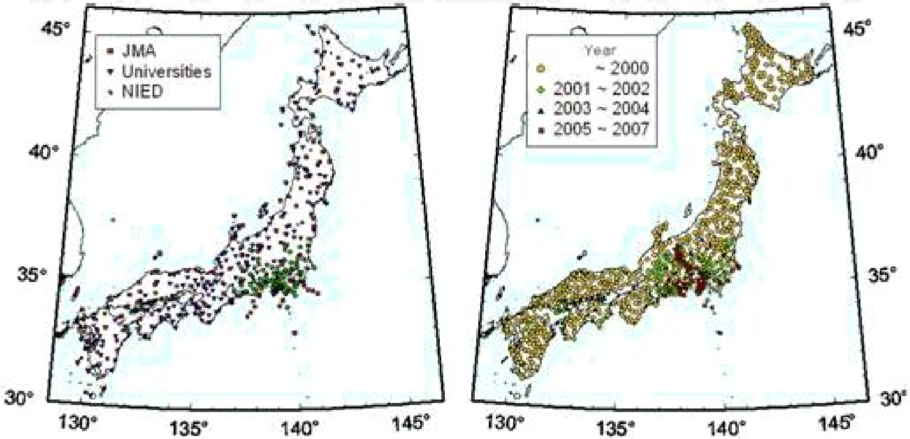
Introduction

Seismic gradiometry is a processing technique utilizing the shape of seismic wavefields captured by ***large seismic array*** to determine fundamental wave propagation characteristics. (*popularized in Langston, 2007a*)

- USArray (http://www.usarray.org/)
- Hi-net (http://www.hinet.bosai.go.jp/), d ~20 km @receivers



courtesy: http://www.usarray.org/



courtesy: http://www.hinet.bosai.go.jp/

- Regularly used techniques ie. Semblance method assumed homogeneous plane wave incidence

Methodology

amplitude term

phase variation as function of time, location and slowness

- Displacement discribed as $u(t,x,y) = \boxed{G(x,y)} f(t - p_x(x - x_0) - p_y(y - y_0))$
- Differentiate, $\boxed{\dfrac{\partial u}{\partial x} = A_x \cdot u + B_x \cdot \dfrac{\partial u}{\partial t}}$ $\boxed{\dfrac{\partial u}{\partial y} = A_y \cdot u + B_y \cdot \dfrac{\partial u}{\partial t}}$

wave gradiometry equation

- where, $A_x = \dfrac{\partial G(x,y)}{\partial x} \cdot \dfrac{1}{G(x,y)}$ $A_y = \dfrac{\partial G(x,y)}{\partial y} \cdot \dfrac{1}{G(x,y)}$

$$B_x = -\left[p_x + \frac{\partial p_x}{\partial x}(x - x_0)\right] \qquad B_y = -\left[p_y + \frac{\partial p_y}{\partial y}(y - y_0)\right]$$

gradiometry parameters

- Integrating B_x and B_y over the interval $[x_0, x]$ gives the **slowness** in the *x* and *y* directions

$$p_x = -\frac{1}{x - x_0}\int_{x_0}^{x} B_x dx \qquad p_y = -\frac{1}{y - y_0}\int_{y_0}^{y} B_y dy$$

- when ***x* --> *x*$_0$** and ***y* --> *y*$_0$** then,

$$p_x(x_0) = -B_x(x_0) \qquad p_y(y_0) = -B_y(y_0)$$

- We assumed that **slowness is nearly constant at a grid point and surrounding stations**

Methodology

If A and B is known, following relations may be used :

- **phase velocity**

$$v = \left(B_x^2 + B_y^2\right)^{-\frac{1}{2}}$$

- **back-azimuth**

$$\theta = \tan^{-1}\left(\frac{B_x}{B_y}\right)$$

B-coefficient related (slowness/velocity)

- **radiation pattern**

$$A_\theta(\theta) = \frac{\partial G}{\partial \theta}\frac{1}{G} = r(A_x \cos\theta - A_y \sin\theta)$$

- **geometrical spreading**

$$A_r(\theta) = \frac{\partial G}{\partial r}\frac{1}{G} = A_x \sin\theta + A_y \cos\theta$$

A-coefficient related (amplitude term)

