Constraints

1. **Constraint 1: Starting Node Constraint**

   - **Description:** Ensures that every vehicle starts its route at its predetermined starting node (parking lot).
   - **Mathematical Formulation:**

$$\sum_{i \in P_d} x_{p0_v, i, v, 1} = 1, \forall v \in V$$

   - **Explanation:** Each vehicle v must depart from its starting parking node $p0_v$ to any demand node i at the first time step.

2. **Constraint 2: Single Edge Traversal Constraint**

   - **Description:** Ensures that no vehicle can traverse more than one edge simultaneously.
   - **Mathematical Formulation:**

$$\sum_{(i,j) \in E} x_{i,j,v,t} \leq 1, \forall v \in V, \forall t \in T$$

   - **Explanation:** A vehicle v can only be on one route between nodes i and j at any given time t.

3. **Constraint 3: Travel Time Constraint**

   - **Description:** Ensures that the travel time between nodes is considered.
   - **Mathematical Formulation:**

$$x_{i,j,v,t} + \sum_{k \in P} \sum_{t'=t+1}^{t+t_{i,j}-1} x_{j,k,v,t'} \leq 1, \forall (i,j) \in E, \forall v \in V, \forall t \in T$$

- **Explanation:** If a vehicle v starts traveling from node i to node j at time t, it cannot start another route from node j to node k until it arrives at node j after $t_{i,j}$ time units.

4. **Constraint 4: Node Departure Constraint**

   - **Description:** Ensures that every vehicle leaves a visited node immediately after arriving, except for parking nodes.
   - **Mathematical Formulation:**

$$x_{i,j,v,t} \leq \sum_{k \in P} x_{j,k,v,t+t_{i,j}}, \forall i \in P, \forall j \in P_d \cup P_c, \forall v \in V, \forall t \in T$$

   - **Explanation:** A vehicle must depart a node j after arriving from node i, ensuring continuous movement.

5. **Constraint 5: Sequential Visit Constraint**

   - **Description:** Ensures that a vehicle can only depart from a node if it has previously arrived there.
   - **Mathematical Formulation:**

$$\sum_{k \in P \setminus \{j\}} x_{j,k,v,t} \leq \sum_{h \in H} x_{h,j,v,t-t_{h,j}}, \forall j \in P_d \cup P_c, \forall v \in V, \forall t \in T$$

   - **Explanation:** A vehicle v can only depart from node j if it has already arrived there at a previous time step $t - t_{h,j}$.

6. **Constraint 6: Non-Stationary Constraint**

   - **Description:** Ensures that a vehicle cannot remain stationary at the same node.
   - **Mathematical Formulation:**

$$\sum_{t \in T} x_{i,i,v,t} = 0, \forall i \in P, \forall v \in V$$

   - **Explanation:** Vehicles must move continuously and cannot stay at a single node i.

7. **Constraint 7: Demand Node Visit Constraint**

   - **Description:** Ensures that every demand node is visited at least once.
   - **Mathematical Formulation:**

$$\sum_{i \in P} \sum_{v \in V} \sum_{t \in T} x_{i,j,v,t} \geq 1, \forall j \in P_d$$

   - **Explanation:** Each demand node j must be visited by some vehicle at least once during the time horizon.

8. **Constraint 8: Parking Node Visit After Disposal**

   - **Description:** Ensures that vehicles visit a parking node immediately after visiting a disposal center.
   - **Mathematical Formulation:**

$$\sum_{p_c \in P_c} \sum_{p_p \in P_p} \sum_{t \in T} x_{p_c,p_p,v,t} = 1, \forall v \in V$$

- **Explanation:** After a vehicle visits a disposal center p_c, it must then visit a parking node p_p to end its route and avoid carrying waste.

9. **Constraint 9: Route End Constraint**

   - **Description:** Ensures that each vehicle's route ends once it has visited a parking node.
   - **Mathematical Formulation:**

$$M\left(1-\sum_{i\in P} x_{i,p_p,v,t'}\right) \geq \sum_{(i,j)\in E}\sum_{t=t'+1}^{t_f} x_{i,j,v,t}, \forall p_p \in P_p, \forall v \in V, \forall t' \in T$$

   - **Explanation:** The route of vehicle v ends once it visits a parking node p_p.

10. **Constraint 10: Initial Waste Amount**

    - **Description:** Specifies the initial amount of waste at each demand node.
    - **Mathematical Formulation:**

$$W_{j,1} = w_i(1), \forall j \in P_d$$

    - **Explanation:** The amount of waste at each demand node j is set at the initial time step.

11. **Constraint 11: Waste Accumulation**

    - **Description:** Manages the accumulation of waste at demand nodes over time.
    - **Mathematical Formulation:**

$$W_{j,t+1} \geq W_{j,t} + \alpha_j - M\sum_{i\in P}\sum_{v\in V} x_{j,i,v,t+1}, \forall j \in P_d, \forall t \in T$$

    - **Explanation:** The waste at demand node j increases with time unless collected by a vehicle.

12. **Constraint 12: Initial Load Constraint**

    - **Description:** Ensures that each vehicle starts its route without load.
    - **Mathematical Formulation:**

$$l_{v,1} = 0, \forall v \in V$$

    - **Explanation:** Vehicles v start their routes with an empty load.

13. **Constraint 13: Load Adjustment**

    - **Description:** Adjusts the load on each vehicle based on waste collected and disposal visits.
    - **Mathematical Formulation:**

$$l_{v,t} - M \left(\sum_{p_c \in P_c} \sum_{i \in P} x_{p_c,i,v,t+1} \right) + \sum_{p_d \in P_d} WX_{p_d,v,t+1} \leq l_{v,t+1}, \forall v \in V, \forall t \in T$$

    - **Explanation:** The load on vehicle v is adjusted based on waste collected from demand nodes and waste disposed of at disposal centers.

14. **Constraint 14: Vehicle Load Capacity**

    - **Description:** Ensures that the load in any vehicle does not exceed its maximum capacity.
    - **Mathematical Formulation:**

$$l_{v,t} \leq c_v, \forall v \in V, \forall t \in T$$

    - **Explanation:** The load $l_{v,t}$ on vehicle v at any time t must be within its maximum capacity c_v.

15. **Constraint 15: Waste Collection**

    - **Description:** Determines the amount of waste that vehicle v collects from demand node i at time t.
    - **Mathematical Formulation:**

$$WX_{i,v,t} \geq W_{i,t-1} + \alpha_i - M \left(1 - \sum_{j \in P} x_{i,j,v,t} \right), \forall i \in P_d, \forall v \in V, \forall t \in T$$

    - **Explanation:** The waste $WX_{i,v,t}$ collected by vehicle v from