Multi-Objective Capacitated Vehicle Routing Problem with Time-Dependent Demands (CVRP-TDD) for Medical Waste Collection

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1 Problem Definition

In an urban area, a set of demand nodes (hospitals, clinics, and other medical institutions) produce medical wastes that must be collected by a vehicle with limited load capacity and disposed of in one of the disposal centers. This set of vehicles needs to start and finish their routes in one of the municipal parking lots. Regularly clearing waste from demand nodes and vehicles to avoid risk build-up is crucial. The risk at the demand nodes increases with each passing moment that the waste remains uncollected.

1.1 Mathematical Notation

$G = (P, E)$ is a complete graph that represents the urban area, where $P = \{p_1, p_2, \ldots, p_n\}$ is the set of nodes (containing demand, parking, and disposal nodes), and $E = \{(i, j) : i, j \in P\}$ the set of edges with weights $d_{i,j}$ denoting the euclidean distance between nodes i and j. P is partitioned as $P = P_d \cup P_p \cup P_c$, where P_d is the set of demand nodes, P_p is the set of parking nodes with unlimited availability, and P_c is the set of disposal centers with unlimited processing capability. Each $p_i \in P_d$ generates waste at a variable rate $w_i(t)$, and the exposure risk factor $r_i^e(t)$ escalates if the waste has not been collected. For this case, we will define $w_i(t)$ and $r_i^e(t)$ as:

$$w_i(t) = \alpha_i \cdot t$$

$$r_i^e(t) = \beta_i \cdot W_{i,t}$$

Where α_i and β_i are coefficients associated with the waste production and risk exposure for each processing node i, and $W_{i,t}$ is the amount of uncollected waste at demand node i at time t.

V is the set of vehicles. Each vehicle $v \in V$ has:

- A maximum load capacity c_v.
- A traveling cost t_v (per unit of time).
- A predetermined starting parking node $p_0^v \in P_p$.

- A velocity s_v (measured in units of distance per unit of time). Using s_i, we can transform the distance matrix $d_{j,k}$ into a time matrix for each vehicle $t^v_{j,k}$ that indicates the units of time that it takes for vehicle v to go from node j to node k, by taking $t^v_{j,k} = d_{j,k}/s_v$.

Finally, we consider a discrete time horizon $T = [1, 2, \dots t_f]$.

2 Mathematical Optimization Model

This section shows the Mixed-Integer Optimization problem for the CVRP-TDD for the Medical Waste Collection Problem.

2.1 Variables

In this section, we show the set of decision variables.

- $x_{i,j,v,t}$: Binary variable indicating if vehicle v departures from node i to node j at time t.
- $W_{i,t}$: Continuous variable denoting the amount of uncollected waste at demand center i at time t.
- $l_{v,t}$: Continuous variable denoting the amount of load at vehicle v in time t.
- $WX_{i,v,t}$: Continuous variable denoting the amount of medical waste that vehicle v would have to load if visited node $i \in P_d$ at time t.

2.2 Objective Functions

The cost objective function is determined by the sum of the traveling costs of every vehicle as:

$$\min : f_1 = \sum_{v \in V} \left[t_v \cdot \sum_{(i,j) \in E} \left(t^v_{i,j} \cdot \sum_{t \in T} x_{i,j,v,t} \right) \right]$$

The risk objective function is determined by the sum of all factors related to risks, as in:

$$\min : f_2 = \sum_{i \in P_d} \left[\beta_i \cdot \sum_{t \in T} W_{i,t} \right]$$

2.3 Constraints

$$\sum_{i \in P_d} x_{p_v^0,i,v,1} = 1, \quad \forall v \in V \tag{1}$$

$$\sum_{(i,j) \in E} x_{i,j,v,t} \leq 1, \quad \forall v \in V, \forall t \in T \tag{2}$$

$$x_{i,j,v,t} + \sum_{k \in P} \sum_{t'=t+1}^{t+t_{i,j}-1} x_{j,k,v,t'} \leq 1, \quad \forall (i,j) \in E, \forall v \in V, \forall t \in T \tag{3}$$

$$x_{i,j,v,t} \leq \sum_{k \in P} x_{j,k,v,t+t^v_{i,j}}, \quad \forall i \in P, \forall j \in P_d \cup P_c, \forall v \in V, \forall t \in T \tag{4}$$

$$\sum_{k \in P \setminus \{j\}} x_{j,k,v,t} \leq \sum_{\substack{h \in H \\ H=\{p \in P \setminus \{j\}: t-t^v_{p,j} \geq 1\}}} x_{h,j,v,t-t^v_{h,j}}, \quad \forall j \in P_d \cup P_c, \forall v \in V, \forall t \in T \tag{5}$$

$$\sum_{t \in T} x_{i,i,v,t} = 0, \quad \forall i \in P, \forall v \in V \tag{6}$$

$$\sum_{i \in P} \sum_{v \in V} \sum_{t \in T} x_{i,j,v,t} \geq 1, \quad \forall j \in P_d \tag{7}$$