- **How can we obtain the spatial gradients? and solve A and B coefficients?**

$$\frac{\partial u}{\partial x} = A_x \cdot u + B_x \cdot \frac{\partial u}{\partial t} \qquad \frac{\partial u}{\partial y} = A_y \cdot u + B_y \cdot \frac{\partial u}{\partial t}$$

Determining the spatial gradients

- Uses Taylor series expansion (1st order) of the seismic wavefield in 2D horizontal space ie. using regular grid

$$\boxed{u^{\text{obs}}(x_S, y_S; t)} \cong \boxed{u(x_G, y_G; t)} \boxed{+ \frac{\partial u(x_G, y_G; t)}{\partial x}(x_S - x_G) + \frac{\partial u(x_G, y_G; t)}{\partial y}(y_S - y_G)}$$

observed location

nearby grid location

1st order Taylor expansion

- Using several observations from stations near target grid to construct an **observation equation** (***N*** is the number of stations used for estimation)

$$\boldsymbol{u}^{\text{obs}} = \boldsymbol{Gm}$$

$$\begin{pmatrix} u^{\text{obs}}(x_{S1}, y_{S1}; t) \\ u^{\text{obs}}(x_{S2}, y_{S2}; t) \\ \vdots \\ u^{\text{obs}}(x_{SN}, y_{SN}; t) \end{pmatrix} = \begin{pmatrix} 1 & x_{S1} - x_G & y_{S1} - y_G \\ 1 & x_{S2} - x_G & y_{S2} - y_G \\ & \vdots & \\ 1 & x_{SN} - x_G & y_{SN} - y_G \end{pmatrix} \begin{pmatrix} \boxed{u(x_G, y_G; t)} \\ \boxed{\begin{matrix} \partial_x u(x_G, y_G; t) \\ \partial_y u(x_G, y_G; t) \end{matrix}} \end{pmatrix}$$

displacement

spatial derivatives

Determining the spatial gradients

- If input data 3 or more stations then it is **over-determined** problem, regularization by damping (*used in Liu & Holt, 2015*) is not necessary, we use least-squares (LSQ) method to estimate the wave amplitude and the spatial gradients

$$\boldsymbol{m} = \boxed{\left(\boldsymbol{G}^T \boldsymbol{W} \boldsymbol{G}\right)^{-1} \boldsymbol{G}^T \boldsymbol{W}} \boldsymbol{u}^{\text{obs}}$$

$$\equiv \boxed{\boldsymbol{K}} \boldsymbol{u}^{obs}$$

W = diagonal weighting matrix

K = kernel matrix

- Computational cost reduced by using a previously computed **kernel matrix** (***K***) for LSQ calculation because the inverse problem **depends only on the station layout**
- Decompossed as follow

$$\boldsymbol{m} = \begin{pmatrix} u(x_G, y_G; t) \\ \partial_x u(x_G, y_G; t) \\ \partial_y u(x_G, y_G; t) \end{pmatrix} = \begin{pmatrix} \boxed{\boldsymbol{k}_{\mathrm{u}} \cdot \boldsymbol{u}^{\text{obs}}} \\ \boldsymbol{k}_{\partial x} \cdot \boldsymbol{u}^{\text{obs}} \\ \boldsymbol{k}_{\partial y} \cdot \boldsymbol{u}^{\text{obs}} \end{pmatrix}$$

displacement

spatial derivatives

- The displacement and its spatial gradient at a grid point are obtained from weighted averages by **Gaussian function** empirically of the observed waveform at nearby stations (*see Jupyter notebook*)

Solving for A- and B-coefficients

- Use time window consisting ***M*** discretized time samples, and assume parameteres $\boldsymbol{A_i}$ and $\boldsymbol{B_i}$ (***i = x,y***) do not change within the time window:

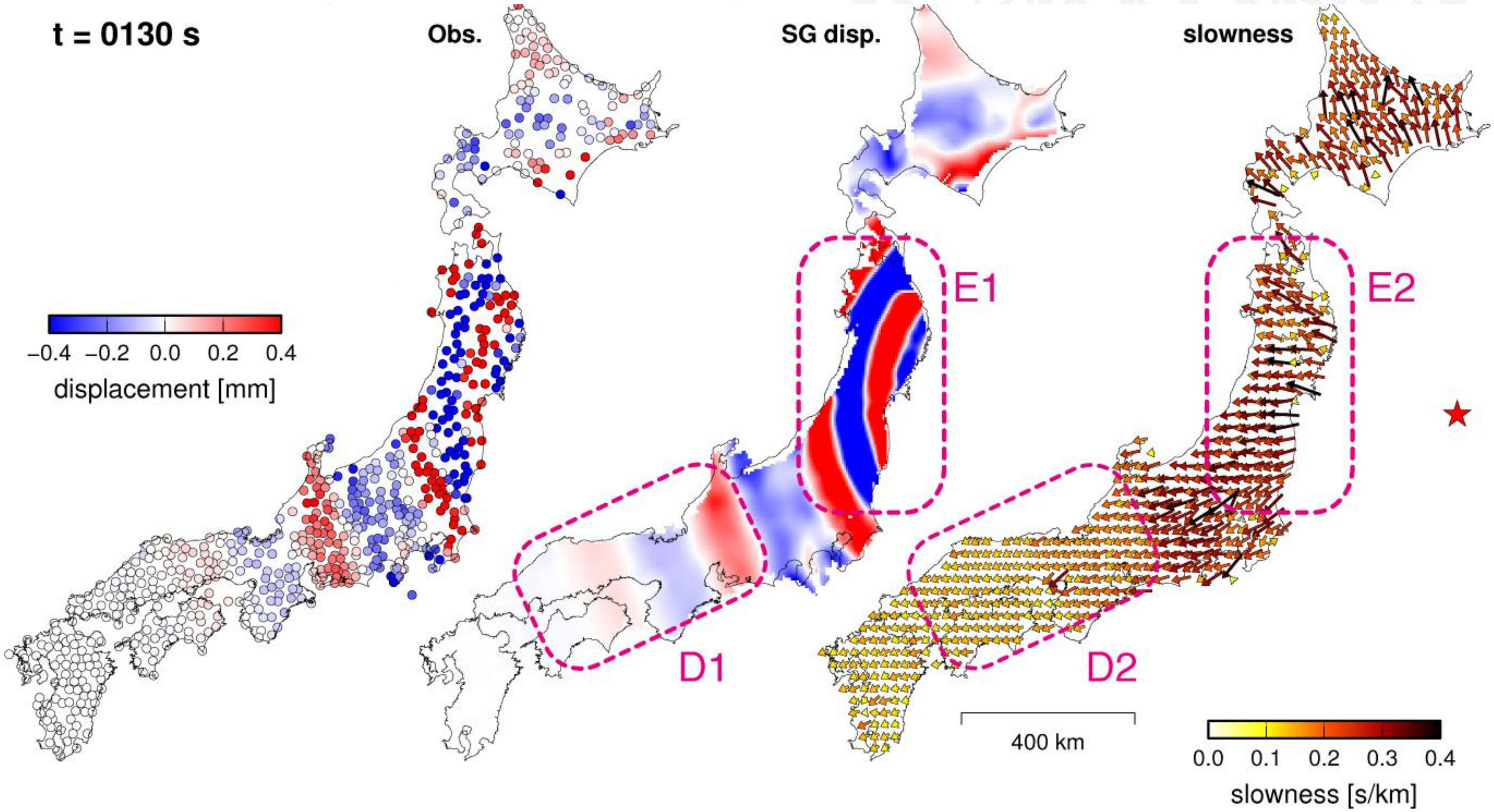
$$\begin{aligned}
\partial_i u(x,y;t_1) &= \boxed{A_i(x)}\, u(x,y;t_1) + \boxed{B_i(x)}\, v(x,y;t_1) \\
\partial_i u(x,y;t_2) &= \boxed{A_i(x)}\, u(x,y;t_2) + \boxed{B_i(x)}\, v(x,y;t_2) \\
&\vdots \\
\partial_i u(x,y;t_M) &= \boxed{A_i(x)}\, u(x,y;t_M) + \boxed{B_i(x)}\, v(x,y;t_M)
\end{aligned}$$