$$\sum_{p_c \in P_c} \sum_{p_p \in P_p} \sum_{t \in T} x_{p_c,p_p,v,t} = 1, \quad \forall v \in V \tag{8}$$

$$M\left(1 - \sum_{i \in P} x_{i,p_p,v,t'}\right) \geq \sum_{(i,j) \in E} \sum_{t=t'+1}^{t_f} x_{i,j,v,t}, \quad \forall p_p \in P_p, \forall v \in V, \forall t' \in T \tag{9}$$

$$W_{j,1} = w_i(1), \quad \forall j \in P_d \tag{10}$$

$$W_{j,t+1} \geq W_{j,t} + \alpha_j - M \sum_{i \in P} \sum_{v \in V} x_{j,i,v,t+1}, \quad \forall j \in P_d, \forall t \in T \tag{11}$$

$$l_{v,1} = 0, \quad \forall v \in V \tag{12}$$

$$l_{v,t} - M\left(\sum_{p_c \in P_c} \sum_{i \in P} x_{p_c,i,v,t+1}\right) + \sum_{p_d \in P_d} WX_{p_d,v,t+1} \leq l_{v,t+1}, \quad \forall v \in V, \forall t \in T \tag{13}$$

$$l_{v,t} \leq c_v, \quad \forall v \in V, \forall t \in T \tag{14}$$

$$WX_{i,v,t} \geq W_{i,t-1} + \alpha_i - M\left(1 - \sum_{j \in P} x_{i,j,v,t}\right), \quad \forall i \in P_d, \forall v \in V, \forall t \in T \tag{15}$$

Constraint (1) states that every vehicle $v \in V$ must start its route in the predetermined starting node p_v^0. Constraint (2) states that no vehicle can traverse more than one edge simultaneously. Constraint (3) states the traveling times between nodes i and j. Constraint (4) states that every vehicle should leave a visited node as soon as it arrives (except for parking nodes). Constraint (5) states that a vehicle can only depart from a node j if it has arrived at that node previously. Constraint (6) states that a vehicle cannot remain stationary at the same node. Constraint (7) states that every demand node must be visited at least once. Constraint (8) states that every vehicle must visit one parking node immediately after visiting a disposal center to end its route to avoid any vehicle ending the route while still carrying medical waste. Constraint (9) states that each vehicle's route ends once it has visited a parking node. Constraint (10) indicates the initial amount of waste for every demand node. Constraint (11) indicates the amount of waste for each time $t \in T$. Constraint (11) manages the load adjustment in each vehicle when it visits either a disposal center or a demand node: it resets the load of a vehicle to zero when it visits a disposal center or adds to the vehicle's load the waste collected from demand nodes. Constraint (12) ensures that each vehicle starts its route without load. Constraint (14) ensures that the load in any vehicle does not exceed its maximum capacity. Constraint (15) determines the amount of waste $WX_{i,v,t}$ that vehicle v would collect from demand node i at time t.

Remark* 1*: In constraints (11) and (15), the term α_j (or α_i) is the increase in uncollected waste at demand node j per time step, assuming a linear waste generation function. However, this constraint can be adapted to any form of the waste generation function by replacing α_j with the forward difference of the chosen waste production function $w_i(t)$, defined as:

$$\Delta w_i(t) = w_i(t+1) - w_i(t)$$

2.4 Normalization Process for Objective Functions

Normalization involves scaling the objective functions so their values fall within a similar range. This is particularly important when combining multiple objectives into a single composite objective function. This ensures that the traveling cost objective function (f_1) and the risk exposure objective function (f_2) have comparable scales, allowing for a fair and balanced optimization.

The normalization process used in this study is based on calculating the maximum possible values for each objective function and then scaling the functions accordingly. The calculated maximum values do not represent real worst-case scenarios and may not be feasible. Instead, they are values that can be fast and efficiently calculated and are used as reference points to understand the ranges of the values of each objective function. The maximum value for each objective function is determined using the following approaches:

2.4.1 Maximum Value for Traveling Cost (f_1^{max})

We consider a worst-case scenario for each vehicle to calculate the maximum possible value for the traveling cost objective function. Each vehicle is assumed to travel back and forth to the furthest node from its starting parking node until the time limit is reached. The steps for each vehicle are as follows:

1. Identify the furthest node from the starting parking node.
2. Calculate the time required for a round trip to the furthest node.
3. Determine the number of round trips possible within the given time limit.
4. Compute the total traveling cost based on the distance and number of trips.

2.4.2 Maximum Value for Risk Exposure (f_2^{max})

The maximum value for the risk exposure objective function is calculated by assuming continuous waste generation at each demand node at a rate defined by α_i over the entire time horizon, summing the total uncollected waste, and then multiplying the accumulated waste by the risk factor β_i for each demand node. This scenario assumes that every demand node produces waste without being picked up by any vehicle.

2.5 Normalized Objective Functions

After calculating the maximum values, the objective functions are normalized by dividing each function by its respective maximum value. Therefore, the normalized objective function is:

$$\min : f = \frac{f_1}{f_1^{max}} + \frac{f_2}{f_2^{max}}$$