- then coeffecients $\boldsymbol{A_i}$ and $\boldsymbol{B_i}$ can be estimated by LSQ method
- Because it is a simple 2-parameter estimation, it has following analytical solution:

$$A_i(x) = \frac{(\boldsymbol{v}_t\cdot\boldsymbol{v}_t)(\partial_i\boldsymbol{u}_t\cdot\boldsymbol{u}_t)-(\boldsymbol{u}_t\cdot\boldsymbol{v}_t)(\partial_i\boldsymbol{u}_t\cdot\boldsymbol{v}_t)}{(\boldsymbol{u}_t\cdot\boldsymbol{u}_t)(\boldsymbol{v}_t\cdot\boldsymbol{v}_t)-(\boldsymbol{u}_t\cdot\boldsymbol{v}_t)^2}$$

$$B_i(x) = \frac{(\boldsymbol{u}_t\cdot\boldsymbol{u}_t)(\partial_i\boldsymbol{u}_t\cdot\boldsymbol{v}_t)-(\boldsymbol{u}_t\cdot\boldsymbol{v}_t)(\partial_i\boldsymbol{u}_t\cdot\boldsymbol{u}_t)}{(\boldsymbol{u}_t\cdot\boldsymbol{u}_t)(\boldsymbol{v}_t\cdot\boldsymbol{v}_t)-(\boldsymbol{u}_t\cdot\boldsymbol{v}_t)^2}$$

Application example (Hi-net, Maeda et.al., 2016)



Other applications

- Other technique to **determining the spatial gradients** is to use displacement field bicubic spline interpolation, reducing velocity method and damped LSQ inversion on USArray data (*see Liu & Holt, 2015*)
- Other technique to **solving A- and B-coefficients** is by using damped LSQ (*see Liu & Holt, 2015*) or Fourier domain (*see Langston, 2007a*) or analytic signal (*Langston, 2007c; Liang & Langston, 2009*)
- Gradiometry parameters link to **Helmholtz equation** (*see Liu & Holt, 2015*)
- Surface wave (**Love and Rayleigh wave**) **decomposition** (*see Maeda et.al., 2016*);
- **Up/down & P/S wave separation**, **Multiples removal** (see VanRenterghem, 2017)
- Make use of **6-components** polarisation analysis, wavefield separation, and ground-roll suppression (*see Sollberger, 2017*)

Solving for structural parameters (link to Helmholtz eq.)

- In Helmholtz equation, we define **structural (corrected) phase velocity** (independent of specific geometry of the wavefield or source properties)
- Solution to **2D eikonal equation** (ray approx. to scalar wave) and **Helmholtz equation**:

$$\frac{1}{c'(x,y)^2} = |\nabla\tau(x,y)|^2 \qquad \frac{1}{c(x,y)^2} = |\nabla\tau(x,y)|^2 - \boxed{\frac{\nabla^2 G(x,y)}{G(x,y)\omega^2}}$$

c' = dynamic (apparent) velocity, ***c*** = structural (corrected) velocity, ***T*** = phase trave time, ***G*** = wave amplitude, ***w*** = angular frequency

- The A-coefficient corresponds to the gradient of logarithmic amplitude:

$$\vec{A} = \nabla \ln G = \frac{\nabla G}{G} \qquad \boxed{\vec{A}^2 + \nabla\cdot\vec{A}} = \left(\frac{\nabla G}{G}\right)^2 + \frac{\nabla^2 G\cdot G - (\nabla G)^2}{G^2} = \boxed{\frac{\nabla^2 G}{G}}$$

- The B-coefficient is identical to the dynamic phase velocities, and can be approximated as

$$|\vec{B}| = |\vec{p}| \approx \left|\frac{1}{c'}\right| = |\nabla\vec{\tau}|$$

- Combining them, then we determine **structural phase velocity** $\frac{1}{c^2} \approx |\vec{B}|^2 - \frac{\boxed{\vec{A}^2 + \nabla\cdot\vec{A}}}{\omega^2}$

Jupyter notebook

- Build simple 3D homogeneous **model** (100x100x100 elements) 200x200x200 km3
- Put a **source** and random **receivers**
- Simulate wave using **Salvus** (*Afanasiev et.al., 2017*), run in geophysik's **cluster computer** (deadlock machine up to ~60 cores), output receiver and grid seismograms (in hdf5 format)
- Determine the **spatial gradients**
- Compute **A- and B-coefficients**
- Show **plots and maps**

References

- Liu, Y., & Holt, W. (2015). Wave gradiometry and its link to Helmholtz equation solutions applied to USArray in the eastern U.S. Journal of Geophysical Research: Solid Earth.
- Maeda, T., Nishida, K., & Obara, K. (2016). Reconstruction of a 2D seismic wavefield by seismic gradiometry. Progress in Earth and Planetary Science.